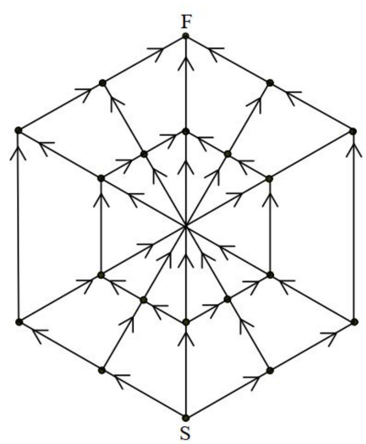
1. A depth-first forest classifies a graph's edges into tree, back, forward, and cross edges. A breadth-first tree can also be used to classify the edges reachable from the source of the search into the same four categories.

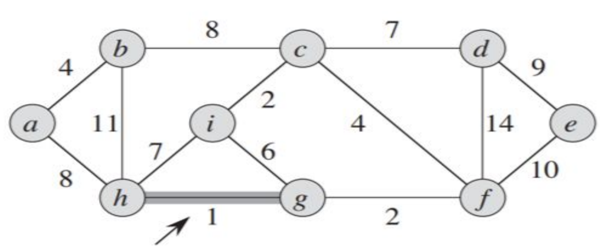
I. Prove that in a breadth-first search of an undirected graph, the following properties hold:
   A. There are no back edges and no forward edges.
   B. For each tree edge (u,v), we have v.d=u.d+1.
   C. For each cross edge (u,v), we have v.d=u.d or v.d=u.d+1.

II. Prove that in a breadth-first search of a directed graph, the following properties hold:
   A. There are no forward edges.
   B. For each tree edge (u,v), we have v.d=u.d+1.
   C. For each tree edge (u,v), we have v.d≤u.d+1.
   D. For each tree edge (u,v), we have 0≤v.d≤u.d.

2. A spider sits at the bottom (point S) of its web, while a fly sits at the top (F). How many different ways can the spider reach the fly by moving along the web's lines in the directions indicated by the arrows?
Hint: Take advantage of topological sorting and the graph's symmetry.

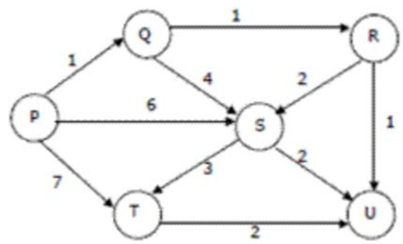


3. I. Execute Kruskal's algorithm on the graph below. Fully explain the process and draw the resulting graph for each step.



II. Suppose that all edge weights in a graph are integers in the range from 1 to |V|. How fast can you make Kruskal's algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W?

4. I. Run Dijkstra's algorithm on the following edge-weighted directed graph with vertex P as the source. Fully explain the process and draw the resulting graph for each step.



II. Suppose that we are given a weighted, directed graph G=(V, E) in which edges that leave the source vertex s may have negative weights, all other edge weights are non-negative, and there are no negative-weight cycles. Argue that Dijkstra's algorithm correctly finds the shortest paths from s in this graph.

5. Solve the all-pairs shortest-path problem for the digraph with the weight matrix:

$$\begin{pmatrix} 0 & 3 & \infty & 2 & 6 \\ 5 & 0 & 4 & 2 & \infty \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & 1 & 0 & 4 \\ 5 & \infty & \infty & \infty & 0 \end{pmatrix}$$