Date

COMPUTER SYSTEMS MODELING (CEP)

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2) Average number of customers in the system ($Ls = E[N] = \sum_{n=1}^{\infty} n P_n$)

3) $$Lq = \sum_{n=0}^{\infty} n P_{n+c}$$

$$Lq = \sum_{n=0} \frac{n \rho^{n+c}}{c!\, c^{j}} P_0 \quad \therefore j = n$$

$$Lq = \rho^{c+1} \frac{\sum n \rho^{n-1}}{c!\, c\, c^{j-n}} P_0 .$$

$$= \sum \rho^{c+1} \frac{\sum n \rho^{n-1}}{c!\, c\, c^{j-1}} P_0 .$$

$$= \sum \frac{\rho^{c+1} P_0\, j \rho^{j-1}}{c!\, c\, c^{j-1}}$$

$$= \frac{\rho^{c+1} P_0}{c!\, c} \sum \frac{d}{d(\rho/c)} \left(\frac{\rho}{c}\right)^j$$

$$= \frac{\rho^{c+1} P_0}{c!\, c} \frac{d}{d(\rho/c)} \frac{1}{1-(\rho/c)}$$

$$= \frac{\rho^{c+1} P_0}{c!\, c} \left[\frac{1}{(1-\rho/c)^2}\right]$$

$$\boxed{Lq = \frac{\rho^{c+1} P_0}{(c-1)!\, (c-\rho)^2}}$$

$$Ls = Lq + \rho$$

$$Ls \quad \frac{\rho^{c+1} P_0}{(c-1)!\, (c-\rho)^2} + \rho$$

Scanned with CamScanner

$$W_s = W_q + \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda}$$

$$\boxed{W_s = \frac{L_q}{\lambda} + \frac{1}{\mu}}$$

Scanned with CamScanner

2) Average number of custumer in the Queue.

$(L_q = \sum_{n=0}^{\infty} n P_{n+c}$

7) $L_q = \sum_{n=0}^{\infty} n P_{n+c}$

$$L_q = \sum_{n=0}^{\infty} n \frac{C^c}{c!} \rho^{n+c} P_0$$

$$L_q = \frac{c^c}{c!} P_0 \sum_{n=1}^{\infty} n \rho^{n+c}$$

$$L_q = \frac{c^c}{c!} P_0 \rho^c \sum_{n=1}^{\infty} n \rho^n$$

$$L_q = \frac{\rho^c c^c}{c!} P_0 \rho \frac{d}{d\rho} \left(\sum_{n=1}^{\infty} \rho^n \right)$$

$$L_q = \frac{\rho^c c^c}{c!} P_0 \rho \frac{d}{d\rho} \left(\frac{\rho}{1-\rho} \right)$$

$$L_q = \frac{(c\rho)^c}{c!} P_0 \rho \cdot \frac{(1-\cancel{\rho}+\cancel{\rho})}{(1-\rho)^2}$$

$$\boxed{L_q = \frac{(c\rho)^c}{c!} \cdot \frac{P_0}{(1-\rho)} \cdot \frac{\rho}{1-\rho}}$$

as we already know $P(N \geq c] =$

$$\boxed{L_q = P[N \geq c] \cdot \frac{\rho}{1-\rho}}$$

$$\boxed{L_q = \frac{K\rho}{1-\rho}}$$

PART : 3

$$L_s = L - L_q$$

$$L_s = \left[\frac{(C\rho)^c}{C!} \cdot \frac{P_0 \rho}{(1-\rho)^2} + C\rho\right] - \left[\frac{(C\rho)^c}{C!} \cdot \frac{P_0 \rho}{(1-\rho)^2}\right]$$

$$L_s = \frac{(C\rho)^c P_0 \rho}{C!(1-\rho)^2}\left[\cancel{1} + \frac{C\rho}{\frac{(C\rho)^c P_0 \rho}{C!(1-\rho)^2}} - \cancel{1}\right]$$

$$L_s = \cancel{\frac{(C\rho)^c P_0 \rho}{C!(1-\rho)^2}}\left[\frac{C\rho}{\cancel{\frac{(C\rho)^c P_0 \rho}{C!(1-\rho)^2}}}\right]$$

$$L_s = C\rho$$

1) Probability of Queuing. $\left(P[N \geq C] = \sum_{n=0}^{\infty} {}^{n}P_{n+c}\right)$

$$\omega = P_c + P_{c+1}$$

$$P[N \geq C] = \sum_{n=c}^{\infty} P_n$$

$$P[N \geq C] = P_c + P_{c+1} + P_{c+2} + \cdots$$

$$P[N \geq C] = \frac{P_c}{1-P}$$

$$P[N \geq C] = \frac{(cp)^c}{C!} P_0 \frac{1}{1-P}$$

$$P[N \geq C] = \frac{(cp)^c}{C!}\left[(1-p)\sum_{n=0}^{c-1} \frac{(cp)^n}{n!} + \frac{(cp)^c}{C!}\right]^{-1}$$

$$P[N \geq C] = \frac{(cp)^c}{C!} \frac{P_0}{1-p}$$

PART: 5

$$E[n_s] = \sum_{n=1}^{m-1} n p_n + \sum_{n=m}^{\infty} m p_n$$

$$= 1 p_0 \frac{(m\rho)}{1!} + 2 p_0 \frac{(m\rho)^2}{2!} + \dots + (m-1) p_0 \frac{(m\rho)^{m-1}}{(m-1)!}$$

$$+ m\left(p_m + p_{m+1} + p_{m+2} + \dots +\right)$$

$$= m\rho\left(p_0 + p_0 \frac{(m\rho)}{1!} + p_0 \frac{(m\rho)^2}{2!} + \dots + p_0 \frac{(m\rho)^{m-2}}{(m-2)!}\right)$$

$$+ m\varrho$$

$$= m\rho\left(p_0 + p_1 + p_2 + \dots + p_{m-2}\right) + m\varrho$$

$$= m\rho\left(1 - p_{m-1} - \varrho\right) + m\varrho$$

$$= m\rho - m\rho p_{m-1} + m\varrho(1-\rho)$$

$$= m\rho, \text{ since } m\varrho(1-\rho) = m p_m = m\rho p_{m-1}$$