Review 1

Comments:
This review is rather like a mixture of major and minor changes. I might ask you to do something as large as rephrasing a paragraph or two or something as small as fixing notation errors. Generally, it sits in the right direction, we just need to put things together, in a more reasonable and logical way. This first review can be overwhelming, but it should help improve the

Note:
For reading this review, if you see abbr's, search for it as I explain what they are short for at its first appearance somewhere in this review.

How to find where these comments come from:
I will put page number and chapter number for the first page where it belongs.

Ch 1
Generally speaking, you might need to add many things. The biggest issue is it lacks a more thorough literature review.

--pp3
1. Your background should start with "situation"
I know why you care: there are upscatterings. But why are there upscatterings. It turns out to be because there are scattering phenomena for certain materials (H2O, D2O, graphite, ZrH etc) are heavily affected by the molecular structure for those compounds allowing bouncing low energy neutrons to higher energies. This is a common phenomenon in thermal reactors and neutron shielding problems. Please write a brief introduction in this direction based on reviewing the introduction and conclusion parts of my previous work as follows which should give a good picture why physically this problem exists. The intension is to make readers follow where and why this becomes the problem so they will continue liscening to you about how to fix it.

The papers:
https://www.sciencedirect.com/science/article/pii/S0306454915003953
https://www.tandfonline.com/doi/abs/10.13182/NSE15-48

--pp5
1. Need a 4\pi beside k in Eq 1.3

2. The equation in narrative part is not correct. Change it to something like A\psi=1/k F\psi where F is the fission operator.
--pp6
1. Change "Useful Approximations" to "Space-angle Approximations of interest" or something similar.

2. Put 1.3.3 as the first subsection instead of third. This is where you get the transport accuracy. Start with how you transform it (rearrange \psi and plug it back in) and why (form a degenerate diffusion-like form that are compatible with CFEM to get a SPD system which can be suitable for algebraic solver and preconditioners.) See the slides in the following link for reference https://www.dropbox.com/s/pxdtwaemofoy5c0/group_talk.pdf?dl=0

--pp10

1. Detail 1.4.3, including CFEM weak forms for SAAF and NDA. This should not be hard, right? It is important as these are what you really solve. You are not solve SAAF and NDA without a spatial discretization. You don't need to derive them but saying they are in the appendix is not proper either. Reviewing them a bit more from where you can find CFEM formulations (morel's HOLO paper and my dissertation, Yaqi Wang's papers are good sources) and giving out the complete weak formulations like: "Given a function space V, \forall function \psi^*\in V, find \psi\in V, s.t. (give your weak form here)".

Ch 2

--pp11

1. The derivation in this chapter is actually another version (maybe refined by one) of the derivation we've published in the ICTT. You should somehow cite it before you start to derive things in this chapter. As I don't think this is a direct reuse of ICTT summary, you may mention it shortly with citation to it before you derive it. Like "In our previous work xxx prototypical derivation [cite it], we in this chapter will rederive the formulation with more detailed explanations."

2. "here, we assume vacuum boundary": please add an equation for what vacuum boundary is. What you can do is add an equation for incident boundary condition (BC) where the transport equation (TE) first appears and then point to that equation when you mention vacuum by explaining how incident BC can be vacuum.

3. Add dot in equation 2.2 and 2.3 after gradient signs. You are doing dot product, not outer product. The same thing applies to the equation on the bottom of pp11.

--pp12

1. add dots for 2.4, 2.5 and wherever necessary. An easy way to think about if you need to add dot is: generally you have scalars in an equation. So every single term in the equation must be a scalar at the end of the day. Then you know if you need to add dots.

2. explain what the meaning of the terms in defined in 2.9 are. They sounds like: error of scalar flux in the multigroup iterations and error source aroused from upscattering in multigroup iterations. Check how the Evans and Morel's articles explain this.

--pp13

1. Need a little bit more explanation on how you transit from 2.12 to 2.13. You are having an equation using \epsilon in 2.12. But now you are having another equation with unknown of \phi_\epsilon. I know you split it but nobody else knows it unless they've read Evans/Morel's papers. Adding an equation saying how specifically you split would help

2. Rephrase the last sentence to sth sounds like "By solving Eq 2.13 using the same spatial discretization used for NDA equations, we can address the scalar flux from previous Gauss Seidel iteration as" and give the last equation on pp13 therein.

Ch 3

--pp14

1. Change the title of the subsection from "Linear Solver" to "Iterative Solver" or something that makes sense. "Linear Solver" generally make people think about algebraic solver like CG, LU, etc.

2. Change this confusing sentence: "**A** is the matrix formed by the approximation of the derivative, $[\hat{\Omega}\cdot\nabla+\Sigma(\vec{r},E)]\psi(\vec{r},\hat{\Omega},E)$, using finite differences or any other discretization method" to sth which sounds like "A is the matrix formed by discretizing the left hand side". The thing is you are using finite element, why are you emphasizing finite difference?

3. I

--pp15

1. First, I don't think you are using CG but I am not sure. Does CG converge for NDA calculations? (NDA equations are non-SPD). If it does, it's fine. You have to mention what equations you are using CG for and why CG would generally work

2. Don't put the pseudocode. You are not implementing CG by yourself but using APIs from SciPy. What you do should be cite SciPy for the usage and cite any literature which explains what CG exactly is.

--pp16 and pp17

1. General comments: you are doing great work explaining how you solve the equations using iterations. What can be greater:

1) add one more pseudocode for how NDA is incorporated in the HOLO calculation

2) add one more pseudocode for how TG-NDA work

For 1), refer to Morel's HOLO paper. For 2), it should be similar to 1) but with one extra step

(solving error equation and addressing scalar flux with the error)

2. Chapter 3 lacks explanations with equations. If possible, at the beginning of Chapter 3, give an equation in operator form (like H\psi=q+S\psi). Per subsection, add corresponding indices. Your pseudocode is good, but hard to follow without showing how the steps reflect back to the equations.

Ch 4

--pp18

I don't think this chapter has started yet on the storytelling part. But if it has, be careful with the words. I don't think you want people to feel "tedious" when they see you say "tedious to compute".

Actually, if you don't have any results to show (or necessity to show the verification for debugging purpose only), shouldn't you just delete 4.1 and 4.2?

And I don't think 4.3 and 4.4 have strong reasons to be in the thesis as they are not research related but just for verification purpose.

What you really need to show but have not shown yet is something like: does TG-NDA agree with NDA? Theoretically, they should give exactly the same results.

--pp19

For NDA/SAAF Agreement: you need several more tests showing that. Put error plots for NDA and SAAF at the same plot in log-log form and change figure 4.1 to log-log scale as well. Then interpret them please. Figures won't talk, but you do.

Ch 6

--pp23

A suggestion before you start this chapter. Start based on the suggestions below. They might not be what we want to do or what we are capable to do, but they can be good suggestions for anyone of interest.

1. Other transport methods:

we are doing SN for angle, but PN can be used for the angular discretization for the high-order equation as well. (cite my dissertation where SN and PN are both discussed in detail)

2. Other space-angle forms and discretizations:

We are using SAAF-CFEM+NDA-CFEM in this work. People can try other second order forms like even parity equation with NDA using CFEM.

The other thing to mention is we can apply this method to first order form by discretizing transport equation with DFEM. In that case, TG-NDA has extra work to do. NDA becomes very complex using interior penalty DFEM (cite this paper https://www.sciencedirect.com/science/article/pii/S0021999117301286). We have extra coefficients living on the interface between spatial cells to perform group collapse as we do for sigma_t or any other things.

3. Efficiency change due to mesh type:

We are doing unstructured mesh in this work. It might be interesting to see the efficiency difference/similarities on structured mesh.

4. Apply to problems with eigen value calculations. Be narrative a bit why this can be useful. (hint: for thermal reactors, there are a lot of highly scattering mediums like water, heavy water and graphite, the upscattering for these materials can be overwhelming.)