Lect 7

Chapter # 3

REVIEW OF PROBABILITY THEORY

Definitions

1) Random Experiment

Also called probabilistic or non deterministic experiment

A random experiment is a process whose outcome is not known in advance but for which the set of all possible individual outcomes is known.

For example a random experiment could be a ready process waiting for processor to become availabe for its execution. Since the waiting time is random here we can say that it is a complete random experiment.

Another example could be realizing of packets for there destination as it could be one of many nodes in the network again the analysis of packets can be consider as a random experiment.

another example could be lets suppose we are examining lot of components for a faulty one. Lets suppose we have got 100 components in a single machine & there's some fault in the machine & i want to examine that particular machine for the faulty component again it's a totally random experiment cause any component can be faulty.

2) Trial

Single performance of a random experiment is called a trial.

For example i want to analyze one packet for its destination this is trial.

OR i want to examine a single component & i want to check whether it is faulty or not.

3) Sample Space

The set of all possible outcomes of a random experiment is called its sample space, usually denoted by S.
clearly sample space depends on how the random experiment is defined. A Random experiment could be anything lets suppose the random experiment is defined as the number of polls required to pull 7 terminals on a communication line. Lets say there are 7 terminals & we are numbering them as 1 2 3 4 5 6 7. They are all in sequence & I want to perform some experiment lets suppose I want to see whether anyone of them is ready to transmit. So what could be the sample space?
The sample space will be from 1 till 7.
Another example of sample space could be Lets suppose i want to toss the coin & the set of all the possible outcome can be its sample space.

4) Event

An event is a subset of sample space.
For example lets consider the random experiment of polling 7 terminal. we can define an event from that particular sample space. Lets say event is maximum number of polls required is 4.
Another example: I have toss 3 coins & i want to define an event that maximum number of heads that are achieved is 3 heads.
so this is actually an event & It's a subset of the complete sample space.

5) Probability

Probability of an event A, denoted P(A), is defined as:
P(A)= |A| / |S|

lets suppose event A is maximum number of poles required is 4 & the sample space of polling is of cardianlity 7
then the probability of event A is calculated as 4/7.

6) Independent Trials

If an experiment involves a sequence of independent but identical stages, we say that we have a sequence of independent trials.
Independent means that the result of one trial or the result of the single performance of random experiment is totally independent of the results of the other experiments.

7) Bernoulli Trials

In the special case where there are only two possible outcomes in each trial, we say that we have a sequence of independent Bernoulli trials.
Examples are checking the condition of if clause in a high level language
or lets suppose there's a binary communicating channel & it is transmitting only two symbols 0 or 1.

8) Random Variable

A random variable X is defined as:
$X: S \rightarrow R$
random variable is a function or some sample space S & the property is that this function assigns a real value to each member of S.
Random variable can be discrete or continous depending upon the nature of range of its value
Discrete random variable assumes finite or countably infinite values,

In tossing of a coin the outcomes are purely discrete. lets say i want to toss two coins together there can be 4 possibilties whereas, a *continuous random variable* can assume infinite values.

i want to know the weight of person & let say the range of the weight is 150 pounds till 180 pounds.

then there will be infinite values in between

one possibility could   150 pounds or 160 pounds or 165 pounds.

Another possibility could 169.1111 pounds.

so continous random variable can assume infinite values.

<u>Example:</u>

Consider two Bernoulli trials.

I have considered the two level cache system. In first level again we have caught the two possibilities either it is going to be a cache hit or chache miss & on the second level again we have caught two possibilities a cache hit or a cache miss. So in total there are 4 possibilities in this case.

Then, S = {HH, HM, MH, MM}.

Let’s define a random variable X giving number of hits in the two trials. X is defining total no. of hits in two trials

s	P (s)	X(s)
HH	¼	2
HM	¼	1
MH	¼	1
MM	¼	0

9) Probability Mass Function

It's also reffered to as Discrete Density function & can be defined as

Probability mass function (pmf) of a discrete random variable X is defined as:

$$p_X(x) \stackrel{\text{def}}{=} P[X = x] = P[s \mid X(s) = x] = \sum_{X(s)=x} P(s)$$

as the event X=x defines that an event contains all those outcomes which maps to a real number x.

From the basic axioms of probability,

$$0 \leq p(x) \leq 1$$

$$\sum_x p(x) = 1$$

we already know that probability value lies in the range of 0-1 & the sum of all probabilities must be equal to 1.

sum of all probability means whether it is a success probability or a it is a failure probability, It is always going to be 1.

Continuing with the example mentioned under random variable, what will be the pmf of X? i.e. p(0), p(1) and p(2)?

s	P (s)	X(s)
HH	¼	2
HM	¼	1
MH	¼	1
MM	¼	0

X was given us the number of hits in two trials.

So the value of p(0) i.e. the 0 hits is going to be 1/4 (see in the table when there are no hits).

value of p(1) i.e. atleast 1 hits.

see in the table when there is 1 hit. (HM & MH)

p(1) will be calculated by adding 1/4 + 1/4 as both has 1 hit.

& the value of p(2) i.e. the 2 hits is going to be 1/4 (see in the table when there are 2 hits). HH

- p(0) = 1⁄4

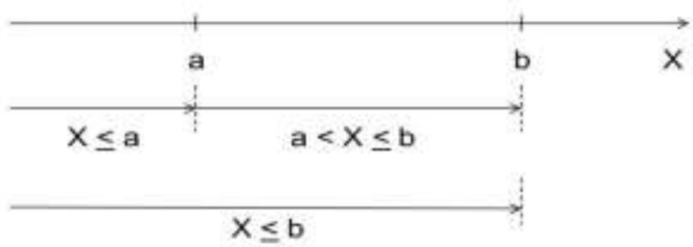
• P(1) = 1⁄2
• P(2) = 1⁄4
sum of all these probabilities must be 1
i.e. 1/4 + 1/2 + 1/4 =1

10) Probability Distribution Function

Probability distribution function $F_X(t)$, also known as cumulative distribution function (CDF) of a discrete random variable X is defined as:

$$F_X(t) \stackrel{\text{def}}{=} P(X \le t) = \sum_{x=-\infty}^{t} p_X(x)$$

It gives the probability of X acquiring a value less than or equal to ‘t’ in its range.



$$P(a < X \le b) = P(X \le b) - P(X \le a)$$
$$= F_X(b) - F_X(a)$$

Previously we have discussed about the pmf, where i actually want to calulate the probability of exactly one event let say a.
probability distribution function said
let say i want to calculate the probability when x<= a ; it means i want to calculate all the probabilities before the point a (i need to sum up all the probabilities before this particular point a).
let suppose i need to calculate the probabilities in a given range & the range lies between a & b where X is a random variable whose value is less than & equal to b & greater than a.
By means of cumulative distribution function i can calculate this

value in a particular range.
understanding it by an example
lets suppose there's a lab network consisting of 100 computers & they all are attacked by a computer virus.
this virus infects each computer with some probability p & is independant of other computers.
first of all i need to find the probability that infected atmost 10 computers.
So here atmost 10 computers means at maximum the virus has hit 10 pcs & we need to calculate all the probability & sum of all the values that whether it has
that whats the probability that it has infected just one computer then we need to compute the probability that it has infected just two computers
till the value 10 (10 computers).
It can be determine by means of CDF.
sometimes we need to compute the probability that the virus has infected atleast 10 computers.
sometime we need to calculate the probability that the virus has infected 10 to 20 computers.
In order to calculate the probabability of random variable, we need to acquire a value less than or equal to or greater than some number 't' in its range then we can use CDF or Probability Distribution function.

<u>Commonly used Discrete Probability Distributions</u>

1) Binomial Distribution

• Consider n independent Bernoulli trials.
• Let p be the probability of success on each trial and q be the probability of failure on each trial. Hence, q = 1 – p.

• As trials are independent, the probability of success will remain constant troughout the experiment So the probability of one sequence of trials with k successes and (n – k) failures is $p^k q^{n-k}$.
where k is the number of successes & n-k is the are remaing sequences with the probability of failure.
• with the counting principle, The number of such sequences of trials with k successes is

$$\binom{n}{k}$$

• If X denotes a random variable indicating the number of successes in n trials, then its pmf is:

$$p_X(k) = P[X = k]$$

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad 0 \le k \le n$$

Here it is important to know that we are intereted in the calculation of number of seccesses.
So the formula is simple but the thing is that after seeing any problem you should be able to identify whether the binomial distribution is applicable on it or not.
The most imp thing is that we will be calculating all the successes in ...
also one imp thing is that some of the experiment can be converted in to binomial experiment. For example if we talk about rolling a die there are 6 possibilities in total but we can convert them into a binomial distribution
like let suppose that we say that we are interested in two outcome like whether we are getting an odd number or we are getting an even number.
In this way an experiment having 6 possibilities or 6 outcomes is converted into an experiment having only 2 results or 2

outcomes.

2) Geometric Distribution

• Consider a sequence of independent Bernoulli trials.

• Let Y be a random variable denoting the number of trials until the first success.

• Hence, pmf of Y is given by:

$$p_Y(k) = P[Y = k]$$

Actually we are interested to find out the probability of first success on k^{th} trial that is, (k – 1) successive failures & then there will be a success. The formula is given by,

• This gives the probability of first success on k^{th} trial. That is, (k – 1) successive failures followed by a success. Thus,

$$p_Y(k) = P[Y = k] = pq^{k-1} = p(1-p)^{k-1} \qquad , k = 1, 2, 3, \ldots$$

This distribution actually finds various applications than waiting time or queing query. Lets suppose student is appearing in exam repeatdly until he gets his first success.

Here we need to understand the difference between binomial & geometric distribution.

It is that in binomial, the number of trials are fixed & we actually counts the number of seccesses

while in geometric distribution, we are counting the number of trials till the first success.

For example the student is appearing in exam & he passes that exam, the probability of passing than particular exam is let say 70%. we need to find out the probability that the student will pass the exam on 4th attempt. So in these types of problems we will apply the geometric distribution.

3) Modified Geometric Distribution

• Consider a sequence of independent Bernoulli trials.

• Let Z be a random variable denoting the number of trials before the first success. Hence, pmf of Z is given by:

$$p_Z(k) = P[Z = k]$$

• This gives the probability of first success on trial number (k + 1).

• That is, k successive failures followed by a success. Thus,

$$p_Z(k) = P[Z = k] = pq^k = p(1-p)^k \qquad , k = 0, 1, 2, 3, \ldots$$

It means that in modified geometric distribution, we want to calculate the number of trials before the first success.
In case of Geometric distribution, we were calculating the number of trials till the first success
but in case of modified geometric distributions, we wont be calculating the or we wont be adding the trial at which we are getting the success.

4) Negative Binomial Distribution

• Consider a sequence of independent Bernoulli trials.

• Let N be a random variable denoting the number of trials until the k^{th} success.

here it is interesting to note that we want to achieve fixed success; the success is fixed & the trials will be variable.
We are interested to calculate the number of trials until some specific successes achieved.

• Let p be the probability of success on each trial and q be the probability of failure on each trial. Hence, q = 1 – p.

• If k^{th} success is achieved on n^{th} trial, then (k – 1) successes must have been scattered in (n – 1) trials. Hence, pmf of N is:

$$p_N(n) = \binom{n-1}{k-1} p^k (1-p)^{n-k}, \qquad n = k, k+1, k+2, \cdots$$

Here it is also interesting to note that if we put the value of k=1 then we can also conclude that the Geometric distribution is the special case of Negative Binomial distribution. Because In Geometric distribution we are interested in only one success i.e. (required in let say) trials.

Tasks

Q.1)Attempts to access a website fail with the probability of 0.10. If all trials are independent, calculate the probability that website will become accessible:

- In 3rd attempt.
- Before 3rd attempt.

first identify probability distribution to be applied here & then use the correct formula to calculate result

Q.2) A communication channel receives independent pulses at the rate of 12 pulses per microsecond. The probability of transmission error is 0.001 for each pulse. Compute the probability of:

- one error per microsecond.
- at least one error per microsecond.

Solve it !

Q.3) What kind of probability distributions do the following random variables have?

dont need to solve in Q3 just identify the type probability distribution , the following random variables are possessing

- No. of CPU time slices required to complete a process in a

time sharing environment?
• The no. of times S is executed in the given loop?
while !B
do S
• The no. of times S is executed ?
repeat S until B

Answers to Q.1 & Q.2

Q.1
a) 0.009
b) 0.0009

Q.2 a) 0.01186
b) 0.01194