Saint Petersburg State University of Information Technologies,

Mechanics and Optics

Laboratory work report 2

Confidence interval for probabilities of discrete choice

on course: Discrete decision making

(course name)

Work completed by:

Bizhumanova A.

(last name, first name)

academic group C4215c

(group number)

Saint-Petersburg

2018

Task 2:

Kevin and June Park (K and J) are in the process of buying a new house. Three houses, A. B, and C are available. The Parks have agreed on two criteria for the selection of the house: yard work (V) and proximity to work (W), and have developed the following comparison matrices. Rank the three houses in order of priority, and compute the consistency ratio for each matrix.

Initial conditions:

A	Kevin	Jane	Normalized
Kevin	1	2	0,66667
Jane	0,5	1	0,33333

Table 1 – Weights of the Kevin and Jane rankings

Kevin	Yard	Work	Normalized
Yard	1	0,333	0,25
Work	3	1	0,75

Jane	Yard	Work	Normalized
Yard	1	4	0,8
Work	0,25	1	0,2

Table 2 – Weights of the decision factors

1)

KY	A	B	C
A	1	2	3
B	0,5	1	2
C	0,333	0,5	1

2)

KW	A	B	C
A	1	2	0,5
B	0,5	1	0,333
C	2	3	1

3)

JY	A	B	C
A	1	4	2
B	0,25	1	3
C	0,5	0,333	1

4)

JW	A	B	C
A	1	0,5	4
B	0,5	1	3
C	0,25	0,333	1

Table 3 – Initial ranks of houses for Kevin on Yard (1), Work proximity (2), Jane on Yard (3), Work Proximity (4)

Summing the columns, we get matrices:

	A	B	C
KY-sum	1,833	3,5	6
JY-sum	1,75	5,333	6

	A	B	C
KW-sum	3,5	6	1,833
JW-sum	1,75	1,833	8

Table 4 – Sum of all columns of the initial decision matrices

The following normalized matrices (table 5 and 6) are determined by dividing all the entries by the respective column-sums:

KY	A	B	C	average
A	0,5456	0,5714	0,5	0,539
B	0,2728	0,2857	0,3333	0,297
C	0,1817	0,1429	0,1667	0,164
sum	1	1	1	1

KW	A	B	C	average
A	0,2857	0,3333	0,2728	0,297274829
B	0,1429	0,1667	0,1817	0,163731068
C	0,5714	0,5	0,5456	0,538994103
sum	1	1	1	1

Table 5: Normalized ranking values for Yard (KY) and Work (KW) for Kevin

JY	A	B	C	average
A	0,5714	0,75	0,3333	0,552
B	0,1429	0,1875	0,5	0,277
C	0,2857	0,0624	0,1667	0,172
sum	1	1	1	1

JW	A	B	C	average
A	0,5714	0,2728	0,5	0,44806848
	0,2857	0,5456	0,375	0,402089341
C	0,1429	0,1817	0,125	0,149842179
sum	1	1	1	1

Table 6: Normalized ranking values for Yard (JY) and Work (JW) for Jane

The values (KY, KW, JY, JW) provide the respective priorities of the houses for J/K for Yard and Work proximity. To calculate the priority based on comparison weight (table 2) we multiply them respectively and get matrices for Jane and Kevin ranging houses:

Kevin	Y	W	Total rank
A	0,1364	0,2143	0,3507
B	0,0682	0,1071	0,1753
C	0,0454	0,4286	0,474

Jane	Y	W	Total rank
A	0,4571	0,1143	0,5714
B	0,1143	0,0571	0,1714
C	0,2286	0,0286	0,2571

Table 7 – Priority for all factors for Kevin and Jane

The final range total rank of each house (table 7, total rank) multiplied by the decision weight of Kevin to Jane (table 1):

	Kevin	Jane	Final house rank
A	0,2338	0,1905	0,4243
B	0,1169	0,0571	0,174
C	0,316	0,0857	0,4017

Table 8 – Overall priority matrix for Kevin and Jane

The best house therefore is A.

The second-best option – C.

Consistency of the Comparison Matrix

For the case where A is not consistent, the relative weight, w_i is approximated by the average of the n elements of row i in the normalized matrix N. Letting $\bar{w}$ be the computed average vector, it can be shown that

$$\mathbf{A}\bar{w} = n_{max}\bar{w}, n_{max} \geq n$$

In this case, the closer n_{max} is to n, the more consistent is the comparison matrix **A.** Based on this observation, **AHP** computes the **consistency ratio** as

$$CR = \frac{CI}{RI} \qquad CI = \text{Consistency index of } \mathbf{A} = \frac{n_{max} - n}{n - 1} \qquad RI = \text{Random consistency of } \mathbf{A} = \frac{1.98(n-2)}{n}$$

The random consistency index, *RI,* was determined empirically as the average *CI* of a large sample of randomly generated comparison matrices, A.

If *CR* <= 0.1, the level of inconsistency is acceptable. Otherwise, the inconsistency is high and the decision maker may need to reestimate the elements of **A** to realize better consistency.

To calculate consistency of each matrix we need to multiply the initial matrices (table 3) to the normalized weights of each rank (tables 5 and 6).

KY	A	B	C	Average normalized priority	KY * Av. Norm. priority
A	1	2	3	0,539	1,62474
B	0,5	1	2	0,297	0,89423
C	0,333	0,5	1	0,164	0,49185
				nmax	3,01082
				CI	0,00541
				CR	0,0082

KW	A	B	C	Average normalized priority	KY * Av. Norm. priority
A	1	2	0,5	0,297274	0,8942
B	0,5	1	0,333	0,163731	0,4919
C	2	3	1	0,538994	1,6247
				nmax	3,0108
				CI	0,0054
				CR	0,0082

Table 9 – Calculation of consistency indexes of Kevin's decision matrices

JY	A	B	C	Average normalized priority	KY * Av. Norm. priority
A	1	4	2	0,552	2,00198
B	0,25	1	3	0,277	0,92951
C	0,5	0,333	1	0,172	0,53958
				nmax	3,47107
				CI	0,23553
				CR	0,35687

JW	A	B	C	Average normalized priority	KY * Av. Norm. priority
A	1	0,5	4	0,448068	1,2485
B	0,5	1	3	0,402089	1,0757
C	0,25	0,333	1	0,1498421	0,3958
				nmax	2,7199
				CI	-0,14
				CR	-0,212

Table 10 – Calculation of consistency indexes of Jane's decision matrices

Final decision:

Resulting CR indexes show that Kevin's decision have an inconsistency level < 0,1 which is acceptable value, while Jane's decision matrixes have rather high inconsistency level, so need to be revised.

Concerning the uncertainty of Jane decision, the higher confidence has house C (the second in original rank).