1

. Consider the directed graph $G = (V, E)$ defined by the following adjacency list:

vertex	*Adj[vertex]*
a	$< b, f >$
b	$< c, f >$
c	$< g >$
d	$< c >$
e	$< d >$
f	$< e >$
g	$< f >$

Draw the graph with d/f values (discovery and finish times) inside each vertex — following the style we used in the example worked out in class.

Make DFS pick the vertices in alphabetical order and make DFS-Visit pick the elements of the adjacency list in the order given above.

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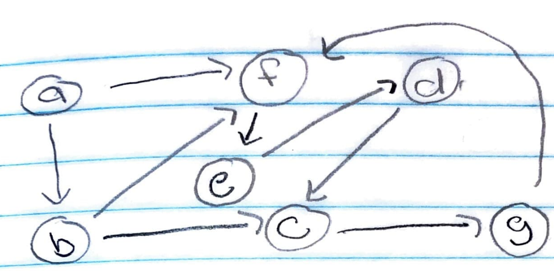
1)



a: 1 / 14
b: 2 / 13
c: 3 / 12
d: 7 / 8
e: 6 / 9
f: 5 / 10
g: 4 / 11

2: A directed graph G = (V, E) contains an edge (u, v). The following is known about an execution of DFS(G) on G: d[v] = 312, f [u] = 622, and f [v] = 1064. Does G contain a cycle? Prove your answer.

According to the Parenthesis Theorem, because $d[v] < f[u] < f[v]$, we know that the u-interval and v-interval are not disjoint. This also tells us that the u-interval does not contain the v-interval. This can only mean that the v-interval contains the u-interval and u is a descendant of v. Therefore we must infer that d[u] is greater than d[v] and less than f[u]. This means that there is a cycle.

3

Show a directed graph $G = (V, E)$ in which there is a path from vertex u to vertex v of length 4 ($u, v \in V$), u is discovered before v in a depth-first search of G and yet v is not a descendant of u in the depth-first forest produced.

(a) Draw your graph. Inside each vertex, indicate the d/f values produced by DFS following the style we used in the example worked out in class. Also draw the depth-first forest produced.

(b) Is this a counter-example to the White-Path Theorem? Explain.

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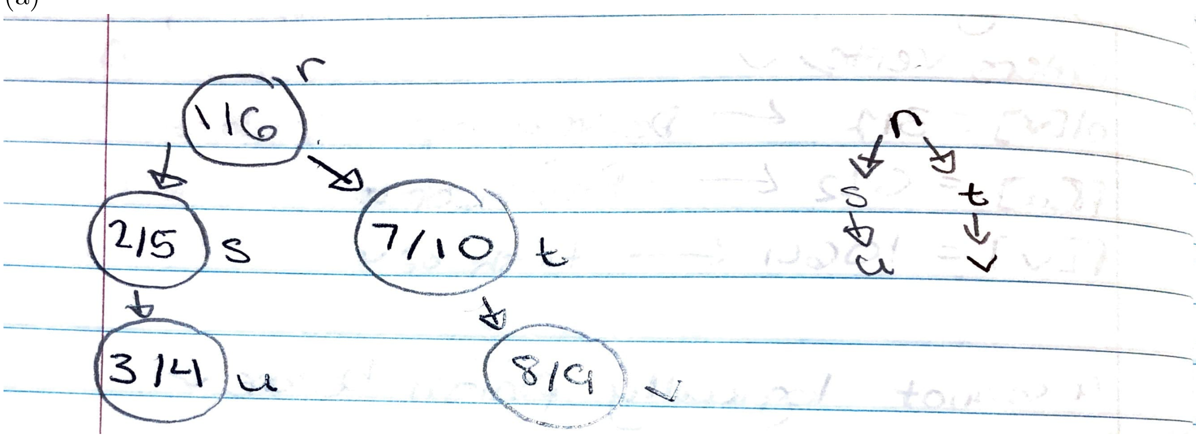
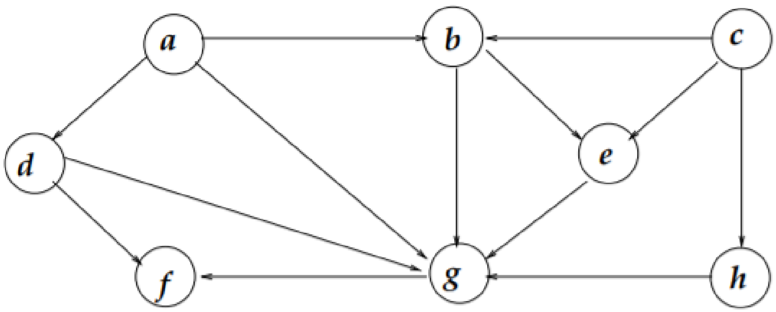
(a)



(b) This is not a counter example of the White Path Theorem. We know that u and v are not descendants of each other, and the white path theorem state that the path from u to v contains only white nodes, but in this example we do not know this. So this is not related to the theorem.

(a) Redraw the following DAG with the vertices laid out linearly from left to right and all edges pointing right (the edges may cross each other).

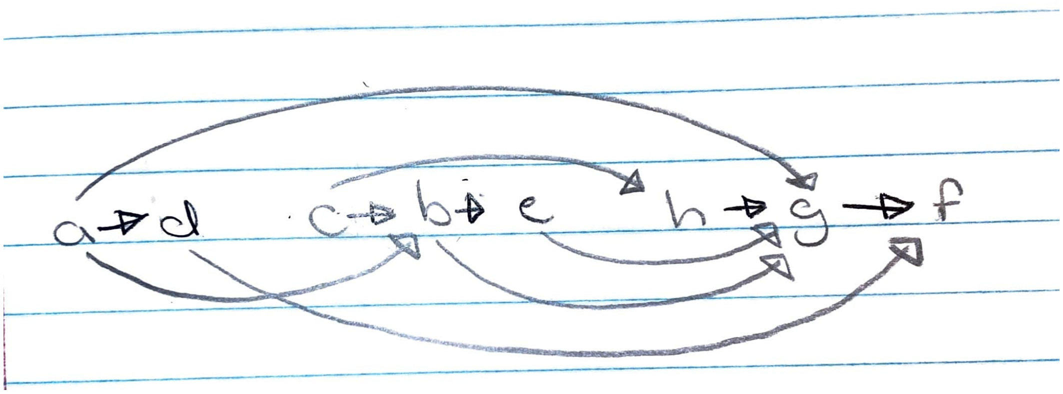


(b) Argue that every DAG can be drawn in the above manner.

(c) How many edges can a DAG have?
Hint: Use part (b) of this question.

4:

(a)



(b) Yes, because a long as there isn't a loop anywhere, there should be no reason for a vertex to have to point left

(c) $\sum_{i=1}^{n-1} i = \frac{n(n-1)}{2}$