

I certiy that I did the assignment by myself

1: 2020 has been a tough year. Next winter, you are going to spend some time bird watching and, more importantly, photographing. Moreover, you have resolved to spend your time optimally! That means you will take pictures of as many birds as possible within D days: starting on day 1, ending on day D. After consulting various birding organizations, you have learned that there are N places in New Mexico and surrounding states. Your plan is to take pictures in one place during the day and travel to another place during the evening, never revisiting the same place. So, for each place p, you have a list of other places R(p) that are reachable (and reachable from) within an evening. Another resource you have obtained from veteran bird watchers is a table B[p, d] that gives you the number of birds you can expect to photograph at place p on day d ($1 \leq p \leq N$; $1 \leq d \leq D$) Your problem is to plan your D days in a way that maximizes the number of birds you can expect to photograph (based on the above information you have collected from the experts). Specifically, given a starting place, your dynamic programming algorithm must return a sequence of places that meets this goal.

- (a) Characterize the structure of your solution: how would you reverse-engineer a solution?
- (b) Recursively define the value of an optimal solution in terms of subsolutions
- (c) Define an array to store subsolutions. Relate it to the function above.
- (d) Outline an algorithm for filling the array
- (e) How would you infer the sequence of places from your array?
- (f) What is the time complexity of your algorithm?

- (a) We would start by looking at the last place on the last day, which we would denote at p_i on day D which should be the place where we are expected to have the maximum amount of photos taken of birds everyday until this point: this would be equal to the maximum number of birds seen up to the previous day (D-1) plus $\max(B[p_i, D])$ for $p_i \in R(p_k)$ where p_k is the place chosen previously
- (b) Let $\text{Opt}[N, D]$ be the optimal number of bird pictures taken after D days and N places available, such that

$$\text{Opt}[N, D] = \begin{cases} \max(B[p_i, D]) & D = 1 \\ \text{Opt}[N, D - 1] + \max(B[p_i, D]) \text{ such that } p_i \in R(p_k) & \text{otherwise} \end{cases}$$

- (c) Let say we have a two dimensional array, where the columns denote the place chosen for that day (p) and the rows denote which day it is (d). For each cell within this array is the amount of birds pictures optimized to be taken on each each day and place, denoted by $\text{Opt}[p_i, d_i]$
- (d) To fill this array, you would take what ever amount of birds is being done on that day ($B[p_i, d_i]$) plus the optimal number of birds to be seen in another place in range on the day previously. The result is put in the designated cell for $\text{Opt}[p_i, d_i]$
- (e) You would be able to infer the sequence of places based on tracking from the last day, you could subtract the number of birds seen on that day and place from its value in $\text{Opt}[]$, and then find the value in the previous day for the place in which is in the same range as p_n . From the beginning you could simply follow which values of p have the highest value possible every day.
- (f) Because we're dealing with a two dimensional array, its logical that the time complexity would be at least $O(n^2)$

2: In order to compute the product of a chain of five matrices A1, A2, A3, A4, A5

(i.e., $A1 \times A2 \times A3 \times A4 \times A5$) whose dimensions are 7×3 , 3×3 , 3×19 , 19×18 , and 18×7 respectively, the algorithm MATCHAINORDER has been called (from MCMULT) with the appropriate parameters

(a) Show the $s[]$ matrix the algorithm obtains. It is an upper triangular matrix; rows and columns signify the start and end indices respectively of subchains. What value of $s[2, 5]$ is returned by MATCHAINORDER ?

(b) What does the number in cell $s[2, 5]$ signify?

(a) The Table $s[]$ is below

	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
$i = 1$		1	2	2	2
$i = 2$			2	2	2
$i = 3$				3	4
$i = 4$					4
$i = 5$					

I've included pictures of my notes, because the math got very convoluted very fast, they are after problem b.

(b) $s[2,5]$ is the cell that tells us that the optimal answer for the problem $s[1,5]$ is also going to be 2.

90

		$3 \times 3 \times 3$	$3 \times 3 \times 19$	$3 \times 19 \times 18$	$19 \times 18 \times 7$	
M		j=1	j=2	j=3	j=4	j=5
i=1	1610	\emptyset	63	462	14 67	1614
i=2			\emptyset	171	1188	1467
i=3				\emptyset	1026	1404
i=4					\emptyset	2,3,4
i=5						\emptyset

S	j=1	j=2	j=3	j=4	j=5
i=1		1	2 3	2	2
i=2			2	2	2 2
i=3				3	4
i=4					4

$m[1,3]$

$(A_1 \cdot A_2 \cdot A_3)$ or $(A_1 \cdot A_2) \cdot A_3$

$$m[1,1] + m[2,3] + 7 \times 3 \times 19 = 570$$

$$m[1,2] + m[3,3] + 7 \times 3 \times 19 = 462$$

$m[2,4]$

$A_2 \cdot (A_3 \cdot A_4)$ or $(A_2 \cdot A_3) \cdot A_4$

$$m[2,2] + m[3,4] + 3 \times 3 \times 18 = 162$$

$$m[2,3] + m[4,4] + 3 \times 19 \times 18 = 1188$$

$m[3,5]$

$(A_3 \cdot A_4 \cdot A_5)$ or $(A_3 \cdot A_4) \cdot A_5$

$$m[3,3] + m[4,5] + 3 \times 19 \times 7 = 2793$$

$$m[3,4] + m[5,5] + 3 \times 18 \times 7 = 1404$$

WICS.CS.N

$$m[1,4]$$

$$m[1,1] + m[2,4] + 7 \times 3 \times 18 = 1566$$

$$m[1,2] + m[3,4] + 7 \times 3 \times 18 = 1467$$

$$m[1,3] + m[4,4] + 7 \times 19 \times 18 = 2856$$

$$m[2,5]$$

$$m[2,2] + m[3,5] + 3 \times 3 \times 7 = 1467$$

$$m[2,3] + m[4,5] + 3 \times 19 \times 7 = 2,964$$

$$m[2,4] + m[5,5] + 3 \times 18 \times 7 = 1566$$

$$m[1,5]$$

$$m[1,1] + m[2,5] + 7 \times 3 \times 7 = 1566$$

$$m[1,2] + m[3,5] + 7 \times 3 \times 7 = 1614$$

$$m[1,3] + m[4,5] + 7 \times 19 \times 7 = 3787$$

$$m[1,4] + m[5,5] + 7 \times 18 \times 7 = 2349$$