Home Work #9
DUE: 2 pm Saturday Nov 8, 2020
(upload portrait-mode PDF on Canvas)

☞ Handwritten assignments will not be accepted.

Start your assignment with the following text provided you can honestly agree with it.

- I certify that every answer in this assignment is the result of my own work; that I have neither copied off the Internet nor from any one else's work; and I have not shared my answers or attempts at answers with anyone else.

1. 2020 has been a tough year. Next winter, you are going to spend some time bird watching and, more importantly, photographing. Moreover, you have resolved to spend your time optimally! That means you will take pictures of as many birds as possible within D days: starting on day 1, ending on day D.

   After consulting various birding organizations, you have learned that there are N places in New Mexico and surrounding states. Your plan is to take pictures in one place during the day and travel to another place during the evening, never revisiting the same place. So, for each place p, you have a list of other places $R(p)$ that are reachable (and reachable from) within an evening. Another resource you have obtained from veteran bird watchers is a table $B[p, d]$ that gives you the number of birds you can expect to photograph at place p on day d ($1 \leq p \leq N$; $1 \leq d \leq D$)

   Your problem is to plan your D days in a way that maximizes the number of birds you can expect to photograph (based on the above information you have collected from the experts). Specifically, given a starting place, your dynamic programming algorithm must return a sequence of places that meets this goal.

   (a) Characterize the structure of your solution: how would you reverse-engineer a solution?

   Hint: Focus on the last place in the given solution. How did the solver arrive at it?

   (b) Recursively define the value of an optimal solution in terms of subsolutions.

   Hint: Think of a function that gives you the maximum number of birds photographed in the first d days.

   (c) Define an array to store subsolutions. Relate it to the function above.

   (d) Outline an algorithm for filling the array.

   (e) How would you infer the sequence of places from your array?

   (f) What is the time complexity of your algorithm?

2. In order to compute the product of a chain of five matrices A_1, A_2, A_3, A_4, A_5 (i.e., $A_1 \times A_2 \times A_3 \times A_4 \times A_5$) whose dimensions are $7 \times 3, 3 \times 3, 3 \times 19, 19 \times 18$, and 18×7 respectively, the algorithm MATCHAINORDER has been called (from MCMULT) with the appropriate parameters.

   (a) Show the $s[\,]$ matrix the algorithm obtains. It is an upper triangular matrix; rows and columns signify the start and end indices respectively of subchains. What value of $s[2,5]$ is returned by MATCHAINORDER ?

   (b) What does the number in cell $s[2,5]$ signify?

   Fill in the entries of the table manually — use a calculator but do not write a program — and you will get a better feel for this technique.