Saint Petersburg State University of Information Technologies,

Mechanics and Optics

Laboratory work report 2

Confidence interval for probabilities of discrete choice

on course: Discrete decision making

(course name)

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## Task 2:

Kevin and June Park (K and J) are in the process of buying a new house. Three houses, A. B, and C are available. The Parks have agreed on two criteria for the selection of the house: yard work (V) and proximity to work (W), and have developed the following comparison matrices. Rank the three houses in order of priority, and compute the consistency ratio for each matrix.

### Initial conditions:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **Kevin** | **Jane** | **Normalized** | |  |  |  |  |  |  |
|  | Kevin | 1 | 2 | 0,66667 |  |  |  |  |  |  |  |
|  | Jane | 0,5 | 1 | 0,33333 |  |  |  |  |  |  |  |
|  | Table 1 – Weights of the Kevin and Jane rankings | | | | | | | | | | |
|  | **Kevin** | **Yard** | **Work** | **Normalized** | |  | **Jane** | **Yard** | **Work** | **Normalized** | |
|  | Yard | 1 | 0,333 | 0,25 |  |  | Yard | 1 | 4 | 0,8 |  |
|  | Work | 3 | 1 | 0,75 |  |  | Work | 0,25 | 1 | 0,2 |  |
|  | Table 2 – Weights of the decision factors | | | | | | | | |  |  |
| **1)** | **KY** | **A** | **B** | **C** |  | **2)** | **KW** | **A** | **B** | **C** |  |
|  | A | 1 | 2 | 3 |  |  | A | 1 | 2 | 0,5 |  |
|  | B | 0,5 | 1 | 2 |  |  | B | 0,5 | 1 | 0,333 |  |
|  | C | 0,333 | 0,5 | 1 |  |  | C | 2 | 3 | 1 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| **3)** | **JY** | **A** | **B** | **C** |  | **4)** | **JW** | **A** | **B** | **C** |  |
|  | A | 1 | 4 | 2 |  |  | A | 1 | 0,5 | 4 |  |
|  | B | 0,25 | 1 | 3 |  |  | B | 0,5 | 1 | 3 |  |
|  | C | 0,5 | 0,333 | 1 |  |  | C | 0,25 | 0,333 | 1 |  |

Table 3 – Initial ranks of houses for Kevin on Yard (1), Work proximity (2), Jane on Yard (3), Work Proximity (4)

Summing the columns, we get matrices:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| KY-sum | 1,833 | 3,5 | 6 |  | KW-sum | 3,5 | 6 | 1,833 |
| JY-sum | 1,75 | 5,333 | 6 |  | JW-sum | 1,75 | 1,833 | 8 |

Table 4 – Sum of all columns of the initial decision matrices

The following normalized matrices (table 5 and 6) are determined by dividing all the entries by the respective column-sums:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **KY** | **A** | **B** | **C** | **average** |  | **KW** | **A** | **B** | **C** | **average** |
| A | 0,5456 | 0,5714 | 0,5 | 0,539 |  | A | 0,2857 | 0,3333 | 0,2728 | 0,297274829 |
| B | 0,2728 | 0,2857 | 0,3333 | 0,297 |  | B | 0,1429 | 0,1667 | 0,1817 | 0,163731068 |
| C | 0,1817 | 0,1429 | 0,1667 | 0,164 |  | C | 0,5714 | 0,5 | 0,5456 | 0,538994103 |
| sum | 1 | 1 | 1 | 1 |  | sum | 1 | 1 | 1 | 1 |
| Table 5: Normalized ranking values for Yard (KY) and Work (KW) for Kevin | | | | | | | | | | |
| **JY** | **A** | **B** | **C** | **average** |  | **JW** | **A** | **B** | **C** | **average** |
| A | 0,5714 | 0,75 | 0,3333 | 0,552 |  | A | 0,5714 | 0,2728 | 0,5 | 0,44806848 |
| B | 0,1429 | 0,1875 | 0,5 | 0,277 |  |  | 0,2857 | 0,5456 | 0,375 | 0,402089341 |
| C | 0,2857 | 0,0624 | 0,1667 | 0,172 |  | C | 0,1429 | 0,1817 | 0,125 | 0,149842179 |
| sum | 1 | 1 | 1 | 1 |  | sum | 1 | 1 | 1 | 1 |
| Table 6: Normalized ranking values for Yard (JY) and Work (JW) for Jane | | | | | | | | | | |

The values (KY, KW, JY, JW) provide the respective priorities of the houses for J/K for Yard and Work proximity. To calculate the priority based on comparison weight (table 2) we multiply them respectively and get matrices for Jane and Kevin ranging houses:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Kevin** | **Y** | **W** | **Total rank** |  |  | **Jane** | **Y** | **W** | **Total rank** |
| A | 0,1364 | 0,2143 | 0,3507 |  |  | A | 0,4571 | 0,1143 | 0,5714 |
| B | 0,0682 | 0,1071 | 0,1753 |  |  | B | 0,1143 | 0,0571 | 0,1714 |
| C | 0,0454 | 0,4286 | 0,474 |  |  | C | 0,2286 | 0,0286 | 0,2571 |

Table 7 – Priority for all factors for Kevin and Jane

The final range total rank of each house (table 7, total rank) multiplied by the decision weight of Kevin to Jane (table 1):

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Kevin** | **Jane** | **Final house rank** |
| A | 0,2338 | 0,1905 | 0,4243 |
| B | 0,1169 | 0,0571 | 0,174 |
| C | 0,316 | 0,0857 | 0,4017 |

Table 8 – Overall priority matrix for Kevin and Jane

The best house therefore is A.

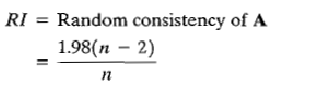
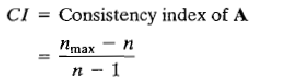
The second-best option – C.

## Consistency of the Comparison Matrix

For the case where A is not consistent, the relative weight, is approximated by the average of the n elements of row i in the normalized matrix N. Letting be the computed average vector, it can be shown that



In this case, the closer ***nmax*** is to ***n****,* the more consistent is the comparison matrix **A.** Based on this observation, **AHP** computes the **consistency ratio** as



The random consistency index, *RI,* was determined empirically as the average *CI* of a large sample of randomly generated comparison matrices, A.

If *CR* <= 0.1, the level of inconsistency is acceptable. Otherwise, the inconsistency is high and the decision maker may need to reestimate the elements of **A** to realize better consistency.

To calculate consistency of each matrix we need to multiply the initial matrices (table 3) to the normalized weights of each rank (tables 5 and 6).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **KY** | **A** | | **B** | **C** | **Average normalized priority** | **KY \* Av. Norm. priority** |  | **KW** | **A** | | **B** | | **C** | | **Average normalized priority** | **KY \* Av. Norm. priority** |
| A | 1 | | 2 | 3 | 0,539 | 1,62474 |  | A | 1 | | 2 | | 0,5 | | 0,297274 | 0,8942 |
| B | 0,5 | | 1 | 2 | 0,297 | 0,89423 |  | B | 0,5 | | 1 | | 0,333 | | 0,163731 | 0,4919 |
| C | 0,333 | | 0,5 | 1 | 0,164 | 0,49185 |  | C | 2 | | 3 | | 1 | | 0,538994 | 1,6247 |
|  | |  | |  | nmax | 3,01082 |  |  | |  | |  | |  | nmax | 3,0108 |  |
|  | |  | |  | CI | 0,00541 |  |  | |  | |  | |  | CI | 0,0054 |  |
|  | |  | |  | CR | 0,0082 |  |  | |  | |  | |  | CR | 0,0082 |  |

Table 9 – Calculation of consistency indexes of Kevin’s decision matrices

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **JY** | **A** | **B** | **C** | **Average normalized priority** | **KY \* Av. Norm. priority** |  | **JW** | **A** | **B** | **C** | **Average normalized priority** | **KY \* Av. Norm. priority** |
| A | 1 | 4 | 2 | 0,552 | 2,00198 |  | A | 1 | 0,5 | 4 | 0,448068 | 1,2485 |
| B | 0,25 | 1 | 3 | 0,277 | 0,92951 |  | B | 0,5 | 1 | 3 | 0,402089 | 1,0757 |
| C | 0,5 | 0,333 | 1 | 0,172 | 0,53958 |  | C | 0,25 | 0,333 | 1 | 0,1498421 | 0,3958 |
|  |  |  |  | nmax | 3,47107 |  |  |  |  |  | nmax | 2,7199 |
|  |  |  |  | CI | 0,23553 |  |  |  |  |  | CI | -0,14 |
|  |  |  |  | CR | 0,35687 |  |  |  |  |  | CR | -0,212 |

Table 10 – Calculation of consistency indexes of Jane’s decision matrices

## Final decision:

Resulting CR indexes show that Kevin’s decision have an inconsistency level < 0,1 which is acceptable value, while Jane’s decision matrixes have rather high inconsistency level, so need to be revised.

Concerning the uncertainty of Jane decision, the higher confidence has house C (the second in original rank).