CS-417
COMPUTER SYSTEMS MODELING

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(LECTURE # 24)

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Recap of Lecture # 23

Queuing Models involving non-exponential distribution

Example Problem

Erlang-n and Hyper-exponential Distribution

Chapter # 6 (Cont'd)

FUNDAMENTALS OF QUEUING MODELS

QUEUING NETWORKS

- A job may receive service at one or more queues before exiting from the system. Such systems are modelled by queueing networks.

- In general, a model in which jobs departing from one queue arrive at another queue (or possibly the same queue) is called a **queuing network.**

- There are two types of queuing networks:
  - Open queuing networks
  - Closed queuing networks

OPEN QUEUING NETWORKS

- An **open queueing network** has external arrivals and departures. The jobs enter the system at "In" and exit at "Out".

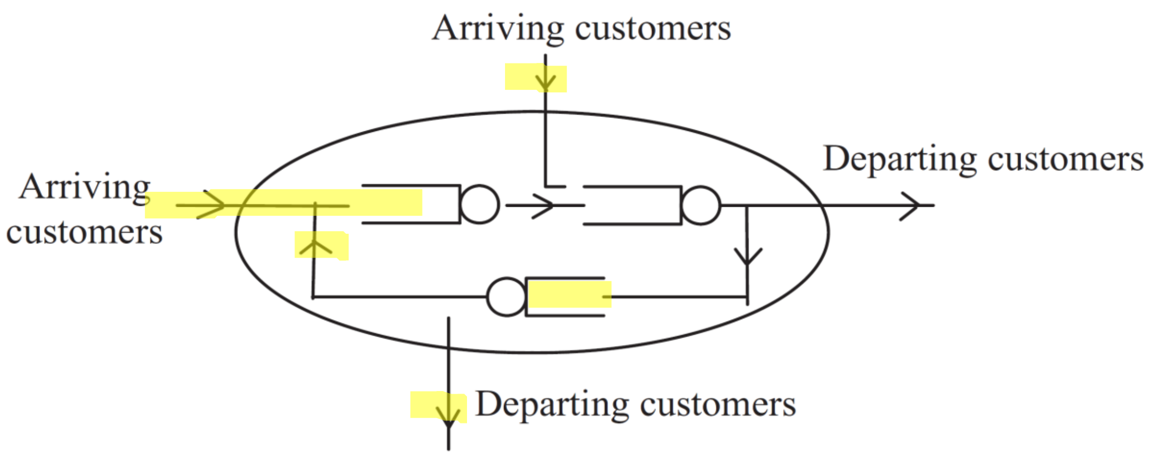

Fig 1: Open Queuing Network

- The number of jobs in the system varies with time.
- In analyzing an open system, we assume that the throughput is known (to be equal to the arrival rate), and the goal is to characterize the distribution of number of jobs in the system.

CLOSED QUEUING NETWORKS

- A **closed queueing network** has no external arrivals or departures.
- The jobs in the system keep circulating from one queue to the next. The total number of jobs in the system is constant.

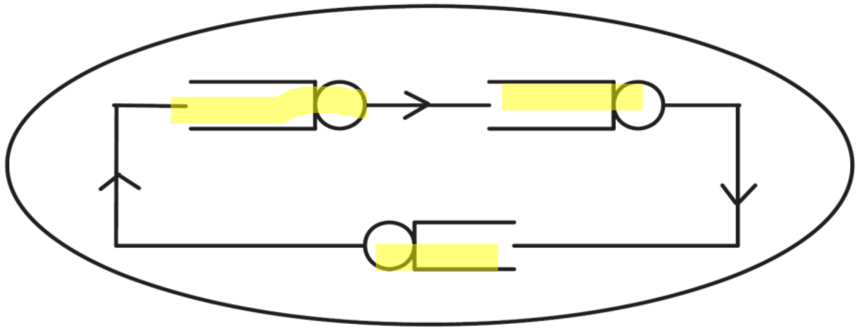

Fig 2: Closed Queuing Network

- It is possible to view a closed system as a system where the Out is connected back to the In.
- The jobs exiting the system immediately re-enter the system.

Jackson's Network

- A Jackson network consists of a number of nodes, where each node represents a queue in which the service rate can be both node-dependent (different nodes have different service rates) and state-dependent (service rates change depending on queue lengths).

- There is no notion of priority in serving the jobs: all jobs at each node are served on a first-come, first-served basis.

Necessary conditions for a Jackson Network

- If the network is open, any external arrivals to node *i* form a Poisson process,

- All service times are exponentially distributed and the service discipline at all queues is first-come, first-served,

- A customer completing service at queue *i* will either move to some new queue *j* with probability P_{ij} or leave the system with the probability 1 - $\sum_{j=1}^{m} P_{ij}$,

- The utilization of all of the queues is less than one.

Jackson's Theorem

- Let $n = (n_1, n_2, \ldots, n_M)$ represent the global state of an open network of M nodes.

- Let n_i = state of node i

(i.e. the total number of customers present in node i)

- Jackson's Theorem states that:

$$p(n) = p(n_1)\, p(n_2) \ldots p(n_M)$$

Burke's Theorem

Consider an M/M/1, M/M/c or M/M/∞ system with arrival rate λ. Suppose that the system is in steady state. Then:

a) The departure process is also Poisson with rate λ.

b) At each time t, the number of customers in the system is independent of the departures prior to t.

Two Stages Tandem Network

Consider the given tandem network consisting of two M/M/1 queues:

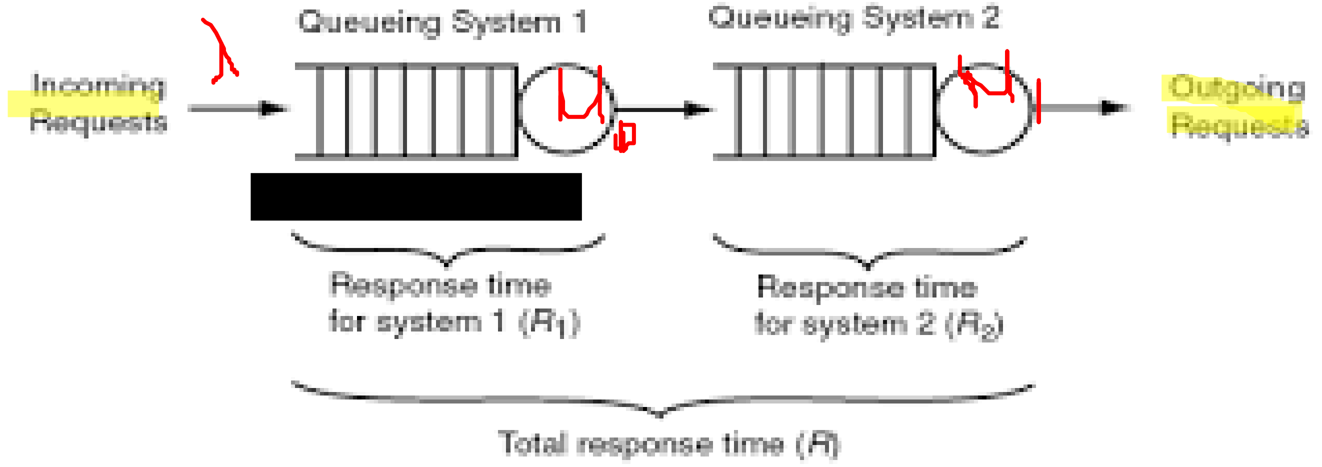

Fig 3: Two Stages Tandem Network

Two stages TN can be modelled as a continuous time Markov Chain.

Two Stages Tandem Network (Cont’d)

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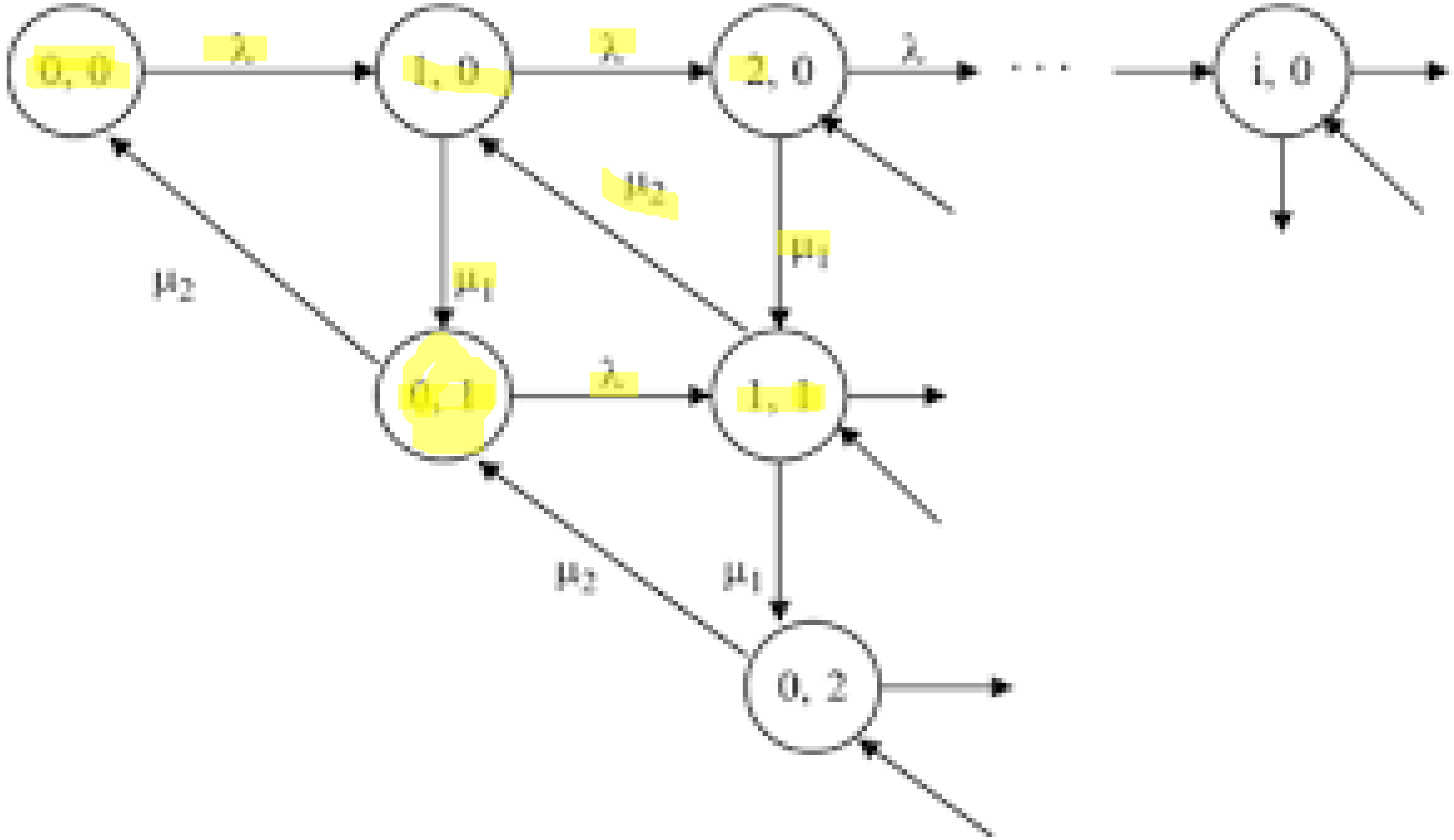


Fig 4: Rate Diagram for 2-Stages Tandem Network

Two Stages Tandem Network (Cont'd)

Applying Jackson's Theorem,

$P(n_0,n_1) = P(n_0)\, P(n_1)$

$= \rho_0^{n_0}(1 - \rho_0)\, \rho_1^{n_1}(1 - \rho_1)$

– This equation can be extended to any number of stages.

For stability, $\rho_0 = \frac{\lambda}{\mu_0} < 1$ & $\rho_1 = \frac{\lambda}{\mu_1} < 1$

Example Problem 1

A repair facility shared by a large number of machines has two sequential stations (with service rates 1/hr & 2/hr). The cumulative failure rate of all the machines is 0.5/h. Assume a 2-stage tandem network with failure rates following Poisson distribution.

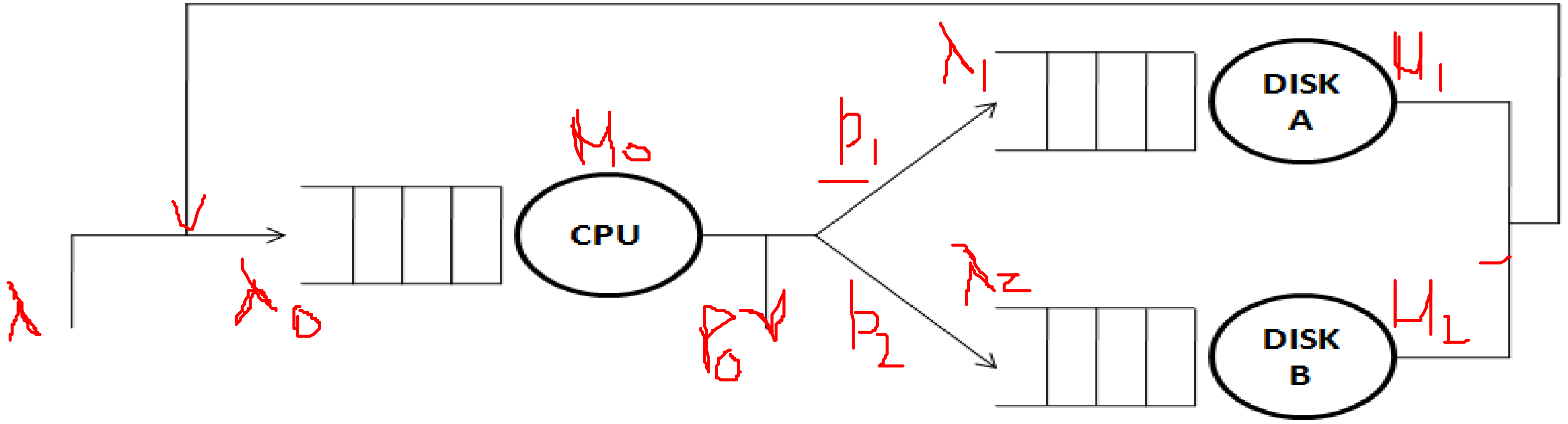
Calculate the average repair time and probability that both the stations are idle.

Answers:

i) 8/3 hours

ii) 0.375

Open Central Server Networks



$$\lambda_j = \begin{cases} \dfrac{\lambda}{p_0}, \; j = 0 \\ \dfrac{p_j}{p_0}\lambda, \; j = 1, 2, 3, \ldots \ldots, m \end{cases}$$

Example Problem 2

Consider an open central server queuing model with a CPU (service rate: 2 programs/sec) and two I/O channels (each with service rate: 1.2/sec). The external job arrival rate is 1/7 programs/sec according to Poisson Process. The branching probabilities are $p_0 = 0.1$, $p_1 = 0.3$ and $p_2 = 0.6$. Assume all the service times are independent exponentially distributed RVs.

Calculate:

a) The probability that there is just a single program in the CPU and Disk B and none in the Disk A.

b) Mean number of programs in the network.

c) Average response time of the overall network.

Answers:

a) *0.02677*
b) *5.5554*
c) *38.88 sec*