CS-417
COMPUTER SYSTEMS MODELING

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(LECTURE # 10)

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Recap of Lecture # 9

Conditional Probability
Rth Percentile Value
Problems related to Exponential Distribution
Bayes’ Theorem

Chapter # 4

RELIABILITY AND AVAILABILITY MODELING

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RELIABILITY

Reliability R(t) of a system is defined as the probability that the system will survive till time t.

Hence, if *T* is a random variable denoting system's lifetime, then

$$R(t)=P[T>t]=1-F_T(t)$$

It should be noted that:

- $\mathrm{R}(0)=1$ (i.e. a system is expected to be operational when it's initially put into operation)
- $\lim_{t\to\infty} R(t)=0$ (i.e. nothing can operate forever)

MATHMATICAL EXPRESSION OF RELIABILITY

Let,

- N_0 = number of identical components under test at t = 0
- $N_s(t)$ = number of components which survived till time t
- $N_f(t)$ = number of components which failed till time t

Clearly,

$$N_s(t)+N_f(t)= N_0$$

Then, using fundamental definitions of reliability and probability, we get,

$$R(t) = \frac{N_s(t)}{N_0} = 1 - \frac{N_f(t)}{N_0}$$

Taking first derivative with respect to time,

$$R'(t) = -N'_f(t)/N_0$$

where, $N'_f(t)$ represents failure rate of components.

Recall the basic definition of reliability,

$$R(t) = 1 - F_T(t)$$

Now, taking first derivative on both sides of with respect to time, we get,

$$R'(t) = -f_X(t)$$

Reliability includes:

- *correctness* (ensuring the system services are as specified),
- *precision* (ensuring information is delivered at an appropriate level of detail), and
- *timeliness* (ensuring that information is delivered when it is required).

HAZARD RATE

$P(a < x \le b) = F_X(b) - F_X(a)$

Let us now calculate the conditional probability that the system will not survive an additional time duration *x*, given that it has already survived till time *t*.

$$P[T \le t + x \mid T > t] = \frac{P[\overset{a}{t} < T \le \overset{b}{t+x}]}{P[T > t]} = \frac{F_X(t+x) - F_X(t)}{R(t)}$$

If we divide this probability by *x* and the interval *x* is shrunk to zero ($x \to 0$), we get the instantaneous failure rate or hazard rate $h(t)$:

$$h(t) = \lim_{x\to 0} \frac{F_X(t+x) - F_X(t)}{xR(t)} = 1 - R(t+x) - 1 + R(t)$$

$$= \frac{1}{R(t)} \boxed{\lim_{x\to 0} \frac{R(t) - R(t+x)}{x}}$$

$$h(t) = -\frac{R'(t)}{R(t)} = \frac{f_X(t)}{R(t)}$$

Calculate h(t) = ?

If X ¬ EXP (λ)

$$h(t) = \frac{f_x(t)}{R(t)}$$

$$h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

h(t) = λ

i.e. the constant failure rate

The *cumulative hazard* H(*t*) is given as:

$$H(t) = \int_0^t h(x)dx$$

$$= -\int_0^t \frac{R'(x)}{R(x)}dx$$

$$= -[lnR(x)]_0^t$$

$$= -lnR(t)$$

This gives

$$R(t) = e^{-H(t)}$$

If $T \sim \text{EXP}(\lambda)$, then

$$R(t) = e^{-\lambda t} \qquad R(t) = e^{-H}$$

$$h(t) = \lambda$$

$$H(t) = \lambda t$$

Clearly, the hazard rate for an exponentially distributed lifetime is constant.

Task

The hazard rate of a certain component is given by:

$$h(t) = \frac{e^{t/4}}{5}$$

1) What are the cumulative hazard function and the reliability function of this component?
2) What is the probability that it survives until t = 2.

Answers

1) H(t) = $\frac{4}{5}\left(e^{t/4} - 1\right)$

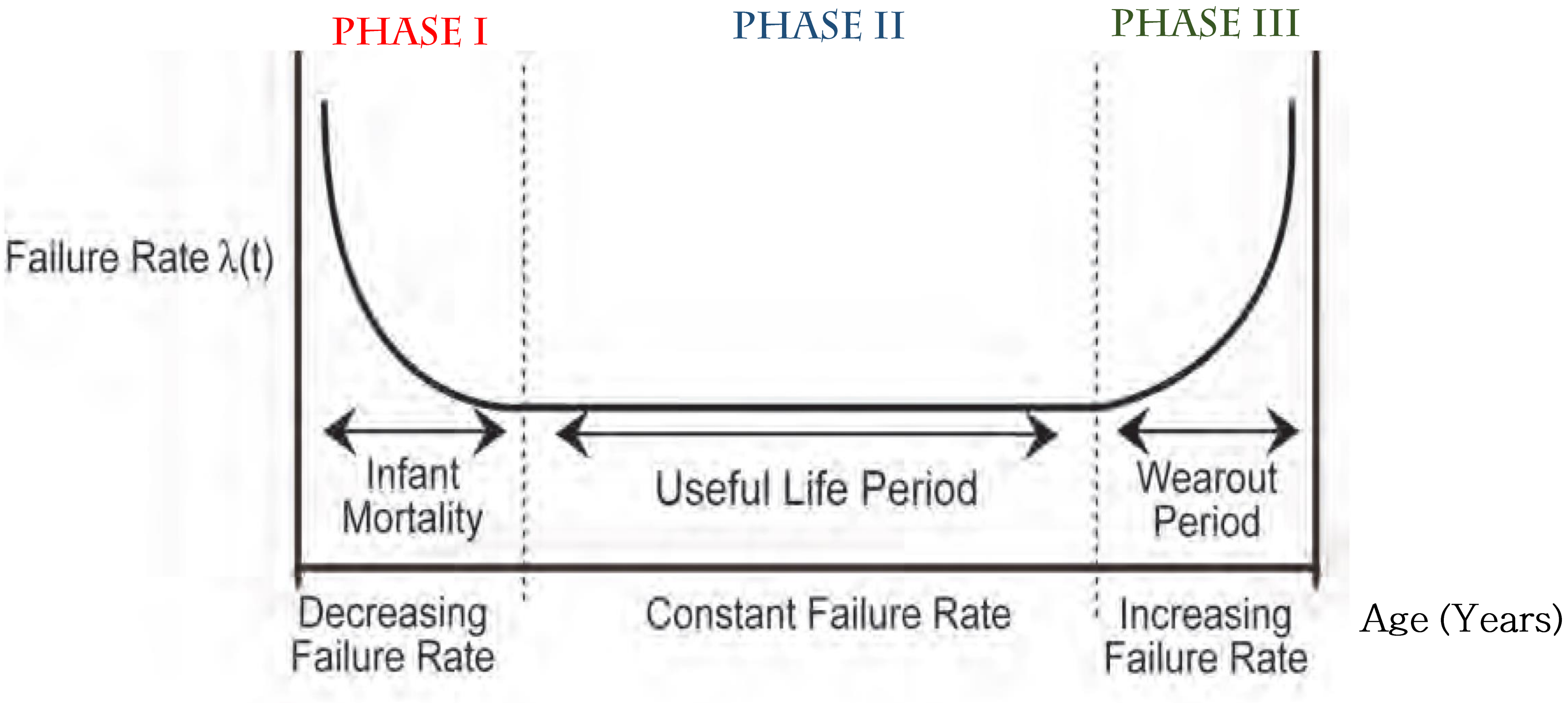
2) R(t) = $e^{-\frac{4}{5}\left(e^{t/4}-1\right)}$

3) R(2) = 0.9591

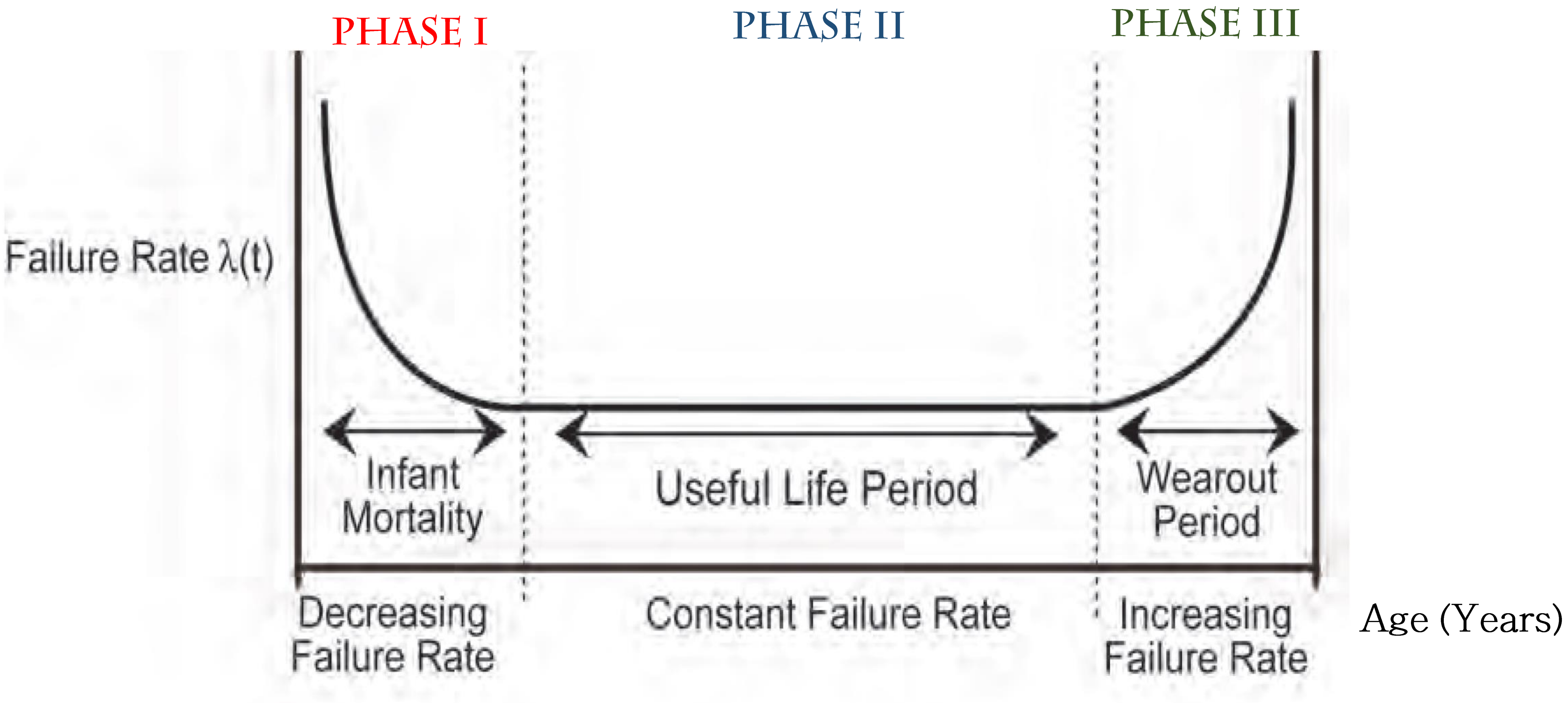
Practice Problem

The failure rate of a certain component is $h(t) = \lambda_0 t$ where $\lambda_0 > 0$ is a constant. Determine the reliability $R(t)$ of the component.

MORTALITY CURVE



MORTALITY CURVE



Simplicity is prerequisite for reliability!!

-Edsger Dijkstra