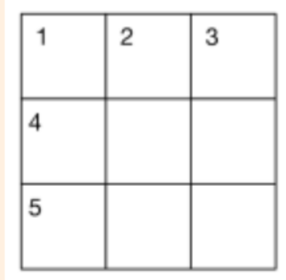
You may (but do not have to) use AISpace and/or csp*.py code to solve these problems.

1. (6p) Exercise 4.1

1.

Consider the crossword puzzle shown in Figure 4.14.



Words:

add, age, aid, aim,
air, are, arm, art, bad,
bat, bee, boa, dim,
ear, eel, eft, lee, oaf

Figure 4.14: A crossword puzzle to be solved with six words

Words:add, age, aid, aim, air, are, arm, art, bad, bat, bee, boa, dim, ear, eel, eft, lee, oafFigure 4.14: A crossword puzzle to be solved with six words

You must find six three-letter words: three words read across (*1-across*, *4-across*, *5-across*) and three words read down (*1-down*, *2-down*, *3-down*). Each word must be chosen from the list of 18 possible words shown. Try to solve it yourself, first by intuition, then by hand using first domain consistency and then arc consistency.

There are at least two ways to represent the crossword puzzle shown in Figure 4.14 as a constraint satisfaction problem.

The first is to represent the word positions (*1-across*, *4-across*, etc.) as variables, with the set of words as possible values. The constraints are that the letter is the same where the words intersect.

The second is to represent the nine squares as variables. The domain of each variable is the set of letters of the alphabet, {a,b,...,z}. The constraints are that there is a word in the word list that contains the corresponding letters. For example, the top-left square and the center-top square cannot both have the value a, because there is no word starting with aa.

1. (a)

   Give an example of pruning due to domain consistency using the first representation (if one exists).

A variable is domain consistent if no value of the domain of the node is ruled impossible by any of the constraints. Therefore a variable is domain consistent if the chosen word for each variable (1 across , 1 down, etc) is possible by the constraint that when the words intersect the intersection has the same letter. An example of this is choosing a word for one across and then choosing a word for 1 down that share the same first letter.

(b)

Give an example of pruning due to arc consistency using the first representation (if one exists).

An arc X,r(X, Y) is arc consistent if, for each value x ∈ dom(X), there is some value y ∈ dom(Y) such that r(x, y) is satisfied. An example of this would be eliminating the words dim, lee, and oaf from one across and one down domains. This is because one across and one down need a word that share the same first letter there are no other words beginning with l, d, or o.

2. (c)

   Are domain consistency plus arc consistency adequate to solve this problem using the first representation? Explain.

   Yes. You can narrow the viable domains using the arc consistency algorithm then use domain splitting to find your answer.

3. (d)

   Give an example of pruning due to domain consistency using the second representation (if one exists).

   Using the second representation an example of pruning due to domain consistency is to remove the value z from the domain of the top left square because no word begins with Z.

4. (e)

   Give an example of pruning due to arc consistency using the second representation (if one exists).

   Using the second representation an example of pruning due to arc consistency is to remove the value z from the domain of the top left square because no word begins with Z.

5. (f)

   Are domain consistency plus arc consistency adequate to solve this problem using the second representation?

   Yes it is. You can narrow the viable domains using the arc consistency algorithm then use domain splitting to find your answer.

6. (g)

   Which representation leads to a more efficient solution using consistency-based techniques? Give the evidence on which you are basing your answer.

   I would recommend the first representation because the variables are whole words rather than letters so there is less time wasted. The small list of words is considered the whole time rather than checking induvial combinations of letters.

2. (7p) Exercise 4.3

3.

Consider the crossword puzzles shown in Figure 4.15.

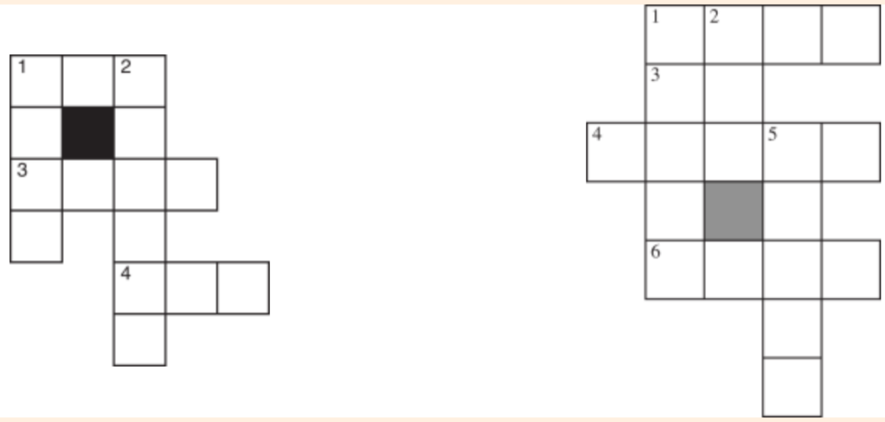


Figure 4.15: Two crossword puzzles

Figure 4.15: Two crossword puzzles

The possible words for (a) are:

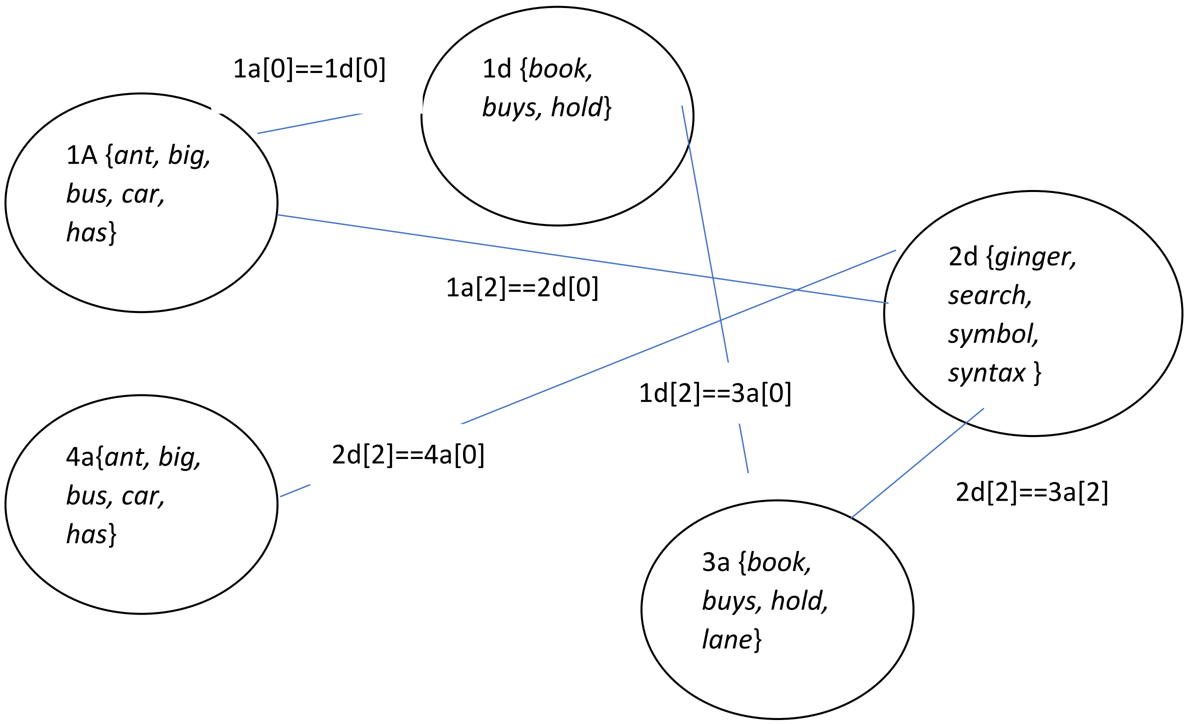
ant, big, bus, car, has, book, buys, hold, lane, year, ginger, search, symbol, syntax.

The available words for (b) are

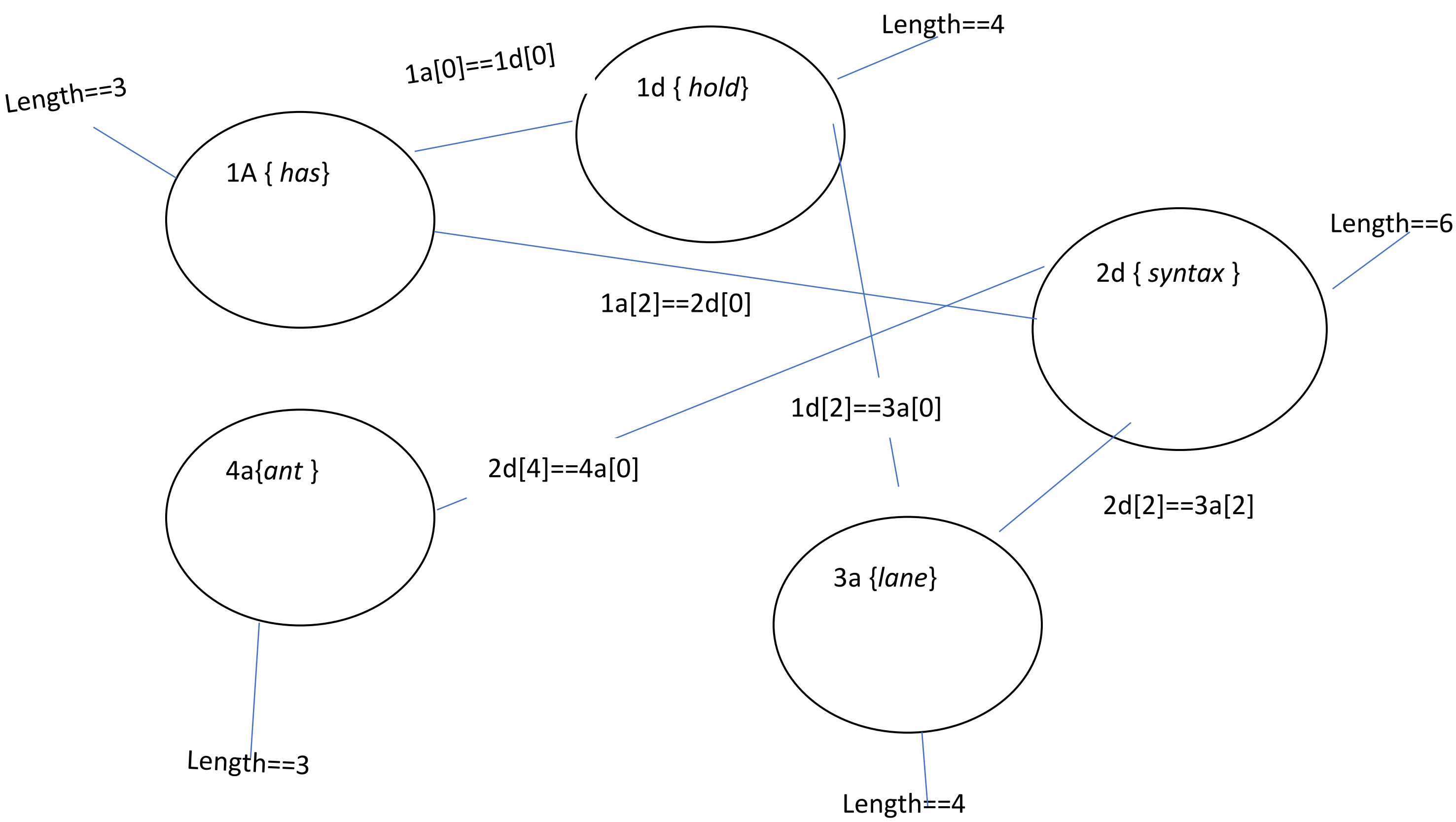
at, eta, be, hat, he, her, it, him, on, one, desk, dance, usage, easy, dove, first, else, loses, fuels, help, haste, given, kind, sense, soon, sound, this, think.

1. (a)

   Draw the constraint graph nodes for the positions (*1-across*, *2-down*, etc.) and words for the domains, after it has been made domain consistent


1a[0]==1d[0]
1d {*book, buys, hold*}
1A {*ant, big, bus, car, has*}
1a[2]==2d[0]
2d {*ginger, search, symbol, syntax* }
4a{*ant, big, bus, car, has*}
2d[2]==4a[0]
1d[2]==3a[0]
2d[2]==3a[2]
3a {*book, buys, hold, lane*}

Solution:


Length==4
Length==3
1a[0]==1d[0]
1d { *hold*}
1A { *has*}
1a[2]==2d[0]
Length==6
2d { *syntax* }
1d[2]==3a[0]
4a{*ant* }
2d[4]==4a[0]
2d[2]==3a[2]
3a {*lane*}
Length==3
Length==4

2. (b)

Give an example of pruning due to arc consistency.

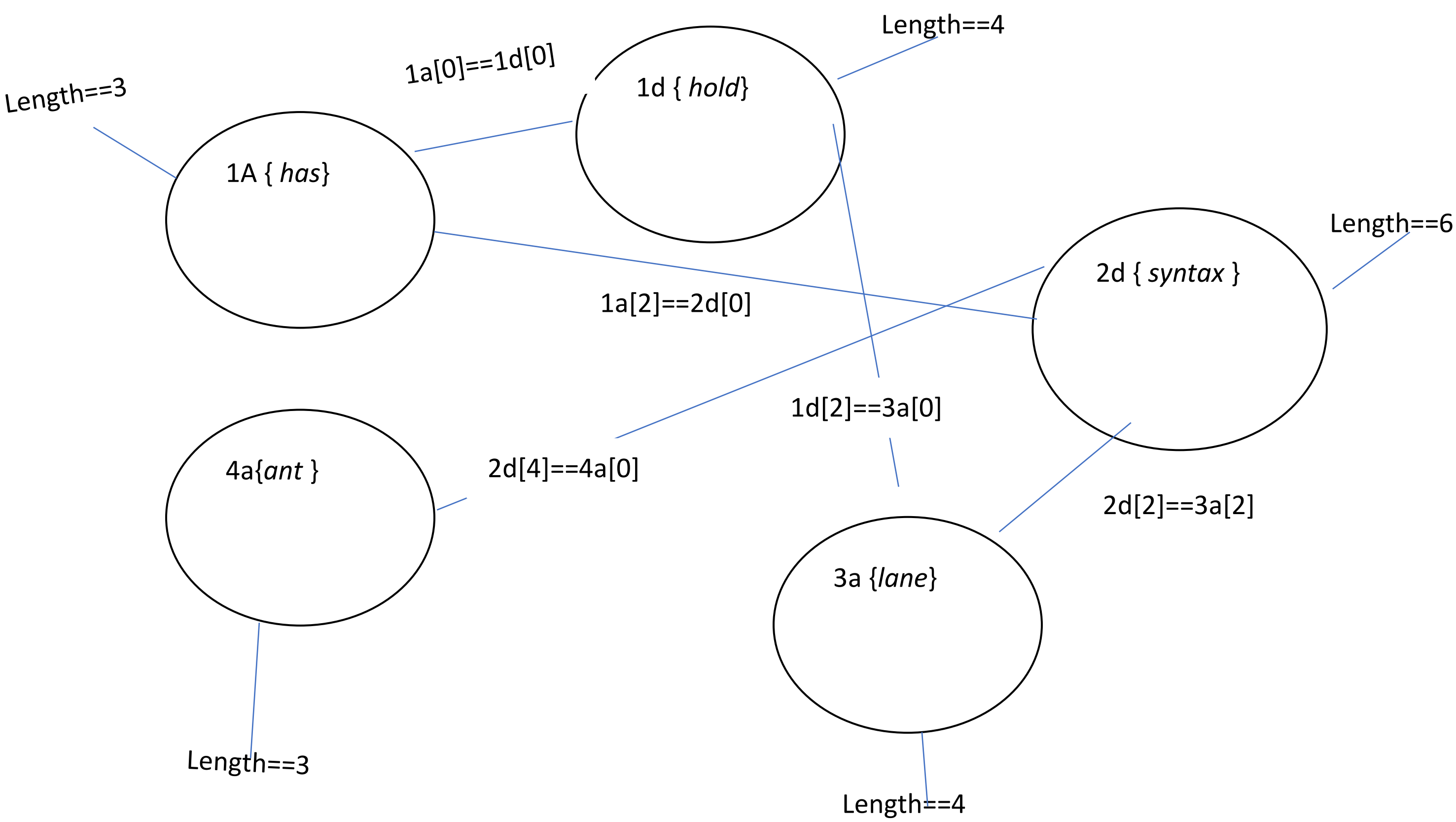
For a an example is removing ginger from the domain of 1 down and 1 across because it is the only word that begins with g. Similarly it is removing the word usage from 1 across and 1 down in puzzle b because that is the only u word.

3. (c)

What are the domains after arc-consistency has halted?

The domains are given below in the solution.

Solution:



4. (d)

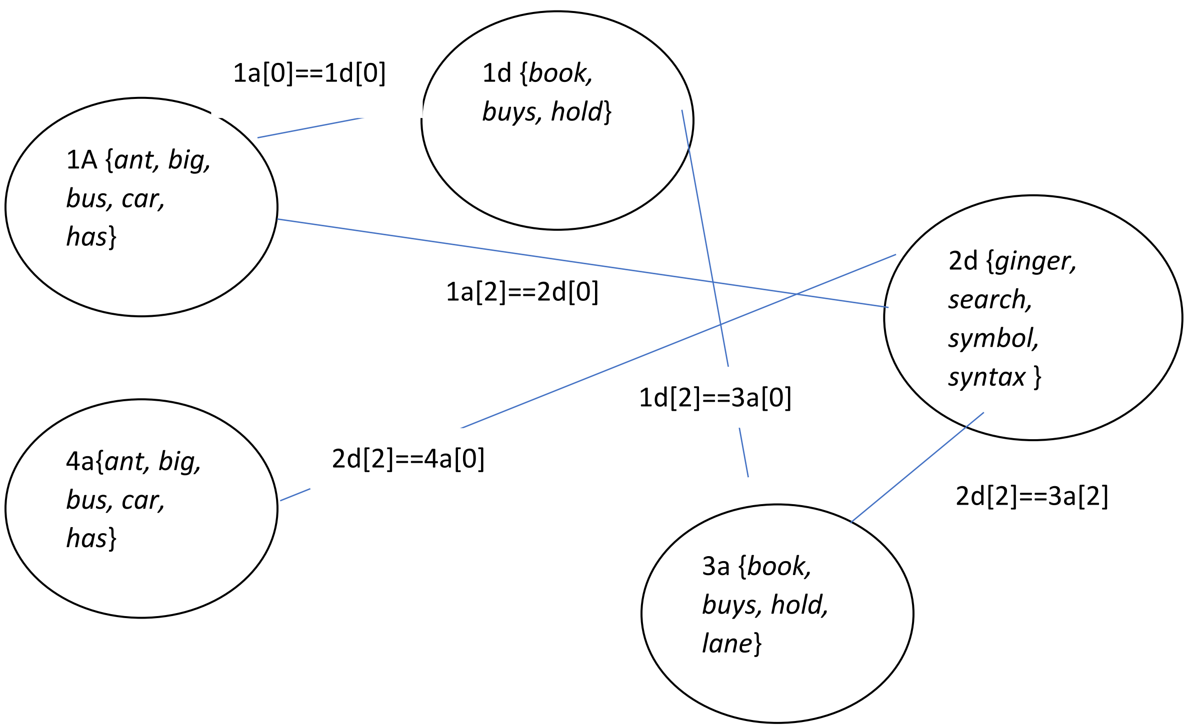
Consider the dual representation, in which the squares on the intersection of words are the variables. The domains of the variable contain the letters that could go in these positions. Give the domains after this network has been made arc consistent. Does the result after arc consistency in this representation correspond to the result in part (c)?

1across&1down: {b, h}

1down&3across: {l}

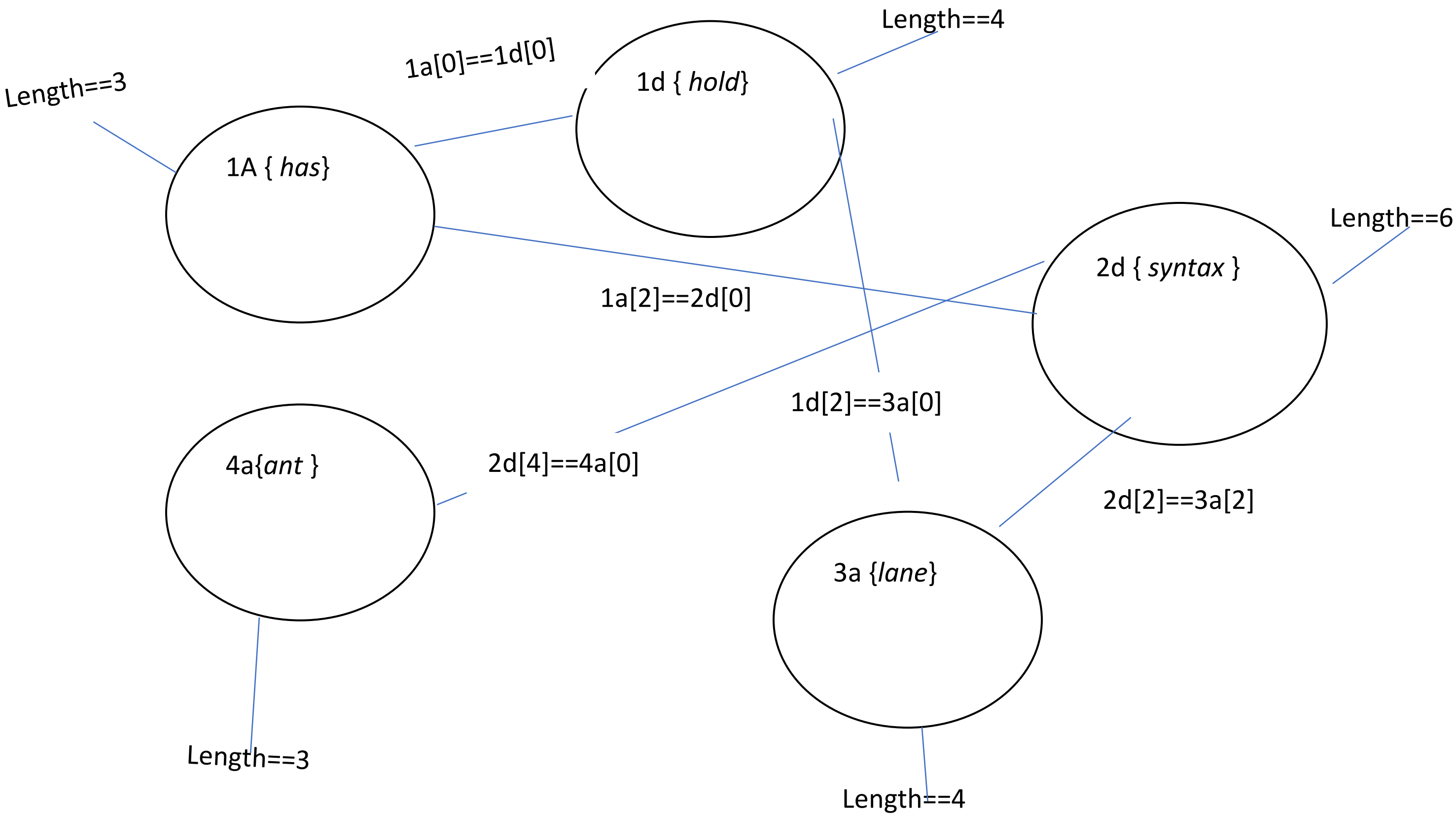
1across&2down: {s}
2down&3across: {n}
2down&4across: {a, c}
Yes these would lead to the same answer

5. (e)
Show how variable elimination solves the crossword problem. Start from the arc-consistent network from part (c).



We can begin by eliminating ant and car from 1across because 1a[0]==1d[0].
We can then can eliminate book and buys from 1 down because 1d[2]==3a[0]
For the same reason(1d[2]==3a[0]) we can eliminate book book,buys, and hold from 3 across.
We can then eliminate ginger, search, and symbol from 2 down because 2d[2]==3a[2]
We can then eliminate big, bus, car, and has from 4 across because 2d[2] =4a[0]
We then can eliminate big and bus from 1a because 1a[0]==1d[0].

Solution:



6. (f)

Does a different elimination ordering affect the efficiency? Explain.

Yes, because if you can choose properly and get a variable down to one possibility early it drastically reduces the possibilities for the other nodes. This will make the algorithm faster.

3. (7p) Exercise 4.6

Consider a scheduling problem, where there are five activities to be scheduled in four time slots. Suppose we represent the activities by the variables A, B, C, D, and E, where the domain of each variable is $\{1,2,3,4\}$ and the constraints are $A>D$, $D>E$, $C\neq A$, $C>E$, $C\neq D$, $B\geq A$, $B\neq C$, and $C\neq D+1$.

[Before you start this, try to find the legal schedule(s) using your own intutions.]

1. (a)

Show how backtracking solves this problem. To do this, you should draw the search tree generated to find all answers. Indicate clearly the valid schedule(s). Make sure you choose a reasonable variable ordering.

To indicate the search tree, write it in text form with each branch on one line. For example, suppose we had variables X, Y, and Z with domains t, f and constraints $X \neq Y$ and $Y \neq Z$. The corresponding search tree is written as:

```
X=t Y=t failure
    Y=f Z=t solution
        Z=f failure
X=f Y=t Z=t failure
        Z=f solution
    Y=f failure
```

[Hint: It may be easier to write a program to generate such a tree for a particular problem than to do it by hand.]

```
A=1 B=1 C=1 failure
        C=2 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=3 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=4 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
    B=2 C=1 failure
        C=2 failure
        C=3 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=4 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
    B=3 C=1 failure
        C=2 D=1 failure
            D=2 failure
            D=3 failure
```

```
          D=4 failure
      C=3 failure
      C=4 D=1 failure
          D=2 failure
          D=3 failure
          D=4 failure
  B=4 C=1 failure
      C=2 D=1 failure
          D=2 failure
          D=3 failure
          D=4 failure
      C=3 D=1 failure
          D=2 failure
          D=3 failure
          D=4 failure
      C=4 failure
A=2 B=1 failure
  B=2 C=1 D=1 failure
          D=2 failure
          D=3 failure
          D=4 failure
      C=2 failure
      C=3 D=1 E=1 failure
              E=2 failure
              E=3 failure
              E=4 failure
          D=2 failure
          D=3 failure
          D=4 failure
      C=4 D=1 E=1 failure
              E=2 failure
              E=3 failure
              E=4 failure
          D=2 failure
          D=3 failure
          D=4 failure
  B=3 C=1 D=1 failure
          D=2 failure
          D=3 failure
          D=4 failure
      C=2 failure
      C=3 failure
      C=4 D=1 E=1 failure
```

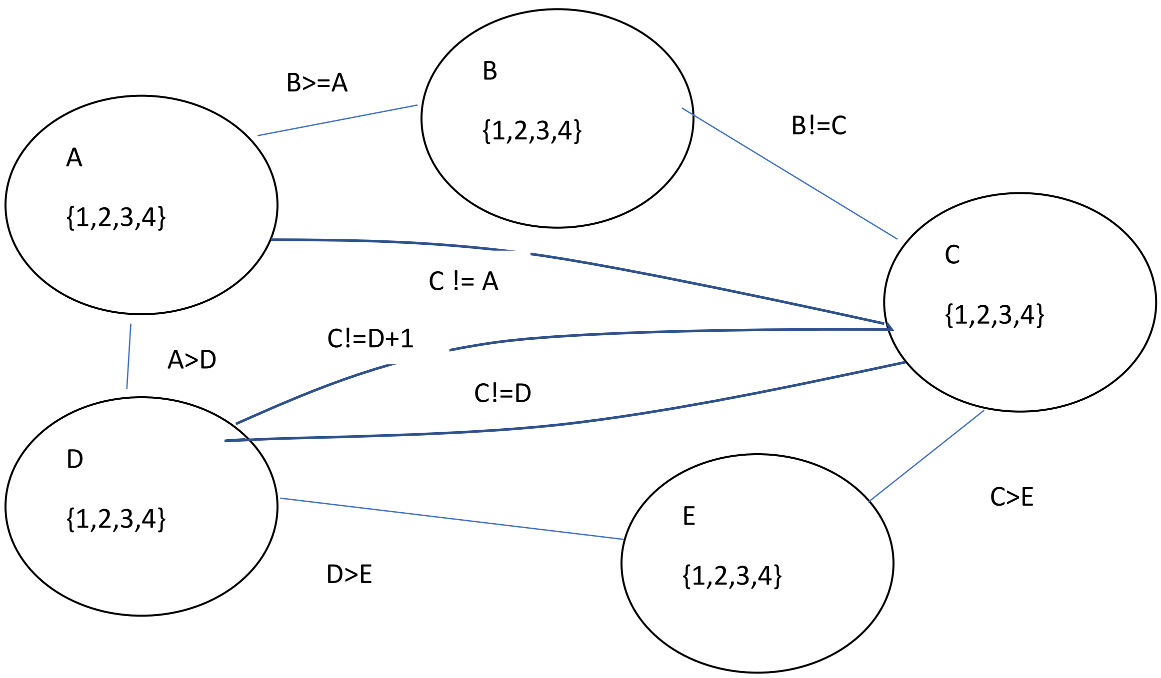
```
                E=2 failure
                E=3 failure
                E=4 failure
            D=2 failure
            D=3 failure
            D=4 failure
    B=4 C=1 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=2 failure
        C=3 D=1 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=4 failure
A=3 B=1 failure
    B=2 failure
    B=3 C=1 D=1 failure
            D=2 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=3 failure
            D=4 failure
        C=2 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=3 failure
        C=4 D=1 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=2 E=1 success
                E=2 failure
                E=3 failure
                E=4 failure
            D=3 failure
            D=4 failure
```

```
    B=4 C=1 D=1 failure
            D=2 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=3 failure
            D=4 failure
        C=2 D=1 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=3 failure
        C=4 failure
A=4 B=1 failure
    B=2 failure
    B=3 failure
    B=4 C=1 D=1 failure
            D=2 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=3 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=4 failure
        C=2 D=1 failure
            D=2 failure
            D=3 E=1 success
                E=2 failure
                E=3 failure
                E=4 failure
            D=4 failure
        C=3 D=1 E=1 failure
                E=2 failure
                E=3 failure
                E=4 failure
            D=2 failure
            D=3 failure
            D=4 failure
        C=4 failure
```

2. (b)

   Show how arc consistency solves this problem. To do this you must

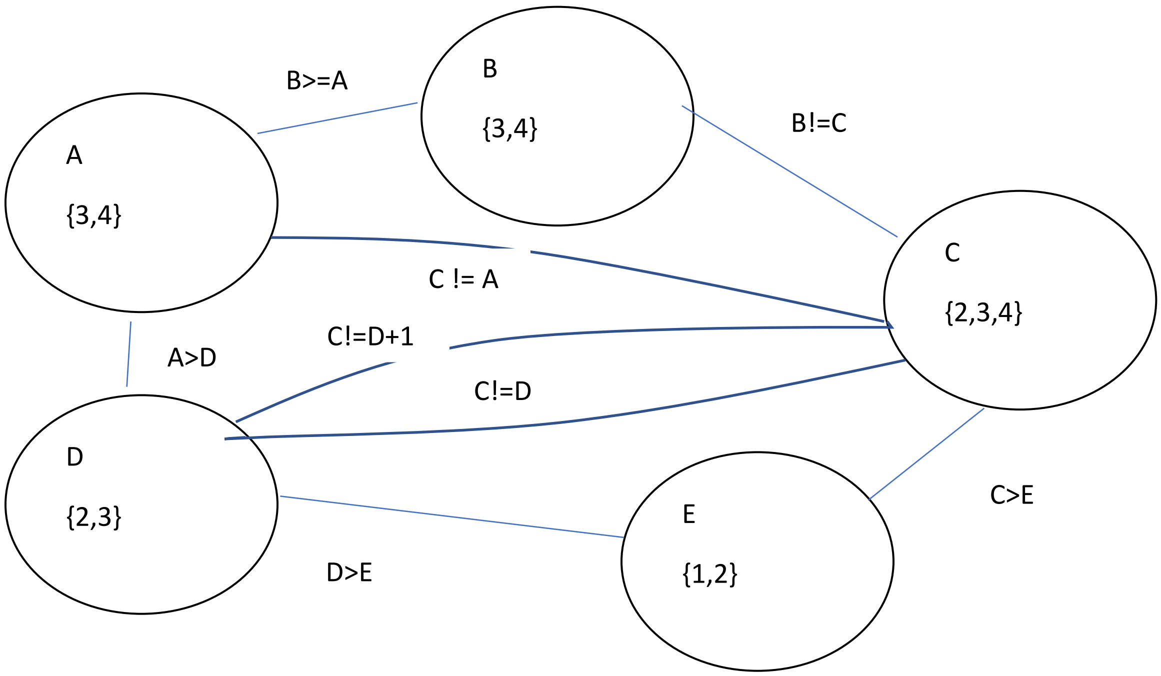
   - draw the constraint graph;



show which elements of a domain are deleted at each step, and which arc is responsible for removing the element;

A>D leads to 4 being deleted from D
D>E leads to 1 being deleted from D
D now is {2,3}
A > D leads to the deletion of A=1 and A=2
A is now {3,4}
B>= A leads to the deletions of B=1 and B=2
B is now {3,4}
D>E leads to 4 and 3 being deleted from E
E is now {1,2}
C>E leads to the deletion of C=1
C is now {2,3,4}

- show explicitly the constraint graph after arc consistency has stopped; and
- show how splitting a domain can be used to solve this problem.



If we use domain splitting to solve the problem we can split the domain in half for D and chose 2

This means E has to be 1 because D>E

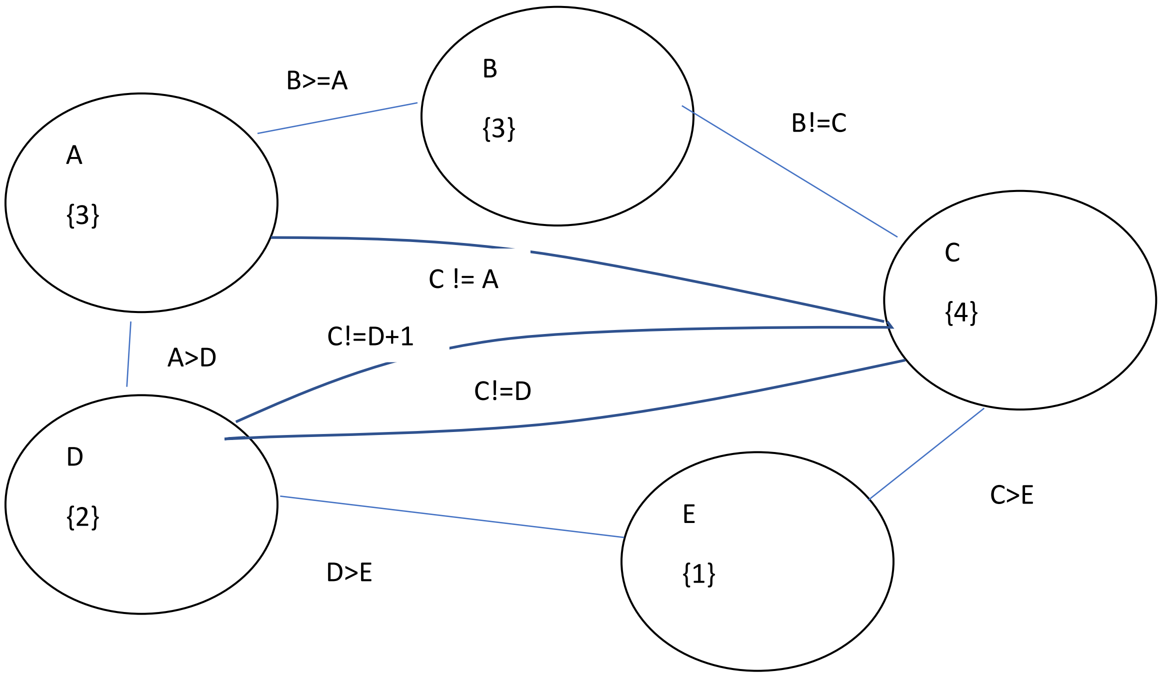
C has to has to be 4 because C!=D+1 and C!=D

B has to be 3 because B!=C

A has to be 3 because B>A

C!=A is satisfied.

See Solution below



If we use domain splitting to solve the problem we can split the domain in half for D and chose 3

This means A has to be 4 because A>D

B has to be 4 because B>=A

C has to be 2 or 1 because c!= A or B or D or D+1

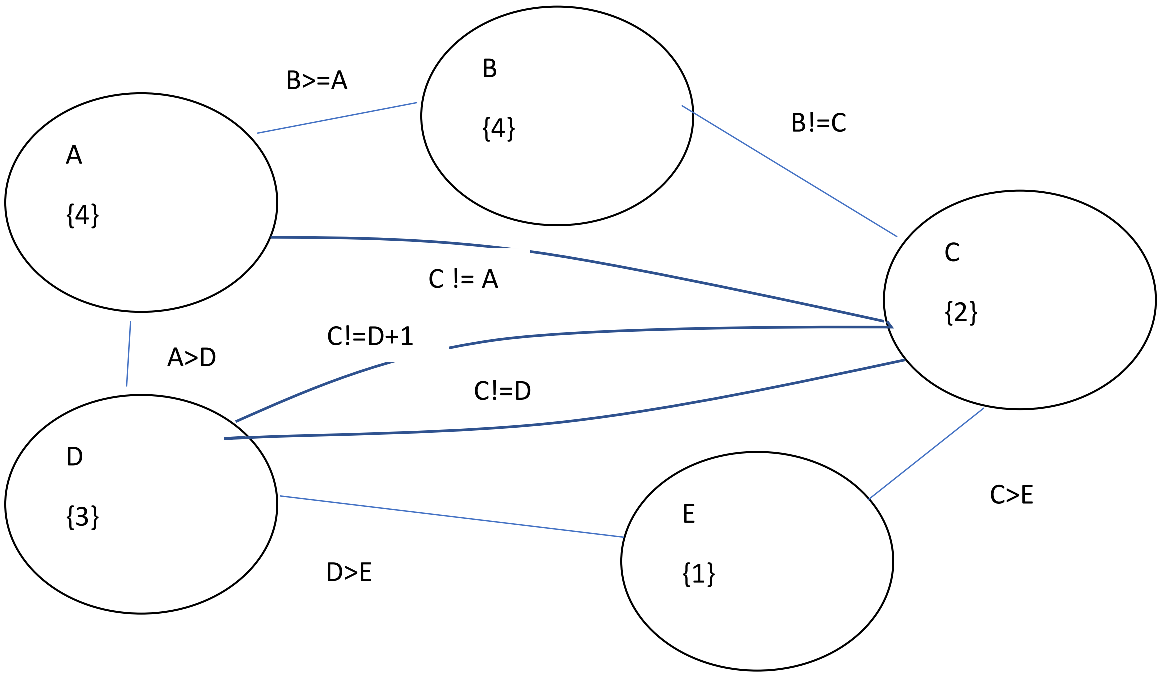
E must be 2 or 1 because D>E

But if we know C>E we can eliminate one for c

C=2

E =1 because C>E

See Solution below


B>=A
B
{4}
B!=C
A
{4}
C
{2}
C != A
C!=D+1
A>D
C!=D
D
{3}
E
{1}
C>E
D>E