Lecture 11

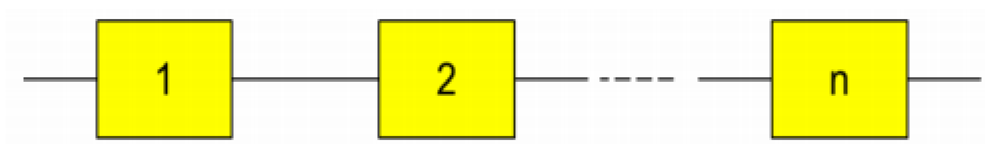
RELIABILITY BLOCK DIAGRAMS (RBD)

Reliability block diagram shows the reliability structure of a system that structure could be either a physical one or a logical one. It says that a system is operational if atleast one path of operational component exists from input to output.
Here we will assume that the failures of components are independant.

1) Series Systems

In a series configuration a failure of any component results in the failure of the entire system.
In most cases when considering complete system at there basic subsystem level it is found that these are arranged reliability wise in a series configuration. i.e.
• When every module (block) in the system must be operational for the entire system to be functional, the blocks are said to be in series interconnection.

• E.g. processor, memory and system bus form a series configuration in a computer system. A failure of any of these sub system will cause a system failure. In other words all of the units in a series system must succeed for a system to succeed.



The reliability of a system is the probabilty that unit 1 succeed, unit 2 succed & all other units in the system succeeds. It means that all the units starting from 1 till n must be operational for the entire system to work successfully.

Let us define an event E_k = block k is operational.
Then, reliability of block k is $R_k = P(E_k)$. Also,

If we caculate the probability that the complete system is working it means that all the blocks i.e. the individual modules are working & if we consider them as mutually exclusive events then probability can be represented by this equation,
P[system is working]= P[all modules working] = $P[E_1 \cap E_2 \cap \cdots \cap E_n]$

Since block failures are independent, therefore, reliability of a series system is given by,

$$R_s=P[E_1]P[E_2]\cdots P[E_n]=R_1R_2\cdots R_n$$

$$R_s = \prod_{i=1}^{n} R_i$$

In other words, For a pure series system, the system reliability is equal to the product of the reliabilities of its consequent components.

For homogeneous modules (i.e. identical reliability),

$$R_s= R^n$$

Remarks

• ***Effect of Component Reliability in a Series System:***
In a series configuration, the component with the least reliability has the biggest effect on the system's reliability.
There's a saying that a chain is only as strong as its weakest link. This is a good example of the effect of a component in a series system; In a chain all the rings are in series & if any of the ring

breaks the system fails. In addition the weakest link in the chain is the one that will break first.
So we can say that the weakest link dictates the strenght of the chain in the same way the weakest component or subsystem dictates the Reliability of a series system.
As a result the reliability of a series system is less than the reliability of the least reliable component.
Clearly,

$$R_s < \min(R_1, R_2, \cdots, R_n)$$

• ***Effect of Number of Components in a Series System***
The number of components is another concern in systems with components connected reliability-wise in series.
As the number of components connected in series increases, the system's reliability decreases.
In order to achieve a high system reliability the component reliability must be high also specially for systems with many components arranged in series.

Example 1

A module of a satellite monitoring system has 500 components in series.
The reliability of each component is 0.999.
• Find the reliability of the module.
• If the number of components is reduced to 200, what is the reliability?
Solution:
For homogeneous modules (i.e. identical reliability),

$$R_s = R^n$$

$$\text{a) } (0.999)^{500} = 0.60637$$

b) $(0.999)^{200} = 0.81864$

Answers:

- 0.60637
- 0.81864

RELIABILITY BLOCK DIAGRAMS (RBD)

2) Parallel System

In a simple parallel system atleast one of the units must succeed for the entire system to succeed.

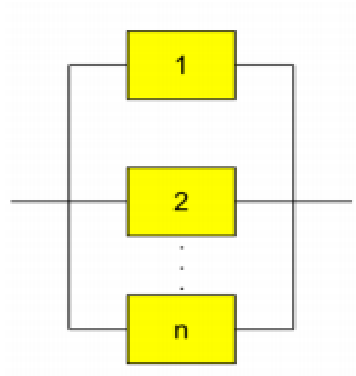
- A parallel system is a kind of configuration wherein functioning of at least one system block is sufficient for the entire system to operate correctly.

The units in parallel are also reffered to as 'redundant units'. Redundancy is a very important aspect of system design & reliability. Adding redundancy is one of the several methods of improving systems reliability.

There are various application of parallel system configuration. It's widely used in arrow space industry & generally used in mission critical system. Other applications includes the rate* computer hard drive systems, the break system, support cables in the bridges & so on.

All in all we can generalize that the parallel systems are widely used where a high

- Application: where a high degree of operation reliability is required to avoid any kind of human, economic, or data loss.



Here if one module is working correctly the entire system operation can work correctly.

In order to derive an expression for reliability of a parallel system, we observe that

$$P[\text{System failing}] = P[\text{all modules failing}] = P[\bar{E}_1 \cap \bar{E}_2 \cap \cdots \cap \bar{E}_n]$$

Since block failures are independent, therefore,

$$1 - P[\text{system working}] = P[\bar{E}_1]P[\bar{E}_2]\cdots P[\bar{E}_n]$$

$$\Rightarrow \quad 1 - R_P = (1-R_1)(1-R_2)\ldots(1-R_n)$$

$$\Rightarrow \quad R_P = 1 - \prod_{i=1}^{n}(1-R_i)$$

So probability of failure or unreliability for a system with statistically independant parallel components is the probability that unit 1 fails & unit 2 fails & all of the other units in the system fails. So in parallel system all of the units must fail for a system to be fail.

In other words; If unit 1 succeed or unit 2 succeed or any of the unit succeed then the overall system will do operation correctly.

For homogeneous modules (i.e. identical reliability)

$$R_P = 1-(1-R)^n$$

Reliability of a parallel system increases with the increase in number of modules.

Remarks

• ***Effect of Component Reliability in a Parallel Configuration***
The component with the highest reliability in a parallel configuration has the biggest effect on the system's reliability, since the most reliable component is the one that will most likely fail last.

• ***Effect of Number of Components in a Parallel System***
For a parallel configuration, as the number of components/subsystems increases, the system's reliability increases.

Example 2

A system has three parallel components, A, B, and C with reliabilities 0.95, 0.92, and 0.90, respectively.

• Find the reliability of the system.
• Determine the reliability if Component C gets out of order.

a) $R_p = 1-[(1-0.95)(1-0.92)(1-0.90)]$

$R_p = 0.9996$

b) $R_p = 1-[(1-0.95)(1-0.92)]$

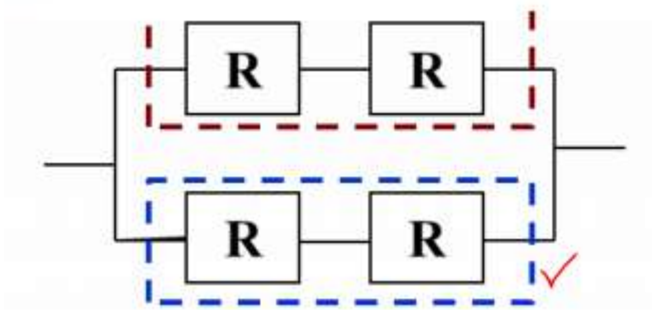
$R_p = 0.996$

Answers:
• 0.9996
• 0.996

After completing example 1 & 2 do comment on answers & verify the remarks. ??

3) SERIES-PARALLEL SYSTEM

While many smaller systems can be accurately represented by either a series system or a simple parallel configuration.
There may be larger system that involves both series & parallel configuration in the overall system.
Such a system can be analyzed by calculating the reliability for the individual series & parallel sections & then combining them in the appropriate manner.
Many systems use a mix of series and parallel configurations as exemplified below:

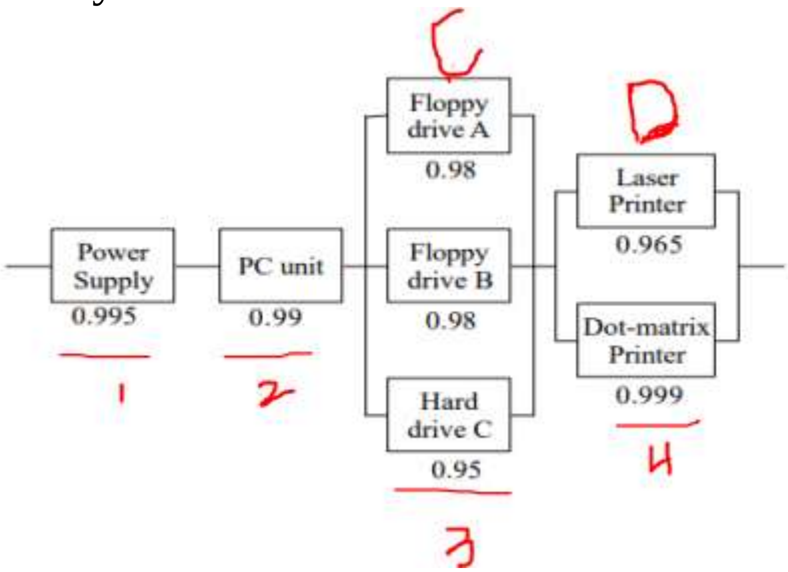


In figure; the two modules inside maroon dotted block are in series & the two more modules inside blue dotted line are in series. But these both dotted blocks (the maroon block & the blue block) are in parallel to one another.
How to find the overall system's Reliability?
by this eq,

$$R_{ov} = 1 - (1 - R * R)(1 - R * R)$$
$$= 1 - (1 - R^2)^2$$

Example 3

Consider the given series-parallel system & determine the overall reliability.



Four individual blocks are connected in series. So overall the reliability can be computed by multiplying all these individual reliabilities.
But the complete block C & the block D is itself in parallel.
First for computing the reliability of block C we need to apply the parallel system configuration formula Again for the block D we need to apply the parallel system configuration formula .
Then after finding the indivudual blocks reliability we can multiply them all to get the overall systems reliability.
Solve!!!

$$R_1 = 0.995$$

$$R_2 = 0.99$$

$$R_3 = 1-[(1-0.98)*(1-0.98)*(1-0.95)] = 0.99998$$

$$R_4 = 1-[(1-0.965)*(1-0.999)] = 0.999965$$

$$R_{system} = R_1*R_2*R_3*R_4$$

$$R_{system} = 0.995*0.99*0.99998*0.999965$$

$$R_{system} = 0.984995$$

Answer:

$R_{system} = 0.984995$

4) K-OUT-OF-N SYSTEM

• k out of n components need to be functional for the system to be functional.

Actually this k out of n components is a special case of parallel redundancy.

This type of configuration requires that atleast k components should succeed out of total n parallel components for the entire system to succeed.

For Example consider an airplane that has 4 engines futhermore, suppose that the design of the aircraft is such that atleast 2 engines are required to work for the aircraft to remain operational. This means that the engines are reliability wise in a k-out-of-n configuration where value of k is 2 (k=2) & the value of n is 4 (n=4) & more specifically they are in a 2-out-of-4 configuration.

As the number of units required to keep the system functioning approached the total number of units in the system, the systems behaviour tends towards that of a series system. It means that if the value of k is n then we are actually talking about series configuration.

& if the number of units required is equal to 1; It means that i am talking about a parallel system.

A parallel system of statistically independant components is a 1-out-of-n system. So

• Please note that parallel (k = 1) and series (k = n) systems are special cases of k-out-of-n system.

• The reliability of such a system is given by binomial distribution: (from week 3 lectures)

$$R_{n|k} = \sum_{i=k}^{n} \binom{n}{i} R^i (1-R)^{n-i}$$

Example 4

Consider a system of 6 pumps of which at least 4 must function properly for system success. Each pump has an 85% reliability for the mission duration.

What is the probability of success of the system for the same mission duration?

Solution:

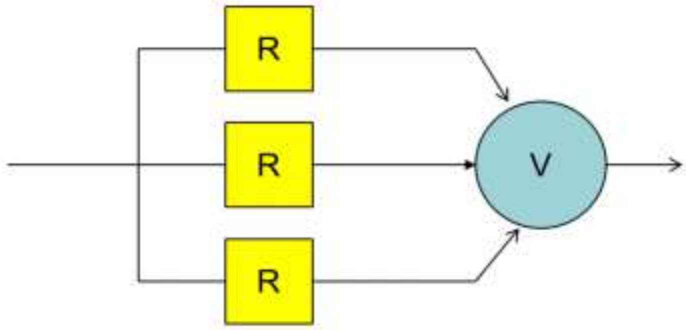
$$n=6$$
$$k=4$$
$$R_i = 85/100 = 0.85$$
$$R = (6C4*0.85^4*(1-0.85)^{6-4})+(6C5*0.85^5*(1-0.85)^{6-5})$$
$$+(6C6*0.85^6*(1-0.85)^{6-6})$$
$$R_{6|4} = 0.1761 + 0.3993 + 0.37714$$
$$R_{6|4} = 0.9546$$

Answer:

$R_{6|4} = 0.9546$

TRIPLE MODULAR REDUNDANCY (TMR)

• A TMR system, also known as a triplex system and a special case of k-out-of-n system (k = 2, n = 3) is illustrated in the following diagram.



There are total 3 modules connected in parallel.
k=2 ; means that atleast 2 out of these 3 modules must work for the complete system to work proerly.

• The ‘V’ block is a majority voter which produces correct output as long as 2 modules are working correctly. Such TMR systems are very common across many scientific disciplines.

We will derived espression for the TMR sytem.
we will start with the formula of binomial distribution & here i have put the value of k & n at the appropriate places. i.e. k=2 & n=3.

$$
\begin{aligned}
R_{TMR} &= \sum_{i=2}^{3} \binom{3}{i} R^i (1-R)^{3-i} \\
&= \binom{3}{2} R^2 (1-R)^{3-2} + \binom{3}{3} R^3 (1-R)^{3-3} \\
&= \frac{3!}{(3-2)! * 2!} R^2 (1-R) + R^3 \\
&= \frac{3!}{2!} R^2 (1-R) + R^3 \\
&= 3R^2 - 3R^3 + R^3 \\
&= 3R^2 - 2R^3
\end{aligned}
$$

$$
R_{TMR} \begin{cases} > R & \text{if } R > 1/2 \\ = R & \text{if } R = 1/2 \\ < R & \text{if } R < 1/2 \end{cases}
$$

Task

Q.1) Three subsystems are reliability-wise in series and make up a system. Subsystem 1 has a reliability of 99.5%, subsystem 2 has a reliability of 98.7% and subsystem 3 has a reliability of 97.3% for a mission of 100 hours.

• What is the overall reliability of the system for a 100-hour mission?
• Now consider that these three sub-systems are arranged in parallel configuration. Compute the overall reliability of the system.

Solution

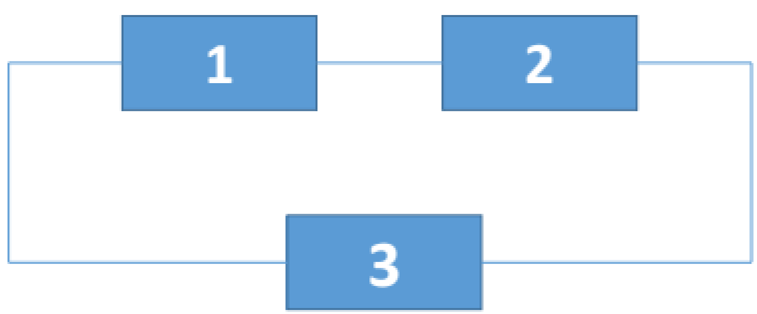
a) $R_s = [(0.995)(0.987)(0.973)]$

$R_p = 0.9555$

b) $R_p = 1-[(1-0.995)(1-0.987)(1-0.973)]$

$R_p = 0.9999982$

Q.2) Consider a system with three components. Units 1 and 2 are connected in series and Unit 3 is connected in parallel with the first two.
• What is the reliability of the system if R1 = 99.5%, R2 = 98.7% and R3 = 97.3% ?



$R_1 = 99.5/100 = 0.995$

$R_2 = 98.7/100 = 0.987$

$$R_3 = 97.3/100 = 0.973$$

$$R_s = R_1 * R_2 = 0.995 * 0.987 = 0.982065$$

$$R_s = 1 - [(1-0.982065)(1-0.973)]$$

$$Rs = 0.99951$$