Computed Tomography (CT) Image **Reconstruction** with **Filter**

SRS Presentation

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Overview

- Terminology, Definition and What is the problem?
- Goal Statements, Theoretical Models, Instance Models and Examples
- System Constraints

Tomography

An imaging technique that reconstructs internal structures without physically cutting the object.

- "Tomos" (τόμος) → meaning **slice**.
- "Graphia" (γραφή) → meaning **representation**.

Figure 1. CT Scanner

Attenuation Coefficient A(x)

Attenuation: How much X-ray are being **blocked** from reaching the detector

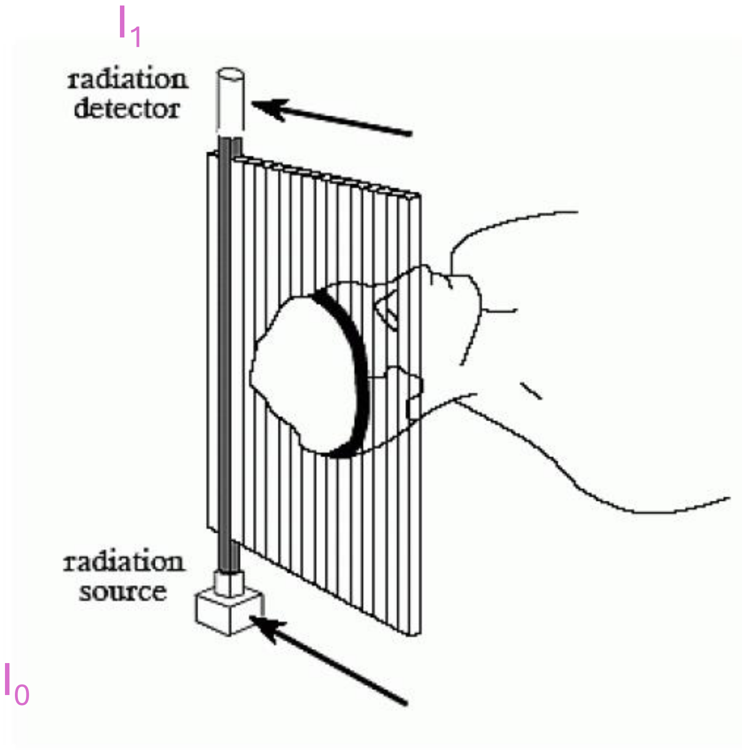


Figure2. CT Scan

Figure 3. CT Image of Brain

What is the **problem** in CT Image Reconstruction with Back-projection (BP) without filter?

- Raw data (scanned slices collected) -> **BP** -> Image view (A(x))
- Natural drawback of BP: blurry image

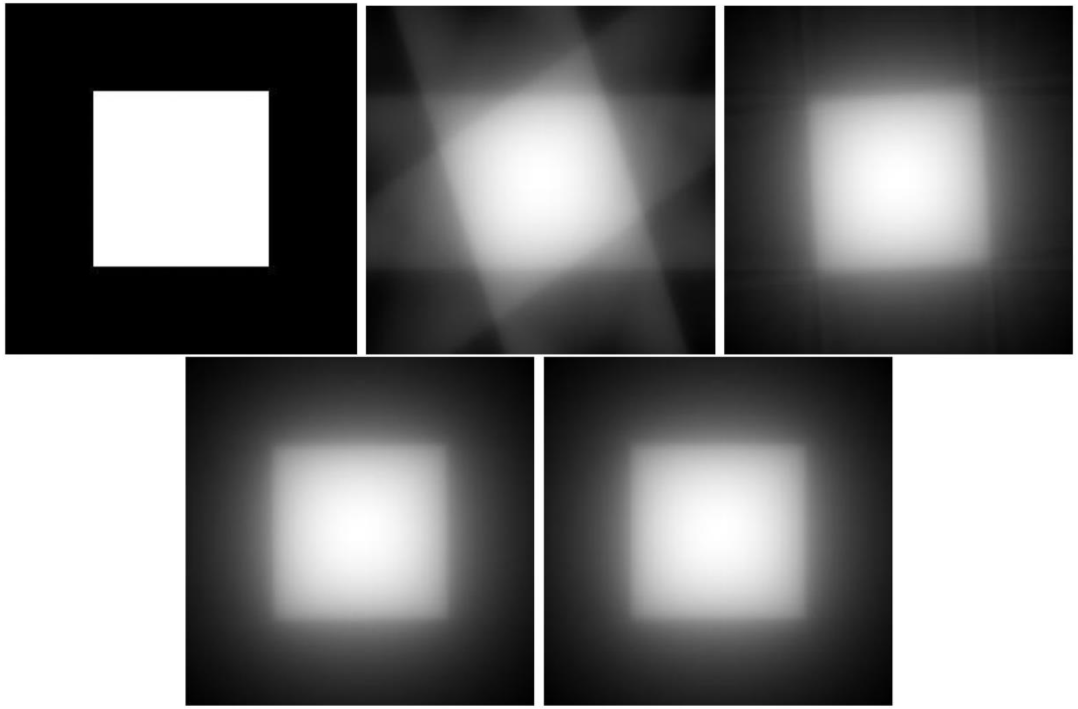


Figure 4. Back projection of a Square in 5, 25, 100, and 1000 directions

Filter Technique: low-pass filter and high-pass filter

All image are made from wave of frequency (a rate of intensity change in space)

Fourier transform split image into continuous frequency (Fourier space)

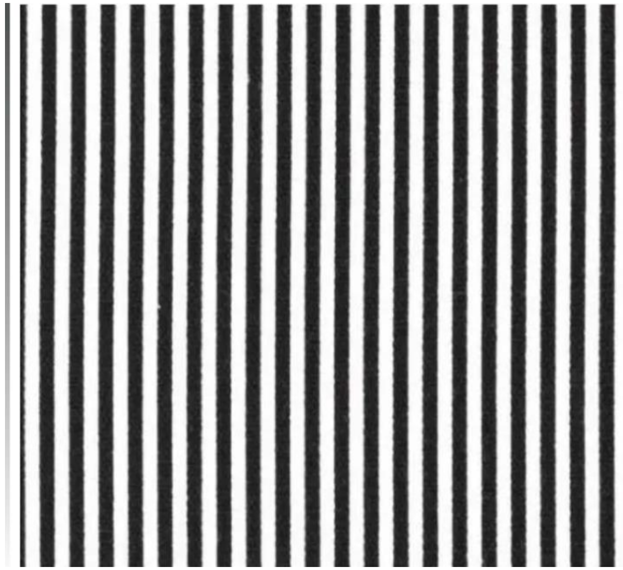


Figure 5. High-frequency image

Figure 6. Low-frequency image

1. High-pass filter removes low-frequency blurring-> image sharper
2. Low-pass filter get rid of high frequency noise -> image clear

Goal Statements

Given a set of raw intensity data measured by a detector:

- GS1: High-Pass Filtered Reconstruct into a sharper edge image.
- GS2: Low-Pass Filtered Reconstruct into an overall smoother image.

Theoretical Models

No assumption needed to refine the scope!

TM 1: Beer-Lambert Law

$$I(x) = e^{-A(x)x}$$

- *I(x) : Intensity of beam at distance x from origin (raw scanned data)*
- *A(x): Attenuation Coefficient at at distance x from origin*

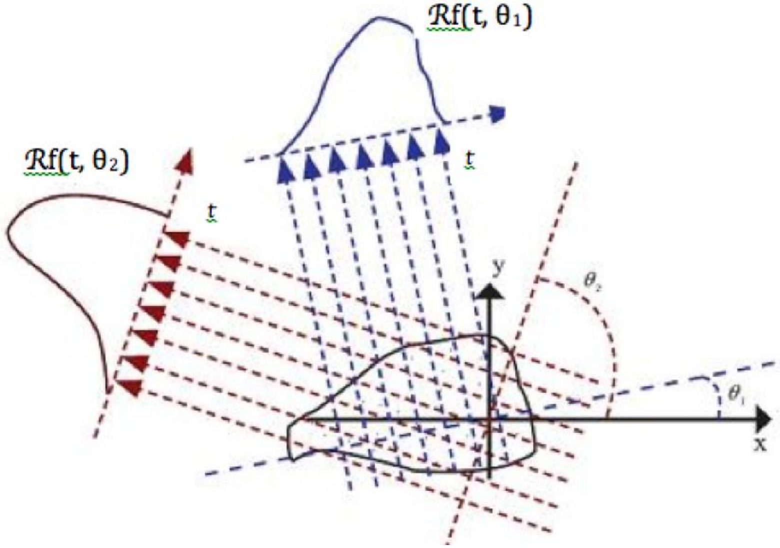
Manipulate to:

$$\ln\left(\frac{I_0}{I_1}\right) = \int_{x_0}^{x_1} A(x)dx$$

TM2: Radon Transform ($\boldsymbol{Rf}$ aka Sinogram)

$$Rf(t,\theta) = \int_{-\infty}^{\infty} f(x(s), y(s))ds$$

- *f(t, θ): A function determines the **density** along a given line l.*
- *x, y: position in the spatial coordinate.*
- *θ: The angle at which the projection is taken.*
- *t: The position along the detector.*



Theoretical Models continue

TM 3: Back Projection (aka Inverse Radon Transform)

$$BRf(x,y) = \frac{1}{\pi}\int_0^{\pi} Rf(x\cos\theta + y\sin\theta, \theta)d\theta$$

- *$BRf(x,y)$: The back projection of Rf in point (x,y) in the spatial coordinate. ->* ***A(x)***

TM4: Ramp Filter (high pass filter)

$$HR\ (k_x, ky) = k = ({k_x}^2 + {k_y}^2)^2$$

- *K_x , K_y : Wave-like pattern in image.*
- *RHS: Determines how much each frequency is amplified.*

TM5: Sheep-logan Filter(low pass filter)

$$S(f) = \frac{2fm}{\pi(\sin|f|\frac{\pi}{2fm})}$$

- *f : Determines how much a given frequency is affected.*
- *f_m : Defines the bandwidth of filter.*

Theoretical Models continue

TM6: Fourier Transform

$$F[Rf(t,\theta)]$$

- *Convert the $Rf(x,y)$ to **Fourier space***

TM7: Inverse Fourier Transform

$$F^{-1}\{F[Rf(t,\theta)] * filter\}$$

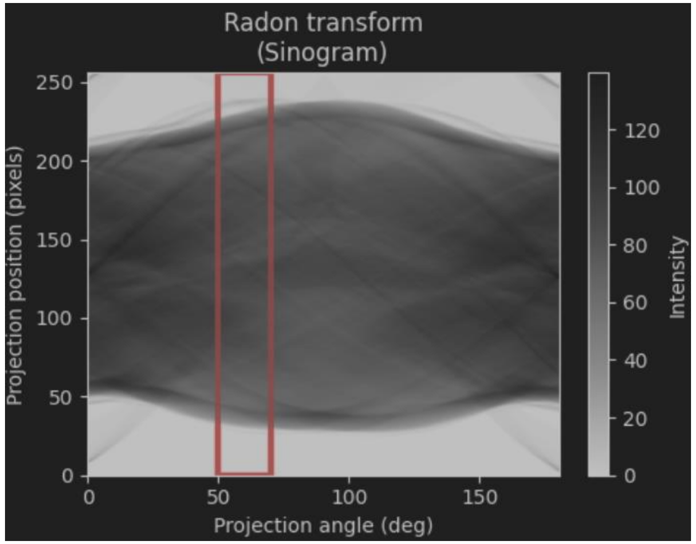
- *Convert the filtered projection back into the **Spatial domain***

Instance Models

Number	IM1 (Ramp FBP)
Equation	$$A(x,y) = \frac{1}{\pi}\int_0^{\pi} \mathcal{F}^{-1}\left\{\mathcal{F}[Rf(s,\theta)] \cdot H_R(k)\right\} d\theta$$
Input	• $Rf(s,\theta)$ (Sinogram) : 2D array (M * N matrix) M: number of detector position s; N: number of projection angle θ • $H_R\ (k_x, ky)$: 1D array consists of **frequency-dependent** values • θ : 1D array containing N angles in degrees or radians
Output	$A(x)$ (reconstructed image): 2D array (P * P) P: Image resolution (e.g. 256 * 256 pixels)
Description	This equation first applies a **ramp filter** in **Fourier space**, then inverse back to spatial domain. The Ramp FBP reconstructs the image and maintains **edge sharpness**.

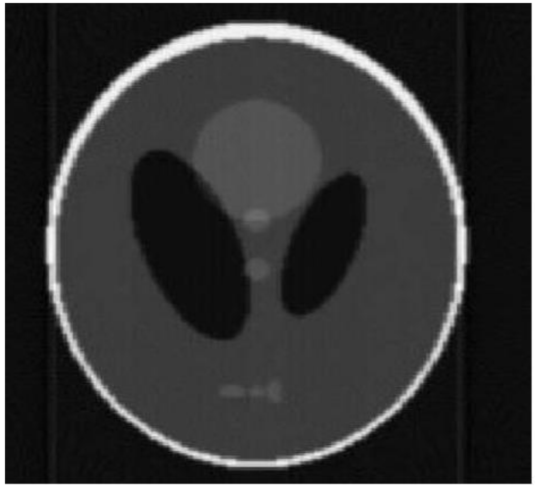
Number	IM2 (Sheep-logan FBP)
Equation	$$A(x,y) = \frac{1}{\pi}\int_0^{\pi} \mathcal{F}^{-1}\left\{\mathcal{F}[Rf(s,\theta)] \cdot S(f)\right\} d\theta$$
Input	• $Rf(s,\theta)$ (Sinogram) : 2D array (M * N matrix) M: number of detector position s; N: number of projection angle θ • $S\ (f)$: 1D array consists of **frequency-dependent** values • θ : 1D array containing N angles in degrees or radians
Output	$A(x)$ (reconstructed image): 2D array (P * P) P: Image resolution (e.g. 256 * 256 pixels)
Description	This equation first applies the **Sheep-Logan** in **Fourier space**, then inverse back to spatial domain. The Sheep-logan FBP reconstructs the image and maintains **an overall smoother image**.

Examples

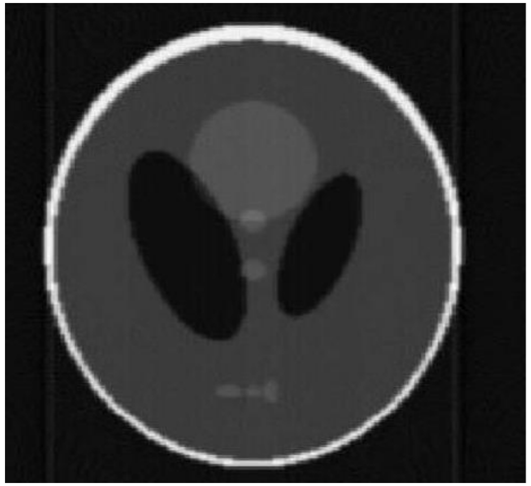


Input

Image reconstructed using simple BP



Ramp filter BP



Sheep-logan filter BP

Output

System Constraints

Modern desktop systems, such as those equipped with Intel 14th generation CPUs and NVIDIA RTX-series GPUs (typically support **32 to 128 GB of RAM**) which is sufficient for standard **512 × 512 resolution reconstructions**.

Higher-resolution imaging (e.g., **1024 × 1024 or 3D volumetric reconstruction**) requires significantly more memory, often necessitating **research computing clusters**.

Any Questions? ☺