

# Software Requirements Specification for FBP CT Image Reconstruction

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## Revision History

Date	Version	Notes
Jan 23, 2025	1.0	Initial Draft
...	...	...

# 1 Reference Material

This section records information for easy reference.

## 1.1 Table of Units

Throughout this document SI (Système International d’Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	unit	SI
m	length	metre
°	angle	degree

## 1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with existing documentation. For its definition, please refer to the Section [4.1.1](#).

symbol	unit	description
$I$	None	Intensity
$I_0$	None	Initial intensity
$A$	None	Attenuation coefficient
$A_{max}$	None	Highest attenuation coefficient
$\theta$	°	Rotation degree
$t$	m	X-ray path length
$\omega$	None	Filter frequency variable
$\omega_{max}$	None	Maximum filter frequency variable

### 1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
FBP CT Image Reconstruction	Filter back projection for CT image reconstruction
TM	Theoretical Model

## 2 Introduction

Reconstruction is the process of turning raw data collected by the detectors into image data which can be viewed on a screen. Both raw data and image data can be manipulated in different ways to create images with differing properties.

The most rudimentary process of turning raw data into image data is called back-projection. This is the process of mathematically mapping the attenuation pathway at every angle measured through a scan to locate where in a patient attenuation is occurring.

However, even if the number of back-projection is increased (say to 1000 directions), there is still a large amount of noise blurring the recreate image. As it turns out, no matter how many directions we try to backproject in, it is not able to perfectly recreate our image using the back-projection. For this process to be at all useful, it is necessary to derive a way to filter out some of the noise the back-projection formula seems to create in our picture and get a smoother representation of our object [1].

The following subsections will outline the primary aspects of this document, ensuring clarity in understanding the purpose, scope, and organization of the system requirements.

### 2.1 Purpose of Document

This document serves to guide software developers and researchers in understanding the technical challenges and requirements involved in the project. It aims to facilitate effective communication between team members and domain experts, ensuring a correct and efficient implementation of Filter Back-projection (FBP) method for CT image reconstruction.

Additionally, this document serves as a reference for users of the software to understand its capabilities, limitations, and assumptions. It also acts as a valuable resource for future maintenance, and verification of the implemented methods, ensuring long-term usability and reliability of the software.

### 2.2 Scope of Requirements

This project focuses on implementing FBP methods for CT image reconstruction, with simplifications applied to manage the complexity of real-world challenges. The scope is constrained to reconstructing 2D images from raw projection data obtained through ideal scanning conditions. It excludes considerations such as non-ideal scanner geometries, variations in detector sensitivity, or artifacts caused by motion during scanning.

The implementation will primarily address noise reduction and basic filtering, without delving into advanced techniques like iterative reconstruction or machine learning-based enhancements. The software assumes uniform spacing between projection angles and consistent image resolution.

## 2.3 Characteristics of Intended Reader

The intended readers of this document are individuals involved in the development, review, and maintenance of the FBP tool for CT image reconstruction. These readers are expected to have a basic understanding of computed tomography principles and image processing techniques such as Fourier transforms, Random transforms, linear algebra and Python.

## 2.4 Organization of Document

The introduction section (Section 2) provides an overview of the project, including the purpose, scope, and intended audience for this document. The General System Description (Section 3) outlines the context and high-level requirements of the system. Detailed problem definitions, theoretical foundations, and specific system functionalities are described in the Specific System Description (Section 4). The Requirements section (Section 5) specifies both functional and non-functional requirements. The document concludes with an overview of potential changes and their implications for future development.

# 3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

## 3.1 System Context

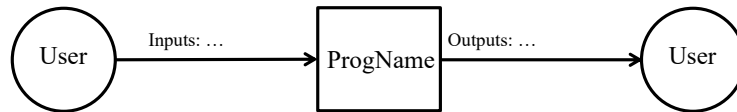


Figure 1: System Context

- User Responsibilities:
  - Provide X-ray projection data in a standardized format.
  - Select appropriate filtering methods.
  - Interpret and analyze the reconstructed images.
- Software Responsibilities:



- Validate input data to ensure compatibility with the processing pipeline.
- Apply transformations and filtering to extract attenuation values.
- Generate reconstructed images based on back-projection techniques.
- Ensure computational efficiency by leveraging optimized libraries.

## 3.2 User Characteristics

The primary users of this tool for CT image reconstruction are individuals with a background in computed tomography, image processing, and numerical methods. They are expected to have at least an undergraduate-level understanding of calculus, linear algebra, and Fourier transforms, as these concepts are fundamental to the reconstruction process. Familiarity with Radon transforms, filtering techniques (such as Ramp and Shepp-Logan filters), and Python programming is also required to effectively use and modify the system. Users may include medical imaging researchers, software engineers developing CT reconstruction algorithms, and graduate students studying image processing techniques.

## 3.3 System Constraints

This tool must operate within specific system constraints to ensure compatibility, performance, and usability. The software must be implemented in Python, leveraging libraries such as NumPy, SciPy, and scikit-image for numerical computations and image processing. It must support grayscale 2D images with a fixed resolution to ensure uniform data processing.

Additionally, the system should be optimized to run efficiently on standard computing hardware, avoiding dependencies on specialized GPU acceleration. Furthermore, the input data format must conform to a standardized structure, ensuring that projection data and filtering parameters align with the expected mathematical models.

# 4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved in FBP CT Image Reconstruction. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

## 4.1 Problem Description

FBP CT Image Reconstruction is a tool designed to address the issue of blurry CT reconstructed images resulting from simple back projection.

#### **4.1.1 Terminology and Definitions**

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

Terminology		Definitions
Intensity		The measure of brightness or signal strength in an image, representing the amount of X-ray energy detected after passing through an object. In CT images, intensity values correspond to the attenuation of X-rays by different tissues.
Attenuation Coefficient $A(x, y)$		measure of how much an X-ray beam is reduced in intensity as it passes through a material, depending on the material's density and composition.
Radon Transform (Sinogram)		A mathematical transformation that converts an image from the spatial domain to the projection domain by integrating values along specific directions.
Fourier Transform		Converting sinogram from the projection domain to the frequency domain.
Inverse Fourier Transform		Converting the projection data back to the spatial projection domain.
Fourier domain		A representation of signals in the frequency domain, where CT images can be reconstructed using Fourier transforms to analyze spatial frequency components.
Spatial domain		The domain in which an image is represented in terms of pixel intensities, as opposed to the frequency domain.
Projection domain		The projection domain is a mathematical representation of an object's line integrals taken from multiple angles, rather than its direct spatial structure.
Low-pass filter		A filter that allows low-frequency components to pass through while reducing high-frequency noise, commonly used in image smoothing.
High-pass filter		A filter that enhances high-frequency components while attenuating low-frequency signals, often used to sharpen images or enhance edges.
Back Projection (BP)		Maps filtered projections to reconstruct the image in spatial domain.
Filter Back Projection (FBP)		A widely used image reconstruction technique in CT that improves upon simple back projection by applying a filtering step in the frequency domain. This enhances image quality by reducing blurring and artifacts.
Monochromatic X-rays		They have a single photon energy without beam hardening artifacts.
Polychromatic X-rays		They contain a range of photon energies, causing beam hardening, where lower-energy photons are absorbed more, making attenuation non-linear and requiring corrections.

### 4.1.2 Physical System Description

The physical system of FBP CT Image Reconstruction, includes the following elements:

**PS1: X-Ray Source**

The CT scanner's X-ray tube emits X-ray beams that pass through the object being scanned.

**PS2: Scanned Object (Patient or Sample)**

The physical object being imaged, which could be a biological tissue, industrial material, or any sample placed in the CT scanner.

**PS3: X-Ray Detectors**

A ring or array of sensors that measure the intensity of X-rays after passing through the scanned object.

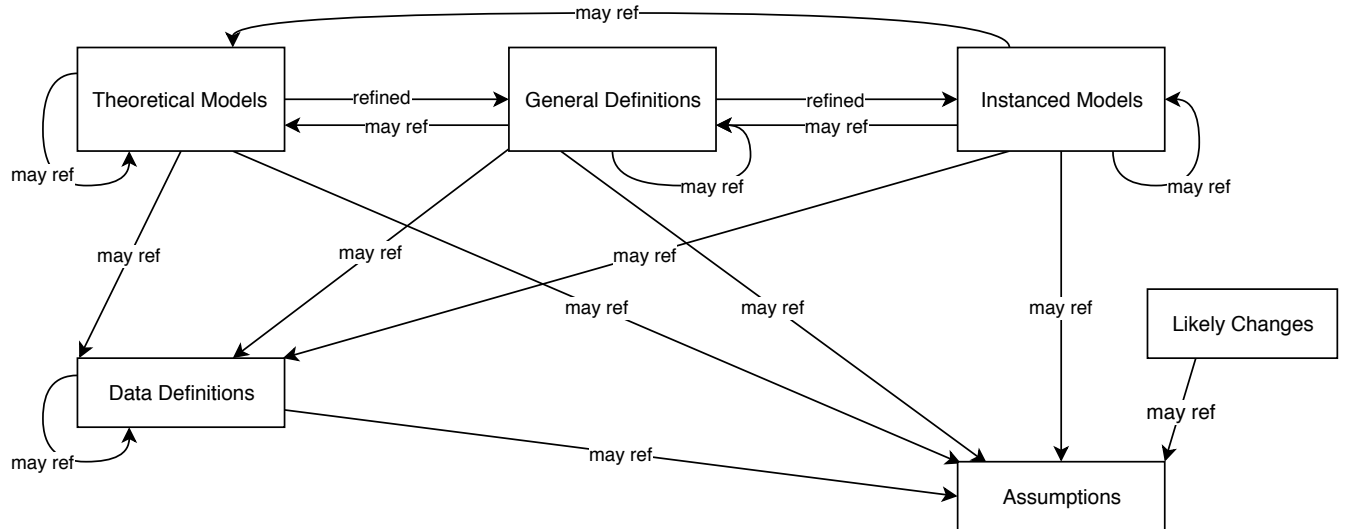
### 4.1.3 Goal Statements

Given a set of raw intensity data measured by a CT scanner detector array, the goal statements are:

GS1: Implement High-Pass Filtered Back Projection to reconstruct a sharper edge CT image.

GS2: Implement Low-Pass Filtered Back Projection to reconstruct an overall smooth CT image.

## 4.2 Solution Characteristics Specification



The instance models that govern FBP CT Image Reconstruction are presented in Subsection 4.2.6. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

### 4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [TM], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

- A1: The Ramp and Shepp-Logan filters are assumed to be representative of high-pass and low-pass filtering techniques. [TM4, TM5, IM1, IM2]
- A2: The system assumes that the CT scanner uses a monochromatic (single-energy) X-ray source to avoid beam hardening artifacts. In reality, CT scanners use polychromatic X-rays, but for mathematical simplicity, this assumption is made. [DD1]

### 4.2.2 Theoretical Models

This section focuses on the general equations and laws that FBP CT Image Reconstruction is based on.

Number	TM1
Label	<b>Log Transformation</b>
Equation	$y = \log_b(x) \iff b^y = x$
Description	It expresses the fundamental relationship between logarithms and exponents, showing their equivalence.
Note	None
Source	None
Ref. By	GD1

Number	TM2
Label	<b>Randon Transform</b>
Equation	$Rf(t, \theta) = \int_{-\infty}^{\infty} f(x(t), y(t)) dt$
Description	<p>The <math>Rf(t, \theta)</math> represents the projection data given the detected intensity data. Where:</p> <ul style="list-style-type: none"> <li>• <math>f(x, y)</math> [DD2] represents the original function describing the intensity absortion coefficient alone the path. In CT image reconstruction, it is attenuation coefficient.</li> <li>• The result, <math>Rf(t, \theta)</math>, provides a function of: <ul style="list-style-type: none"> <li>– <math>t</math>: The perpendicular distance from the origin to the projection line.</li> <li>– <math>\theta</math>: The projection angle used during scanning.</li> </ul> </li> </ul>
Note	None
Source	[1]
Ref. By	TM3, IM1, IM2

Number	TM3
Label	<b>Back Projection</b>
Equation	$BRf(x, y) = 1/\pi \int_0^\pi Rf(t, \theta) d\theta$
Description	<p>The Back Projection, also known as the Inverse Radon Transform [TM2], is a technique used to reconstruct an image from its projections. It is widely applied in CT and other imaging techniques to recover the original function from its Radon transform.</p> <ul style="list-style-type: none"> <li>• <math>BRf(x, y)</math> represents the Inverse Radon transform of the function <math>f(x, y)</math>. Its result is an approximation of <math>f(x, y)</math> [DD2], which is the attenuation coefficient. <ul style="list-style-type: none"> <li>– <math>x, y</math>: Cartesian coordinates in the reconstructed image space.</li> <li>– <math>\theta</math>: The projection angle used during scanning.</li> <li>– <math>Rf(t, \theta)</math>: The projection data mapped back to the image domain. [GD1]</li> </ul> </li> </ul>
Note	None
Source	[1]
Ref. By	IM1, IM2, GD1

Number	TM4
Label	<b>Ramp Filter (high pass filter)</b>
Equation	$H(\omega) =  \omega $
Description	<p>The Ramp filter belongs to the high pass filter [A1]. It helps to counteract the blurring introduced by simple back projection, improving image sharpness and accuracy.</p> <ul style="list-style-type: none"> <li>• <math>H(\omega)</math> represents the Ramp Filter function.</li> <li>• <math>\omega</math> is the frequency variable.</li> <li>• <math> \omega </math> behaves as a high-pass filter, amplifying high frequencies and suppressing low frequencies.</li> </ul>
Note	None
Source	[1]
Ref. By	IM1

Number	TM5
Label	<b>Shepp-Logan Filter (low pass filter)</b>
Equation	$S(\omega) =  \omega  \cdot \text{sinc}(\frac{\omega}{2\omega_{max}})$
Description	<p>The Shepp-Logan filter is a modified version of the Ramp Filter used in low-pass filter [A1] for CT image reconstruction. It helps reduce noise by attenuating high frequencies smoothly.</p> <ul style="list-style-type: none"> <li>• <math>S(\omega)</math> represents the Shepp-Logan Filter function.</li> <li>• <math>\omega</math> is the frequency variable.</li> <li>• <math>\omega_{max}</math> is the maximum frequency (cutoff frequency).</li> <li>• <math>\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}</math> is the sinc function, which introduces smooth attenuation of high frequencies.</li> </ul>
Note	None
Source	[1]
Ref. By	IM2



Number	TM6
Label	<b>Fourier Transform</b>
Equation	$F(x) = \mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{-j\omega x} dx$
Description	<p>Fourier Transform (<math>\mathcal{F}\{f(x)\}</math>) Converts a function from the spatial domain to the frequency domain.</p> <ul style="list-style-type: none"> <li>• <math>F(\omega)</math> is the function in the frequency domain.</li> <li>• <math>\omega</math> is the frequency variable.</li> <li>• <math>f(x)</math> is the function in the spatial domain.</li> <li>• <math>e^{-j\omega x}</math> represents the complex exponential basis function (where <math>j = \sqrt{-1}</math>).</li> </ul>
Note	None
Source	None
Ref. By	IM1, IM2

Number	TM7
Label	<b>Inverse Fourier Transform</b>
Equation	$f(x) = \mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega x} d\omega$
Description	<p>Inverse Fourier Transform (<math>\mathcal{F}^{-1}\{F(\omega)\}</math>) Converts a function from the frequency domain back to the spatial domain.</p> <ul style="list-style-type: none"> <li>• <math>F(\omega)</math> is the function in the frequency domain.</li> <li>• <math>\omega</math> is the frequency variable.</li> <li>• <math>f(x)</math> is the function in the spatial domain.</li> <li>• <math>e^{j\omega x}</math> represents the inverse transformation basis.</li> </ul>
Note	None
Source	None
Ref. By	IM1, IM2

### 4.2.3 General Definitions

This section collects the laws and equations that will be used in building the instance models.

Number	GD1
Label	<b>Intensity data projection</b>
SI Units	None
Equation	$P(t, \theta) = -\ln(\frac{I(t, \theta)}{I_0}) = \int_{x-raypath} f(x, y)ds = Rf(t, \theta)$
Description	Taking the logarithm [TM1] linearizes the exponential relationship, converting the intensity data[DD1] from spacial domain into projection domain. This transformation allows the Radon Transform [TM2] to directly represent line integrals of the attenuation coefficient [DD2]. $P(t, \theta) = -\ln(\frac{I(t, \theta)}{I_0})$ is the value used as an input of back projection. [TM3]
Source	None
Ref. By	IM1, IM2

### Detailed derivation of converting intensity from spatial domain to projection domain

The X-ray intensity [DD1] after passing through an object follows:

$$I(t, \theta) = I_0 e^{-\int_{x-raypath} f(x, y)ds} \quad (1)$$

where:

- $I(t, \theta)$  is the measured intensity at detector position  $t$  for angle  $\theta$ ,
- $I_0$  is the initial X-ray intensity,
- $f(x, y)$  is the attenuation coefficient of the object, [DD2]
- The integral sums the attenuation along the X-ray path  $ds$ .

To linearize this equation, we take the natural logarithm on both sides:

$$\ln I(t, \theta) = \ln I_0 - \int_{x-raypath} f(x, y)ds \quad (2)$$

Rearranging the equation:

$$-\ln\left(\frac{I(t, \theta)}{I_0}\right) = \int_{x\text{-raypath}} f(x, y) ds \quad (3)$$

Thus, defining the projection data  $P(t, \theta)$ :

$$P(t, \theta) = -\ln\left(\frac{I(t, \theta)}{I_0}\right) \quad (4)$$

This equation represents the Radon Transform [TM2], which maps the intensity from spatial domain  $I(t, \theta)$  into the projection domain  $P(t, \theta)$ .

#### 4.2.4 Data Definitions

This section collects and defines all the data types needed to document the models.

Number	DD1
Label	<b>Intensity Data</b>
Symbol	$I(t, \theta)$
SI Units	None
Equation	$I(t, \theta) = I_0 e^{-\int_{x\text{-raypath}} f(x, y) ds}$
Description	<p>This projection data represents the attenuated linear X-ray [A2] intensity after passing through the object. Where:</p> <ul style="list-style-type: none"> <li>• <math>I(t, \theta)</math> is the measured intensity at detector position.</li> <li>• <math>I_0</math> is the initial intensity.</li> <li>• <math>f(x, y)</math> is the unknown attenuation coefficient [DD2].</li> <li>• <math>s</math>: The length of x-ray path. The integral sums the attenuation along the X-ray path.</li> </ul>
Source	[1]
Ref. By	GD1, IM1, IM2

Number	DD2
Label	<b>Attenuation Coefficient</b>
Symbol	$A(x, y)$
SI Units	None
Equation	$A(x, y) = f(x, y)$
Description	It denoted as $f(x, y)$ , represents the degree to which X-ray intensity [DD1] is reduced as it passes through a material at a given location $(x, y)$ . It quantifies the fraction of the X-ray beam that is absorbed or scattered per unit distance and is a fundamental property used in CT image reconstruction to characterize tissue composition.
Source	[1]
Ref. By	IM1, IM2, TM2, TM3, GD1

#### 4.2.5 Data Types

This section collects and defines all the data types needed to document the models. In this system, it isn't necessary, since the inputs and outputs are straightforward types, like reals, integers, and sequences of reals and integers. Please refer to IM1 and IM2.

#### 4.2.6 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in Sections 4.2.2.

The goals GS1 are solved by IM1 and he goals GS2 are solved by IM2.

Number	IM1
Label	<b>Ramp filter back projection to reconstruct Attenuation Coefficient</b>
Input	<ul style="list-style-type: none"> <li>• <math>I(t, \theta)</math>: A 2D array (M * N) with value measured X-ray intensity where: <ul style="list-style-type: none"> <li>– M: Number of detector positions t.</li> <li>– N: Number of projection angles <math>\theta</math>.</li> </ul> </li> <li>• <math>\theta</math>: projection angles in degree.</li> <li>• <math>H(\omega)</math>: 1D array of filtering variable representing the high-pass filter.</li> </ul>
Equation	$A(x, y) = \frac{1}{\pi} \int_0^\pi \mathcal{F}^{-1} \mathcal{F}[-\ln \frac{I(t, \theta)}{I_0} \cdot H(\omega)]$
Output	$A(x, y)$ : 2D array (P × P), where P is the image resolution.
Description	<p>This instance model describes the Filtered Back Projection, which reconstructs the <math>A(x, y)</math> from projection data. The method consists of:</p> <ul style="list-style-type: none"> <li>• Log transformation of intensity data <math>I(t, \theta)</math> to obtain projection data <math>P(t, \theta)</math>. [DD1] <i>Radon Transform interpretation, where <math>P(t, \theta) = Rf(t, \theta)</math>.</i> [GD1]</li> <li>• Fourier filtering with Ramp filter <math>H(\omega)</math> to enhance reconstruction accuracy. [TM4, TM6, TM7]</li> <li>• Back Projection over all projection angles <math>\theta</math> to reconstruct the final image. [TM3, DD2]</li> </ul>
Source	[1]
Ref. By	None

Number	IM2
Label	<b>Shepp-Logan back projection to reconstruct Attenuation Coefficient</b>
Input	<ul style="list-style-type: none"> <li>• <math>I(t, \theta)</math>: 2D array (M * N) with value measured X-ray intensity where: <ul style="list-style-type: none"> <li>– M: Number of detector positions t.</li> <li>– N: Number of projection angles <math>\theta</math>.</li> </ul> </li> <li>• <math>\theta</math>: projection angles in degree.</li> <li>• <math>S(\omega)</math>: 1D array of filtering variable representing the low-pass filter.</li> </ul>
Equation	$A(x, y) = \frac{1}{\pi} \int_0^\pi \mathcal{F}^{-1} \mathcal{F}[-\ln \frac{I(t, \theta)}{I_0} \cdot S(\omega)]$
Output	$A(x, y)$ : 2D array (P × P), where P is the image resolution.
Description	<p>This instance model describes the Filtered Back Projection, which reconstructs the <math>A(x, y)</math> from projection data. The method consists of:</p> <ul style="list-style-type: none"> <li>• Log transformation of intensity data <math>I(t, \theta)</math> to obtain projection data <math>P(t, \theta)</math>. [DD1]</li> <li>• Radon Transform interpretation, where <math>P(t, \theta) = Rf(t, \theta)</math>. [GD1]</li> <li>• Fourier filtering with Shepp-Logan filter <math>S(\omega)</math> to enhance reconstruction accuracy. [TM5, TM6, TM7]</li> <li>• Back Projection over all projection angles <math>\theta</math> to reconstruct the final image. [TM3, DD2]</li> </ul>
Source	[1]
Ref. By	None

## Derivation of Filter Back Projection

The Filtered Back Projection (FBP) reconstructs an image from X-ray projections by first applying a log transformation to convert intensity measurements into projection data. The Fourier Transform is then used to filter the projections with a ramp or shepp-logan filter, which enhances high-frequency details. The filtered projections are then transformed back using the Inverse Fourier Transform. Finally, integration over all projection angles  $\theta$  performs the back-projection, reconstructing the  $A(x, y)$ .

#### 4.2.7 Input Data Constraints

Table 1 shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 1 are listed in Table 2.

Table 1: Input Variables

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
$I(t, \theta)$	$I(t, \theta) > 0$	$0 \leq I(t, \theta) \leq I_0$	None	5%
$H(\omega)$	Bandwidth-limit	frequency domain	None	None
$S(\omega)$	Bandwidth-limit	frequency domain	None	None
$\theta$	$0^\circ \leq \theta \leq 180^\circ$	discrete angles	None	None

Table 2: Specification Parameter Values

Var	Value
$I_0$	1

#### 4.2.8 Properties of a Correct Solution

A correct Filtered Back Projection reconstruction must adhere to physical correctness and computational stability. The reconstructed image should preserve the total attenuation observed in the sinogram, ensuring consistency with the projection data. Additionally, the attenuation values must remain non-negative, as negative attenuation is not physically meaningful. The reconstruction must also respect constraints derived from the X-ray attenuation model, ensuring values stay within a reasonable range.

A sample table is shown in Table 3

Table 3: Output Variables

Var	Physical Constraints
$A(x, y)$	$0 \leq A(x, y) \leq A_{max}$

$A_{max}$  depends on the highest attenuation material in the scan.

## 5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

### 5.1 Functional Requirements

- R1: The system must accept a grayscale 2D input image, apply the Radon Transform to generate projection data, and ensure that intensity values are normalized between 0 and 1 (from IM1 and IM2).
- R2: The system must perform log transformation on intensity values to obtain projection data, ensuring linear attenuation and consistency with the Radon Transform (from IM1 and IM2).
- R3: The system must implement Fourier filtering using either a Ramp or Shepp-Logan filter (from IM1 and IM2).
- R4: The system must perform the back-projection over all projection angles to reconstruct the attenuation coefficient after filtering (from IM1 and IM2).
- R5: The reconstructed attenuation values must be non-negative and consistent with input projections.

### 5.2 Nonfunctional Requirements

- NFR1: **Accuracy** The reconstructed attenuation values must have an error margin within tolerance compared to the ground truth.



- NFR2: **Usability** The system shall provide an interface that allows users to input projection data and visualize both sinograms and reconstructed images, with instructions detailed in the user guide.
- NFR3: **Maintainability** Any modification to the image reconstruction algorithm or filtering process should require less than 10% of the original development time, ensuring ease of updates and enhancements.
- NFR4: **Portability** The system shall be executable on Windows, macOS, and Linux without modification.
- NFR5: **Reusability** The system shall provide a well-documented API that allows developers to integrate and apply additional filtering methods beyond Ramp and Shepp-Logan filters.

### 5.3 Rationale

The rationale behind the design of this CT Image Reconstruction System is based on balancing accuracy, computational efficiency, and extensibility. The Filtered Back Projection method was chosen due to its well-established effectiveness in reconstructing images from projection data while maintaining computational feasibility. The inclusion of both Ramp and Shepp-Logan filters provides flexibility in handling noise and resolution trade-offs. The log transformation of intensity data was incorporated to ensure the attenuation model aligns with the Radon Transform formulation.

The system is designed with modularity in mind, allowing future integration of additional filtering techniques through an API, enhancing reusability. The implementation adheres to standard non-functional requirements, including accuracy validation via MSE and PSNR, ensuring usability through a clear user interface, and guaranteeing portability across multiple platforms. These decisions collectively ensure a robust, adaptable, and scientifically grounded reconstruction framework.

## 6 Likely Changes

- LC1: Future improvements may introduce additional filtering methods beyond Ramp and Shepp-Logan filters, requiring the API to support more flexible and adaptive reconstruction techniques (A1).

## 7 Unlikely Changes

UC1: The fundamental mathematical model for Radon Transform and Filtered Back Projection is unlikely to change, as it is a well-established method in CT image reconstruction and widely validated in medical imaging.

## 8 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an “X” may have to be modified as well. Table 5 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 6 shows the dependencies of instance models, requirements, and data constraints on each other. Table 4 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	A1	A2
TM1		
TM2		
TM3		
TM4	X	
TM5	X	
TM6		
TM7		
GD1		
DD1		
DD2		X
IM1	X	
IM2	X	

Table 4: Traceability Matrix Showing the Connections Between Assumptions and Other Items

	TM1	TM2	TM3	TM4	TM5	TM6	TM7	GD1	DD1	DD2	IM1	IM2
TM1								X				
TM2			X								X	X
TM3								X			X	X
TM4											X	
TM5												X
TM6											X	X
TM7											X	X
GD1											X	X
DD1								X			X	X
DD2		X	X					X			X	X
IM1		X	X	X		X	X	X	X	X	X	X
IM2		X	X		X	X	X	X	X	X	X	X

Table 5: Traceability Matrix Showing the Connections Between Items of Different Sections

	IM1	IM2	R1	R2	R3	R4	R5
IM1			X	X	X	X	
IM2			X	X	X	X	
R1	X	X					
R2	X	X					
R3	X	X					
R4	X	X					
R5							

Table 6: Traceability Matrix Showing the Connections Between Requirements and Instance Models

## 9 Development Plan

The development of the CT Image Reconstruction Tool will be carried out in a structured manner to ensure accuracy, efficiency, and future extensibility. Initially, the focus will be on implementing the core functionality, including input validation, Radon Transform computation, and the application of log transformation to convert intensity measurements into a linear attenuation model. Once this foundation is established, the tool will integrate Fourier

filtering using both Ramp and Shepp-Logan filters, followed by back projection to reconstruct the original image from the sinogram.

After the core functionality is implemented, the focus will shift to enhancements and optimizations. This includes improving numerical stability, optimizing computational efficiency, and refining filtering techniques. Additionally, usability improvements, such as better error handling and visualization tools, will be incorporated. Finally, an API extension will be developed to allow the integration of new filtering methods, making the tool scalable and adaptable for future research and applications.

## **10 Values of Auxiliary Constants**

There is not values of the symbolic parameters introduced in the report.

## References

- [1] Jen Beatty. “The Radon Transform and the Mathematics of Medical Imaging”. In: (2012). Paper 646. URL: <https://digitalcommons.colby.edu/honorstheses/646>.
- [2] W. Spencer Smith. “Systematic Development of Requirements Documentation for General Purpose Scientific Computing Software”. In: *Proceedings of the 14th IEEE International Requirements Engineering Conference, RE 2006*. Minneapolis / St. Paul, Minnesota, 2006, pp. 209–218. URL: <http://www.ifi.unizh.ch/req/events/RE06/>.
- [3] W. Spencer Smith, John McCutchan, and Jacques Carette. *Commonality Analysis for a Family of Material Models*. Technical Report CAS-17-01-SS. McMaster University, Department of Computing and Software, 2017.