

$$I = \int_C z^n \bar{z} \cos(z) dz \quad |z|=1$$

(سوال 2)

$$\Rightarrow z = e^{i\theta} \Rightarrow dz = ie^{i\theta} d\theta$$

$$\bar{z} = e^{-i\theta}, \quad \cos(z) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

By using $\int_C f(z) dz = \int_0^{2\pi} f(z(\theta)) z'(\theta) d\theta$

$$I = \int_0^{2\pi} e^{in\theta} e^{-i\theta} \frac{e^{i\theta} + e^{-i\theta}}{2} (ie^{i\theta}) d\theta$$

$$= \frac{i}{2} \int_0^{2\pi} [e^{i(n+1)\theta} + e^{i(n-1)\theta}] d\theta$$

$$= \begin{cases} \pi i & n = -1 \\ \pi i & n = 1 \\ 0 & n \neq \pm 1 \end{cases}$$