Lecture 10
Chapter # 4
RELIABILITY AND AVAILABILITY MODELING

RELIABILITY

System's reliability is the probability that the system will function as per specification under a given environment & during a specified period of time.

Reliability $R(t)$ of a system is defined as the probability that the system will survive till time t.

Hence, if T is a random variable denoting system's lifetime, then reliability can be calculated by using this formula

$$R(t)=P[T>t]=1-F_T(t)$$

As we have already discussed that exponential distribution is used to model the lifetime of various system & components. So here we have used exponential distribution to model the reliability of the system.

It should be noted that:

At time t=0 the reliability of the system would be qual to 1.

- $R(0) = 1$ (i.e. a system is expected to be operational when it's initially put into operation)
- $\lim_{t\to\infty} R(t)=0$ (i.e. nothing can operate forever)

Here the reliability of the system also states that the environment in which the system is used & the purpose that it is used for must be taken into account.

Lets suppose we want to measure the reliability of a word processor in an office environment where most users are interested only in the currently required features of the software. So they do not try to experiment with the system & only work with the availaibe or required features. But if we then measure the reliability of the same system in the university environment it may be quite different. Here students may explore the boundaries of the system & use the system in unexpected ways so it may result in system failures that did not occur in the more constraint office environment.

MATHMATICAL EXPRESSION OF RELIABILITY

Let,

• N_0 = number of identical components under test at t = 0
• $N_s(t)$ = number of components which survived till time t
• $N_f(t)$ = number of components which failed till time t

Clearly,

$$N_s(t)+N_f(t)= N_0$$

Then, using fundamental definitions of reliability and probability, we get,

$$R(t) = \frac{N_s(t)}{N_0} = 1 - \frac{N_f(t)}{N_0}$$

means reliability is the ratio of number of components that are survived till time t i.e. $N_s(t)$ and the total number of identical components that are under test i.e. N_0.

Taking first derivative with respect to time,

$$R'(t) = -N'_f(t)/N_0$$

Whenever i want to calculate the rate of change of anything i actually take its derivative.

where, N'f(t) represents failure rate of components.

Recall the basic definition of reliability,

$$R(t) = 1 - F_T(t)$$

It is actually representing the cummulative distributive function of exponential distribution.

Now, taking first derivative on both sides of with respect to time, we get, the probability density function represented by $f_X(t)$.

$$R'(t) = -f_X(t)$$

In this way we have already known that if we calculate the probability density function we will be able to calculate the probabilities in a given range easily. & since this function is not exactly a probability it's a density function so its ok to get its value greater than 1 at some point in time or at some value of x.

Reliability includes:

• <u>correctness :</u> means ensuring the system services are as specified,
• <u>precision :</u> means ensuring information is delivered at an appropriate level of detail, and
• <u>timeliness :</u> means ensuring that information is delivered when it is required.

HAZARD RATE

Let us now calculate the conditional probability that the system will not survive an additional time duration x, given that it has already survived till time t.
It means that the system is hazardous; it can not be expected to survive any additional time duration. If the system is not expected to survive any additional time duration it means that itis hazardous & it can be stop working after time duration t.
Here we will use simply the conditional probability phenomena, Lets suppose there's a random variable T

$$P[T \le t + x \mid T > t] = \frac{P[\overset{a}{t} < T \le t \overset{b}{+} x]}{P[T > t]} = \frac{F_X(t+x) - F_X(t)}{R(t)}$$

Recall the formula that we disscussed in the session of exponential distribution,

$$P(a < x \le b) = F_x(b) - F_x(a)$$

$P(a<x \le b) = F_x(b) - F_x(a)$

at some time we need to calculate the probability of some random variable in a particular range. & The points of range are a & b.

If we divide this probability by x and the interval x is shrunk to zero ($x \to 0$),
we get the instantaneous failure rate or hazard rate h(t):
converting this statement into this equation,

$$h(t) = \lim_{x \to 0} \frac{F_X(t+x) - F_X(t)}{xR(t)}$$

Here we can actually add one more step for understanding; we are simply using the basic definition of reliability i.e.

$=1-R(t+x)-(1 + R(t))$

then we will achieve,

$$h(t) = \lim_{x \to 0} \frac{F_X(t+x) - F_X(t)}{xR(t)} = 1 - R(t+x) - 1 + R(t)$$

$$= \frac{1}{R(t)} \boxed{\lim_{x \to 0} \frac{R(t) - R(t+x)}{x}}$$

$$h(t) = -\frac{R'(t)}{R(t)} = \frac{f_X(t)}{R(t)}$$

by mean of this particular term we can actually calculate; the derivative of this partiular function by means of the first

principle method or the ab initio method.
you must have learn in your undergrad about the first principle method.
Since we are interested in calculation of Hazard rate
whenever we are intereted in calculation of Hazard rate or failure rate or rate of change of anything, we have to calculate its derivative. So here I am simply writing the R'(t) which is the representing the derivative of R(t) with the negative sign because in the actual formula of ab initio method it should be R(t+x)-R(x) /x so this particular formula gives the result of positive first derivative.
& this R(t) is constant because the system has already survived till time t that is why it is not included in the derivative.

we have just derived the formula for an hazard rate or failure rate of the system.
Hazard rate is expressed in failures/10000 hr
or it is also expressed as failures in time i.e. failures/10^9 hr.

Calculate h(t) = ?

provided that the random variable follow the exponential distrbution.

If X ¬ EXP (λ)

$h(t) = \frac{f_x(t)}{R(t)}$ — formula from previous.

Exponential distribution is used to represent the reliability of various components & reliability of systems.

If X ¬ EXP (λ)

$$h(t) = \frac{f_X(t)}{R(t)}$$

$$h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

$h(t) = \lambda$

i.e. the constant failure rate

The *cumulative hazard* H(*t*) is given as:

$$H(t) = \int_0^t h(x)dx$$

$$= -\int_0^t \frac{R'(x)}{R(x)}dx$$

$$= -[lnR(x)]_0^t$$

$$= -lnR(t)$$

$\frac{dy}{y}$

This gives

$$R(t) = e^{-H(t)}$$

here for this integral you have to recall the chain rule;
dy/y = lny so R'(x)/R(x) = lnR(x).
Here H(t) is the cumulative hazard.

If $T \sim$ EXP(λ), then

$$R(t) = e^{-\lambda t}$$

$R(t) = e^{-H}$

$$h(t) = \lambda$$

$$H(t) = \lambda t$$

by comparing these two eq we got $h(t) = \lambda$

Clearly, the hazard rate for an exponentially distributed lifetime is constant.

Task

The hazard rate of a certain component is given by:

$$h(t) = \frac{e^{t/4}}{5}$$

1) What are the cumulative hazard function and the reliability function of this component?

2) What is the probability that it survives until t = 2.

Answers

1) H(t) = $\frac{4}{5}\left(e^{t/4} - 1\right)$

2) R(t) = $e^{-\frac{4}{5}\left(e^{t/4}-1\right)}$

3) R(2) = 0.9591

Practice Problem

The failure rate of a certain component is $h(t) = \lambda_0 t$ where $\lambda_0 > 0$ is a constant. Determine the reliability R(t) of the component.

It is similar to the above task. Solve!!

$H(t) = \lambda_0 t^2/2$??

$R(t) = e^{-\lambda 0t2/2}$??

MORTALITY CURVE

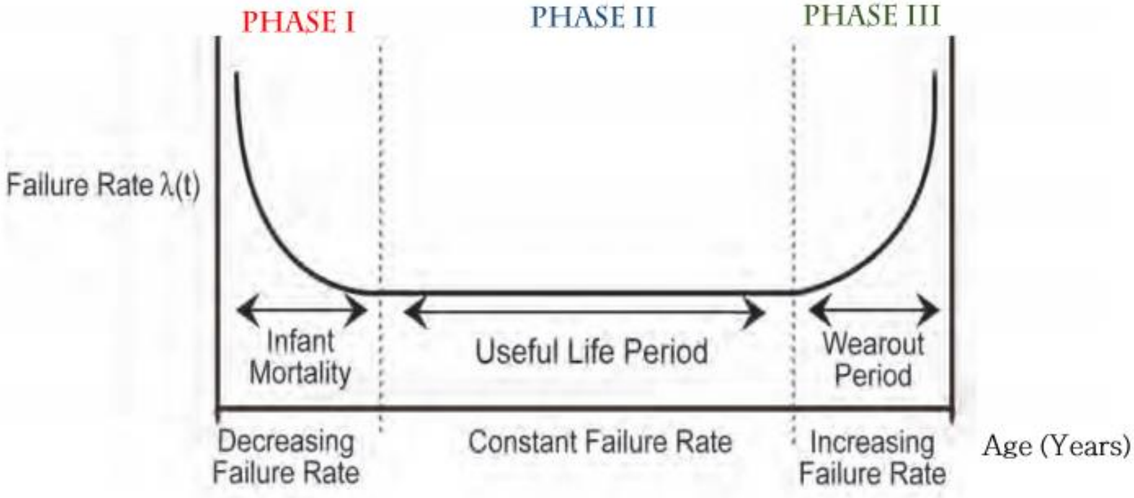
The behavior of the failure rate as a function of age is known as the *mortality curve*, *hazard function*, *life characteristic*, or *lambda characteristic*. The mortality curve is empirically observed to have the so-called bathtub shape shown in Figure 3.6.

During the early life period (infant mortality phase, burnin period, debugging period, or breakin period), failures are of the **endogenous** type and arise from inherent defects in the system attributed to faulty design, manufacturing, or assembly. During this period, the failure rate is expected to drop with age.

When the system has been debugged, it is prone to chance or random failure (also called **exogenous** failure). Such failures are usually associated with environmental conditions under which the component is operating. They are the results of severe, unpredictable stresses arising from sudden environmental shocks; the failure rate is determined by the severity of the environment.

During this useful-life phase, failure rate is approximately constant and the exponential model is usually acceptable. -book 3 pg 149

The practical operation of a component is a time based function & if one wants to graph the failure rate of a component population versus time, it is likely the graph would take the bath tub shape .



In the given figure; y-axis represents the Failure Rate $\lambda(t)$ & the

x-axis is the time.
form the given shape the curve can be divided into 3 distincts portions or phases.
the First phase is called 'Infant Mortality' or the 'Burnin phase'.
the 2nd phase is called the 'Useful Life Period' & the 3rd phase is called the 'Wearout Period'.
The initial Infant Mortality period of bath tub curve is characterized by the high failure rate followed by a period of decresing failure.
This decreasing failure rate period is also regarded as debugging phase & it is attributed to endogenous failures like faulty design, defected parts, damaging in handling, poor installation or misapplication.
These factors show there affect early in life resulting in the burnin mode.
To correct this situation one may resort to design improvement, care & material selection & tighten production quality control.
The instant Infant Mortality period is followed by a nearly constant failure rate period which is also known as the 'Useful Life period'.
The failures occuring in this phase are due to exogenous factors from the environment.
As the Failure rate is constant, this period is modeled using exponential distribution.
From the product sale point of view this phase is often used to formulate the pricing, warranties & the serving policies.
Its aspect is particularly important in consumer goods such as laptop, computer, cellphone etc.
It is also generally agreed that exceptional maintainance packages* encompassing* compentive* & predictive can actually extend this feature.

The 3rd phase is called the aging or wearout period & it is characterize by a rapid increasing failure rate with time.
In most cases this period encompasses the normal distribution of design like failures.
The aging is usually due to material petty*, corosion, in .. etc.
The Wearout mode is often encountered in mechanical system with moving part such as walls*, pumps, engine, cutting tools, bearing balls & joints .. etc.
So the design life of most equipment requires periodic maintance. For example the allignment need to be maintain, proper lubrication on some parts is required & so on.
In some cases certain components need replacement to ensure the main piece of equipment lasts for its design life.
Anytime we fail to perform maintaince activities intended by the equipment designer we shortened the operating life of the equipment.

Simplicity is prerequisite for reliability!!

-Edsger Dijkstra
