CS-417
COMPUTER SYSTEMS MODELING
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(LECTURE # 11)

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Recap of Lecture # 10

Reliability
Mathematical Expression of Reliability
Hazard Rate
Mortality Curve

Chapter # 4 (Cont'd)

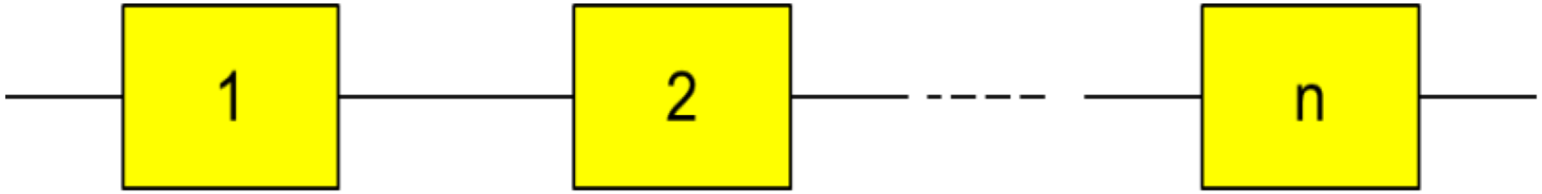
RELIABILITY AND AVAILABILITY MODELING

RELIABILITY BLOCK DIAGRAMS (RBD)

1) Series Systems

- When *every* module (block) in the system must be operational for the entire system to be functional, the blocks are said to be in series interconnection.

- E.g. processor, memory and system bus form a series configuration in a computer system.



Let us define an event E_k = block *k* is operational.

Then, reliability of block *k* is $R_k = P(E_k)$. Also,

P[system is working] = P[all modules working] = $P[E_1 \cap E_2 \cap \cdots \cap E_n]$

Since block failures are independent, therefore, reliability of a series system is given by,

$$R_s = P[E_1]P[E_2]\cdots P[E_n] = R_1 R_2 \cdots R_n$$

$$R_s = \prod_{i=1}^{n} R_i$$

For homogeneous modules (i.e. identical reliability),

$$R_s = R^n$$

Remarks

- **Effect of Component Reliability in a Series System**

In a series configuration, the component with the least reliability has the biggest effect on the system's reliability.

Clearly,

$$R_s < min(R_1, R_2, \cdots, R_n)$$

- **Effect of Number of Components in a Series System**

The number of components is another concern in systems with components connected reliability-wise in series.

As the number of components connected in series increases, the system's reliability decreases.

Example 1

A module of a satellite monitoring system has 500 components in series. The reliability of each component is 0.999.

- Find the reliability of the module.
- If the number of components is reduced to 200, what is the reliability?

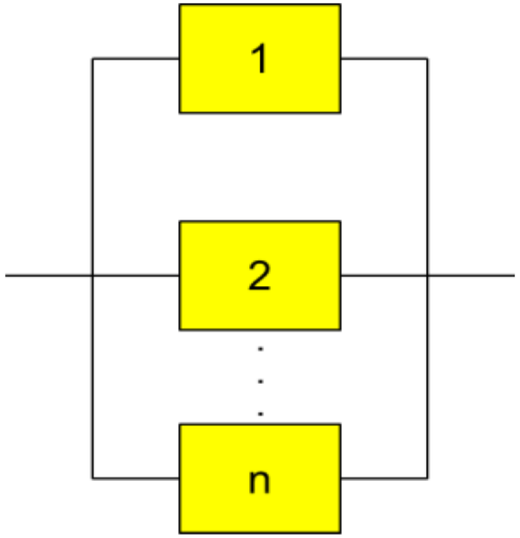
Answers:

- 0.60637
- 0.81864

RELIABILITY BLOCK DIAGRAMS (RBD)

2) Parallel System

- A parallel system is a kind of configuration wherein functioning of *at least one* system block is sufficient for the entire system to operate correctly.
- Application: where a high degree of operation reliability is required to avoid any kind of human, economic, or data loss.



In order to derive an expression for reliability of a parallel system, we observe that

$$\text{P[System failing]} = \text{P[all modules failing]} = P[\overline{E_1} \cap \overline{E_2} \cap \cdots \cap \overline{E_n}]$$

Since block failures are independent, therefore,

$$1 - \text{P[system working]} = P[\overline{E1}]P[\overline{E2}] \cdots P[\overline{En}]$$

$$\Rightarrow \quad 1 - R_P = (1 - R_1)(1 - R_2) \ldots (1 - R_n)$$

$$\Rightarrow \quad R_P = 1 - \prod_{i=1}^{n} (1 - R_i)$$

For homogeneous modules (i.e. identical reliability)

$$R_P = 1 - (1 - R)^n$$

Reliability of a parallel system increases with the increase in number of modules.

Remarks

- **Effect of Component Reliability in a Parallel Configuration**

The component with the highest reliability in a parallel configuration has the biggest effect on the system's reliability, since the most reliable component is the one that will most likely fail last.

- **Effect of Number of Components in a Parallel System**

For a parallel configuration, as the number of components/subsystems increases, the system's reliability increases.

Example 2

A system has three parallel components, A, B, and C with reliabilities 0.95, 0.92, and 0.90, respectively.

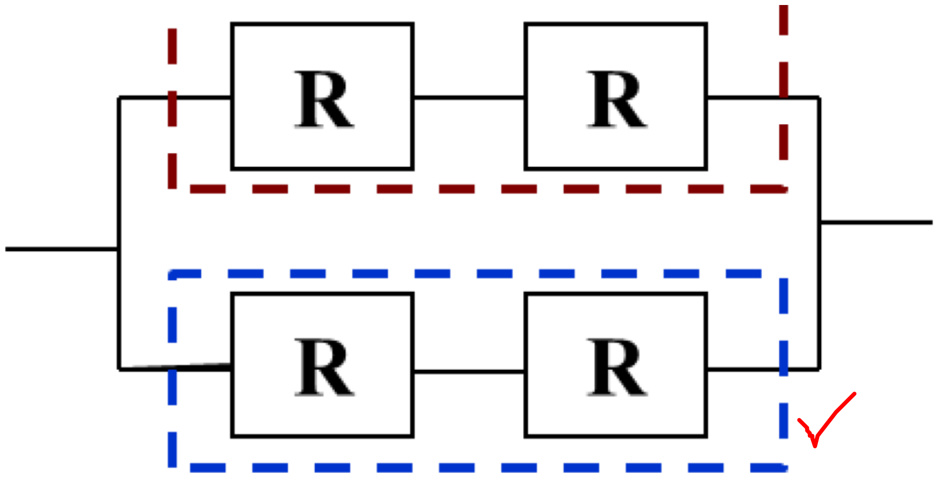
- Find the reliability of the system.
- Determine the reliability if Component C gets out of order.

Answers:

- 0.9996
- 0.996

3) SERIES-PARALLEL SYSTEM

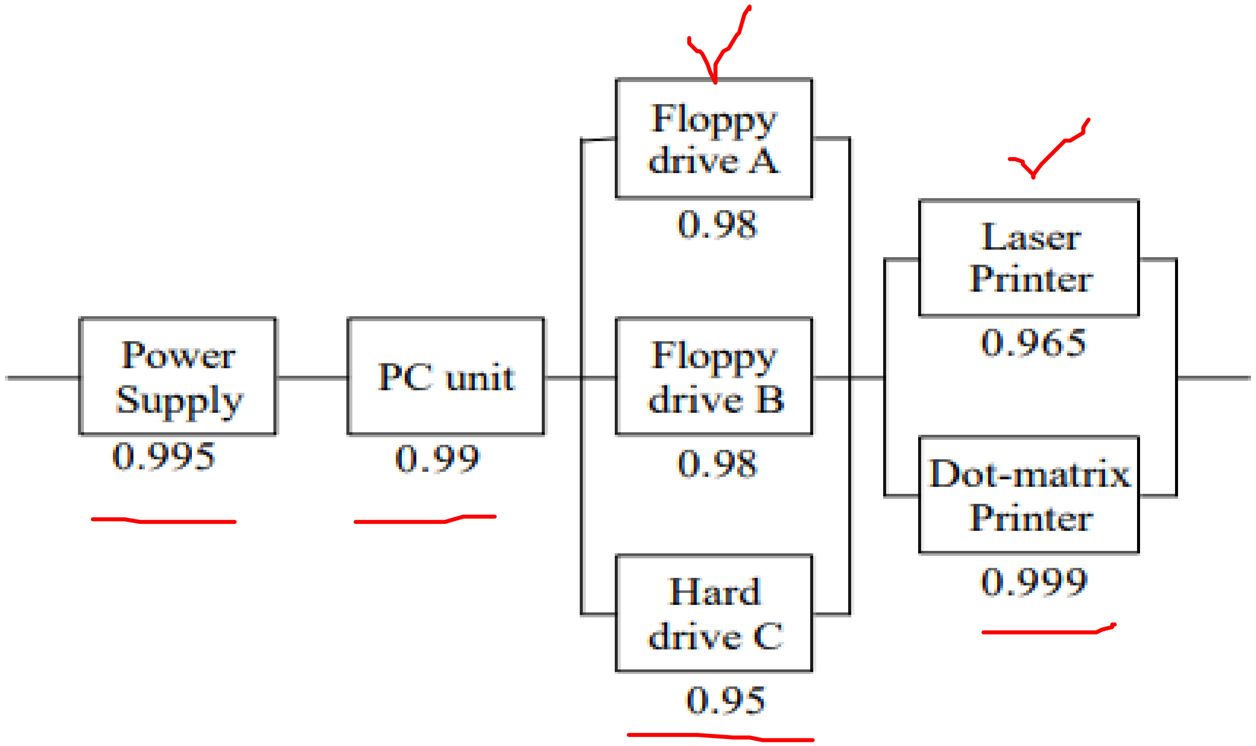
Many systems use a mix of series and parallel configurations as exemplified below:



$$R_{ov} = 1 - (1 - R * R)(1 - R * R)$$
$$= 1 - (1 - R^2)^2$$

Example 3

Consider the given series-parallel system & determine the overall reliability.



Answer:

$R_{system} = 0.984995$

4) *K-OUT-OF-N* SYSTEM

- *k* out of *n* components need to be functional for the system to be functional.

- Please note that parallel (*k* = 1) and series (*k* = *n*) systems are special cases of *k*-out-of-*n* system.

- The reliability of such a system is given by binomial distribution:

$$R_{n|k} = \sum_{i=k}^{n} \binom{n}{i} R^{i} (1-R)^{n-i}$$

Example 4

Consider a system of 6 pumps of which at least 4 must function properly for system success. Each pump has an 85% reliability for the mission duration.

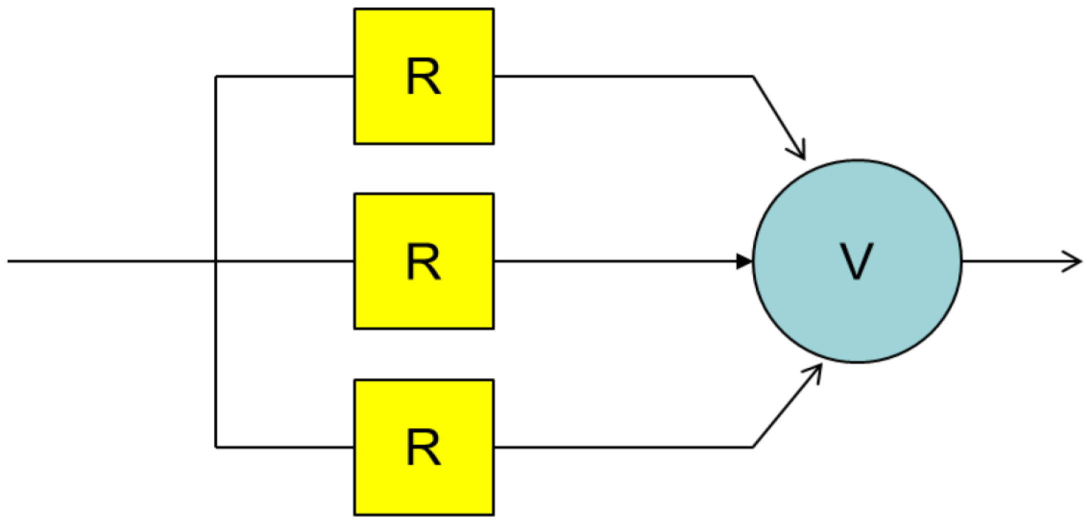
What is the probability of success of the system for the same mission duration?

Answer:

$R_{6|4} = 0.9546$

TRIPLE MODULAR REDUNDANCY (TMR)

- A TMR system, also known as a triplex system and a special case of *k*-out-of-*n* system ($k = 2$, $n = 3$) is illustrated in the following diagram.



- The 'V' block is a majority voter which produces correct output as long as 2 modules are working correctly. Such TMR systems are very common across many scientific disciplines.

$$R_{TMR} = \sum_{i=2}^{3} \binom{3}{i} R^i (1-R)^{3-i}$$

$$= \binom{3}{2} R^2 (1-R)^{3-2} + \binom{3}{3} R^3 (1-R)^{3-3}$$

$$= \frac{3!}{(3-2)! * 2!} R^2 (1 - R) + R^3$$

$$= \frac{3!}{2!} R^2 (1 - R) + R^3$$

$$= 3R^2 - 3R^3 + R^3$$

$$= 3R^2 - 2R^3$$

$$R_{TMR} \begin{cases} > R & \text{if } R > 1/2 \\ = R & \text{if } R = 1/2 \\ < R & \text{if } R < 1/2 \end{cases}$$

Task

Q.1) Three subsystems are reliability-wise in series and make up a system. Subsystem 1 has a reliability of 99.5%, subsystem 2 has a reliability of 98.7% and subsystem 3 has a reliability of 97.3% for a mission of 100 hours.

- What is the overall reliability of the system for a 100-hour mission?
- Now consider that these three sub-systems are arranged in parallel configuration. Compute the overall reliability of the system.

Task

Q.2) Consider a system with three components. Units 1 and 2 are connected in series and Unit 3 is connected in parallel with the first two.

- What is the reliability of the system if R_1 = 99.5%, R_2 = 98.7% and R_3 = 97.3% ?