UNC | GILLINGS SCHOOL OF GLOBAL PUBLIC HEALTH

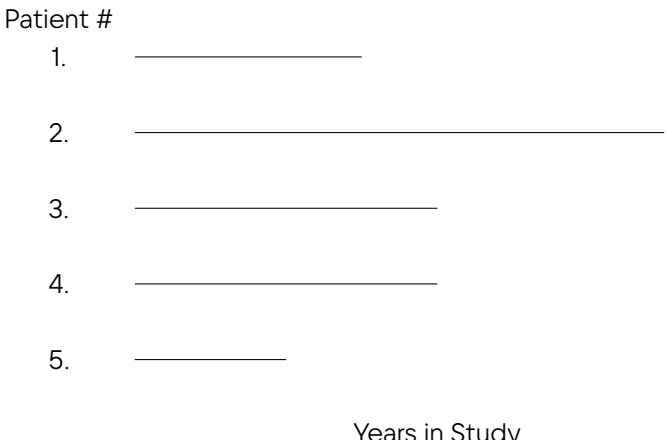
# An Introduction to Censored Covariates

Marissa Ashner

STA779: *Applied Survival Analysis (Spring 2023)*

February 28, 2023

# REVIEW: Censoring



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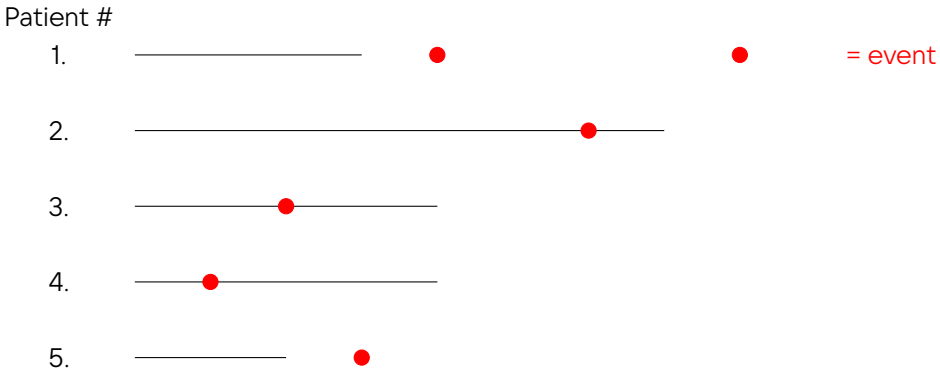

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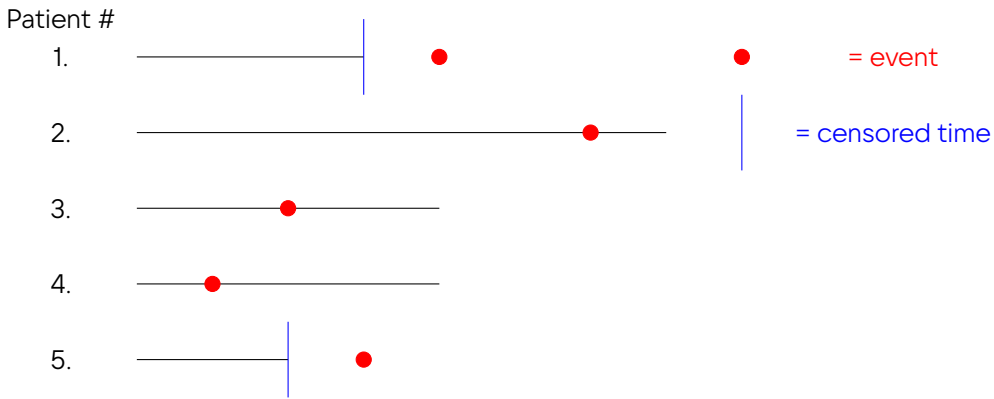

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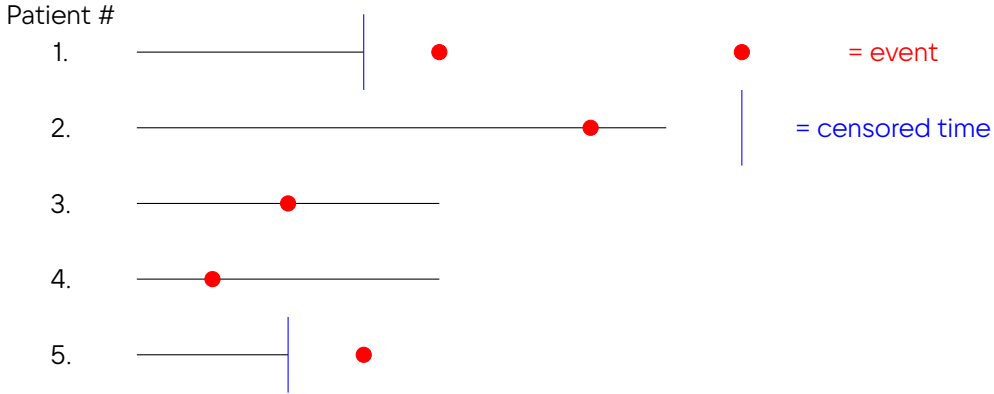


**What kind of censoring is this?:** *Right?, Left?, Interval?*

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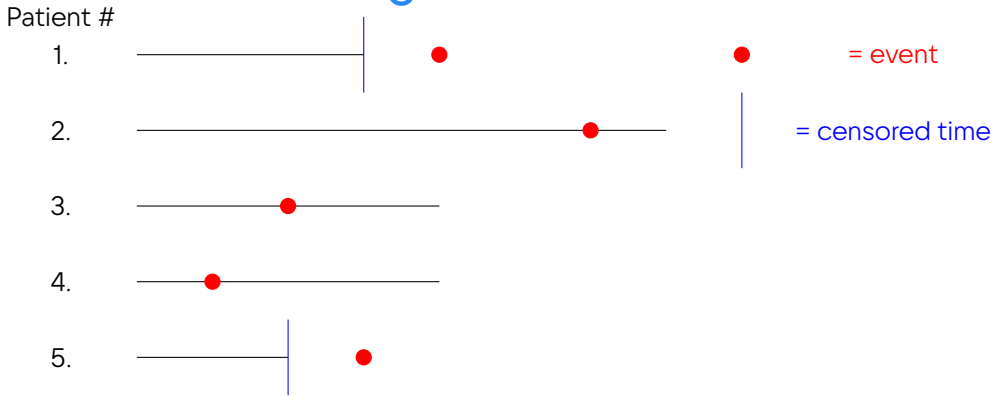


**What kind of Type I censoring could this be?:** *Fixed? Progressive? Generalized?*

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- The whole idea around survival analysis, where the *outcome* you are interested in is some 'time until event' (or some function of that time)
- It is often the case that you will adjust for some other factors, or covariates, that could affect this outcome
- or maybe you want to see which factors determine survival of this event

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- What if I want to predict height based on someone's weight?

# How do you determine where to put variables in a model?

- What if I want to measure the linear association between height and weight?
- This depends heavily on your **research question and analysis goal**

# Censored Covariates

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- The patient's diagnosis time here, is a covariate, based on our research question
- AND it is right-censored, since not all patients will be diagnosed during a Huntington disease observational study
- Another example: What if I want to measure the impact of income on college matriculation but income is top-coded, meaning anyone with an income over a certain number, let's say $\$250K$, is simply labeled as $\geq \$250K$.

# Types of censored covariates

- **Limit of Detection Censoring**
  - Also called top-coded or bottom-coded, like the income example
  - A lot of biological markers often have a lower limit of detection, meaning the device to measure the factor cannot pick up levels below a certain threshold
  - Often, observations have the same limit of detection, but they can also have different limits
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  - Similar to Type I censoring that you discussed last week
- **Random Censoring**
  - Like the Huntington disease example
  - Censoring values differ for each observation, and we don't consider these values known a priori
  - Instead, we consider the set of these values as a random variable, with it's own distribution

# Some Notation

- Let $X$ denote the censored covariate of interest
- Let $C$ denote the censoring value (considering random censoring)
- We observe $W = \min(X, C)$ and $\Delta = I(X \leq C)$
- Let $Y$ be the outcome of interest and $Z$ be the set of fully-observed covariates

# Missing Data vs. Censored Data

- Censored Data $\subset$ Missing Data

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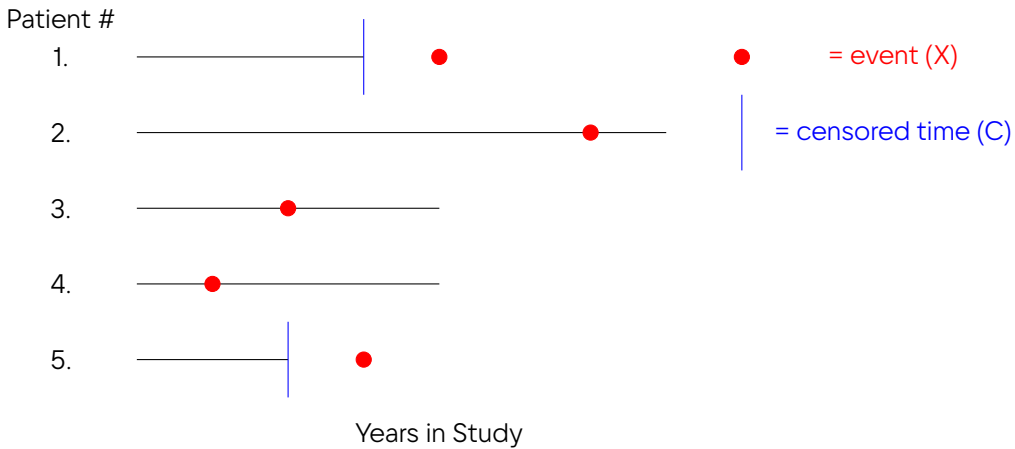
- $\Delta \not\perp\!\!\!\perp X$

# What happens when we replace $X$ with $W$?



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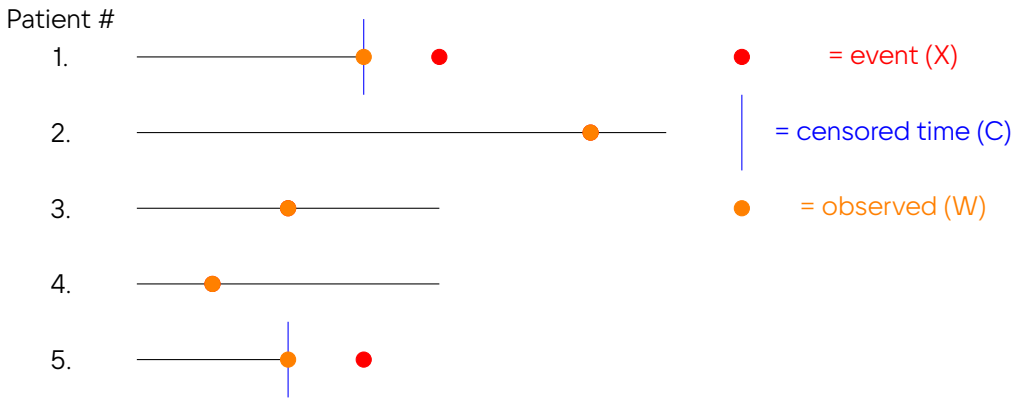

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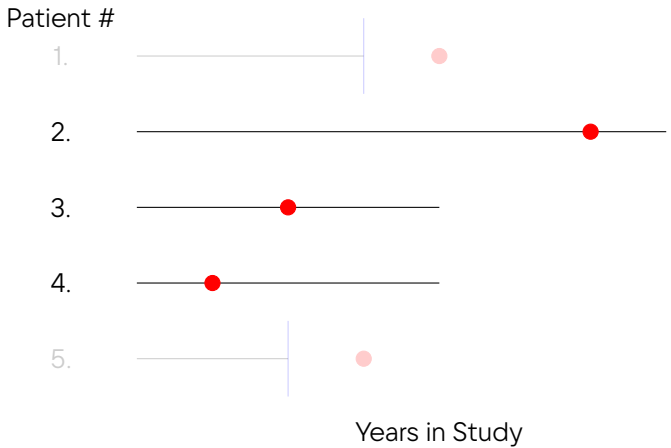

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- i.e. $\Delta \perp\!\!\!\perp Y | X, Z$
- What happens if our COVARIATE is censored?
- What happens if our OUTCOME is censored (i.e., switch the roles of X and Y?)

# Let’s Try it!

Click here for the code on my GitHub!

# Thank you! Any questions?

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