CSM.

"COMPLEX ENGINEERING ACTIVITY".

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SECTION: A

SUBJECT: COMPUTER SYSTEMS MODELLING (CS-417)

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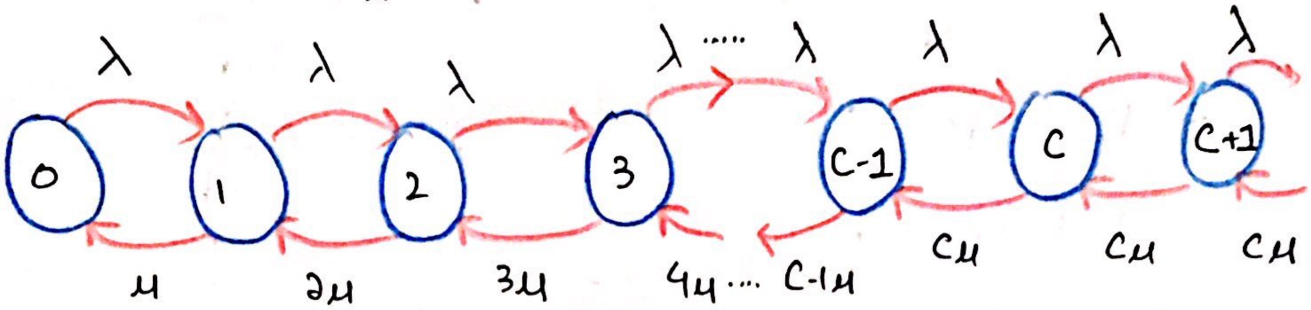
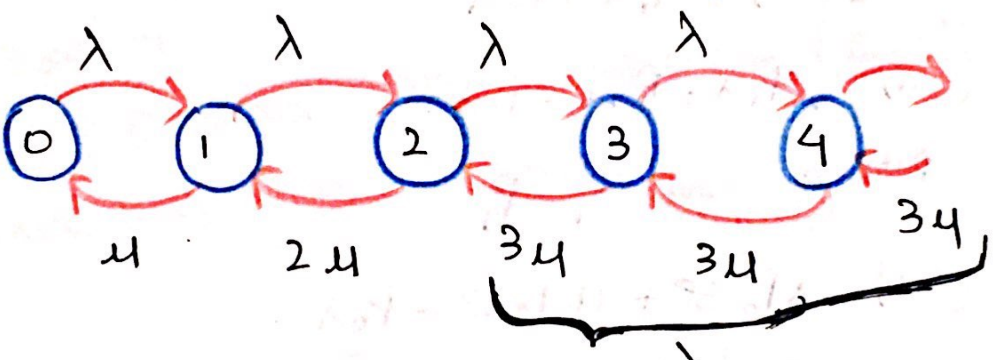
TASK ASSIGNMENT:



$$\rho = \frac{\lambda}{c\mu} < 1$$

Now, in our case $C = 3$ therefore our diagram will reduced to.



Here it will be 3μ Because diagram shows at "C" server and after "C" servers "μ" value will be equal to "$C\mu$"

NOW, DERIVATIONS FOR P_n:

At State ⓪:

In flows = out-flows.

$$P_1 \mu = P_0 \lambda$$

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$$\Rightarrow P_1 = P_0 \frac{\lambda}{\mu}$$

$$\because \rho = \frac{\lambda}{c\mu}$$

$$\therefore c\rho = \frac{\lambda}{\mu}$$

$$\therefore \boxed{P_1 = P_0 \, c\rho}$$

But in our case $c=3$

$$\therefore \boxed{P_1 = P_0 \, 3\rho}$$

At state 1:

Inflows = outflows.

$$P_0\lambda + P_2(2\mu) = P_1\lambda + P_1\mu$$

$$P_2(2\mu) = P_1\lambda + P_1\mu - P_0\lambda.$$

putting value of P_1

$$P_2(2\mu) = \lambda P_0 \, 3\rho + \mu \, P_0 3\rho - P_0\lambda.$$

$$P_2(2\mu) = P_0\left[\lambda 3\rho + \mu 3\rho - \lambda\right]$$

putting value of $\rho = \frac{\lambda}{3\mu}$

$$P_2(2\mu) = P_0\left[\lambda . \cancel{3} . \frac{\lambda}{\cancel{3}\mu} + \cancel{\mu 3} . \frac{\lambda}{\cancel{3\mu}} - \lambda\right]$$

$$= P_0\left[\frac{\lambda^2}{\mu} + \cancel{\lambda} - \cancel{\lambda}\right]$$

$$P_2 = \frac{P_0 \cdot \lambda^2}{2\mu \cdot \mu}$$

$$P_2 = \frac{P_0 \, \lambda^2}{2\mu^2}$$

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$$P_2 = \frac{P_0}{2}\left(\frac{\lambda}{\mu}\right)^2 \qquad \because \frac{\lambda}{\mu} = c\rho$$

$$\therefore \boxed{P_2 = \frac{P_0}{2} c^2\rho^2}$$

At state 2:

Inflows = Outflows.

$$P_3(3\mu) + P_1\lambda = P_2\lambda + P_2(2\mu).$$

putting values of P_1 & P_2

$$P_3(3\mu) + P_0 3\rho \cdot \lambda = \lambda \frac{P_0}{2}(3^2\rho^2) + \frac{P_0}{\cancel{2}}(3^2\rho^2)(\cancel{2}\mu)$$

$$P_3(3\mu) = \lambda \frac{P_0}{2}(3^2\rho^2) + P_0 3^2\rho^2\mu - P_0 3\rho.\lambda$$

$$\because \rho = \frac{\lambda}{3\mu}$$

$$\therefore P_3(3\mu) = \lambda.\frac{P_0}{2}\left(\cancel{3^2}.\frac{\lambda^2}{\cancel{3^2}\mu^2}\right) + P_0.\cancel{3^2}\left(\frac{\lambda^2}{\cancel{3^2}\cancel{\mu^2}}\cancel{\mu}\right) - P_0\cancel{3}\frac{\lambda}{\cancel{3}\mu}\lambda.$$

$$P_3(3\mu) = \frac{P_0}{2}\frac{\lambda^3}{\mu^2} + \cancel{\frac{P_0\lambda^2}{\mu}} - \cancel{\frac{P_0\lambda^2}{\mu}}$$

$$P_3 = \frac{P_0}{2\times 3} \cdot \frac{\lambda^3}{\mu^3}$$

$$P_3 = \frac{P_0}{3\times 2} \cdot \frac{\lambda_3}{\mu_3}$$

$$\boxed{P_3 = \frac{P_0}{6} c^3\rho^3} \quad \text{or} \quad \because \frac{\lambda}{\mu} = c\rho.$$

$$\Rightarrow P_3 = \frac{P_0}{6}(3\rho)^3.$$

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At State 3:

inflows = outflows.

$$P_4(3\mu) + P_2\lambda = P_3\lambda + P_3(3\mu)$$

Putting values of P_2 & P_3.

$$P_4(3\mu) + \frac{P_0}{2}(3\rho)^2\lambda = \frac{P_0}{6}(3\rho)^3\lambda + \frac{P_0}{6}(3\rho)^3(3\mu).$$

$$P_4(3\mu) = \frac{P_0}{6}\left(\cancel{3^3}\cdot\frac{\lambda^3}{\cancel{3^3}\cdot\mu^3}\right)\lambda + \frac{P_0}{\cancel{6}_2}\left(\cancel{3^3}\cdot\frac{\lambda^3}{\cancel{3^3}\mu^{\cancel{3}2}}\right)\cancel{3}\mu\Big) - \frac{P_0}{2}\left(\cancel{3^2}\cdot\frac{\lambda^3}{\cancel{3^2}\mu^2}\right)$$

$$P_4(3\mu) = \frac{P_0}{6}\frac{\lambda^4}{\mu^3} + \cancel{\frac{P_0}{2}\frac{\lambda^3}{\mu^2}} - \cancel{\frac{P_0}{2}\frac{\lambda^3}{\mu^2}}$$

$$P_4 = \frac{P_0}{3\times6}\frac{\lambda^4}{\mu^4}$$

$$P_4 = \frac{P_0}{3\times6}(c\rho)^4$$

$$\Rightarrow \boxed{P_4 = \frac{P_0}{3\times6}(c\rho)^4}$$

→ This can be written as.
$c^{n-c}\cdot c!$ Because in this case
$n=4 \therefore (3)^{4-3} = 3.$

At State 4:

Inflows = outflows.

$$P_5(3\mu) + P_3\lambda = P_4\lambda + P_4(3\mu)$$

putting values of P_3 & P_4

$$P_5(3\mu) + \frac{P_0}{6}(3\rho)^3\lambda = \frac{P_0}{3\times6}(3\rho)^4\lambda + \frac{P_0}{3\times6}(3\rho)^4(3\mu)$$

$$P_5(3\mu) = \frac{P_0}{3\times6}\left(\frac{3^4 \cdot \lambda^4}{3^4 \cdot \mu^4}\right)\lambda + \frac{P_0}{\cancel{3}\times6}\left(\frac{\cancel{3^4}\lambda^4}{\cancel{3^4}\mu^4{}^3}\right)\cancel{3\mu} - \frac{P_0}{6}\left(\frac{\cancel{3^3}\cdot\lambda^3}{\cancel{3^3}\mu^3}\right)\lambda$$

$$P_5(3\mu) = \frac{P_0 \cdot \lambda^5}{3\times6\cdot\mu^4} + \cancel{\frac{P_0}{6}\frac{\lambda^4}{\mu^3}} - \cancel{\frac{P_0}{6}\frac{\lambda^4}{\mu^3}}.$$

$$P_5 = \frac{P_0}{3\times6\times3} \cdot \frac{\lambda^5}{\mu^5}$$

$$\boxed{P_5 = \frac{P_0}{3^2\times6}(ce)^5}$$

$$\Rightarrow \boxed{P_5 = \frac{P_0}{3^2\times 3\times2}(3e)^5}$$

$\rightarrow 3!_0$. (i.e $c!_0$)

Here $n=5$, & $c=3$

$\therefore n-c = 5-3 = 2$.

$\rightarrow c^{n-c}$.

Finding P_n:

Now, Here we can conclude with the above derivations that.

→For $0 \leq n \leq c$ Here in our case $c=3$

we noticed a generic pattern which can be written as.

$$\boxed{P_n = \frac{P_0}{n!_0} \cdot (ce)^n}$$

But.

For $n > c$

we obsewed a different pattern for $n>c$ which we can write in general form as

$$P_n = \frac{P_0}{c^{n-c} \cdot c!} (c^n \rho^n)$$

$$= \frac{P_0 \cdot c^{-n+c} \cdot c^n \rho^n}{c!}$$

$$\boxed{P_n = P_0 \cdot \frac{\rho^n c^c}{c!}}$$

Now, Finding

* L_q: Average Number of Customers in the queue $\left(L_q = \sum_{n=0}^{\infty} n P_{n+c} \right)$

$$L_q = \sum_{n=0}^{\infty} n \; P_{n+c}$$

∵ Here it is an indication of $(n+c)$ i.e $n > c$.

$$\therefore \quad L_q = \sum_{n=0}^{\infty} n \cdot P_0 \cdot \frac{\rho^{n+c} c^c}{c!}$$

$$= \frac{c^c \cdot P_0}{c!} \sum_{n=0}^{\infty} n \; \rho^{n+c}$$

$$L_q = \frac{c^c \; P_0}{c!} \sum_{n=0}^{\infty} n \; \rho^n \cdot \rho^c$$

$$L_q = \frac{(c\rho)^c \cdot P_0}{c!} \sum_{n=0}^{\infty} n \rho^n$$

Now, using Lecture concept

$\because \frac{d}{dx} x^n = n x^{n+1}$.

$\therefore L_q = \frac{(c\rho)^c . P_0 \rho}{c!} . \frac{d}{d\rho} \left(\sum_{n=0}^{\infty} \rho^n \right)$

$L_q = \frac{(c\rho)^c}{c!} P_0 . \rho \frac{d}{d\rho} \left(\frac{\rho}{1-\rho} \right)$

$= \frac{(c\rho)^c}{c!} P_0 . \rho \left(\frac{1 - \cancel{\rho} + \cancel{\rho}}{(1-\rho)^2} \right)$

$$\boxed{L_q = \frac{(c\rho)^c}{c!} P_0 \frac{\rho}{(1-\rho)^2}}$$

*** L: Average Number of customers in the System.** $\left(L = E[N] = \sum_{n=1}^{\infty} n P_n \right)$.

As we know that

$L = \lambda w$.

$\because w = w_q + \frac{1}{\mu}$.

$\therefore L = \lambda \left[w_q + \frac{1}{\mu} \right]$

$L = \lambda w_q + \frac{\lambda}{\mu}$

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$\because \lambda w_q = L_q$.

& $\dfrac{\lambda}{\mu} = c\rho$

$\therefore \quad L = L_q + c\rho$

Putting the value of L_q.
in above eq. we get.

$$\boxed{L = \frac{(c\rho)^c}{c!} P_o \frac{\rho}{(1-\rho)^2} + c\rho}$$

*** L_s: Average No. of Customers in the Server:**

$\because L_s = L - L_q$

$$L_s = \cancel{\left[\frac{(c\rho)^c P_o}{c!} \frac{\rho}{(1-\rho)^2}\right]} + c\rho - \cancel{\left[\frac{(c\rho)^c P_o}{c!} \frac{\rho}{(1-\rho)^2}\right]}$$

$$\boxed{L_s = c\rho}$$

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* Probability of Queuing. $\left(P[N \geq c] = \sum_{n=c}^{\infty} P_n\right)$

$$P[N \geq c] = \sum_{n=c}^{\infty} P_n. \quad \text{--- (i)}$$

As we can see that we have derived the equation of P_n (for $n \geq c$)

is $P_n = P_o \dfrac{\rho^n c^c}{c!}$

So we will put this value in eq (i)

$$P[N \geq c] = \sum_{n=c}^{\infty} \cdot P_o \frac{\rho^n c^c}{c!}$$

$$= P_o \cdot \frac{c^c}{c!} \sum_{n=c}^{\infty} \rho^n$$

$$= P_o \cdot \frac{c^c}{c!} \left[\rho^c + \rho^{c+1} + \rho^{c+2} + \cdots\cdots \infty\right]$$

$$= P_o \frac{c^c}{c!} \cdot \left(\frac{\rho^c}{1-\rho}\right)$$

$$\boxed{P[N \geq c] = \frac{(c\rho)^c}{c!\,(1-\rho)} P_o}$$