Lecture 10

- Reliability → $R(t) = P[T>t] = 1 - F_T(t)$
- $R(0) = 1$
- $\lim_{t\to\infty} R(t)=0$
- $N_s(t)+N_f(t)= N_0$
- $R(t) = \frac{N_s(t)}{N_0} = 1 - \frac{N_f(t)}{N_0}$
- $N'f(t)$ → failure rate of components.
- $R'(t) = -f_X(t)$
- Correctness, precision, timeliness
- HAZARD RATE → system is not expected to survive any additional time x after t
- $$P[T \le t + x \mid T > t] = \frac{P[t < T \le t + x]}{P[T > t]} = \frac{F_X(t+x) - F_X(t)}{R(t)}$$
- failure rate or hazard rate h(t) → $h(t) = \lim_{x\to 0} \frac{F_X(t+x) - F_X(t)}{xR(t)}$ = $h(t) = \frac{f_X(t)}{R(t)}$
- Hazard rate → failures/10000 hr → failures in time == failures/10^9 hr.

  If X ¬ EXP (λ)

  $h(t) = \frac{f_X(t)}{R(t)}$

  $h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$
- **$h(t) = \lambda$** → constant failure rate / hazard rate for an exponentially distributed lifetime is constant

  The *cumulative hazard* H(t)

  $$\begin{aligned} H(t) &= \int_0^t h(x)dx \\ &= -\int_0^t \frac{R'(x)}{R(x)} dx \\ &= -[\ln R(x)]_0^t \\ &= -\ln R(t) \end{aligned}$$

  This gives
- $$R(t) = e^{-H(t)}$$
- 2 PP → hazard rate or reliability
- MORTALITY CURVE

Lecture 11

- Reliability block diagrams (RBD) → E_k = block k is operational.
  →reliability of block k is $R_k = P(E_k)$
- Series Systems →$R_s = P[E_1]\, P[E_2] \cdots P[E_n] = R_1 R_2 \cdots R_n$ = = $R_s = \prod_{i=1}^{n} R_i$
  →For homogeneous = identical reliability → $R_s = R^n$
  →$R_s < \min(R_1, R_2, \cdots, R_n)$
  → number of components increases; the system's reliability decreases
  →PP
  →n-out-of-n system

$$1 - R_P = (1 - R_1)(1 - R_2) \ldots (1 - R_n)$$

$$R_P = 1 - \prod_{i=1}^{n}(1 - R_i)$$

- Parallel System →'redundant units' →
  → homogeneous modules = identical reliability → $R_P = 1-(1-R)^n$
  →Effects & PP
  →1-out-of-n system
- SERIES-PARALLEL SYSTEM →PP
- K-OUT-OF-N SYSTEM → $R_{n|k} = \sum_{i=k}^{n} \binom{n}{i} R^i (1-R)^{n-i}$
  →PP
- Triple modular redundancy (TMR) → triplex system →2-out-of-3 system
  →Derivation → $R_{TMR} = 3R^2 - 2R^3$
  →2 PP

Lecture 12

- Task
- System availability → availability & reliability
  → Theoretical Example
  → $$A = \frac{Up\ Time}{Up\ Time + Down\ Time}$$
  $$A = \frac{MTTF}{MTTF + MTTR} = \frac{MTTF}{MTBF}$$
- Unavailability → $$U = 1 - A$$
  $$U = \frac{MTTR}{MTTF + MTTR} = \frac{MTTR}{MTBF}$$
- PP

- Fault, Error, Failure
- Software Reliability vs Hardware Reliability
- Probability of Failure on Demand (POFOD):
- Rate of Occurrence of Failures (ROCOF) → Failure Intensity
- Mean time to failure (MTTF)
- Availability (AVAIL)
- Reliability validation → The reliability measurement process
- statistical testing.
- PP of MTTF & etc.

Lecture 18

- Queuing Theory → inter-arrival times & service time
- Input Source → Population size → mostly infinity
- Queuing Behavior → jockey, balk, renege
- Service time / holding time
- Cost optimization model → total cost = waiting cost + cost of providing service
- Graph
- PP

Lecture 19

- Kendall Notation → A/B/C/X/Y/Z → default; ∞, ∞ and FIFO
- PP
- Job flow balance
- One-step behavior
- Homogeneity
- Exclusivity
- Non-blocking
- Independence
- Terminology and Notation → λ, μ, state of the sys., queue length
- Little's Law → $L = \lambda W$ → Derivation
- Utilization Law → $\rho = \lambda/(s\mu)$; server ; CPU ;etc.
- system's service capacity → $s\mu$
- Operational Analysis & Stochastic Analysis

Lecture 20

- Transient State & Steady State
- Steady State Notations → P_n, $L = \sum_{n=0}^{\infty} nP_n$, $L_q = \sum_{n=s} (n-s)P_n$, W
- **M / m / 1** queue analysis → the birth-and-death process
- mean entering rate = mean leaving rate → balance equation → conversation of flow
- $P_n = \rho^n P_0$ &
- $\therefore P_0 = 1 - \rho$ & reason

Lecture 21

- M/M/1 Analysis → at least k customers in the system; $P[N \geq k] = \rho^k$
  - → expected number of customers in the system,
    $$L = E[N] = \frac{\rho}{1-\rho}$$
  - → expected time spent in the system,
    $$W = \frac{1}{\mu-\lambda} \quad W = \frac{1}{\mu(1-\rho)} = \frac{W_s}{(1-\rho)}$$
  - → mean waiting time,
    $$W_q = W - Ws \quad \& \quad W_q = \frac{L}{\mu}$$
  - → number of customers in the queue,
    $$L_q = \frac{\rho^2}{1-\rho}$$
  - → number of customers in the service facility
    $$L_s = L - L_q \quad \& \quad L_s = \rho$$
- distribution of total time, service time & waiting time
  $W_s(t) = P\{s \leq t\} = 1 - e^{-\mu t} = 1 - e^{-t/Ws}$
  $W(t) = P\{W \leq t\} = 1 - e^{-t/W}$
  $W_q(t) = P\{q \leq t\} = 1 - \rho e^{-t/W}$ (When queue discipline is FCFS)
- PASTA (Poisson Arrivals See Time Averages) Theorem
- 2 PP

Lecture 22

- M/M/1/K System (Finite Buffer Capacity)
  - → $\therefore P_n = \rho^n \frac{1-\rho}{1-\rho^{k+1}}$
  - → $P_0 = \frac{1}{1+k} \Rightarrow P_n = \frac{1}{1+k}$
  - → $L = \frac{\rho}{1-\rho} - (k+1)\frac{\rho^{k+1}}{1-\rho^{k+1}}$ & $L = k/2$ iff $\rho = 1$
  - → Average Number of customers in the Service & in the Queue,
    $L_s = 1 - P_0$ & $L_q = L - L_s$ → $L_q = L - (1 - P_0)$
- Effective Arrival Rate → $\lambda_a = \lambda(1 - \mathbf{P_k})$
  - → $W = \frac{L}{\lambda_a} = \frac{L}{\lambda(1-Pk)}$ & $W_q = \frac{L_q}{\lambda(1-Pk)}$
- Server Utilization (U),
  - → $U = \rho(1 - P_k)$
- 2 PP

Lecture 23

- Queuing models involving nonexponential dist. →mean = $1/\mu$ & variance = σ^2
- The M/G/1 Model,

$$P_0 = 1 - \rho,$$

$$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)},$$

$$L = \rho + L_q,$$

$$W_q = \frac{L_q}{\lambda},$$

$$W = W_q + \frac{1}{\mu}.$$

- Pollaczek-Khintchine formula → above formula of L_q
- service-time distribution is exponential → $\sigma^2 = 1/\mu^2$
- UNIFORM DISTRIBUTION,
  - → The expected value of X is E[X] = **(b+a)/2** **= $1/\mu$**
  - → The variance of X is Var[X] = **(b – a)^2/12** **= σ^2**
  - → PP
- M/D/s Model → fixed constant / degenerate service- time distribution
  - → special case of the *M/G/1* model where $\sigma^2 = 0$,
  - → $L_q = \dfrac{\rho^2}{2(1-\rho)},$
- Erlang-n Distribution → exponential distribution in series
  - → service time = n/μ for n stages
  - → population mean = E[X] = $n\alpha$
  - →variance V [X] = $n\alpha^2$ & α is given by $1/\mu$
- hypo-exponential/generalized Erlang dist. → diff. service rate for every server
- Hyper-Exponential Distribution →exponential distribution in parallel

  In k cumulative k formulae lecture main h

Lecture 24

- QUEUING NETWORKS
- Open queuing networks
- Closed queuing networks
- Jackson's Network → service rate → node-dependent & state-dependent
  - → $\rho < 1$ & service jobs by FCFS
  - →n_i = state of node i & total no. of customers in node i
  - → $p(n) = p(n_1)\, p(n_2) \ldots\, p(n_M)$

- Burke's Theorem →
- Two Stages Tandem Network → rate diagram

  →$P(n_0,n_1) = P(n_0)\, P(n_1)$

  $= \rho_0^{n0}(1 - \rho_0)\, \rho_1^{n1}(1 - \rho_1)$; Jackson theorem on TN

  → For stability, $\rho_0 = \frac{\lambda}{\mu_0} < 1$ & $\rho_1 = \frac{\lambda}{\mu_1} < 1$
- PP remaining formulas of M/M/1 lec: 21
- Open Central Server Networks →

$$\lambda_j = \begin{cases} \frac{\lambda}{p_0}, & j = 0 \\ \frac{p_i}{p_0}\lambda, & j = 1, 2, 3, \dots \dots, m \end{cases}$$

- PP → see W formula

Lecture 27

- Petri nets (PNs) →contention for resources &

  →synchronization b/w concurrent various activities.
- Representation using set notation → M = (P,T,I,O,MP/μ) 7& example
- Dynamic Behavior of Petri-Nets → t ki sb i/p P k pass token hoga tuh hi t will fire
- Dual of a petri-net → Places & Transitions replaced & Example

Lecture 28

- Inverse of Petri Net → switches input functions with output functions.
- Petri Nets as Multi-graph → bold input output lines
- State of a Petri Net → cardinality, μ → marking function

  →Condition/Events nets, Parallelism/Concurrency, Synchronization
- The Bounded Buffer Producer/Consumer Problem

Lecture 29

- Mutual Exclusion (Conflict)
- Inhibitor Arcs, conflict→if t_x fires, Crucial→ t absorbs one t
- Reachability, Reversibility→comes back to μ_0,
- K-bounded Petri-Net → 3-bounded Petri-Net if max no. of tokens is 3
- PP, Deadlock PN
- Properties of PN → liveness, safeness, boundedness, conservation

Lecture 30

- 2 PP
- Matrix analysis → $D = D^+ - D^-$ → incidence matrix, pre-incidence→input matrix

  →$M' = M - e_j D^- + e_j D^+ = \mathbf{M + e_j D}$

  →$\mathbf{M_{fin} = M_1 + (\underline{e_i} + \underline{e_j} + \underline{e_k} + \underline{e_j} + \underline{e_i})\, D}$
- PP
- Timed Petri Nets & The Semantics of the Firing

Lecture 25

- Simulation → Advantages, state variables, event
  → Continuous-Time and Discrete-Time Models
  → Continuous-State and Discrete-State Models
  or continuous event model & discrete event model
  →Open and Closed Models
- Simulation Efficiency Considerations → level of details important
- Types of Simulations → 1) Emulation → enables host PC to behave like guest PC
  → 2) Static (Monte Carlo) Simulation →PP in z lc 25 pdf

Lecture 26

→ 3) Discrete-Event Simulations →

a) An event scheduler

b) Simulation Clock and a Time-advancing Mechanism
   fixed-increment approach & event-increment approach

c) Event processing routines
   Cache-hit & Cache-miss

d) Initialization Routines

e) Event-generation
   Execution Driven
   Trace Driven
   Distribution Driven

f) Recording and summarization of data

→4) Continuous-Event Simulations
   Simulation Algorithm

- Generation of random numbers →
  1. Inverse Transformation method.
  2. Convolution method.
  3. Acceptance-rejection method.

PP → see HW numerical