Linear Programming for Nash Equilibria

Find a resource that has a solver for linear programs.

- MATLAB uses the linprog function -- type help linprog at the MATLAB prompt.
- A Java applet for solving linear programs.
- Another Java applet solver

Use the solver to find the Nash equilibrium for the following games:

- Battle of the Sexes (page 58-59 of text).
- Rock, Paper Scissors (pages 57-58 of the text).
- A 3x3 zero-sum game that you design. The solution to this game must be a mixed strategy and the payoff matrix cannot be symmetric.

Do your solutions make sense? Justify that the solutions are indeed equilbria for the zero-sum games, and discuss whether the solution returned by your program is correct for the Battle of the Sexes game.

Midterm 1

Select one of the following papers:

- Equilibrium behavior and repeated play of the prisoner's dilemma (http://www.sciencedirect.com/science/article/pii/0022249678900305#)
- Behavioral Game Theory: Thinking, Learning and Teaching (http://papers.ssrn.com/sol3/papers.cfm?abstract_id=295585)
- Game strategies in network security (http://link.springer.com/article/10.1007/s10207-004-0060-x)
- Satisficing and Learning Cooperation in the Prisoner's Dilemma (http://faculty.cs.byu.edu/~mike/mikeg/papers/IJCAI.pdf)

Read the paper, annotate as you go, and write a two-page summary addressing the following issues:

- What problem is being modeled?
- What is the payoff matrix for the game?
- Is this game a stage game or a repeated game? Is this choice appropriate for the game?
- Which solution concepts from the class were applied to the game? Were these appropriate choices? Why or why not?

- Which solution concepts from the class were not applied to the game? How would the results or methods have changed if other solution concepts were applied?
- What should be the next step in this research?

Submit your report and a copy of your annotations.

Lab 1 (Repeated Play Prisoner's Dilemma)

The purposes of this lab are:

- To let you play with ideas from game theory.
- To teach you about how repeated play games differ from the normal game theory context.
- To let you think about developing a utility-based strategic agent for a competitive environment.
- What you should learn:
- When you complete this lab, you should understand the following:
- Why the tit-for-tat strategy is such a good idea.
- Where game theoretic assumptions fail in repeated games.
- The issues involved in designing a competitive agent (such as inter-agent modeling, anticipation, forgiveness, commitment, etc.).
- You should also gain practice writing technical documents. Check out the grading guidelines at the bottom of the description.

You will conduct a tournament that tests how different strategies perform in repeated-play prisoner's dilemma game. You will create your own strategy, and play against the following agents:

- An agent who employs the one-shot equilibrium solution (always defect)
- An agent who chooses randomly
- An agent who always cooperates with you (and never confesses)
- An agent who employs the tit-for-tat strategy (reviewed below), and
- An agent who employs the tit-for-two-tats strategy (also reviewed below).
- An agent who uses the Pavlov strategy (reviewed below).
- An agent who uses the Win-Stay/Lose-Shift strategy (reviewed below).
- An agent who uses the Never Forgive strategy (reviewed below).

You should try out how well each strategy works when there are 5 trials, 100 trials, and 200 trials. You should also play a variant where, after each interaction, you flip a biased coin to decide if you continue to another round; the coin turns up heads p percent of the time, and whenever the coin turns up heads you continue. Conduct the experiment for p = 0.75, 0.99, and 0.9. Each agent will compete against all other agents, and the total scores (e.g., the sum of an agent's score across all head-to-head meetings -- one sum for each value of p) will be recorded. You should use the prisoner's dilemma payoff matrix with the following payoffs.

P1/P2	Cooperate 2	Defect 2
Cooperate 1	(4,4)	(1,5)
Defect 1	(5,1)	(2,2)

In the payoff matrix, cooperate means cooperating with the other prisoner by remaining silent, and defect means defecting from the team strategy by confessing to the police. If you play the game 100 trials, your score is the sum of all of the payoffs you received so, for example, if you and your opponent each cooperated, then your score would be 3*100. High scores win.

When you are done, you should turn in:

- One page describing and justifying your strategy.
- A description of the results for each number of trials listed above (5, 100, and 200); since this is a data intensive lab, take some time to present the information in an intelligible format.
- A description of the results for each of the probabilities above (0.75, 0.9, and 0.99); since this is a data intensive lab, take some time to present the information in an intelligible format.
- One page explaining why the winner won, and discussing how their strategy performed and why.

Your report will be graded on four criteria: style, grammar, analysis, content.

- Style: Is there an adequate introduction? Does the conclusions section make conclusions justified by the data? Is there a logical flow to the organization of the paper?
- Grammar: Is the language adequate?
- Analysis: Does the report present not only what is found but why? Have you made conclusions that are not justified? Do you include multiple trials to account for uncertainty? Have you reported both means and variances of data?
- Content: Have you completed what was assigned? Is there some originality in the work?

Lab 2 (Evolutionary Games)

The purposes of this lab are:

- To let you experiment game theoretic concepts in an evolutionary setting.
- To teach you the relationship between repeated play and evolutionary stable strategies.
- To learn the differences between Nash Equilibria and evolutionary stable strategies.

When you complete this lab, you should understand the following:

- The effect that various kinds of interaction models have on which solutions are selected.
- The effects of various kinds of selection dynamics on which solutions evolve.
- The issues involved in designing a competitive agent (such as inter-agent modeling, anticipation, forgiveness, commitment, etc.).

You will conduct a series of experiments that determine which types of strategies are likely to evolve under various selection dynamics and interaction models for a couple of interesting games.

In evolutionary games, the two main factors that contribute to what is learned are: The types of interactions that occur between the agents in a population. The rules that are applied to determine which strategies within the population are fit and therefore likely to be learned by the population. Descriptions of these concepts can be found in the lecture notes. For this lab, we will evaluate replicator dynamics using random pairings and imitator dynamics using neighborhood pairings on a lattice (and the N, NE, E, SE, S, SW, W, NW neighborhood definition).

You will perform experiments on the following games:

- Prisoner's Dilemma
- Stag Hunt
- Battle of the Sexes

Use cardinal values for the entries in the payoff matrices (e.g., use T=5, R=3, P=2, and S=1). In your report, describe what values you used for each game.

You will consider two types of selection and interaction dynamics: (replicator dynamics, random pairings) (imitator dynamics, lattice pairings) Use 900 agents in all of your simulations (this translates to a 30 by 30 lattice). In your report on the imitator dynamics, be sure to describe how you define an agent's neighborhood, that is, tell me who plays whom.

Use agents that have a single state. In other words, use agents that remember the previous action of the other agent and then use this action to determine their next action. Thus, the set of all agents are

	My action on current round			
Action of other agent on previous round	Agent 1 (AC)	Agent 2 (AD)	Agent 3 (TfT)	Agent 4 (~TfT)
C	C	D	C	D

D	C	D	D	C

Note that this precludes agents like Win Stay Lose Shift, since I can't have actions that depend on both your previous choice and my previous choice. Assume that Agent 1 and Agent 3 play C on the first round, and that Agent 2 and Agent 4 play D on the first round.

You will consider iterated games for gamma = 0.95 and gamma = 0.99. For the iterated games, compute V(A|B) prior to running the games. This allows you to turn the iterated games into payoff matrices. For example, if agent 1 plays agent 2 in the iterated prisoner's dilemma, then the payoff to agent 1 is S / (1-gamma).

Begin with various mixes of agents. Be sure to explore spatial effects under imitator dynamics in the spirit of Skyrms' book. For example, how do the initial locations of agent types affect which strategies survive under imitator dynamics?

What to Turn in:

You will be conducting a very large experiment. There are two types of selection/interaction dynamics, three games, two types of game durations (gamma=0.99 and gamma = 0.95), and four types of agents. This means that you will be doing 2x3x2x4 experiments, and you will do these multiple times to account for variabilities in the initial populations. Turn in a summary of your data, and discuss the more interesting results. When you make an interesting observation, do more than simply describe the observation -- also use the concepts from Skyrms, Axelrod (especially invasion), class discussion, and your own experiments to explain why the observation occurs.

I suggest comparing and contrasting the effects of selection/interaction dynamics over the various games. I also suggest writing your code so that you can do hundreds of experiments with different initial populations and then show what evolves as a function of the balance between agents in the initial populations.

Additionally, do something that you think will be cool, like add some mutations, add mixed strategies, or try a different interaction dynamic.

Write a report on the experiments that you performed, but only report on interesting results. The rubric from the first lab will be used on this lab too.

Note that many students have found it useful to use the NetLogo simulation framework to help visualize spatial effects.

Consensus and topology

The purpose of this assignment is to have you explore how topological dynamics affect connectivity.

Part 1: Probable Connectivity

- Randomly place agents in a two dimensional world. Place the agents at uniformly random x locations in the range $[-M,M]$ and similary for random y locations.
- Create the adjacency graph that results when an agent is connected to all agents within a fixed metric distance, R. What is the probability that the resulting graph is connected as a function of metric distance? (Report your results as a function of R/M so that all results from all students are on a similar scale.)
- Create the adjacency graph that results when an agent is connected to its N nearest neighbors. What is the probability that the resulting graph is connected as a function of N? For different values of N, what is the average (across some sample of random graphs) distance of the nearest neighbor that is farthest from the agent?
- Which is more likely to be connected, a topological neighborhood or a metric neighborhood? Justify your answer.

Part 2: Dynamics

- Create a simulation of a group of somewhere between 20 and 50 agents in a 2D world.
- Set a fixed radius of repulsion based on the Euclidean distance between agents. Create the graph Laplacian LR for the resulting graph. Note that, since agents move, LR changes over time.
- Choose two values for a radius of attraction, both of which must be greater than the radius of repulsion and both based on Euclidean distance. Create the graph Laplacian LA for the resulting graph. Note that, since agents move, LA changes over time.
- Simulate agent dynamics as x{dot} = -LAx + LRx + noise.
- Create plots of the Fiedler eigenvalue and average degree for all agents as a function of time, and explain the results.
- Now, repeat the above but instead of basing the attraction graph on Euclidean distance instead do it based on nearest neighbors. Do this for two values of nearest neighbors (e.g., 5 nearest neighbors, 10 nearest neighbors). Note that, since agents move, LA changes over time.
- Discuss why the nearest neighbor and metric-base (distance) neighbor methods have different characteristics over time.

Note that you'll need to make some assumptions or decisions because there is some intentional ambiguity in the above description, especially when there is both attraction and repulsion forces. Make an informed decision and tell me what you decided.

Midterm 2

Select one of the following papers:

- Consensus and Coordination paper (http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=4118472).
- Sections 1-3 from Jackson's paper (http://web.stanford.edu/~jacksonm/netect.pdf).
- Valentini et al's paper (http://www.aamas-conference.org/AAMAS/aamas2014/proceedings/aamas/p45.pdf).

Annotate the paper and write a two or three page report that analyzes the paper. The evaluation should include a description of who plays whom (interaction dynamics), how agents/nodes change their state (selection or behavior dynamics), relevant equilibrium concepts, and any key graph theory properties that affect the efficiency of the model (e.g., number of neighbors, sensitivity, Fiedler eigenvalue, etc.). Identify any errors in the paper. Include a discussion of whether the model is appropriate for the phenomena that is described & what you would do next to make the model more interesting or to learn something cool from it.

Fictitious play

The purpose of this homework is to have you learn fictitious play. Program up the fictitious play algorithm to do the following.

Battle of the sexes.

1. What solution occurs if initial weights are set as K_{man}(Cooperate)=K_{woman}(Defect)=1, **and K_{woman}(Cooperate)=K_{man}(Defect)=0?**
2. What solution occurs if initial weights are set as K_{man}(Cooperate)=K_{woman}(Defect)=0, **and K_{woman}(Cooperate)=K_{man}(Defect)=1?**

The coordination game

Row/Column	C	D
C	(2,2)	(0,0)
D	(0,0)	(1,1)

3. What solution is chosen?
4. Does the solution depend on initial weights?

Matching pennies

Row/Column	Heads	Tails
Heads	(1,-1)	(-1,1)
Tails	(-1,1)	(1,-1)

5. What do the phase trajectories look like when initial weights are K(Heads)=K(Tails)=1? **Use these initial weights for both players.**
6. What do the phase trajectories look like when initial weights are K(Heads)=1000 and K(Tails)=1? **Use these initial weights for both players.**
7. What solution is chosen for each of the initial conditions?

Shapley's game

Row/Column	L	M	R
T	(0,0)	(1,0)	(0,1)
M	(0,1)	(0,0)	(1,0)
D	(1,0)	(0,1)	(0,0)

1. Does this game converge? Justify your answer.
2. What do the phase trajectories look like?

Since I won't look at your code, make sure that your discussion justifies your answers. Pretend that you are writing an essay on an exam and that you must convince me that not only can you generate an answer but also that your answer is right.

Lab 3 (Swarms and Flocks)

Implement Couzin's equations from "Collective memory and spatial sorting in animal groups" (http://csim.scu.edu.tw/~chiang/course/ComputerGameAdvance/Collective%20Memory%20and%20Spatial%20Sorting%20in%20Animal%20Groups.pdf). Only implement the algorithm for two dimensions. You will likely find it easier to create the torus if you have more than one hundred agents, depending on the other parameters. *Hint: when I did the equivalent of the lab, I did a systematic search through parameter space by stepping through different parameter sets and measuring momentum and polarization. I then visually inspected sets of parameters that produced momentum and polarization values that mimicked what was in Couzin's paper.*

- What parameters produce a swarm, torus, highly parallel group, and dynamic parallel group?
- What are the average Fiedler eigenvalues for each of the groups above? (Use the topology that assumes that agents in the radius of attraction are connected bidirectionally.)
- How do the values of the Fiedler eigenvalues relate to the structures above and the parameters that produce them? Why?
- What happens if you change the organization above from one that interacts with only five nearest neighbors rather than all neighbors within the repulsion, alignment, or attraction radius? Can you produce a swarm? torus? highly parallel group? dynamic parallel group? In the report, be sure to explain how you meshed the neighborhood model with the metric-based zones. Hint, eliminate the zone of attraction so that an agent is always attracted to any agent that is outside of the radius of repulsion.
- Modify something in one of the models above, changing fewer than 50% of the agents so that they don't behave according to the equations, and tell me what cool thing you were able to get them to do.

Write a report on the experiments that you performed. The rubric from the first lab will be used on this lab too. Be sure to describe not only what happened but why it happened.

Use data to support your observations. For example, how do Fiedler eigenvalues change over time from randomly specified initial conditions through the time that the structures emerge? What are the average Fiedler eigenvalues over multiple trials, and what are the variances?

Final Exam

Read "Should we Compete or Should we Cooperate? Applying Game Theory to Task Allocation in Drone Swarms" and write 3-5 pages that analyzes the paper using concepts from class. Include some reasonable subset from the following prompts:

- Who talks with whom, that is, what are the interaction dynamics? Why or why aren't this choice of interaction dynamics desirable for the problem?
- How are decisions made, that is, what solution concepts and/or social choice mechanisms are used by agents to make decisions. Why or why aren't these decision dynamics desirable for the problem?
- Some of the problem modeling and solution concepts in the paper are used incorrectly; there are a number of conceptual errors. What is wrong and why?
- What axioms from Arrow's Impossibility Theorem are violated by the voting scheme? Justify.

- Are the resulting solutions equilibria? Are they stable equilibria? What happens if an agent decides to not comply with the solution? What happens if a group of adversarial agents join the team to try and cause problems?
- What problems do you see with the conclusions? Why or why aren't they problems?
- What would you have done differently?