MARKOV RELIABILITY MODEL

CLASS EXERCISE

The intranet system of a large organization is supported by a primary naming server (1) along with a standby naming server (2). The system works only if one of the two machines is operating. The primary server fails (assuming) at the end of a given day with probability 0.08 whereas the standby machine has failure probability of 0.1. Whenever any machine fails, it will certainly be repaired and operational by the end of next day.

Thus for t=0, 1, 2, …, the random variable X_t takes on the values,

$$Xt = \begin{cases} 0 & Both\ machines\ working \\ 1 & Primary\ server\ 1\ failed \\ 2 & Standby\ server\ 2\ failed \\ 3 & Both\ machines\ failed \end{cases}$$

The cost of repair (in 1000s Rs) is charged as follows:-

$$C(X_t) = \begin{cases} 0 & Both\ machines\ working \\ 3 & Primary\ server\ failed \\ 2.5 & Standby\ server\ failed \\ 4 & Both\ machines\ failed \end{cases}$$

Now answer the following questions:-

a) Construct the (one-step) transition matrix for this Markov chain.

b) Calculate the steady-state probabilities of the state of this Markov chain?

c) What is the probability that the intranet will be inoperable after *n* periods, for n = 2, 5? [Hint: find $\mathbf{p_{ij}^n}$ using software]

d) Find out the (long-run) expected average cost per period?