CS-417 CSM Mid Term

0.55
0.45
1.00

Q.3) (03)

$\lambda = 8$ req/min

a) $P(X \le 2) = ?$

$\alpha = \lambda t = (8)(\tfrac{1}{2}) = 4$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$
$$= \sum_{k=0}^{2} \frac{e^{-\alpha} \alpha^k}{k!} = \sum_{k=0}^{2} \frac{e^{-4} (4)^k}{k!}$$

$$\boxed{P(X \le 2) = 0.2381}$$

b) $P(X=0)$

$\alpha = \lambda t = 8(1) = 8$

$$P(X=0) = \frac{e^{-8} (8)^0}{0!}$$

$$\boxed{P(X=0) = 3.3546 \times 10^{-4}}$$

Q.4) (03)

$P_A = \frac{90}{150} = 0.6$

$P_B = \frac{35}{150} = 0.233$

$P_C = \frac{25}{150} = 0.1667$

a) $P[X_A = 10, X_B = 6, X_C = 8] = \frac{24!}{10!\,6!\,8!} \times P_A^{10} P_B^{6} P_C^{8}$

$= 0.00344$

b) $P[X_A + X_B = 24] = \frac{24!}{24!\,0!} (p_A + p_B)^{24} p_c^{0}$

$= 0.01245$

Q.5) (04)

$R = e^{-\lambda t} = e^{-0.000388(300)}$

$= 0.8901$

$$R_s = \sum_{x=2}^{4} \binom{4}{x} R^x (1-R)^{4-x}$$

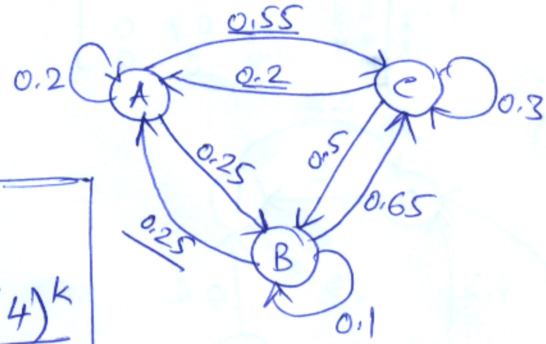
$R_s = 0.995$

$$MTTF = \sum_{i=2}^{4} \frac{1}{i\lambda} = \frac{1}{(2)\lambda} + \frac{1}{3(\lambda)} + \frac{1}{4(\lambda)}$$

$$\boxed{MTTF = 2792}$$

Q.6) (03)

a) $$P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0.2 & 0.25 & 0.55 \\ B & 0.25 & 0.10 & 0.65 \\ C & 0.2 & 0.5 & 0.30 \end{array}$$



$\pi = \pi P$

$$[\pi_A \ \pi_B \ \pi_C] = [\pi_A \ \pi_B \ \pi_C] \begin{bmatrix} 0.2 & 0.25 & 0.55 \\ 0.25 & 0.1 & 0.65 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}$$

$\pi_A + \pi_B + \pi_C = 1$ —(i)

$0.2\pi_A + 0.25\pi_B + 0.2\pi_C = \pi_A$ —(ii)

$0.25\pi_A + 0.1\pi_B + 0.5\pi_C = \pi_B$ —(iii)

$0.55\pi_A + 0.65\pi_B + 0.3\pi_C = \pi_C$

$\pi_A = \frac{122}{565} = 0.2159$, $\boxed{\pi_B = \frac{36}{113} = 0.3185}$, $\pi_C = \frac{263}{565} = 0.4654$

Q.7) $S = 15$ (02)

$$S = \lim_{n\to\infty} \frac{1}{1-f}$$

$\Rightarrow 15 = \frac{1}{1-f}$

$\Rightarrow 1-f = 1/15 \Rightarrow f = 1 - 1/15$

$\Rightarrow f = 14/15$

$\Rightarrow \boxed{f = 0.9333}$

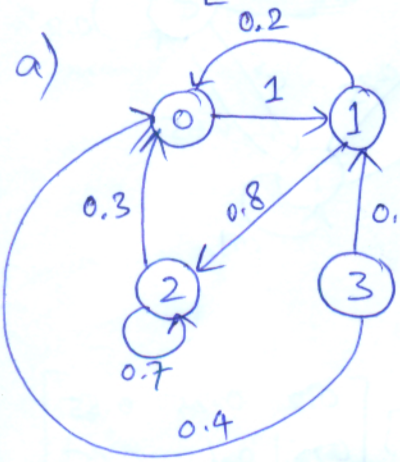
Q.1 & Q.10) See lectures

(04)

$$
\begin{array}{c|cccc}
 & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 1 & 0 & 0 \\
1 & 0.2 & 0 & 0.8 & 0 \\
2 & 0.3 & 0 & 0.7 & 0 \\
3 & 0.4 & 0.6 & 0 & 0
\end{array}
$$

$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 \\ 0.3 & 0 & 0.7 & 0 \\ 0.4 & 0.6 & 0 & 0 \end{bmatrix}$

a)



b) Reducible

c) $\{0, 1, 2\} \rightarrow$ Recurrent

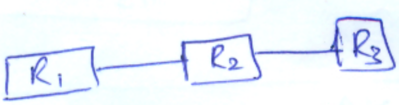
$\{3\} \rightarrow$ Transient

Q.9) (03)

$R_1 =$ Card Reader $= 0.89$

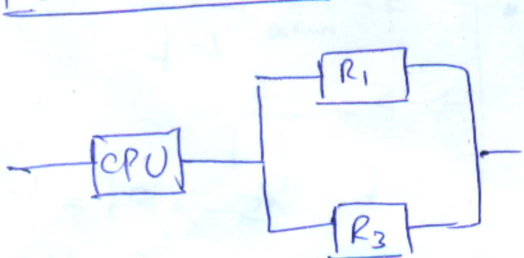
R_2 CPU $= 0.98$

$R_3 =$ Line Printer $= 0.85$



$R_s = (0.89)(0.98)(0.85)$

$\boxed{R_s = 0.74137}$



$\Rightarrow R_s = (0.98) * [1 - (1-0.89)(1-0.85)]$

$\Rightarrow \boxed{R_s = 0.96383}$

Mid Term Solution

Q.6)

$$P = \begin{array}{c|ccc} & FF & DP & UR \\ FF=0 & 0.7 & 0.2 & 0.1 \\ DP=1 & 0.6 & 0.2 & 0.2 \\ UR=2 & 0.5 & 0.3 & 0.2 \end{array}$$

a)

$$P^{(2)} = \begin{bmatrix} 0.66 & 0.21 & 0.13 \\ 0.64 & 0.22 & 0.14 \\ 0.63 & 0.22 & 0.15 \end{bmatrix}$$

$$\boxed{P^{(2)}_{21} = 0.22}$$

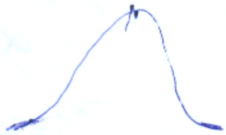
b) $\pi_0 + \pi_1 + \pi_2 = 1$ — i,

$\pi = \pi P$; $[\pi_0 \ \pi_1 \ \pi_2] = [\pi_0 \ \pi_1 \ \pi_2] \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.2 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}$

$\pi_0 = 0.7\pi_0 + 0.6\pi_1 + 0.5\pi_2$ — ii

$\pi_1 = 0.2\pi_0 + 0.2\pi_1 + 0.3\pi_2$ — iii,

$\pi_2 = 0.1\pi_0 + 0.2\pi_1 + 0.2\pi_2$



$\pi_0 = \frac{58}{89} = 0.6516$, $\boxed{\pi_1 = \frac{19}{89} = 0.2134}$, $\pi_2 = \frac{12}{89} = 0.1348$

Q.7)

$T_{imp} = \frac{8888}{2} = 4444$ sec

$T_{unaff} = 5555$ sec

$$T_{imp} = \frac{T_{aff}}{\text{Imp factor}} + T_{unaff}$$

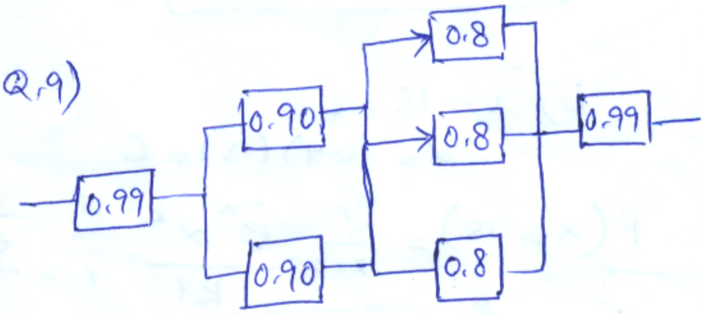
$$4444 = \frac{3333}{IF} + 5555$$

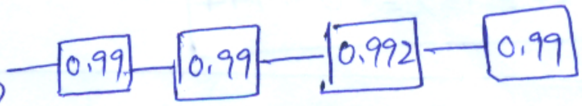
$$-1111 = 3333/IF$$

$$\boxed{IF = -3}$$

Since the unaffected part takes 5555 sec to execute therefore it is not possible to improve 'mult' instruction to ~~achieve~~ make the overall execution of program improved by a factor of 2.

Q.9)



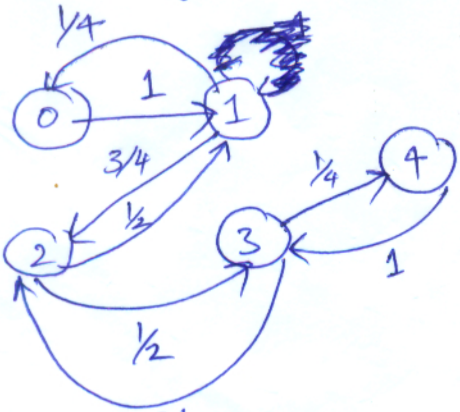


$\Rightarrow$ $\boxed{R_{overall} = 0.9625}$

For Q#1 & Q#10 → see lectures

Q.8)

a)



c) All are re-current. None are transient or absorbing.

b) All the states are communicating so chain is irreducible

Q.4) $n=18$

$p = 0.2$

$q = 1-0.2 = 0.8$

a) $P(X=1) = {}^nC_x \, p^x q^{n-x}$

$= {}^{18}C_1 \, (0.2)^1 (0.8)^{18-1}$

$= 0.0810$

b) $P(X \geq 1) = 1 - P[X=0]$

$= 1 - [{}^{18}C_0 (0.2)^0 (0.8)^{18}]$

$= 0.9819$

Q.3)

$\alpha = 0.4t$

a) $t = 5 \sec$

$\therefore \alpha = (0.4)(5) = 2$

$x = 2$

$$P[X=2] = \frac{e^{-\alpha}\alpha^k}{k!} = \frac{e^{-2}(2)^2}{2!}$$

$$\boxed{P[X=2] = 0.27067}$$

b) $t = 15 \sec$

$\therefore \alpha = (0.4)(15) = 6$

$$P(x \le 15) = \sum_{k=0}^{15} \frac{e^{-\alpha}\alpha^k}{k!} = \sum_{k=0}^{15} \frac{e^{-6}(6)^k}{k!}$$

$$\boxed{P(x \le 15) = 0.9994}$$

Q.5) $R_1 = e^{-0.0001t}$

$R_2 = e^{-0.0002t}$

$R_3 = e^{-0.0004t}$

a)

$$R_{ov} = 1 - (1 - e^{-0.0001t})(1 - e^{-0.0002t})(1 - e^{-0.0004t})$$

Put $t = 1000$

$$\boxed{R_{ov} = 0.99431}$$

b)

$F_x(t) = 1 - R(t) = 1 - 0.99431$

$F_x(t) = 5.686 \times 10^{-3}$

Solution (CS-417) CSM Mid Term

Q.1) App areas of kernel & micro bench marks ......

Q.10) pitfalls of MIPS & MFLOPS.

Q.3) $p = 0.09$

(01)

a) $$P[X=0] = \binom{n}{0} p^0 (1-p)^{n-0}$$
$$= (1-0.09)^{15}$$
$$= 0.2430$$

b) $$P[X \geq 2] = 1 - P[X < 2] \quad (02)$$
$$= 1 - \sum_{i=0}^{1} \binom{n}{i} p^i (1-p)^{n-i}$$
$$= 1 - \binom{15}{0}(0.09)^0(1-0.09)^{15} - \binom{15}{1}(0.09)^1(1-0.09)^{15-1}$$
$$= 0.3965$$

Q.4)
$\lambda = 7/\text{month}$; $t = 20 \text{ days} = 20/30 = 2/3$ month , $\alpha = \lambda t$

$\alpha = (7)\left(\frac{2}{3}\right) = 14/3$

(03)

$$P(N \geq 3) = 1 - P(N \leq 2)$$
$$= 1 - [P(N=0) + P(N=1) + P(N=2)]$$
$$= \cancel{1 - \left[\frac{\alpha^k e^{-\alpha}}{k!} \right]}$$
$$= 1 - \left[\frac{(14/3)^0 e^{-14/3}}{0!} + \frac{(14/3)^1 e^{-14/3}}{1!} + \frac{(14/3)^2 e^{-14/3}}{2!} \right]$$
$$= 0.8443$$

Q.5) $R = e^{-\lambda t}$

$R_1 = e^{-0.002t}$, $R_2 = e^{-0.002t}$, $R_3 = e^{-0.001t}$, $R_4 = e^{-0.003t}$

$$R_s = \prod_{i=1}^{4} R_i = (e^{-0.002t})(e^{-0.002t})(e^{-0.001t})(e^{-0.003t})$$
$$R_s = e^{-0.008t}$$

At $t = 100$

$$R_s = e^{-0.008(100)}$$
$$R_s = 0.44932$$

Q.6)

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.8 & 0.1 & 0.1 \\ 1 & 0.3 & 0.5 & 0.2 \\ 2 & 0.7 & 0.2 & 0.1 \end{array}$$

$0 \to FF$
$1 \to$ Deg. Perf
$2 \to$ Urg. Repair

a)
$$P^{(2)} = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.74 & 0.15 & 0.11 \\ 1 & 0.53 & 0.32 & 0.15 \\ 2 & 0.69 & 0.19 & 0.12 \end{array}$$

$$\boxed{P_{20}^{(2)} = 0.69}$$

b) $\pi_0 + \pi_1 + \pi_2 = 1$

$\cancel{0.8\pi_0 + 0.1\pi_1 + 0.1\pi_2 = \pi_0}$

$\cancel{0.3\pi_0 + 0.5\pi_1 + 0.2\pi_2 = \pi_1}$

$\cancel{0.7\pi_0 + 0.2\pi_1 + 0.1\pi_3 = \pi_2}$

$\cancel{\pi_0 = \pi_1 = \pi_2 = 1/3}$

$\pi = \pi P$

$$[\pi_0\ \pi_1\ \pi_2] = [\pi_0\ \pi_1\ \pi_2] \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}$$

$0.8\pi_0 + 0.3\pi_1 + 0.7\pi_2 = \pi_0$

$0.1\pi_0 + 0.5\pi_1 + 0.2\pi_2 = \pi_1$

$0.1\pi_0 + 0.2\pi_1 + 0.1\pi_2 = \pi_2$

$\boxed{\pi_0 = 0.695}$, $\boxed{\pi_1 = 0.18644}$, $\boxed{\pi_2 = 0.1186}$

Probability for system to remain fully functional in long run = 0.695

Q.7)

$$T_{imp} = \frac{T_{aff}}{\text{Imp. factor}} + T_{unaff}$$

$$3 = \frac{13}{x} + 4$$

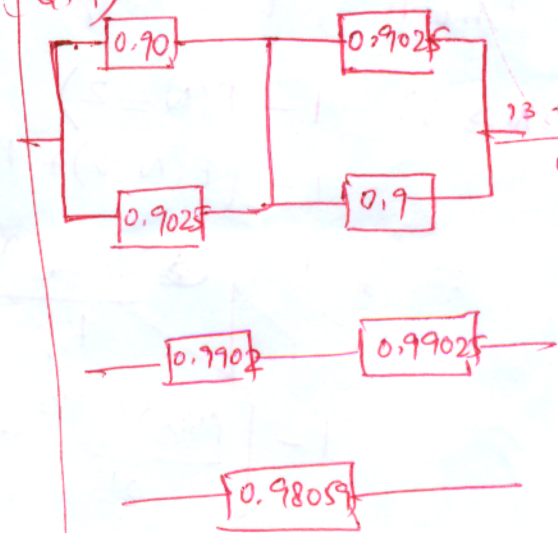
$$3 - 4 = \frac{13}{x}$$

$$-1 = 13/x$$

$$\boxed{x = -13}$$

Program can't run in 3 seconds. It will take at least 4 seconds for execution of un-improved part

Q.9)



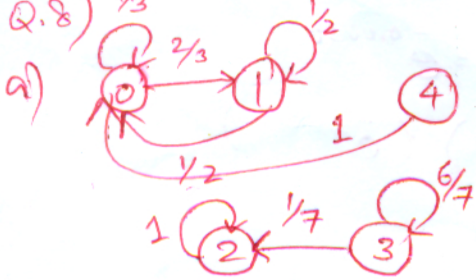
$13 + 4x = 3x$

$13 = 3x - 4x$

$13 = -x$

Q.8)

a)



c) $\{0,1\} \to$ communicating states

$\{3,4\} \to$ Transient

$\{2\} \to$ Absorbing.

b) Chain is not irreducible as all states are not communicating with one another.