CS-417
COMPUTER SYSTEMS MODELING
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(LECTURE # 7)

FAKHRA AFTAB
LECTURER
DEPARTMENT OF COMPUTER & INFORMATION SYSTEMS ENGINEERING
NED UNIVERSITY OF ENGINEERING & TECHNOLOGY

Recap of Lecture # 6

Example of Hardware Monitoring
Software Monitoring
Design issues in Software Monitoring
Hybrid Monitoring
Set up of hybrid measurement

Chapter # 3

REVIEW OF PROBABILITY THEORY

Definitions

1) Random Experiment

A *random* experiment is a process whose outcome is not known in advance but for which the set of all possible individual outcomes is known.

2) Trial

Single performance of a random experiment is called a trial.

3) Sample Space

The set of all possible outcomes of a random experiment is called its sample space, usually denoted by S.

Definitions (Cont'd)

4) Event

An *event* is a subset of sample space.

5) Probability

Probability of an event A, denoted P(A), is defined as:

$$P(A)=\frac{|A|}{|S|}$$

6) Independent Trials

If an experiment involves a sequence of independent but identical stages, we say that we have a sequence of independent trials.

Definitions (Cont'd)

7) Bernoulli Trials

In the special case where there are only two possible outcomes in each trial, we say that we have a sequence of independent *Bernoulli* trials.

8) Random Variable

A random variable X is defined as:

$$X: S \rightarrow \mathbb{R}$$

Discrete random variable assumes finite or countably infinite values, whereas, a continuous random variable can assume infinite values.

Definitions (Cont'd)

Example:

Consider two Bernoulli trials.

Then, S = {HH, HM, MH, MM}.

Let's define a random variable X giving number of hits in the two trials.

s	*P (s)*	*X(s)*
HH	¼	2
HM	¼	1
MH	¼	1
MM	¼	0

Definitions (Cont'd)

9) Probability Mass Function

Probability mass function (pmf) of a discrete random variable X is defined as:

$$p_X(x) \stackrel{\text{def}}{=} P[X = x] = P[s \mid X(s) = x] = \sum_{X(s)=x} P(s)$$

From the basic axioms of probability,

$$0 \leq p(x) \leq 1$$

$$\sum_x p(x) = 1$$

Continuing with the example mentioned under random variable, what will be the pmf of X? i.e. p(0), p(1) and p(2)?

- p(0) = ¼
- P(1) = ½
- P(2) = ¼

s	*P (s)*	*X(s)*
HH	¼	2
HM	¼	1
MH	¼	1
MM	¼	0

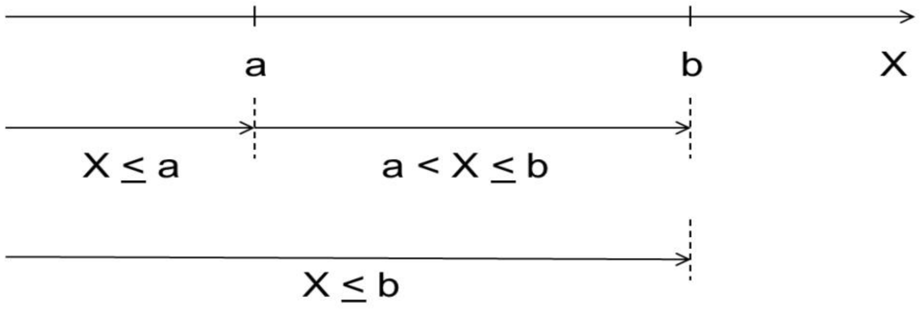
Definitions (Cont’d)

10) Probability Distribution Function

Probability distribution function $F_X(t)$, also known as cumulative distribution function (CDF) of a discrete random variable X is defined as:

$$F_X(t) \stackrel{\text{def}}{=} P(X \le t) = \sum_{x=-\infty}^{t} p_X(x)$$

It gives the probability of X acquiring a value less than or equal to ‘*t*’ in its range.



$$P(a < X \le b) = P(X \le b) - P(X \le a)$$
$$= F_X(b) - F_X(a)$$

Commonly used Discrete Probability Distributions

1) Binomial Distribution

- Consider n independent Bernoulli trials.
- Let p be the probability of success on each trial and q be the probability of failure on each trial. Hence, $q = 1 – p$.
- As trials are independent, the probability of one sequence of trials with k successes and $(n – k)$ failures is $p^k q^{n-k}$.
- The number of such sequences of trials with k successes is $\binom{n}{k}$
- If X denotes a random variable indicating the number of successes in n trials, then its *pmf* is:

$$p_X(k) = P[X = k]$$

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad 0 \le k \le n$$

Commonly used Discrete Probability Distributions (Cont'd)

2) Geometric Distribution

- Consider a sequence of independent Bernoulli trials.
- Let Y be a random variable denoting the number of trials until the first success.
- Hence, pmf of Y is given by:

$$p_Y(k) = P[Y = k]$$

- This gives the probability of first success on kth trial. That is, $(k - 1)$ successive failures followed by a success. Thus,

$$p_Y(k) = P[Y = k] = pq^{k-1} = p(1-p)^{k-1} \qquad , k = 1, 2, 3, \ldots$$

Commonly used Discrete Probability Distributions (Cont'd)

3) Modified Geometric Distribution

- Consider a sequence of independent Bernoulli trials.
- Let Z be a random variable denoting the number of trials before the first success. Hence, pmf of Z is given by:

$$p_Z(k) = P[Z = k]$$

- This gives the probability of first success on trial number (k + 1).
- That is, *k* successive failures followed by a success. Thus,

$$p_Z(k) = P[Z = k] = pq^k = p(1-p)^k \qquad , k = 0, 1, 2, 3, \ldots$$

Commonly used Discrete Probability Distributions (Cont'd)

4) Negative Binomial Distribution

- Consider a sequence of independent Bernoulli trials.
- Let N be a random variable denoting the number of trials until the kth success.
- Let p be the probability of success on each trial and q be the probability of failure on each trial. Hence, $q = 1 - p$.
- If kth success is achieved on nth trial, then $(k - 1)$ successes must have been scattered in $(n - 1)$ trials. Hence, *pmf* of N is:

$$p_N(n) = \binom{n-1}{k-1} p^k (1-p)^{n-k}, \qquad n = k, k+1, k+2, \cdots$$

Tasks

Q.1)Attempts to access a website fail with the probability of 0.10. If all trials are independent, calculate the probability that website will become accessible:

- In 3rd attempt.
- Before 3rd attempt.

Q.2) A communication channel receives independent pulses at the rate of 12 pulses per microsecond. The probability of transmission error is 0.001 for each pulse. Compute the probability of:

- one error per microsecond.
- at least one error per microsecond.

Q.3) What kind of probability distributions do the following random variables have?

- No. of CPU time slices required to complete a process in a time sharing environment?

- The no. of times S is executed in the given loop?

while !B do S

- The no. of times S is executed ?

repeat S until B

Answers to Q.1 & Q.2

Q.1 a) 0.009

b) 0.0009

Q.2 a) 0.01186

b) 0.01194