Home Work #6
DUE: 1pm Tuesday Oct 6, 2020
(upload portrait-mode PDF on Canvas)

☞ Handwritten assignments will not be accepted.

Start your assignment with the following text provided you can honestly agree with it.

- I certify that every answer in this assignment is the result of my own work; that I have neither copied off the Internet nor from any one else's work; and I have not shared my answers or attempts at answers with anyone else.

1. Show how to implement a FIFO queue by means of two ordinary LIFO stacks so that the amortized cost of each ENQUEUE and DEQUEUE operation is $O(1)$ (i.e., a sequence of n operations cost $O(n)$). Assume the actual costs for PUSH and POP are 1 and all other operations are free.

   (a) First, write pseudo-code for the two operations **using our notation** and **with adequate comments**.

   (b) Next, demonstrate the amortized costs using the accounting method.

   (c) Now, demonstrate the amortized costs using the potential method.

2. A data structure supports the following two operations for a set of integers S:

   (a) INSERT(S, x), which inserts x into S; and

   (b) DELETELARGERHALF(S), which deletes the largest $\lceil \frac{|S|}{2} \rceil$ elements from S.

   You can assume (a) that the algorithm MEDIAN computes the median of an array of n integers in time n and (b) the algorithm PARTITION (the PARTITION we have seen is modified to accept an input pivot) on such an array takes time n.

   Using the potential method of analysis, prove that it is possible to implement the data structure using an array (with a variable size) such that a sequence of n operations on the data structure will execute in $O(n)$ time.

   Hint:

   - Implement DELETELARGERHALF using MEDIAN and PARTITION.
   - Let the array be associated with a *size* attribute that records the current number of valid elements in it (the array is indexed from 1).
   - Design the potential function based on the number of elements in the array so that the amortized cost of DELETELARGERHALF is zero.