Home Work #10
DUE: See Canvas
(upload portrait-mode PDF on Canvas)

☞ Handwritten assignments will not be accepted.

Start your assignment with the following text provided you can honestly agree with it.

- I certify that every answer in this assignment is the result of my own work; that I have neither copied off the Internet nor from any one else's work; and I have not shared my answers or attempts at answers with anyone else.

1. Given: a directed graph $G = (V, E)$ represented with an **adjacency-matrix**.
   (Convention: $Adj[i, j] = 1$ if there is an edge **from** vertex i **to** vertex j; 0 otherwise.)

   (a) Write an algorithm for determining whether or not G contains a vertex with in-edges from every other vertex but no out-edge. (*Definition*: A directed edge (u, v) is an *out-edge* for vertex u and *in-edge* for vertex v. In other words, an edge that leaves vertex u is an out-edge for u; one that enters vertex v is an in-edge for v.)

   Your algorithm **must** run in time $O(V)$.

   Hint: Focus on the Adj matrix. You are looking for a vertex with no out-edges. What entries would you expect in the row corresponding to such a vertex? What about the column?

   (b) Outline the idea behind your algorithm in a few sentences.

   (c) Explain why its time complexity is $O(V)$.

2. You are given a list of n professional wrestlers. Between any pair of professional wrestlers, there may or may not be a rivalry. You are also given a list of r pairs of wrestlers for which there *are* rivalries. Give an algorithm that determines whether or not it is possible to partition the set of wrestlers into *good guys* and *bad guys* such that each rivalry is between a good guy and a bad guy. Your algorithm must run in $O(n + r)$-time.

   If it is possible to perform such a partition, then your algorithm should output the designations.

   (*Partition* implies that *each* wrestler has to be designated a good guy or a bad guy and no wrestler can be *both* a good guy and a bad guy.)

   Hint: Consider Breadth-first search,