Part I: Crypto

Chapter 2: Crypto Basics

MXDXBVTZWVMXNSPBQXLIMSCCSGXSCJXBOVQXCJZMOJZCVC
TVWJCZAAXZBCSSCJXBQCJZCOJZCNSPOXBXSBTVWJC
JZDXGXXMOZQMSCSCJXBOVQXCJZMOJZCNSPJZHGXXMOSPLH
JZDXZAAXZBXHCSCJXTCSGXSCJXBOVQX

— plaintext from Lewis Carroll, *Alice in Wonderland*

The solution is by no means so difficult as you might
be led to imagine from the first hasty inspection of the characters.
These characters, as any one might readily guess,
form a cipher — that is to say, they convey a meaning…

— Edgar Allan Poe, *The Gold Bug*

Crypto

- **Cryptology** — The art and science of making and breaking "secret codes"
- **Cryptography** — making "secret codes"
- **Cryptanalysis** — breaking "secret codes"
- **Crypto** — all of the above (and more)

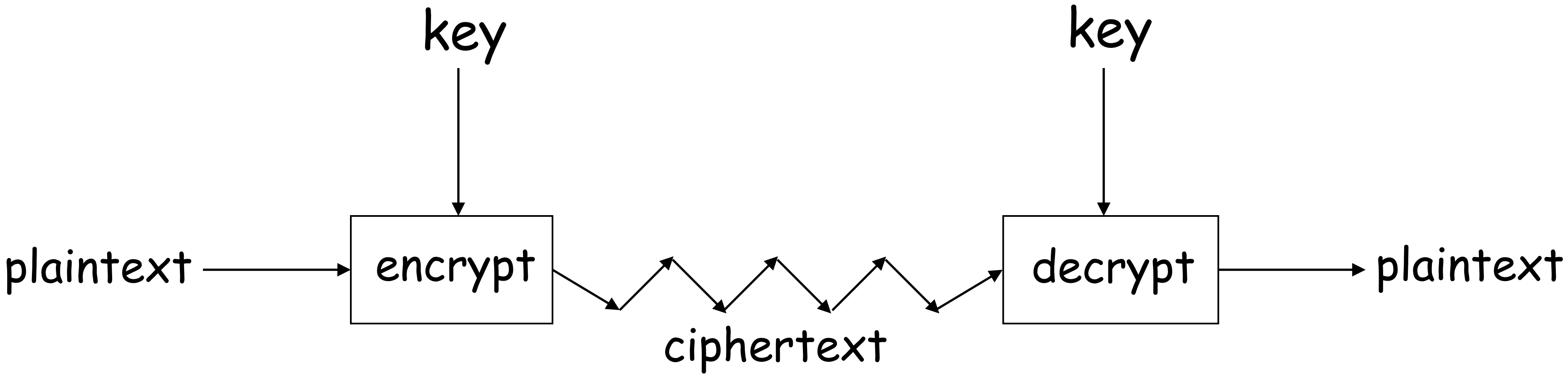
How to Speak Crypto

- A *cipher* or *cryptosystem* is used to *encrypt* the *plaintext*
- The result of encryption is *ciphertext*
- We *decrypt* ciphertext to recover plaintext
- A *key* is used to configure a cryptosystem
- A *symmetric key* cryptosystem uses the same key to encrypt as to decrypt
- A *public key* cryptosystem uses a *public key* to encrypt and a *private key* to decrypt

Crypto

- Basic assumptions
  - The system is completely known to the attacker
  - Only the key is secret
  - That is, crypto algorithms are not secret
- This is known as **Kerckhoffs' Principle**
- Why do we make such an assumption?
  - Experience has shown that secret algorithms tend to be weak when exposed
  - Secret algorithms never remain secret
  - Better to find weaknesses beforehand

Crypto as Black Box



A generic view of symmetric key crypto

Simple Substitution

- Plaintext: fourscoreandsevenyearsago
- Key:

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Ciphertext:
  IRXUVFRUHDQGVHYHQBHDUVDJR
- Shift by 3 is "Caesar's cipher"

Ceasar's Cipher Decryption

- Suppose we know a Caesar's cipher is being used:

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Given ciphertext:
  VSRQJHEREVTXDUHSDQWV
- Plaintext: spongebobsquarepants

Not-so-Simple Substitution

- Shift by n for some n ∈ {0,1,2,…,25}
- Then key is n
- Example: key n = 7

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G

Cryptanalysis I: Try Them All

- A simple substitution (shift by n) is used
    - But the key is unknown
- Given ciphertext: CSYEVIXIVQMREXIH
- How to find the key?
- Only 26 possible keys — try them all!
- **Exhaustive key search**
- Solution: key is n = 4

Simple Substitution: General Case

- In general, simple substitution key can be any **permutation** of letters
  - Not necessarily a shift of the alphabet
- For example

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	J	I	C	A	X	S	E	Y	V	D	K	W	B	Q	T	Z	R	H	F	M	P	N	U	L	G	O

- Then $26! > 2^{88}$ possible keys

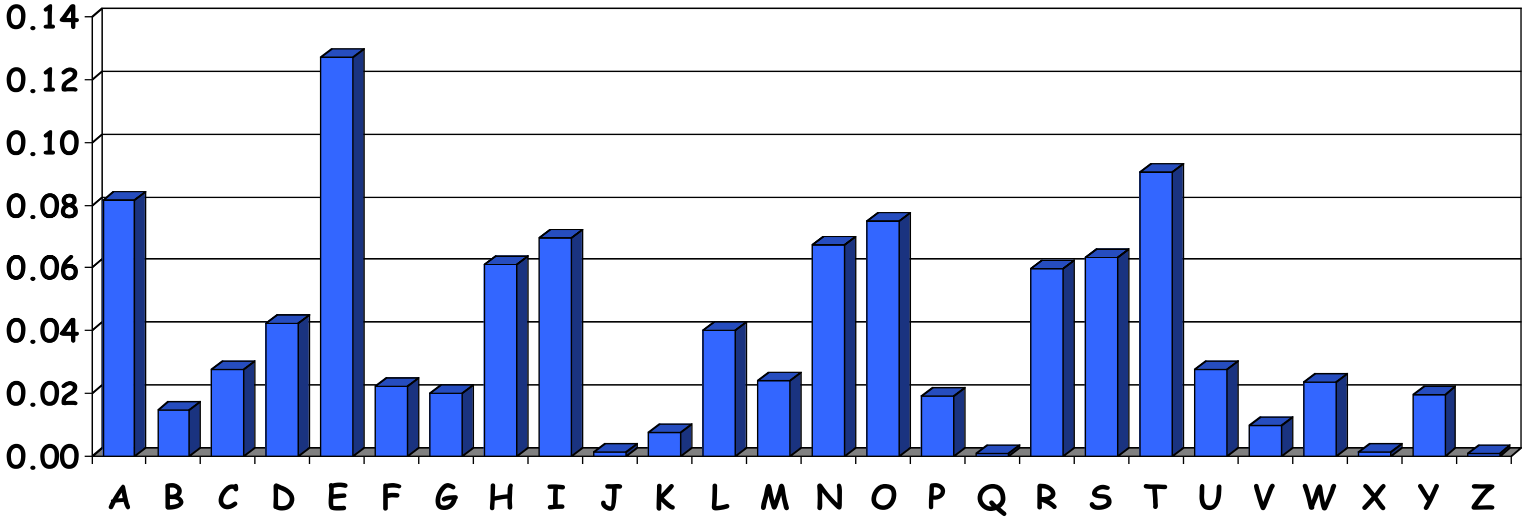
Cryptanalysis II: Be Clever

- We know that a simple substitution is used
- But not necessarily a shift by n
- Find the key given the ciphertext:

PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOX
BTFXQWAXBVCXQWAXFQJVWLEQNTOZQGGQLFXQWAKVWLXQ
WAEBIPBFXFQVXGTVJVWLBTPQWAEBFPBFHCVLXBQUFEVWLXGD
PEQVPQGVPPBFTIXPFHXZHVFAGFOTHFEFBQUFTDHZBQPOTHXTY
FTODXQHFTDPTOGHFQPBQWAQJJTODXQHFOQPWTBDHHIXQV
APBFZQHCFWPFHPBFIPBQWKFABVYYDZBOTHPBQPQJTQOTOGHF
QAPBFEQJHDXXQVAVXEBQPEFZBVFOJIWFFACFCCFHQWAUVWF
LQHGFXVAFXQHFUFHILTTAVWAFFAWTEVOITDHFHFQAITIXPFH
XAFQHEFZQWGFLVWPTOFFA

Cryptanalysis II

- Cannot try all 2^{88} simple substitution keys
- Can we be more clever?
- English letter frequency counts…



Cryptanalysis II

- Ciphertext:

PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTFXQ
WAXBVCXQWAXFQJVWLEQNTOZQGGQLFXQWAKVWLXQWAEBIPBFXFQ
VXGTVJVWLBTPQWAEBFPBFHCVLXBQUFEVWLXGDPEQVPQGVPPBFTIXPFH
XZHVFAGFOTHFEFBQUFTDHZBQPOTHXTYFTODXQHFTDPTOGHFQPBQW
AQJJTODXQHFOQPWTBDHHIXQVAPBFZQHCFWPFHPBFIPBQWKFABVYY
DZBOTHPBQPQJTQOTOGHFQAPBFEQJHDXXQVAVXEBQPEFZBVFOJIWFF
ACFCCFHQWAUVWFLQHGFXVAFXQHFUFHILTTAVWAFFAWTEVOITDHFH
FQAITIXPFHXAFQHEFZQWGFLVWPTOFFA

- Analyze this message using statistics below

Ciphertext frequency counts:

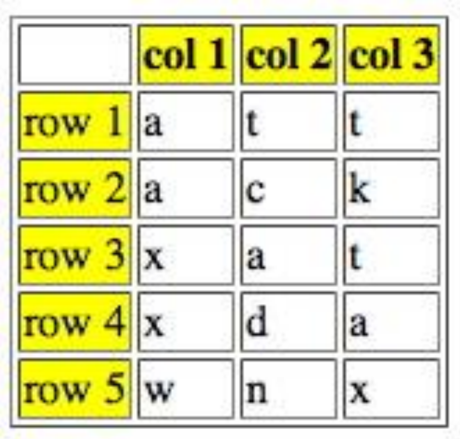
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
21	26	6	10	12	51	10	25	10	9	3	10	0	1	15	28	42	0	0	27	4	24	22	28	6	8

Cryptanalysis: Terminology

- Cryptosystem is **secure** if best know attack is to try all keys
  - Exhaustive key search, that is
- Cryptosystem is **insecure** if ***any*** shortcut attack is known
- But then insecure cipher might be harder to break than a secure cipher!
  - What the … ?

Double Transposition

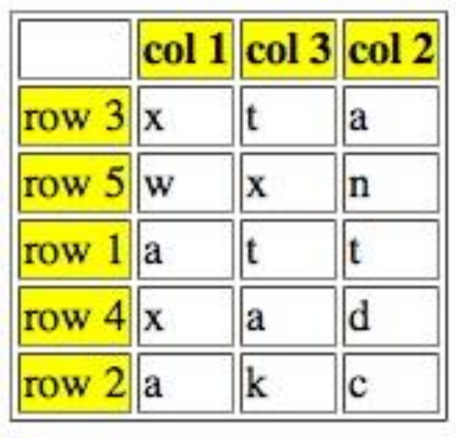
- Plaintext: attackxatxdawn



	col 1	col 2	col 3
row 1	a	t	t
row 2	a	c	k
row 3	x	a	t
row 4	x	d	a
row 5	w	n	x

Permute rows and columns

⇒



	col 1	col 3	col 2
row 3	x	t	a
row 5	w	x	n
row 1	a	t	t
row 4	x	a	d
row 2	a	k	c

- Ciphertext: xtawxnattxadakc
- Key is matrix size and permutations: (3,5,1,4,2) and (1,3,2)

One-Time Pad: Encryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

Encryption: Plaintext ⊕ Key = Ciphertext

	h	e	i	l	h	i	t	l	e	r
Plaintext:	001	000	010	100	001	010	111	100	000	101
Key:	111	101	110	101	111	100	000	101	110	000
Ciphertext:	110	101	100	001	110	110	111	001	110	101
	s	r	l	h	s	s	t	h	s	r

One-Time Pad: Decryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

Decryption: Ciphertext ⊕ Key = Plaintext

	s	r	l	h	s	s	t	h	s	r
Ciphertext:	110	101	100	001	110	110	111	001	110	101
Key:	111	101	110	101	111	100	000	101	110	000
Plaintext:	001	000	010	100	001	010	111	100	000	101
	h	e	i	l	h	i	t	l	e	r

One-Time Pad

Double agent claims following “**key**” was used:

	s	r	l	h	s	s	t	h	s	r
Ciphertext:	110	101	100	001	110	110	111	001	110	101
“**key**”:	101	111	000	101	111	100	000	101	110	000
“Plaintext”:	011	010	100	100	001	010	111	100	000	101
	k	i	l	l	h	i	t	l	e	r

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

One-Time Pad

Or claims the key is…

	s	r	l	h	s	s	t	h	s	r
Ciphertext:	110	101	100	001	110	110	111	001	110	101
"key":	111	101	000	011	101	110	001	011	101	101
"Plaintext":	001	000	100	010	011	000	110	010	011	000
	h	e	l	i	k	e	s	i	k	e

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

One-Time Pad Summary

- **Provably** secure
  - Ciphertext gives **no** useful info about plaintext
  - All plaintexts are ***equally likely***
- BUT, only when be used correctly
  - Pad must be random, used only once
  - Pad is known only to sender and receiver
- Note: pad (key) is same size as message
- So, why not distribute msg instead of pad?

Real-World One-Time Pad

- Project VENONA
  - Soviet spies encrypted messages from U.S. to Moscow in 30's, 40's, and 50's
  - Nuclear espionage, etc.
  - Thousands of messages
- Spy carried one-time pad into U.S.
- Spy used pad to encrypt secret messages
- Repeats within the "one-time" pads made cryptanalysis possible

VENONA Decrypt (1944)

[C% Ruth] learned that her husband [v] was called up by the army but he was not sent to the front. He is a mechanical engineer and is now working at the ENORMOUS [ENORMOZ] [vi] plant in SANTA FE, New Mexico. [45 groups unrecoverable]

detain VOLOK [vii] who is working in a plant on ENORMOUS. He is a FELLOWCOUNTRYMAN [ZEMLYaK] [viii]. Yesterday he learned that they had dismissed him from his work. His active work in progressive organizations in the past was cause of his dismissal. In the FELLOWCOUNTRYMAN line LIBERAL is in touch with CHESTER [ix]. They meet once a month for the payment of dues. CHESTER is interested in whether we are satisfied with the collaboration and whether there are not any misunderstandings. He does not inquire about specific items of work [KONKRETNAYa RABOTA]. In as much as CHESTER knows about the role of LIBERAL's group we beg consent to ask C. through LIBERAL about leads from among people who are working on ENOURMOUS and in other technical fields.

- "Ruth" == Ruth Greenglass
- "Liberal" == Julius Rosenberg
- "Enormous" == the atomic bomb

Codebook Cipher

- Literally, a book filled with "codewords"
- Zimmerman Telegram encrypted via codebook

Februar	13605
fest	13732
finanzielle	13850
folgender	13918
Frieden	17142
Friedenschluss	17149
:	:

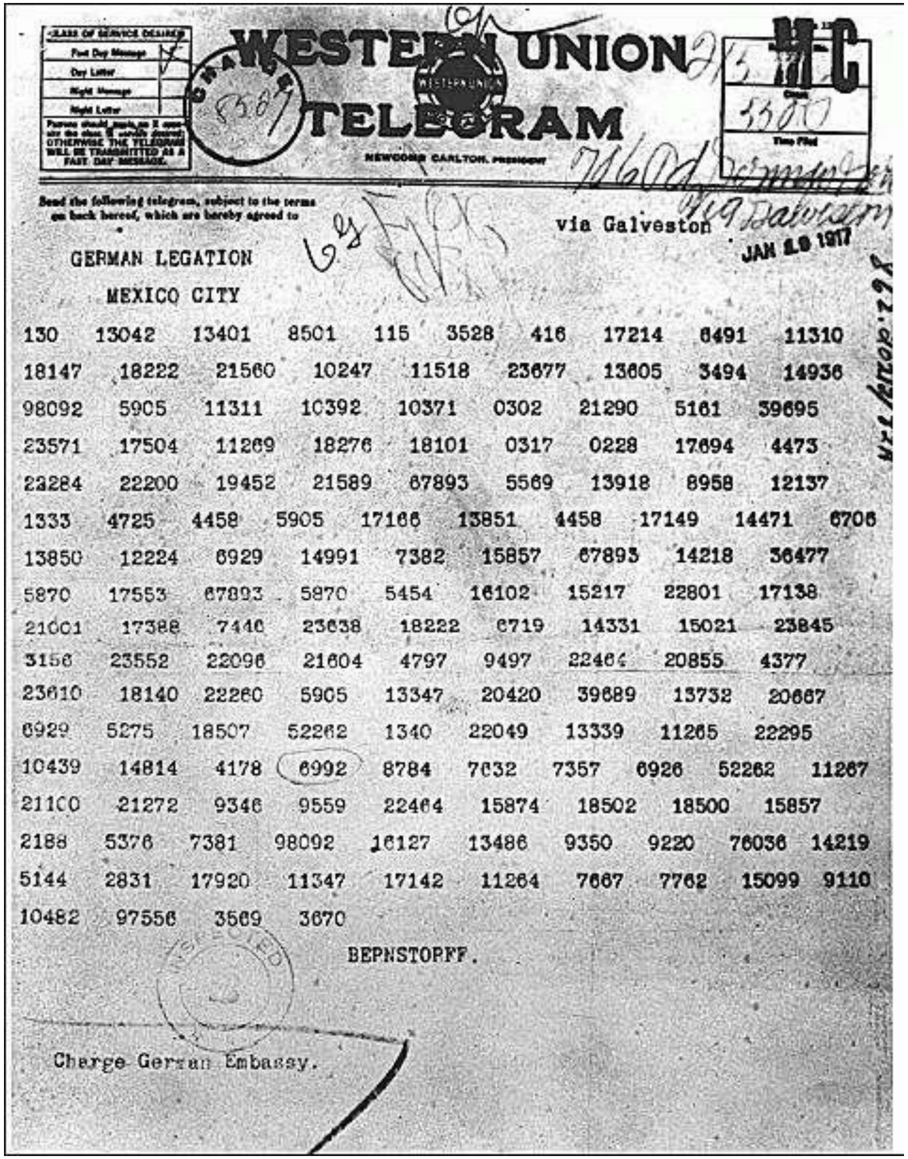
- Modern block ciphers are codebooks!
- More about this later…

Codebook Cipher: Additive

- Codebooks also (usually) use **additive**
- Additive — book of "random" numbers
  - Encrypt message with codebook
  - Then choose position in additive book
  - Add in additives to get ciphertext
  - Send ciphertext and additive position (MI)
  - Recipient subtracts additives before decrypting
- Why use an additive sequence?

Zimmerman Telegram

- Perhaps most famous codebook ciphertext ever
- A major factor in U.S. entry into World War I



WESTERN UNION TELEGRAM

NEWCOMB CARLTON, PRESIDENT

Send the following telegram, subject to the terms on back hereof, which are hereby agreed to

via Galveston

JAN 19 1917

GERMAN LEGATION

MEXICO CITY

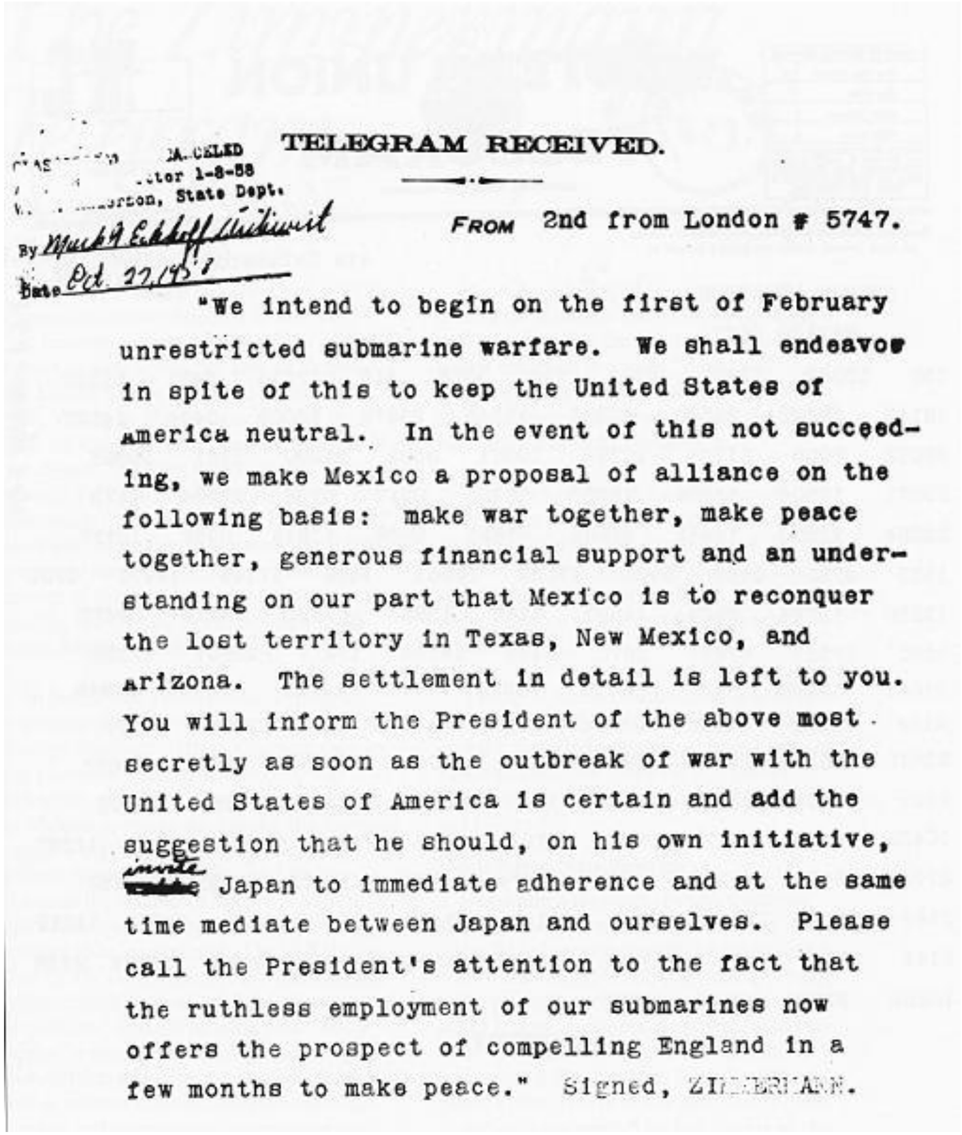
130 13042 13401 8501 115 3528 416 17214 6491 11310
18147 18222 21560 10247 11518 23677 13605 3494 14936
98092 5905 11311 10392 10371 0302 21290 5161 39695
23571 17504 11269 18276 18101 0317 0228 17694 4473
23284 22200 19452 21589 67893 5569 13918 8958 12137
1333 4725 4458 5905 17166 13851 4458 17149 14471 6706
13850 12224 6929 14991 7382 15857 67893 14218 36477
5870 17553 67893 5870 5454 16102 15217 22801 17138
21001 17388 7446 23638 18222 6719 14331 15021 23845
3156 23552 22096 21604 4797 9497 22464 20855 4377
23610 18140 22260 5905 13347 20420 39689 13732 20667
6929 5275 18507 52262 1340 22049 13339 11265 22295
10439 14814 4178 6992 8784 7632 7357 6926 52262 11267
21100 21272 9346 9559 22464 15874 18502 18500 15857
2188 5376 7381 98092 16127 13486 9350 9220 76036 14219
5144 2831 17920 11347 17142 11264 7667 7762 15099 9110
10482 97556 3569 3670

BERNSTORFF.

Charge German Embassy.

Zimmerman Telegram Decrypted

- British had recovered partial codebook
- Then able to fill in missing parts



…CELED
…tor 1-8-58
…rton, State Dept.
By
Date Oct. 22,1957

TELEGRAM RECEIVED.

FROM 2nd from London # 5747.

"We intend to begin on the first of February unrestricted submarine warfare. We shall endeavor in spite of this to keep the United States of America neutral. In the event of this not succeeding, we make Mexico a proposal of alliance on the following basis: make war together, make peace together, generous financial support and an understanding on our part that Mexico is to reconquer the lost territory in Texas, New Mexico, and Arizona. The settlement in detail is left to you. You will inform the President of the above most secretly as soon as the outbreak of war with the United States of America is certain and add the suggestion that he should, on his own initiative, invite Japan to immediate adherence and at the same time mediate between Japan and ourselves. Please call the President's attention to the fact that the ruthless employment of our submarines now offers the prospect of compelling England in a few months to make peace." Signed, ZIMMERMANN.

Random Historical Items

- Crypto timeline
- Spartan Scytale — transposition cipher
- Caesar's cipher
- Poe's short story: ***The Gold Bug***
- Election of 1876

Election of 1876

- “Rutherfraud” Hayes vs “Swindling” Tilden
    - Popular vote was virtual tie
- Electoral college delegations for 4 states (including Florida) in dispute
- Commission gave all 4 states to Hayes
    - Voted on straight party lines
- Tilden accused Hayes of bribery
    - Was it true?

Election of 1876

- Encrypted messages by Tilden supporters later emerged
- Cipher: Partial codebook, plus transposition
- Codebook substitution for important words

ciphertext	**plaintext**
Copenhagen	Greenbacks
Greece	Hayes
Rochester	votes
Russia	Tilden
Warsaw	**telegram**
:	:

Election of 1876

- Apply codebook to original message
- Pad message to multiple of 5 words (total length, 10,15,20,25 or 30 words)
- For each length, a fixed permutation applied to resulting message
- Permutations found by comparing several messages of same length
- Note that the **same key** is applied to all messages of a given length

Election of 1876

- Ciphertext: **Warsaw they read all unchanged last are idiots can't situation**
- Codebook: Warsaw == telegram
- Transposition: 9,3,6,1,10,5,2,7,4,8
- Plaintext: **Can't read last telegram. Situation unchanged. They are all idiots.**
- A weak cipher made worse by reuse of key
- Lesson? Don't overuse keys!

Early 20th Century

- WWI — Zimmerman Telegram
- "Gentlemen do not read each other's mail"
  - Henry L. Stimson, Secretary of State, 1929
- WWII — golden age of cryptanalysis
  - Midway/Coral Sea
  - Japanese **Purple** (codename **MAGIC**)
  - German **Enigma** (codename **ULTRA**)

Post-WWII History

- Claude Shannon — father of the science of information theory
- Computer revolution — lots of data to protect
- Data Encryption Standard (DES), 70's
- Public Key cryptography, 70's
- CRYPTO conferences, 80's
- Advanced Encryption Standard (AES), 90's
- The crypto genie is out of the bottle…

Claude Shannon

- The founder of Information Theory
- 1949 paper: *Comm. Thy. of Secrecy Systems*
- Fundamental concepts
    - **Confusion** — obscure relationship between plaintext and ciphertext
    - **Diffusion** — spread plaintext statistics through the ciphertext
- Proved one-time pad is secure
- One-time pad is confusion-only, while double transposition is diffusion-only

Taxonomy of Cryptography

- **Symmetric Key**
  - Same key for encryption and decryption
  - Modern types: Stream ciphers, Block ciphers
- **Public Key** (or "asymmetric" crypto)
  - Two keys, one for encryption (public), and one for decryption (private)
  - And digital signatures — nothing comparable in symmetric key crypto
- **Hash algorithms**
  - Can be viewed as "one way" crypto

Taxonomy of Cryptanalysis

- From perspective of info available to Trudy…
  - Ciphertext only — Trudy's worst case scenario
  - Known plaintext
  - Chosen plaintext
    - "Lunchtime attack"
    - Some protocols will encrypt chosen data
  - Adaptively chosen plaintext
  - Related key
  - Forward search (public key crypto)
  - And others…

Chapter 3:
Symmetric Key Crypto

The chief forms of beauty are order and symmetry…
— Aristotle

"You boil it in sawdust: you salt it in glue:
You condense it with locusts and tape:
Still keeping one principal object in view —
To preserve its symmetrical shape."
— Lewis Carroll, *The Hunting of the Snark*

Symmetric Key Crypto

- Stream cipher — generalize one-time pad
  - Except that key is relatively short
  - Key is stretched into a long **keystream**
  - Keystream is used just like a one-time pad
- Block cipher — generalized codebook
  - Block cipher key determines a codebook
  - Each key yields a different codebook
  - Employs both "confusion" and "diffusion"

Stream Ciphers

Stream Ciphers

- Once upon a time, not so very long ago… stream ciphers were the king of crypto
- Today, not as popular as block ciphers
- We'll discuss two stream ciphers:
- A5/1
  - Based on shift registers
  - Used in GSM mobile phone system
- RC4
  - Based on a changing lookup table
  - Used many places

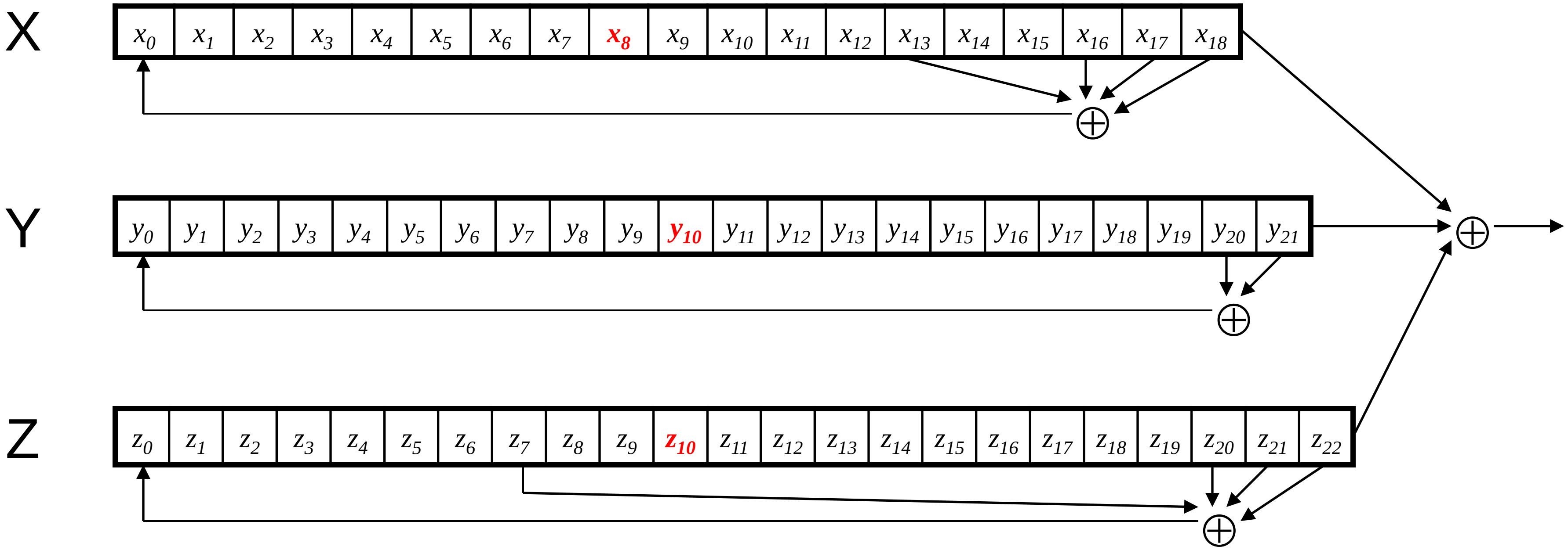
A5/1: Shift Registers

- A5/1 uses 3 *shift registers*
    - X: 19 bits $(x_0,x_1,x_2, \ldots,x_{18})$
    - Y: 22 bits $(y_0,y_1,y_2, \ldots,y_{21})$
    - Z: 23 bits $(z_0,z_1,z_2, \ldots,z_{22})$

A5/1: Keystream

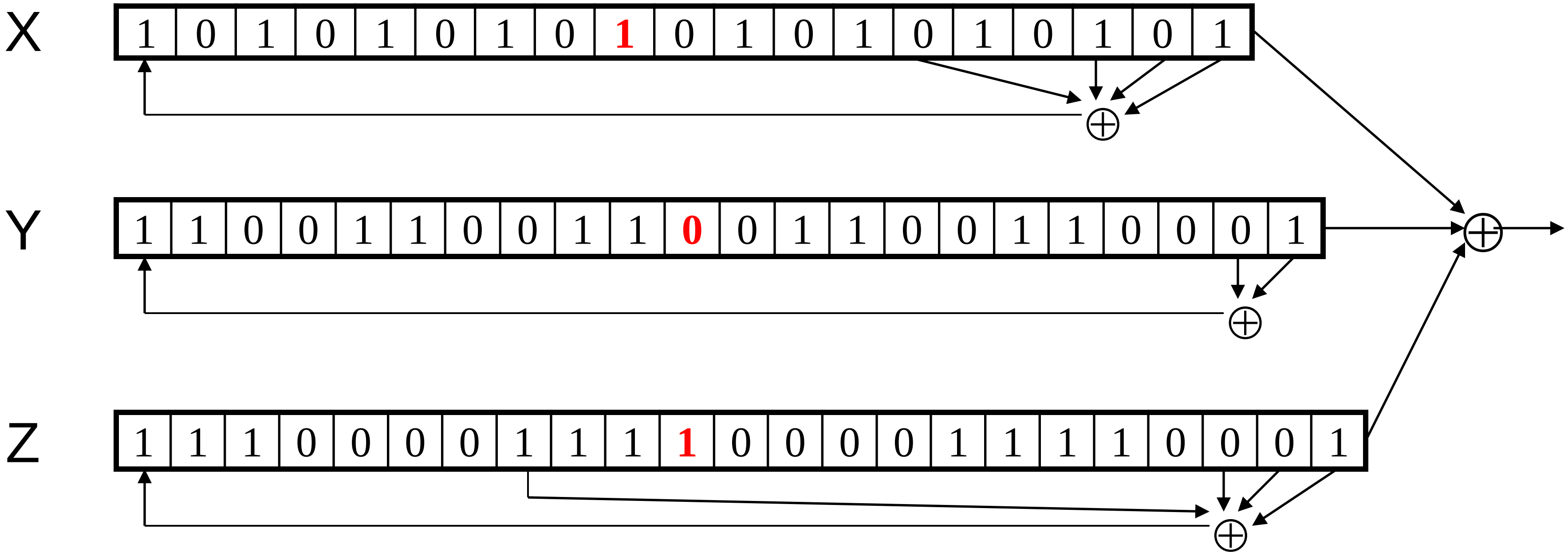
- At each iteration: $m = \mathrm{maj}(x_8, y_{10}, z_{10})$
  - Examples: $\mathrm{maj}(0,1,0) = 0$ and $\mathrm{maj}(1,1,0) = 1$
- If $x_8 = m$ then X *steps*
  - $t = x_{13} \oplus x_{16} \oplus x_{17} \oplus x_{18}$
  - $x_i = x_{i-1}$ for $i = 18,17,\ldots,1$ and $x_0 = t$
- If $y_{10} = m$ then Y *steps*
  - $t = y_{20} \oplus y_{21}$
  - $y_i = y_{i-1}$ for $i = 21,20,\ldots,1$ and $y_0 = t$
- If $z_{10} = m$ then Z *steps*
  - $t = z_7 \oplus z_{20} \oplus z_{21} \oplus z_{22}$
  - $z_i = z_{i-1}$ for $i = 22,21,\ldots,1$ and $z_0 = t$
- Keystream **bit** is $x_{18} \oplus y_{21} \oplus z_{22}$

A5/1



- Each variable here is a single bit
- Key is used as **initial fill** of registers
- Each register steps (or not) based on $\mathrm{maj}(x_8, y_{10}, z_{10})$
- Keystream bit is XOR of rightmost bits of registers

A5/1



- In this example, $m = \text{maj}(x_8, y_{10}, z_{10}) = \text{maj}(\mathbf{1},\mathbf{0},\mathbf{1}) = \mathbf{1}$
- Register X steps, Y does not step, and Z steps
- Keystream bit is XOR of right bits of registers
- Here, keystream bit will be $0 \oplus 1 \oplus 0 = 1$

Shift Register Crypto

- Shift register crypto efficient in hardware
- Often, slow if implemented in software
- In the past, very, very popular
- Today, more is done in software due to fast processors
- Shift register crypto still used some
  - Especially in resource-constrained devices

RC4

- A self-modifying lookup table
- Table always contains a permutation of the byte values 0,1,…,255
- Initialize the permutation using key
- At each step, RC4 does the following
  - Swaps elements in current lookup table
  - Selects a keystream byte from table
- Each step of RC4 produces a **byte**
  - Efficient in software
- Each step of A5/1 produces only a bit
  - Efficient in hardware

RC4 Initialization

- S[] is permutation of 0,1,...,255
- key[] contains N bytes of key

```
for i = 0 to 255
        S[i] = i
        K[i] = key[i (mod N)]
next i
j = 0
for i = 0 to 255
        j = (j + S[i] + K[i]) mod 256
        swap(S[i], S[j])
next i
i = j = 0
```

RC4 Keystream

- At each step, swap elements in table and select keystream byte

```
i = (i + 1) mod 256
j = (j + S[i]) mod 256
swap(S[i], S[j])
t = (S[i] + S[j]) mod 256
keystreamByte = S[t]
```

- Use keystream bytes like a one-time pad
- **Note:** first 256 bytes should be discarded
  - Otherwise, related key attack exists

Stream Ciphers

- Stream ciphers were popular in the past
  - Efficient in hardware
  - Speed was needed to keep up with voice, etc.
  - Today, processors are fast, so software-based crypto is usually more than fast enough
- Future of stream ciphers?
  - Shamir declared "the death of stream ciphers"
  - May be greatly exaggerated…

Block Ciphers

(Iterated) Block Cipher

- Plaintext and ciphertext consist of fixed-sized blocks
- Ciphertext obtained from plaintext by iterating a **round function**
- Input to round function consists of ***key*** and ***output*** of previous round
- Usually implemented in software

Feistel Cipher: Encryption

- **Feistel cipher** is a type of block cipher
  - *Not* a specific block cipher
- Split plaintext block into left and right halves: $P = (L_0, R_0)$
- For each round $i = 1, 2, ..., n$, compute

  $L_i = R_{i-1}$

  $R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$

  where F is **round function** and K_i is **subkey**
- Ciphertext: $C = (L_n, R_n)$

Feistel Cipher: Decryption

- Start with ciphertext $C = (L_n, R_n)$
- For each round $i = n, n-1, \ldots, 1$, compute

  $R_{i-1} = L_i$
  $L_{i-1} = R_i \oplus F(R_{i-1}, K_i)$
  where F is round function and K_i is subkey
- Plaintext: $P = (L_0, R_0)$
- Decryption works for any function F
  - But only secure for certain functions F

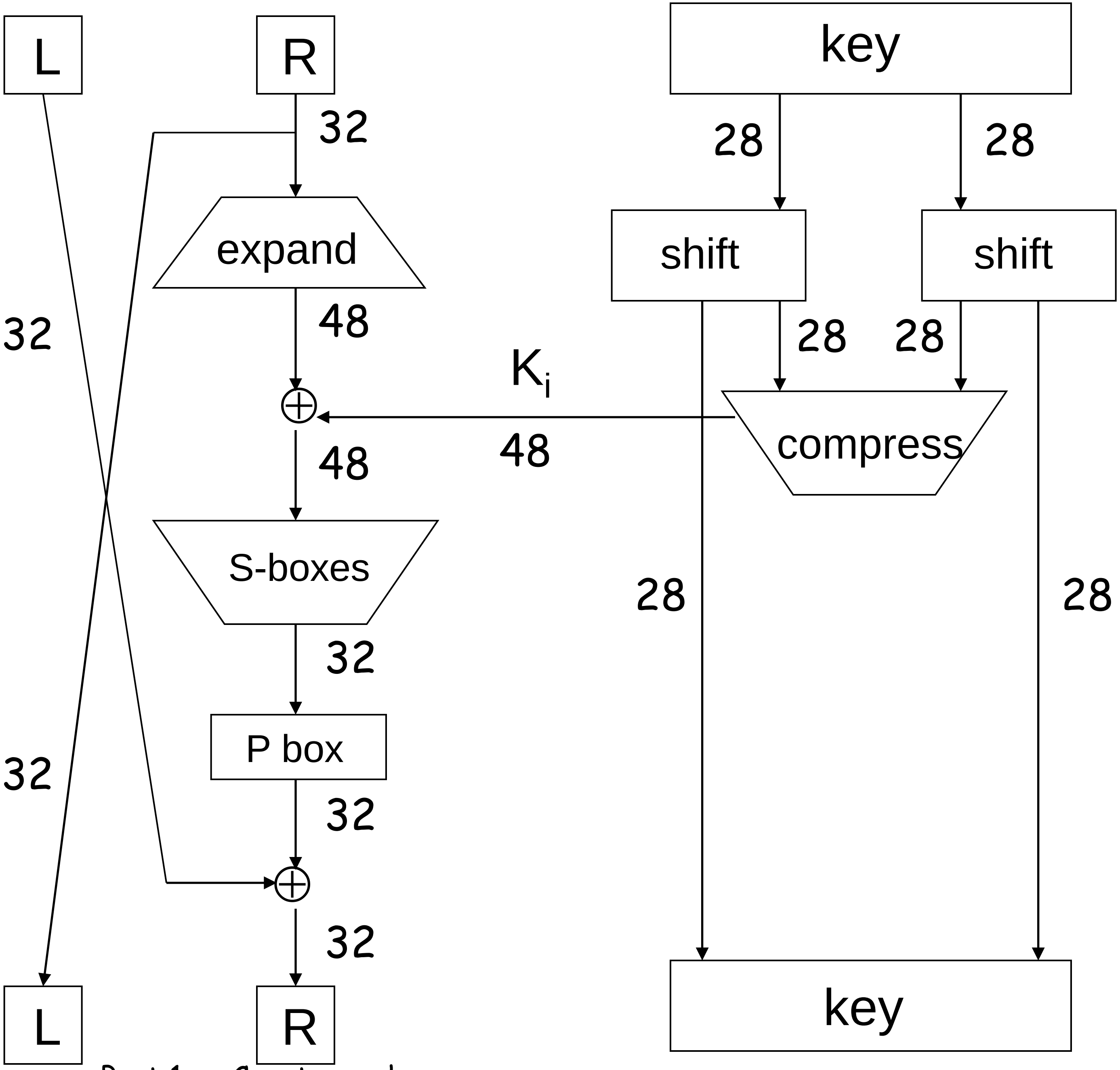
Data Encryption Standard

- **DES** developed in 1970's
- Based on IBM's Lucifer cipher
- DES was U.S. government standard
- Development of DES was controversial
  - NSA secretly involved
  - Design process was secret
  - Key length reduced from 128 to 56 bits
  - Subtle changes to Lucifer algorithm

DES Numerology

- DES is a Feistel cipher with…
  - 64 bit block length
  - 56 bit key length
  - 16 rounds
  - 48 bits of key used each round (subkey)
- Round function is simple (for block cipher)
- Security depends heavily on “S-boxes”
  - Each S-box maps 6 bits to 4 bits

One Round of DES



DES Expansion Permutation

- Input 32 bits

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

- Output 48 bits

31 0 1 2 3 4 3 4 5 6 7 8
7 8 9 10 11 12 11 12 13 14 15 16
15 16 17 18 19 20 19 20 21 22 23 24
23 24 25 26 27 28 27 28 29 30 31 0

DES S-box

- 8 "substitution boxes" or S-boxes
- Each S-box maps 6 bits to 4 bits
- Here is S-box number 1

input bits (0,5)
↓
input bits (1,2,3,4)

	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
00	1110	0100	1101	0001	0010	1111	1011	1000	0011	1010	0110	1100	0101	1001	0000	0111
01	0000	1111	0111	0100	1110	0010	1101	0001	1010	0110	1100	1011	1001	0101	0011	1000
10	0100	0001	1110	1000	1101	0110	0010	1011	1111	1100	1001	0111	0011	1010	0101	0000
11	1111	1100	1000	0010	0100	1001	0001	0111	0101	1011	0011	1110	1010	0000	0110	1101

DES P-box

- Input 32 bits

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

- Output 32 bits

15 6 19 20 28 11 27 16 0 14 22 25 4 17 30 9
1 7 23 13 31 26 2 8 18 12 29 5 21 10 3 24

DES Subkey

- 56 bit DES key, numbered 0,1,2,…,55
- Left half key bits, LK

49	42	35	28	21	14	7
0	50	43	36	29	22	15
8	1	51	44	37	30	23
16	9	2	52	45	38	31

- Right half key bits, RK

55	48	41	34	27	20	13
6	54	47	40	33	26	19
12	5	53	46	39	32	25
18	11	4	24	17	10	3

DES Subkey

- For rounds i=1,2,...,16
  - Let LK = (LK circular shift left by r_i)
  - Let RK = (RK circular shift left by r_i)
  - Left half of subkey K_i is of LK bits

    13 16 10 23 0 4 2 27 14 5 20 9
    22 18 11 3 25 7 15 6 26 19 12 1
  - Right half of subkey K_i is RK bits

    12 23 2 8 18 26 1 11 22 16 4 19
    15 20 10 27 5 24 17 13 21 7 0 3

DES Subkey

- For rounds 1, 2, 9 and 16 the shift r_i is 1, and in all other rounds r_i is 2
- Bits 8,17,21,24 of LK omitted each round
- Bits 6,9,14,25 of RK omitted each round
- **Compression permutation** yields 48 bit subkey K_i from 56 bits of LK and RK
- **Key schedule** generates subkey

DES Last Word (Almost)

- An initial permutation before round 1
- Halves are swapped after last round
- A final permutation (inverse of initial perm) applied to (R_{16}, L_{16})
- None of this serves any security purpose

Security of DES

- Security depends heavily on S-boxes
  - Everything else in DES is linear
- 35+ years of intense analysis has revealed no back door
- Attacks, essentially exhaustive key search
- **Inescapable conclusions**
  - Designers of DES knew what they were doing
  - Designers of DES were way ahead of their time (at least wrt certain cryptanalytic techniques)

Block Cipher Notation

- P = plaintext block
- C = ciphertext block
- Encrypt P with key K to get ciphertext C
  - $C = E(P, K)$
- Decrypt C with key K to get plaintext P
  - $P = D(C, K)$
- Note: $P = D(E(P, K), K)$ and $C = E(D(C, K), K)$
  - But $P \neq D(E(P, K_1), K_2)$ and $C \neq E(D(C, K_1), K_2)$ when $K_1 \neq K_2$

Triple DES

- Today, 56 bit DES key is too small
  - Exhaustive key search is feasible
- But DES is everywhere, so what to do?
- **Triple DES** or **3DES** (112 bit key)
  - $C = E(D(E(P,K_1),K_2),K_1)$
  - $P = D(E(D(C,K_1),K_2),K_1)$
- Why Encrypt-Decrypt-Encrypt with 2 keys?
  - Backward compatible: $E(D(E(P,K),K),K) = E(P,K)$
  - And 112 is a lot of bits

3DES

- Why not $C = E(E(P,K),K)$ instead?
  - Trick question — still just 56 bit key
- Why not $C = E(E(P,K_1),K_2)$ instead?
- A (semi-practical) **known plaintext** attack
  - Pre-compute table of $E(P,K_1)$ for every possible key K_1 (resulting table has 2^{56} entries)
  - Then for each possible K_2 compute $D(C,K_2)$ until a match in table is found
  - When match is found, have $E(P,K_1) = D(C,K_2)$
  - Result gives us keys: $C = E(E(P,K_1),K_2)$

Advanced Encryption Standard

- Replacement for DES
- AES competition (late 90's)
  - NSA openly involved
  - Transparent selection process
  - Many strong algorithms proposed
  - Rijndael Algorithm ultimately selected (pronounced like "Rain Doll" or "Rhine Doll")
- Iterated block cipher (like DES)
- Not a Feistel cipher (unlike DES)

AES: Executive Summary

- **Block size:** 128 bits (others in Rijndael)
- **Key length:** 128, 192 or 256 bits (independent of block size in Rijndael)
- 10 to 14 rounds (depends on key length)
- Each round uses 4 functions (3 “layers”)
    - ByteSub (nonlinear layer)
    - ShiftRow (linear mixing layer)
    - MixColumn (nonlinear layer)
    - AddRoundKey (key addition layer)

AES ByteSub

- Treat 128 bit block as 4x4 byte array

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow \texttt{ByteSub} \longrightarrow \begin{bmatrix} b_{00} & b_{01} & b_{02} & b_{03} \\ b_{10} & b_{11} & b_{12} & b_{13} \\ b_{20} & b_{21} & b_{22} & b_{23} \\ b_{30} & b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

- ByteSub is AES's "S-box"
- Can be viewed as nonlinear (but invertible) composition of two math operations

AES "S-box"

Last 4 bits of input

First 4 bits of input

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

AES ShiftRow

- Cyclic shift rows

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow \texttt{ShiftRow} \longrightarrow \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{11} & a_{12} & a_{13} & a_{10} \\ a_{22} & a_{23} & a_{20} & a_{21} \\ a_{33} & a_{30} & a_{31} & a_{32} \end{bmatrix}$$

AES MixColumn

- Invertible, linear operation applied to each column

$$\begin{bmatrix} a_{0i} \\ a_{1i} \\ a_{2i} \\ a_{3i} \end{bmatrix} \longrightarrow \texttt{MixColumn} \longrightarrow \begin{bmatrix} b_{0i} \\ b_{1i} \\ b_{2i} \\ b_{3i} \end{bmatrix} \quad \text{for } i = 0, 1, 2, 3$$

- Implemented as a (big) lookup table

AES AddRoundKey

- XOR subkey with block

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \oplus \begin{bmatrix} k_{00} & k_{01} & k_{02} & k_{03} \\ k_{10} & k_{11} & k_{12} & k_{13} \\ k_{20} & k_{21} & k_{22} & k_{23} \\ k_{30} & k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} b_{00} & b_{01} & b_{02} & b_{03} \\ b_{10} & b_{11} & b_{12} & b_{13} \\ b_{20} & b_{21} & b_{22} & b_{23} \\ b_{30} & b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Block Subkey

- RoundKey (subkey) determined by **key schedule** algorithm

AES Decryption

- To decrypt, process must be invertible
- Inverse of MixAddRoundKey is easy, since "$\oplus$" is its own inverse
- MixColumn is invertible (inverse is also implemented as a lookup table)
- Inverse of ShiftRow is easy (cyclic shift the other direction)
- ByteSub is invertible (inverse is also implemented as a lookup table)

A Few Other Block Ciphers

- Briefly…
  - IDEA
  - Blowfish
  - RC6
- More detailed…
  - TEA

IDEA

- Invented by James Massey
  - One of the giants of modern crypto
- IDEA has 64-bit block, 128-bit key
- IDEA uses **mixed-mode arithmetic**
- Combine different math operations
  - IDEA the first to use this approach
  - Frequently used today

Blowfish

- Blowfish encrypts 64-bit blocks
- Key is variable length, up to 448 bits
- Invented by Bruce Schneier
- Almost a Feistel cipher

$$R_i = L_{i-1} \oplus K_i$$
$$L_i = R_{i-1} \oplus F(L_{i-1} \oplus K_i)$$

- The round function F uses 4 S-boxes
  - Each S-box maps 8 bits to 32 bits
- **Key-dependent S-boxes**
  - S-boxes determined by the key

RC6

- Invented by Ron Rivest
- Variables
  - Block size
  - Key size
  - Number of rounds
- An AES finalist
- Uses **data dependent rotations**
  - Unusual for algorithm to depend on plaintext

Time for TEA…

- Tiny Encryption Algorithm (TEA)
- 64 bit block, 128 bit key
- Assumes 32-bit arithmetic
- Number of rounds is variable (32 is considered secure)
- Uses "weak" round function, so large number of rounds required

TEA Encryption

Assuming 32 rounds:

```
(K[0], K[1], K[2], K[3]) = 128 bit key
(L,R) = plaintext (64-bit block)
delta = 0x9e3779b9
sum = 0
for i = 1 to 32
    sum += delta
    L += ((R<<4)+K[0])^(R+sum)^((R>>5)+K[1])
    R += ((L<<4)+K[2])^(L+sum)^((L>>5)+K[3])
next i
ciphertext = (L,R)
```

TEA Decryption

Assuming 32 rounds:

```
(K[0], K[1], K[2], K[3]) = 128 bit key
(L,R) = ciphertext (64-bit block)
delta = 0x9e3779b9
sum = delta << 5
for i = 1 to 32
    R –= ((L<<4)+K[2])^(L+sum)^((L>>5)+K[3])
    L –= ((R<<4)+K[0])^(R+sum)^((R>>5)+K[1])
    sum –= delta
next i
plaintext = (L,R)
```

TEA Comments

- **"Almost"** a Feistel cipher
  - Uses + and - instead of ⊕ (XOR)
- Simple, easy to implement, fast, low memory requirement, etc.
- Possibly a "related key" attack
- eXtended TEA (XTEA) eliminates related key attack (slightly more complex)
- Simplified TEA (STEA) — insecure version used as an example for cryptanalysis

Block Cipher Modes

Multiple Blocks

- How to encrypt multiple blocks?
- Do we need a new key for each block?
  - If so, as impractical as a one-time pad!
- Encrypt each block independently?
- Is there any analog of codebook "additive"?
- How to handle partial blocks?
  - We won't discuss this issue

Modes of Operation

- Many modes — we discuss 3 most popular
- Electronic Codebook (**ECB**) mode
  - Encrypt each block independently
  - Most obvious approach, but a bad idea
- Cipher Block Chaining (**CBC**) mode
  - Chain the blocks together
  - More secure than ECB, virtually no extra work
- Counter Mode (**CTR**) mode
  - Block ciphers acts like a stream cipher
  - Popular for random access

ECB Mode

- Notation: $C = E(P, K)$
- Given plaintext $P_0, P_1, \ldots, P_m, \ldots$
- Most obvious way to use a block cipher:

Encrypt	**Decrypt**
$C_0 = E(P_0, K)$	$P_0 = D(C_0, K)$
$C_1 = E(P_1, K)$	$P_1 = D(C_1, K)$
$C_2 = E(P_2, K)$ …	$P_2 = D(C_2, K)$ …

- For fixed key K, this is "electronic" version of a codebook cipher (without additive)
  - With a different codebook for each key

ECB Cut and Paste

- Suppose plaintext is

  Alice digs Bob. Trudy digs Tom.

- Assuming 64-bit blocks and 8-bit ASCII:

  P_0 = “Alice di”, P_1 = “gs Bob. ”,

  P_2 = “Trudy di”, P_3 = “gs Tom. ”

- Ciphertext: C_0, C_1, C_2, C_3
- Trudy cuts and pastes: C_0, C_3, C_2, C_1
- Decrypts as

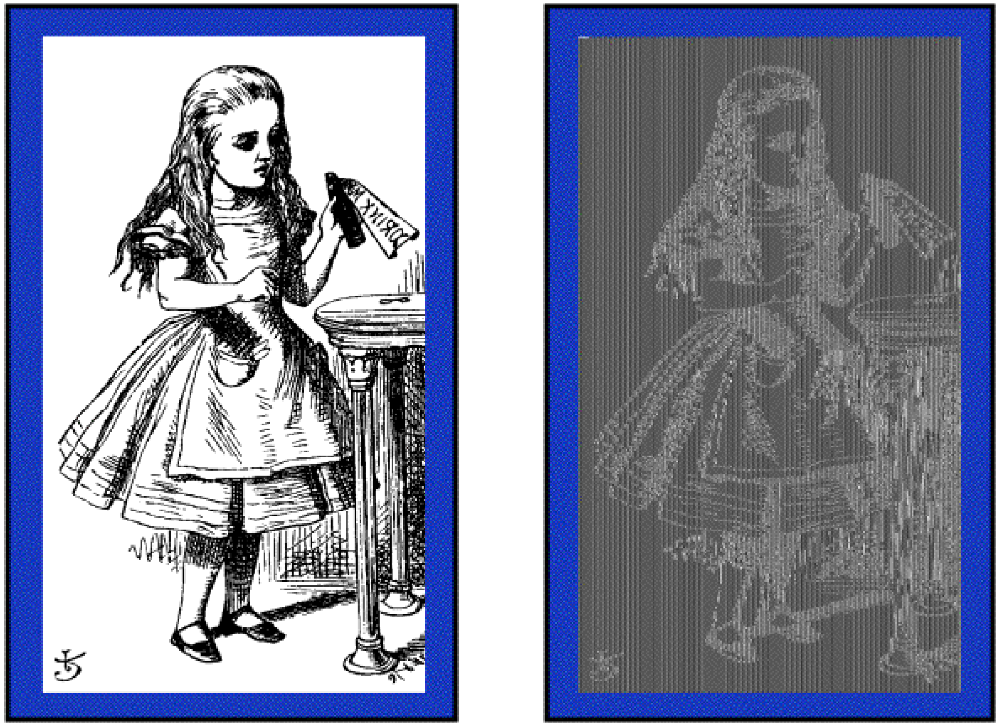
  Alice digs Tom. Trudy digs Bob.

ECB Weakness

- Suppose $P_i = P_j$
- Then $C_i = C_j$ and Trudy knows $P_i = P_j$
- This gives Trudy some information, even if she does not know P_i or P_j
- Trudy might know P_i
- Is this a serious issue?

Alice Hates ECB Mode

- Alice's uncompressed image, and ECB encrypted (TEA)



- Why does this happen?
- Same plaintext yields same ciphertext!

CBC Mode

- Blocks are "chained" together
- A random initialization vector, or IV, is required to initialize CBC mode
- IV is random, but not secret

Encryption

$C_0 = E(IV \oplus P_0, K),$

$C_1 = E(C_0 \oplus P_1, K),$

$C_2 = E(C_1 \oplus P_2, K),\ldots$

Decryption

$P_0 = IV \oplus D(C_0, K),$

$P_1 = C_0 \oplus D(C_1,$

$P_2 = C_1 \oplus D(C_2, K),\ldots$

- Analogous to classic codebook *with additive*

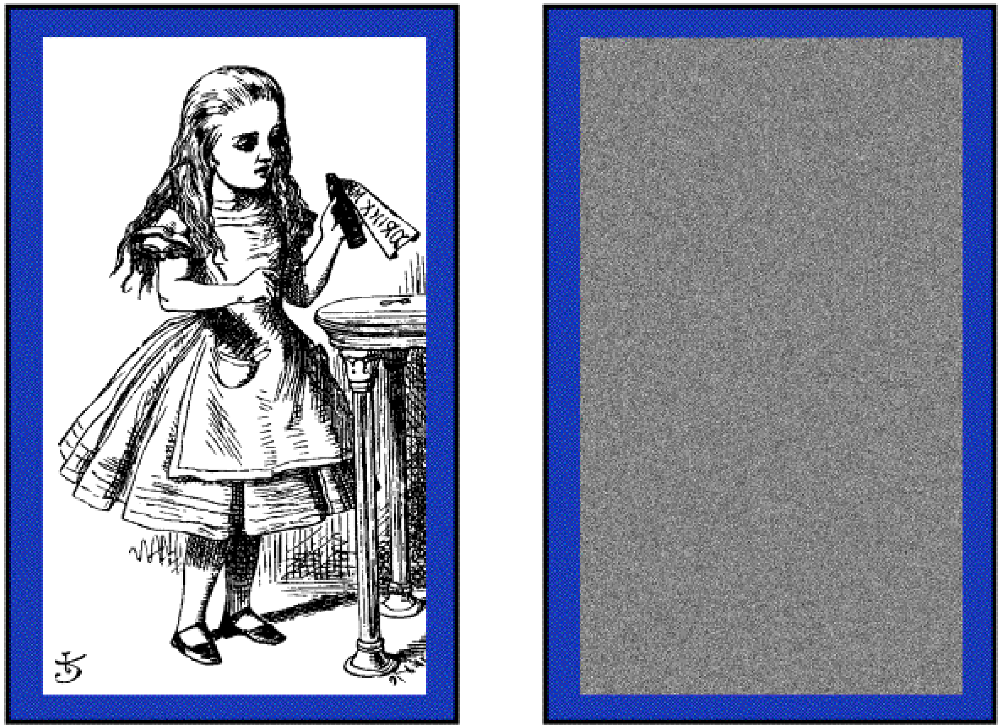
CBC Mode

- Identical plaintext blocks yield different ciphertext blocks — this is very good!
- But what about errors in transmission?
  - If C_1 is garbled to, say, G then

    $P_1 \neq C_0 \oplus D(G, K), P_2 \neq G \oplus D(C_2, K)$
  - But $P_3 = C_2 \oplus D(C_3, K), P_4 = C_3 \oplus D(C_4, K), \ldots$
  - Automatically recovers from errors!
- Cut and paste is still possible, but more complex (and will cause garbles)

Alice Likes CBC Mode

- Alice's uncompressed image, Alice CBC encrypted (TEA)



- Why does this happen?
- Same plaintext yields different ciphertext!

Counter Mode (CTR)

- CTR is popular for random access
- Use block cipher like a stream cipher

Encryption	**Decryption**
$C_0 = P_0 \oplus E(IV, K)$,	$P_0 = C_0 \oplus E(IV, K)$,
$C_1 = P_1 \oplus E(IV+1, K)$,	$P_1 = C_1 \oplus E(IV+1, K)$,
$C_2 = P_2 \oplus E(IV+2, K)$,…	$P_2 = C_2 \oplus E(IV+2, K)$,…

- Note: CBC also works for random access
  - But there is a significant limitation…

Integrity

Data Integrity

- **Integrity** — detect unauthorized writing (i.e., detect unauthorized mod of data)
- Example: Inter-bank fund transfers
  - Confidentiality may be nice, integrity is ***critical***
- Encryption provides **confidentiality** (prevents unauthorized disclosure)
- Encryption alone does **not** provide integrity
  - One-time pad, ECB cut-and-paste, etc., etc.

MAC

- Message Authentication Code (MAC)
  - Used for data **integrity**
  - Integrity **not** the same as confidentiality
- MAC is computed as **CBC residue**
  - That is, compute CBC encryption, saving only final ciphertext block, the MAC
  - The MAC serves as a cryptographic checksum for data

MAC Computation

- MAC computation (assuming N blocks)

$C_0 = E(IV \oplus P_0, K),$

$C_1 = E(C_0 \oplus P_1, K),$

$C_2 = E(C_1 \oplus P_2, K),\ldots$

$C_{N-1} = E(C_{N-2} \oplus P_{N-1}, K) = MAC$

- Send IV, P_0, P_1, …, P_{N-1} and MAC
- Receiver does same computation and verifies that result agrees with MAC
- Both sender and receiver must know K

Does a MAC work?

- Suppose Alice has 4 plaintext blocks
- Alice computes

  $\mathbf{C_0} = E(IV \oplus P_0, K)$, $\mathbf{C_1} = E(\mathbf{C_0} \oplus P_1, K)$,

  $\mathbf{C_2} = E(\mathbf{C_1} \oplus P_2, K)$, $\mathbf{C_3} = E(\mathbf{C_2} \oplus P_3, K) = \mathbf{MAC}$
- Alice sends IV, P_0, P_1, P_2, P_3 and **MAC** to Bob
- Suppose Trudy changes P_1 to X
- Bob computes

  $\mathbf{C_0} = E(IV \oplus P_0, K)$, $\boldsymbol{C_1} = E(\mathbf{C_0} \oplus X, K)$,

  $\boldsymbol{C_2} = E(\boldsymbol{C_1} \oplus P_2, K)$, $\boldsymbol{C_3} = E(\boldsymbol{C_2} \oplus P_3, K) = \boldsymbol{MAC} \neq \mathbf{MAC}$
- It works since error propagates into **MAC**
- Trudy can't make ***MAC*** == **MAC** without K

Confidentiality and Integrity

- Encrypt with one key, MAC with another key
- Why not use the same key?
  - Send last encrypted block (MAC) twice?
  - This cannot add any security!
- Using different keys to encrypt and compute MAC works, even if keys are related
  - But, twice as much work as encryption alone
  - Can do a little better — about 1.5 “encryptions”
- Confidentiality *and* integrity with same work as one encryption is a research topic

Uses for Symmetric Crypto

- Confidentiality
  - Transmitting data over insecure channel
  - Secure storage on insecure media
- Integrity (MAC)
- Authentication protocols (later…)
- Anything you can do with a hash function (upcoming chapter…)

Chapter 4: Public Key Cryptography

You should not live one way in private, another in public.
— Publilius Syrus

Three may keep a secret, if two of them are dead.
— Ben Franklin

Public Key Cryptography

- Two keys, one to encrypt, another to decrypt
  - Alice uses Bob's **public key** to encrypt
  - Only Bob's **private key** decrypts the message
- Based on "trap door, one way function"
  - "One way" means easy to compute in one direction, but hard to compute in other direction
  - Example: Given p and q, product N = pq easy to compute, but hard to find p and q from N
  - "Trap door" is used when creating key pairs

Public Key Cryptography

- Encryption
  - Suppose we **encrypt** M with Bob's public key
  - Bob's private key can **decrypt** C to recover M
- Digital Signature
  - Bob **signs** by "encrypting" with his private key
  - Anyone can **verify** signature by "decrypting" with Bob's public key
  - But only Bob could have signed
  - Like a handwritten signature, but much better…

Knapsack

Knapsack Problem

- Given a set of n weights $W_0, W_1, \ldots, W_{n-1}$ and a sum S, find $a_i \in \{0,1\}$ so that

  $$S = a_0W_0+a_1W_1 + \ldots + a_{n-1}W_{n-1}$$

  (technically, this is the *subset sum* problem)
- **Example**
  - Weights (62,93,26,52,166,48,91,141)
  - Problem: Find a subset that sums to S = 302
  - Answer: 62 + 26 + 166 + 48 = 302
- The (general) knapsack is NP-complete

Knapsack Problem

- **General knapsack** (GK) is hard to solve
- But **superincreasing knapsack** (SIK) is easy
- SIK — each weight greater than the *sum of all previous weights*
- **Example**
  - Weights (2,3,7,14,30,57,120,251)
  - Problem: Find subset that sums to S = 186
  - Work from largest to smallest weight
  - Answer: 120 + 57 + 7 + 2 = 186

Knapsack Cryptosystem

1. Generate superincreasing knapsack (SIK)
2. Convert SIK to "general" knapsack (GK)
3. **Public Key:** GK
4. **Private Key:** SIK and conversion factor

- Goal…
  - Easy to encrypt with GK
  - With private key, easy to decrypt (solve SIK)
  - Without private key, Trudy has no choice but to try to solve GK

Example

- Start with (2,3,7,14,30,57,120,251) as the SIK
- Choose m = 41 and n = 491 (m, n relatively prime, n exceeds sum of elements in SIK)
- Compute "general" knapsack
  - 2 · 41 mod 491 = 82
  - 3 · 41 mod 491 = 123
  - 7 · 41 mod 491 = 287
  - 14 · 41 mod 491 = 83
  - 30 · 41 mod 491 = 248
  - 57 · 41 mod 491 = 373
  - 120 · 41 mod 491 = 10
  - 251 · 41 mod 491 = 471
- "General" knapsack: (82,123,287,83,248,373,10,471)

Knapsack Example

- **Private key:** (2,3,7,14,30,57,120,251)

  $m^{-1} \bmod n = 41^{-1} \bmod 491 = 12$
- **Public key:** (82,123,287,83,248,373,10,471), n=491
- Example: Encrypt 10010110

  $82 + 83 + 373 + 10 = 548$
- To decrypt, use private key…
  - $548 \cdot 12 = 193 \bmod 491$
  - Solve (easy) SIK with S = 193
  - Obtain plaintext 10010110

Knapsack Weakness

- **Trapdoor:** Convert SIK into "general" knapsack using modular arithmetic
- **One-way:** General knapsack easy to encrypt, hard to solve; SIK easy to solve
- This knapsack cryptosystem is **insecure**
  - Broken in 1983 with Apple II computer
  - The attack uses **lattice reduction**
- "General knapsack" is not general enough!
  - This special case of knapsack is easy to break

RSA

RSA

- Invented by Clifford Cocks (GCHQ) and **R**ivest, **S**hamir, and **A**dleman (MIT)
  - RSA is the *gold standard* in public key crypto
- Let p and q be two large prime numbers
- Let N = pq be the **modulus**
- Choose e relatively prime to (p−1)(q−1)
- Find d such that ed = 1 mod (p−1)(q−1)
- **Public key** is (N,e)
- **Private key** is d

RSA

- Message M is treated as a number
- To encrypt M we compute

  $C = M^e \bmod N$
- To decrypt ciphertext C, we compute

  $M = C^d \bmod N$
- Recall that e and N are public
- If Trudy can factor $N = pq$, she can use e to easily find d since $ed = 1 \bmod (p-1)(q-1)$
- So, **factoring the modulus breaks RSA**
  - Is factoring the only way to break RSA?

Does RSA Really Work?

- Given $C = M^e \bmod N$ we want to show that $M = C^d \bmod N = M^{ed} \bmod N$
- We'll need **Euler's Theorem:**
  If x is relatively prime to n then $x^{\varphi(n)} = 1 \bmod n$
- Facts:
  1) $ed = 1 \bmod (p - 1)(q - 1)$
  2) By definition of "mod", $ed = k(p - 1)(q - 1) + 1$
  3) $\varphi(N) = (p - 1)(q - 1)$
- Then $ed - 1 = k(p - 1)(q - 1) = k\varphi(N)$
- So, $C^d = M^{ed} = M^{(ed - 1) + 1} = M \cdot M^{ed - 1} = M \cdot M^{k\varphi(N)}$
  $= M \cdot (M^{\varphi(N)})^k \bmod N = M \cdot 1^k \bmod N =$ **$M \bmod N$**

Simple RSA Example

- Example of ***textbook*** RSA
  - Select "large" primes p = 11, q = 3
  - Then N = pq = 33 and (p − 1)(q − 1) = 20
  - Choose e = 3 (relatively prime to 20)
  - Find d such that ed = 1 mod 20
    - We find that d = 7 works
- **Public key:** (N, e) = (33, 3)
- **Private key:** d = 7

Simple RSA Example

- **Public key:** (N, e) = (33, 3)
- **Private key:** d = 7
- Suppose message to encrypt is M = 8
- Ciphertext C is computed as

  $C = M^e \bmod N = 8^3 = 512 = 17 \bmod 33$
- Decrypt C to recover the message M by

  $M = C^d \bmod N = 17^7 = 410{,}338{,}673$

  $= 12{,}434{,}505 * 33 + 8 = 8 \bmod 33$

More Efficient RSA (1)

- Modular exponentiation example
  - $5^{20} = 95367431640625 = 25 \bmod 35$
- A better way: **repeated squaring**
  - 20 = 10100 base 2
  - (1, 10, 101, 1010, 10100) = (1, 2, 5, 10, 20)
  - Note that $2 = 1 \cdot 2$, $5 = 2 \cdot 2 + 1$, $10 = 2 \cdot 5$, $20 = 2 \cdot 10$
  - $5^1 = 5 \bmod 35$
  - $5^2 = (5^1)^2 = 5^2 = 25 \bmod 35$
  - $5^5 = (5^2)^2 \cdot 5^1 = 25^2 \cdot 5 = 3125 = 10 \bmod 35$
  - $5^{10} = (5^5)^2 = 10^2 = 100 = 30 \bmod 35$
  - $5^{20} = (5^{10})^2 = 30^2 = 900 = 25 \bmod 35$
- No huge numbers and it's efficient!

More Efficient RSA (2)

- Use **$e = 3$** for all users (but not same N or d)
  - + Public key operations only require 2 multiplies
  - o Private key operations remain expensive
  - - If $M < N^{1/3}$ then $C = M^e = M^3$ and **cube root attack**
  - - For any M, if C_1, C_2, C_3 sent to 3 users, cube root attack works (uses Chinese Remainder Theorem)
- Can prevent cube root attack by padding message with random bits
- Note: $e = 2^{16} + 1$ also used ("better" than $e = 3$)

Diffie-Hellman

Diffie-Hellman Key Exchange

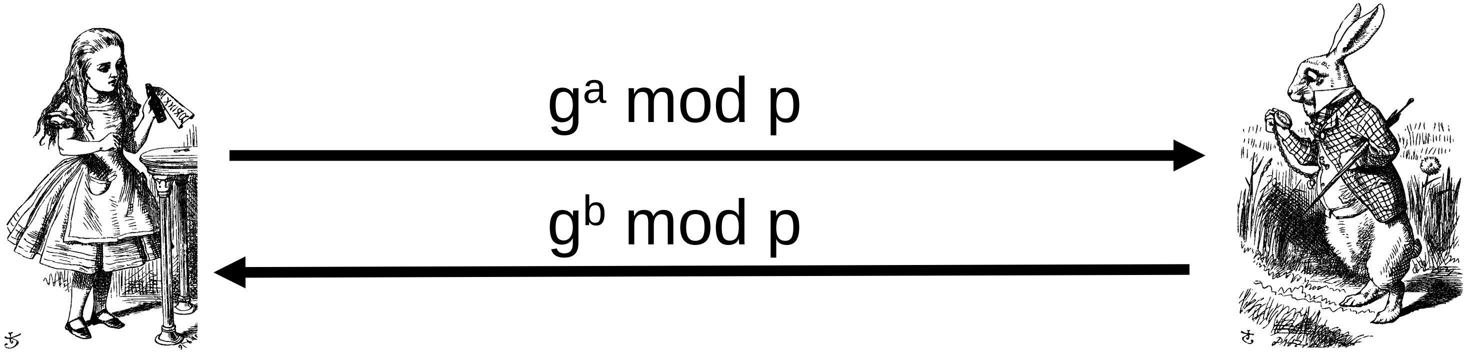
- Invented by Williamson (GCHQ) and, independently, by D and H (Stanford)
- A “key exchange” algorithm
  - Used to establish a shared symmetric key
  - ***Not*** for encrypting or signing
- Based on **discrete log** problem
  - **Given:** g, p, and g^k mod p
  - **Find:** exponent k

Diffie-Hellman

- Let p be prime, let g be a **generator**
  - For any $x \in \{1,2,\ldots,p-1\}$ there is n s.t. $x = g^n \bmod p$
- Alice selects her private value a
- Bob selects his private value b
- Alice sends $g^a \bmod p$ to Bob
- Bob sends $g^b \bmod p$ to Alice
- Both compute shared secret, $g^{ab} \bmod p$
- Shared secret can be used as symmetric key

Diffie-Hellman

- **Public:** g and p
- **Private:** Alice's exponent a, Bob's exponent b



g^a mod p

g^b mod p

Alice, a

Bob, b

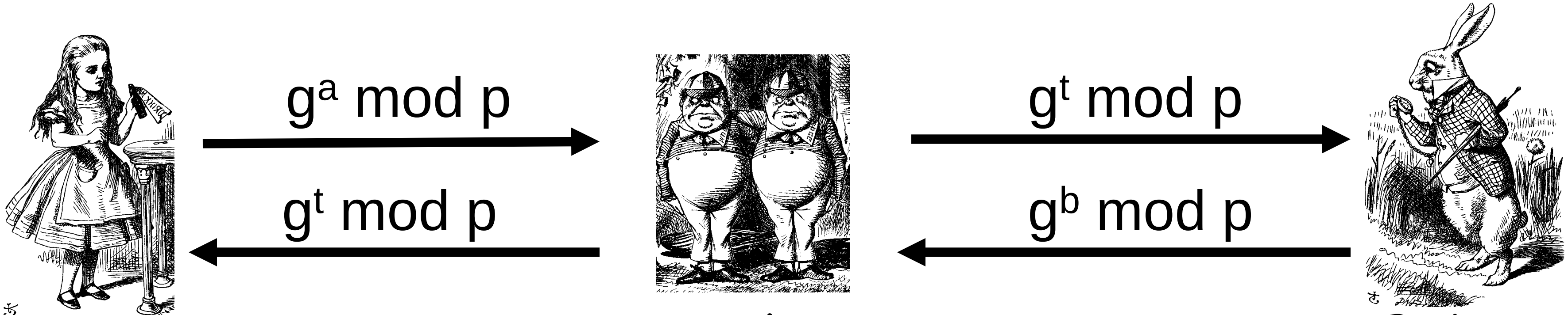
- Alice computes $(g^b)^a = g^{ba} = g^{ab}$ mod p
- Bob computes $(g^a)^b = g^{ab}$ mod p
- They can use $K = g^{ab}$ mod p as symmetric key

Diffie-Hellman

- Suppose Bob and Alice use Diffie-Hellman to determine symmetric key K = g^{ab} mod p
- Trudy can see g^a mod p and g^b mod p
  - But… $g^a g^b$ mod p = g^{a+b} mod p ≠ g^{ab} mod p
- If Trudy can find a or b, she gets K
- If Trudy can solve **discrete log** problem, she can find a or b

Diffie-Hellman

- Subject to man-in-the-middle (MiM) attack



g^a mod p

g^t mod p

g^t mod p

g^b mod p

Alice, a

Trudy, t

Bob, b

- Trudy shares secret g^{at} mod p with Alice
- Trudy shares secret g^{bt} mod p with Bob
- Alice and Bob don't know Trudy is MiM

Diffie-Hellman

- How to prevent MiM attack?
    - Encrypt DH exchange with symmetric key
    - Encrypt DH exchange with public key
    - Sign DH values with private key
    - Other?
- At this point, DH may look pointless…
    - …but it's not (more on this later)
- You **MUST** be aware of MiM attack on Diffie-Hellman

Elliptic Curve Cryptography

Elliptic Curve Crypto (ECC)

- "Elliptic curve" is **not** a cryptosystem
- Elliptic curves provide different way to do the math in public key system
- Elliptic curve versions of DH, RSA, …
- Elliptic curves are more efficient
  - Fewer bits needed for same security
  - But the operations are more complex, yet it is a big "win" overall

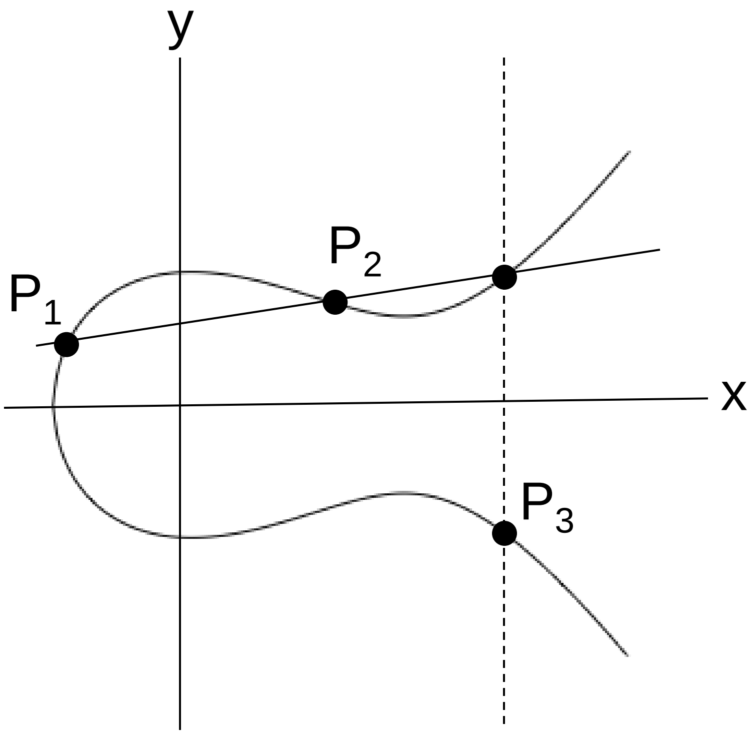
What is an Elliptic Curve?

- An elliptic curve E is the graph of an equation of the form

$$y^2 = x^3 + ax + b$$

- Also includes a "point at infinity"
- What do elliptic curves look like?
- See the next slide!

Elliptic Curve Picture



- Consider elliptic curve

  E: $y^2 = x^3 - x + 1$
- If P_1 and P_2 are on E, we can define addition,

  $$P_3 = P_1 + P_2$$

  as shown in picture
- Addition is all we need…

Points on Elliptic Curve

- Consider $y^2 = x^3 + 2x + 3 \pmod{5}$

  $x = 0 \Rightarrow y^2 = 3 \Rightarrow$ no solution (mod 5)

  $x = 1 \Rightarrow y^2 = 6 = 1 \Rightarrow y = 1,4 \pmod{5}$

  $x = 2 \Rightarrow y^2 = 15 = 0 \Rightarrow y = 0 \pmod{5}$

  $x = 3 \Rightarrow y^2 = 36 = 1 \Rightarrow y = 1,4 \pmod{5}$

  $x = 4 \Rightarrow y^2 = 75 = 0 \Rightarrow y = 0 \pmod{5}$

- Then points on the elliptic curve are

  (1,1) (1,4) (2,0) (3,1) (3,4) (4,0) and the point at infinity: ∞

Elliptic Curve Math

- Addition on: $y^2 = x^3 + ax + b \pmod{p}$

  $P_1=(x_1,y_1)$, $P_2=(x_2,y_2)$

  $P_1 + P_2 = P_3 = (x_3,y_3)$ where

  $x_3 = m^2 - x_1 - x_2 \pmod{p}$

  $y_3 = m(x_1 - x_3) - y_1 \pmod{p}$

  And $m = (y_2-y_1)*(x_2-x_1)^{-1} \bmod p$, if $P_1 \neq P_2$

  $m = (3x_1{}^2+a)*(2y_1)^{-1} \bmod p$, if $P_1 = P_2$

  Special cases: If m is infinite, $P_3 = \infty$, and

  $\infty + P = P$ for all P

Elliptic Curve Addition

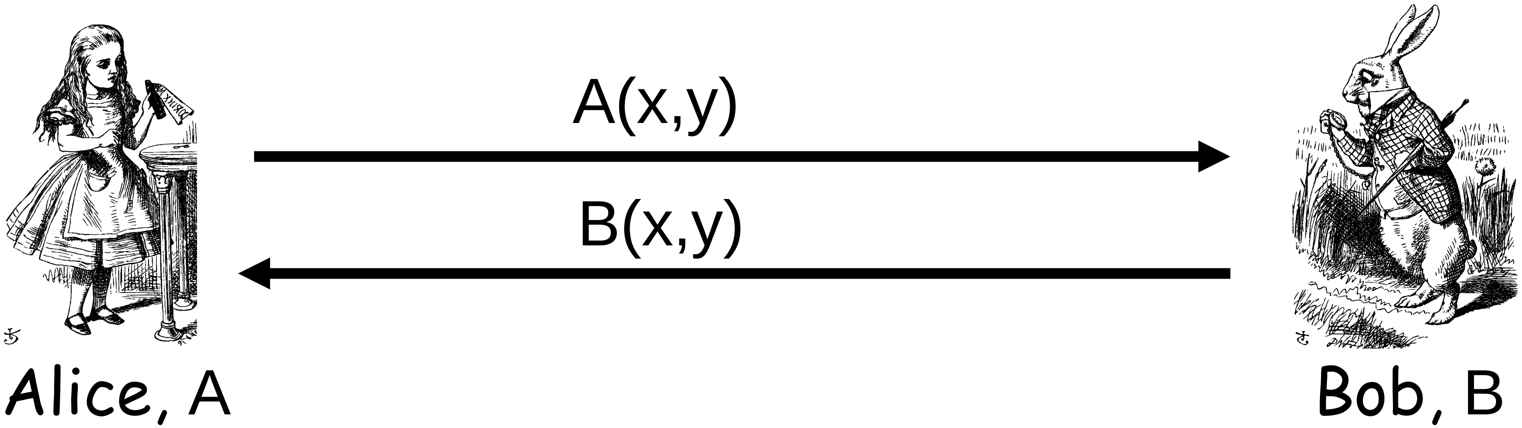
- Consider $y^2 = x^3 + 2x + 3 \pmod 5$. Points on the curve are (1,1) (1,4) (2,0) (3,1) (3,4) (4,0) and ∞
- What is (1,4) + (3,1) = $P_3 = (x_3,y_3)$?

$$m = (1-4)*(3-1)^{-1} = -3*2^{-1}$$
$$= 2(3) = 6 = 1 \pmod 5$$
$$x_3 = 1 - 1 - 3 = 2 \pmod 5$$
$$y_3 = 1(1-2) - 4 = 0 \pmod 5$$

- On this curve, (1,4) + (3,1) = (2,0)

ECC Diffie-Hellman

- **Public:** Elliptic curve and point (x,y) on curve
- **Private:** Alice's A and Bob's B



A(x,y)

B(x,y)

Alice, A

Bob, B

- Alice computes A(B(x,y))
- Bob computes B(A(x,y))
- These are the same since AB = BA

ECC Diffie-Hellman

- **Public:** Curve $y^2 = x^3 + 7x + b \pmod{37}$ and point $(2,5) \Rightarrow b = 3$
- **Alice's private:** A = 4
- **Bob's private:** B = 7
- Alice sends Bob: 4(2,5) = (7,32)
- Bob sends Alice: 7(2,5) = (18,35)
- Alice computes: 4(18,35) = (22,1)
- Bob computes: 7(7,32) = (22,1)

Larger ECC Example

- Example from Certicom ECCp-109
  - Challenge problem, solved in 2002
- Curve E: $y^2 = x^3 + ax + b \pmod{p}$
- Where

$$p = 564538252084441556247016902735257$$

$$a = 321094768129147601892514872825668$$

$$b = 430782315140218274262276694323197$$

- Now what?

ECC Example

- The following point P is on the curve E

(x,y) = (97339010987059066523156133908935,
149670372846169285760682371978898)

- Let k = 281183840311601949668207954530684
- The kP is given by

(x,y) = (44646769697405861057630861884284,
522968098895785888047540374779097)

- And this point is also on the curve E

Really Big Numbers!

- Numbers are big, but not big enough
  - ECCp-109 bit (32 digit) solved in 2002
- Today, ECC DH needs bigger numbers
- But RSA needs way bigger numbers
  - Minimum RSA modulus today is 1024 bits
  - That is, more than 300 decimal digits
  - That's about 10x the size in ECC example
  - And 2048 bit RSA modulus is common…

Uses for Public Key Crypto

Uses for Public Key Crypto

- Confidentiality
  - Transmitting data over insecure channel
  - Secure storage on insecure media
- Authentication protocols (later)
- Digital signature
  - Provides integrity and **non-repudiation**
  - No non-repudiation with symmetric keys

Non-non-repudiation

- Alice orders 100 shares of stock from Bob
- Alice computes **MAC** using symmetric key
- Stock drops, Alice claims she did ***not*** order
- Can Bob prove that Alice placed the order?
- **No!** Bob also knows the symmetric key, so he could have forged the **MAC**
- **Problem:** Bob knows Alice placed the order, but he can't prove it

Non-repudiation

- Alice orders 100 shares of stock from Bob
- Alice **signs** order with her private key
- Stock drops, Alice claims she did not order
- Can Bob prove that Alice placed the order?
- **Yes!** Alice's private key used to sign the order — only Alice knows her private key
- This assumes Alice's private key has not been lost/stolen

Public Key Notation

- **Sign** message M with *Alice's* **private key:** $[M]_{\text{Alice}}$
- **Encrypt** message M with *Alice's* **public key:** $\{M\}_{\text{Alice}}$
- Then

$$\{[M]_{\text{Alice}}\}_{\text{Alice}} = M$$

$$[\{M\}_{\text{Alice}}]_{\text{Alice}} = M$$

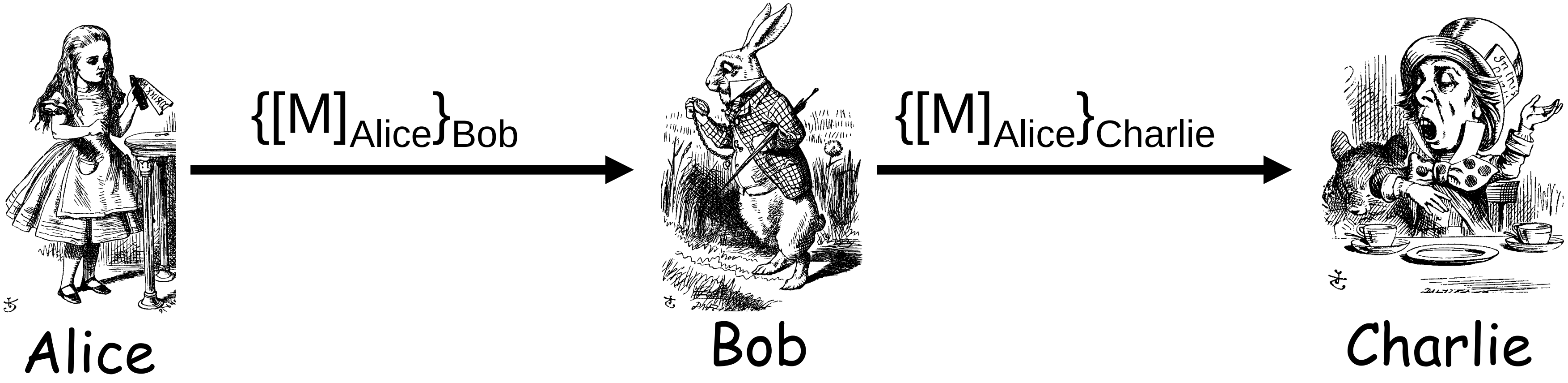
Sign and Encrypt
vs
Encrypt and Sign

Confidentiality and Non-repudiation?

- Suppose that we want confidentiality and integrity/non-repudiation
- Can public key crypto achieve both?
- Alice sends message to Bob
  - **Sign and encrypt:** $\{[M]_{\text{Alice}}\}_{\text{Bob}}$
  - **Encrypt and sign:** $[\{M\}_{\text{Bob}}]_{\text{Alice}}$
- Can the order possibly matter?

Sign and Encrypt

- M = “I love you”



$\{[M]_{\text{Alice}}\}_{\text{Bob}}$

$\{[M]_{\text{Alice}}\}_{\text{Charlie}}$

Alice

Bob

Charlie

- **Q:** What’s the problem?
- **A:** No problem — public key is public

Encrypt and Sign

- M = "My theory, which is mine…."

$[\{M\}_{Bob}]_{Alice}$

$[\{M\}_{Bob}]_{Charlie}$

Alice

Charlie

Bob

- **Note** that Charlie cannot decrypt M
- **Q:** What is the problem?
- **A:** No problem — public key is public

Public Key Infrastructure

Public Key Certificate

- Digital **certificate** contains name of user and user's public key (possibly other info too)
- It is ***signed*** by the issuer, a ***Certificate Authority*** (CA), such as VeriSign

  M = (Alice, Alice's public key), $S = [M]_{CA}$

  Alice's Certificate = (M, S)

- Signature on certificate is verified using CA's public key

  Must verify that $M = \{S\}_{CA}$

Certificate Authority

- Certificate authority (CA) is a trusted 3rd party (TTP) — creates and signs certificates
- Verify signature to verify **integrity** & identity of **owner of corresponding private key**
    - Does **not** verify the identity of the **sender** of certificate — certificates are public!
- Big problem if CA makes a mistake
    - CA once issued Microsoft cert. to someone else
- A common format for certificates is X.509

PKI

- Public Key Infrastructure (PKI): the stuff needed to securely use public key crypto
    - Key generation and management
    - Certificate authority (CA) or authorities
    - Certificate revocation lists (CRLs), etc.
- No general standard for PKI
- We mention 3 generic "trust models"
    - We only discuss the CA (or CAs)

PKI Trust Models

- Monopoly model
  - One universally trusted organization is the CA for the known universe
  - Big problems if CA is ever compromised
  - Who will act as CA ???
    - System is useless if you don't trust the CA!

PKI Trust Models

- Oligarchy
  - Multiple (as in, “a few”) trusted CAs
  - This approach is used in browsers today
  - Browser may have 80 or more CA certificates, just to verify certificates!
  - User can decide which CA or CAs to trust

PKI Trust Models

- Anarchy model
  - Everyone is a CA…
  - Users must decide who to trust
  - This approach used in PGP: "Web of trust"
- Why is it anarchy?
  - Suppose certificate is signed by Frank and you don't know Frank, but you do trust Bob and Bob says Alice is trustworthy and Alice vouches for Frank. Should you accept the certificate?
- **Many** other trust models/PKI issues

Confidentiality in the Real World

Symmetric Key vs Public Key

- Symmetric key +'s
  - **Speed**
  - No public key infrastructure (PKI) needed (but have to generate/distribute keys)
- Public Key +'s
  - **Signatures** (non-repudiation)
  - No ***shared*** secret (but, do have to get private keys to the right user…)

Notation Reminder

- Public key notation
  - Sign M with Alice's **private key**

    $[M]_{Alice}$
  - Encrypt M with Alice's **public key**

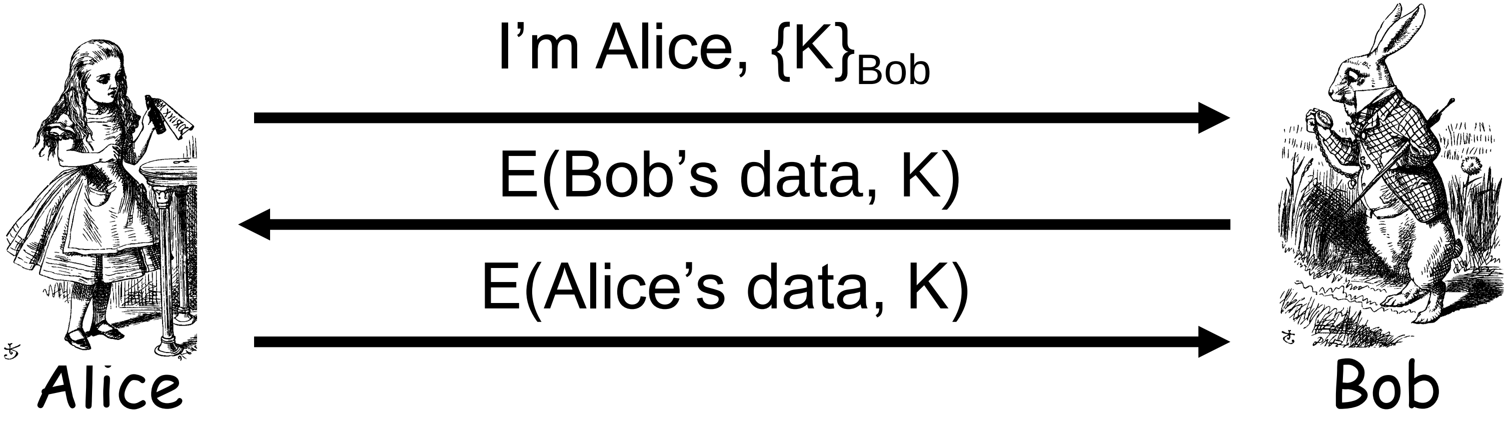
    $\{M\}_{Alice}$
- Symmetric key notation
  - Encrypt P with **symmetric key** K

    $C = E(P,K)$
  - Decrypt C with **symmetric key** K

    $P = D(C,K)$

Real World Confidentiality

- **Hybrid cryptosystem**
  - Public key crypto to establish a key
  - Symmetric key crypto to encrypt data…



I'm Alice, $\{K\}_{Bob}$

E(Bob's data, K)

E(Alice's data, K)

Alice

Bob

- Can Bob be sure he's talking to Alice?

Chapter 5: Hash Functions++

“I'm sure [my memory] only works one way.” Alice remarked.
“I can't remember things before they happen.”
“It's a poor sort of memory that only works backwards,”
the Queen remarked.
“What sort of things do you remember best?" Alice ventured to ask.
“Oh, things that happened the week after next,"
the Queen replied in a careless tone.
— Lewis Carroll, *Through the Looking Glass*

Chapter 5: Hash Functions++

A boat, beneath a sunny sky
Lingering onward dreamily
In an evening of July —
Children three that nestle near,
Eager eye and willing ear,
...
— Lewis Carroll, *Through the Looking Glass*

Hash Function Motivation

- Suppose Alice signs M
  - Alice sends M and $S = [M]_{\text{Alice}}$ to Bob
  - Bob verifies that $M = \{S\}_{\text{Alice}}$
  - Can Alice just send S?
- If M is big, $[M]_{\text{Alice}}$ costly to ***compute*** & ***send***
- Suppose instead, Alice signs h(M), where h(M) is a much smaller "fingerprint" of M
  - Alice sends M and $S = [h(M)]_{\text{Alice}}$ to Bob
  - Bob verifies that $h(M) = \{S\}_{\text{Alice}}$

Hash Function Motivation

- So, Alice signs h(M)
    - That is, Alice computes $S = [h(M)]_{\text{Alice}}$
    - Alice then sends (M, S) to Bob
    - Bob verifies that $h(M) = \{S\}_{\text{Alice}}$
- What properties must h(M) satisfy?
    - Suppose Trudy finds M’ so that h(M) = h(M’)
    - Then Trudy can replace (M, S) with (M’, S)
- Does Bob detect this tampering?
    - No, since $h(M') = h(M) = \{S\}_{\text{Alice}}$

Crypto Hash Function

- Crypto hash function h(x) must provide
  - **Compression** — output length is small
  - **Efficiency** — h(x) easy to compute for any x
  - **One-way** — given a value y it is infeasible to find an x such that h(x) = y
  - **Weak collision resistance** — given x and h(x), infeasible to find y ≠ x such that h(y) = h(x)
  - **Strong collision resistance** — infeasible to find ***any*** x and y, with x ≠ y such that h(x) = h(y)
- Lots of collisions exist, but hard to find ***any***

Pre-Birthday Problem

- Suppose N people in a room
- How large must N be before the probability someone has same birthday as me is $\geq 1/2$?
    - Solve: $1/2 = 1 - (364/365)^N$ for N
    - We find $N = 253$

Birthday Problem

- How many people must be in a room before probability is ≥ 1/2 that any two (or more) have same birthday?
  - 1 – 365/365 · 364/365 · · ·(365–N+1)/365
  - Set equal to 1/2 and solve: **N = 23**
- Surprising? A paradox?
- Maybe not: "Should be" about sqrt(365) since we compare all **pairs** x and y
  - And there are 365 possible birthdays

Of Hashes and Birthdays

- If h(x) is N bits, then 2^N different hash values are possible
- So, if you hash about $\mathrm{sqrt}(2^N) = 2^{N/2}$ values then you expect to find a collision
- **Implication?** "Exhaustive search" attack...
  - Secure N-bit hash requires $2^{N/2}$ work to "break"
  - Recall that secure N-bit symmetric cipher has work factor of 2^{N-1}
- Hash output length vs cipher key length?

Non-crypto Hash (1)

- Data $X = (X_1,X_2,X_3,\ldots,X_n)$, each X_i is a byte
- Define $h(X) = (X_1+X_2+X_3+\ldots+X_n) \bmod 256$
- Is this a secure cryptographic hash?
- Example: X = (10101010, 00001111)
- Hash is h(X) = 10111001
- If Y = (00001111, 10101010) then h(X) = h(Y)
- Easy to find collisions, so **not** secure…

Non-crypto Hash (2)

- Data $X = (X_0,X_1,X_2,\ldots,X_{n-1})$
- Suppose hash is defined as

  $h(X) = (nX_1+(n-1)X_2+(n-2)X_3+\ldots+2\cdot X_{n-1}+X_n) \bmod 256$
- Is this a secure cryptographic hash?
- Note that

  $h(10101010, 00001111) \neq h(00001111, 10101010)$
- But hash of (00000001, 00001111) is same as hash of (00000000, 00010001)
- Not "secure", but this hash is used in the (non-crypto) application rsync

Non-crypto Hash (3)

- Cyclic Redundancy Check (CRC)
- Essentially, CRC is the remainder in a long division calculation
- Good for detecting burst **errors**
  - Such random errors unlikely to yield a collision
- But easy to ***construct*** collisions
  - In crypto, Trudy is the enemy, not "random"
- CRC has been mistakenly used where crypto integrity check is required (e.g., WEP)

Popular Crypto Hashes

- **MD5** — invented by Rivest (of course…)
  - 128 bit output
  - MD5 collisions easy to find, so it's broken
- **SHA-1** — A U.S. government standard, inner workings similar to MD5
  - 160 bit output
- Many other hashes, but MD5 and SHA-1 are the most widely used
- Hashes work by hashing message in blocks

Crypto Hash Design

- Desired property: **avalanche effect**
  - Change to 1 bit of input should affect about half of output bits
- Crypto hash functions consist of some number of rounds
- Want security and speed
  - "Avalanche effect" after few rounds
  - But simple rounds
- Analogous to design of block ciphers

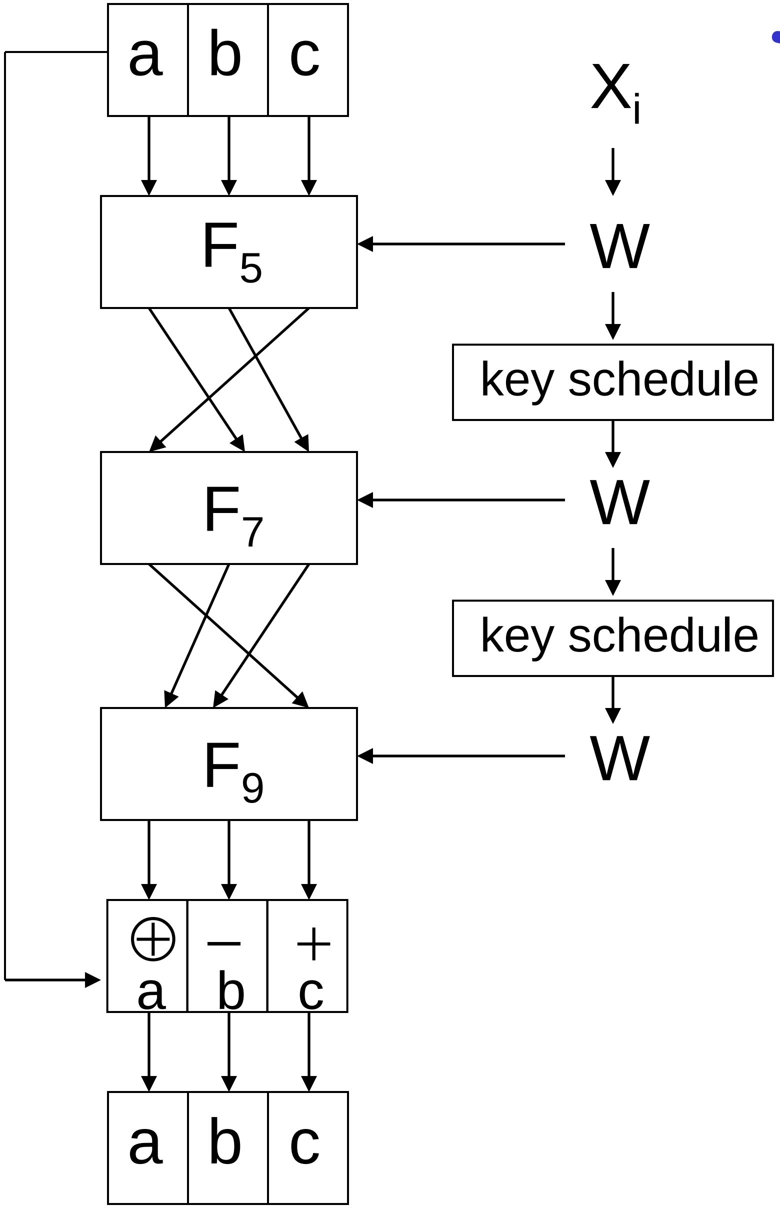
Tiger Hash

- "Fast and strong"
- Designed by Ross Anderson and Eli Biham — leading cryptographers
- Design criteria
  - Secure
  - Optimized for **64-bit** processors
  - Easy replacement for MD5 or SHA-1

Tiger Hash

- Like MD5/SHA-1, input divided into 512 bit blocks (padded)
- Unlike MD5/SHA-1, output is **192 bits** (three 64-bit words)
  - Truncate output if replacing MD5 or SHA-1
- Intermediate rounds are all 192 bits
- 4 S-boxes, each maps 8 bits to 64 bits
- A "key schedule" is used

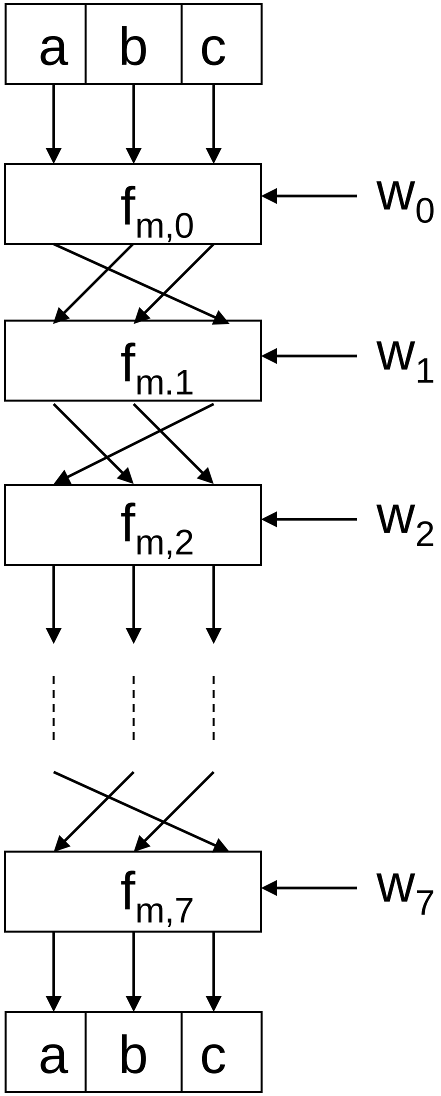
Tiger Outer Round



- Input is X
    - $X = (X_0,X_1,\ldots,X_{n-1})$
    - X is padded
    - Each X_i is 512 bits
- There are n iterations of diagram at left
    - One for each input block
- Initial (a,b,c) constants
- Final (a,b,c) is hash
- Looks like block cipher!

Tiger Inner Rounds

- Each F_m consists of precisely **8 rounds**
- 512 bit input W to F_m
  - $W=(w_0,w_1,\ldots,w_7)$
  - W is one of the input blocks X_i
- All lines are 64 bits
- The $f_{m,i}$ depend on the S-boxes (next slide)



Tiger Hash: One Round

- Each $f_{m,i}$ is a function of a,b,c,w_i and m
  - Input values of a,b,c from previous round
  - And w_i is 64-bit block of 512 bit W
  - Subscript m is multiplier
  - And $c = (c_0,c_1,\ldots,c_7)$
- Output of $f_{m,i}$ is
  - $c = c \oplus w_i$
  - $a = a – (S_0[c_0] \oplus S_1[c_2] \oplus S_2[c_4] \oplus S_3[c_6])$
  - $b = b + (S_3[c_1] \oplus S_2[c_3] \oplus S_1[c_5] \oplus S_0[c_7])$
  - $b = b * m$
- Each S_i is **S-box**: 8 bits mapped to 64 bits

Tiger Hash Key Schedule

- Input is X
  - $X=(x_0,x_1,\ldots,x_7)$
- Small change in X will produce large change in key schedule output

$x_0 = x_0 - (x_7 \oplus \text{0xA5A5A5A5A5A5A5A5})$

$x_1 = x_1 \oplus x_0$

$x_2 = x_2 + x_1$

$x_3 = x_3 - (x_2 \oplus ((\sim x_1) << 19))$

$x_4 = x_4 \oplus x_3$

$x_5 = x_5 + x_4$

$x_6 = x_6 - (x_5 \oplus ((\sim x_4) >> 23))$

$x_7 = x_7 \oplus x_6$

$x_0 = x_0 + x_7$

$x_1 = x_1 - (x_0 \oplus ((\sim x_7) << 19))$

$x_2 = x_2 \oplus x_1$

$x_3 = x_3 + x_2$

$x_4 = x_4 - (x_3 \oplus ((\sim x_2) >> 23))$

$x_5 = x_5 \oplus x_4$

$x_6 = x_6 + x_5$

$x_7 = x_7 - (x_6 \oplus \text{0x0123456789ABCDEF})$

Tiger Hash Summary (1)

- Hash and intermediate values are 192 bits
- 24 (inner) rounds
  - **S-boxes:** Claimed that each input bit affects a, b and c after 3 rounds
  - **Key schedule:** Small change in message affects many bits of intermediate hash values
  - **Multiply:** Designed to ensure that input to S-box in one round mixed into many S-boxes in next
- S-boxes, key schedule and multiply together designed to ensure strong **avalanche** effect

Tiger Hash Summary (2)

- Uses lots of ideas from block ciphers
  - S-boxes
  - Multiple rounds
  - Mixed mode arithmetic
- At a higher level, Tiger employs
  - Confusion
  - Diffusion

HMAC

- Can compute a MAC of the message M with key K using a "hashed MAC" or **HMAC**
- HMAC is a ***keyed*** hash
  - Why would we need a key?
- How to compute HMAC?
- Two obvious choices: h(K,M) and h(M,K)
- Which is better?

HMAC

- Should we compute HMAC as $h(K,M)$?
- Hashes computed in blocks
  - $h(B_1,B_2) = F(F(A,B_1),B_2)$ for some F and constant A
  - Then $h(B_1,B_2) = F(h(B_1),B_2)$
- Let $M' = (M,X)$
  - Then $h(K,M') = F(h(K,M),X)$
  - Attacker can compute HMAC of M' without K
- Is $h(M,K)$ better?
  - Yes, but… if $h(M') = h(M)$ then we might have $h(M,K)=F(h(M),K)=F(h(M'),K)=h(M',K)$

Correct Way to HMAC

- Described in RFC 2104
- Let B be the block length of hash, in bytes
  - B = 64 for MD5 and SHA-1 and Tiger
- ipad = 0x36 repeated B times
- opad = 0x5C repeated B times
- Then

$$\text{HMAC}(M,K) = h(K \oplus \text{opad}, h(K \oplus \text{ipad}, M))$$

Hash Uses

- Authentication (HMAC)
- Message integrity (HMAC)
- Message fingerprint
- Data corruption detection
- Digital signature efficiency
- Anything you can do with symmetric crypto
- Also, many, many clever/surprising uses…

Online Bids

- Suppose Alice, Bob and Charlie are bidders
- Alice plans to bid A, Bob B and Charlie C
- They don't trust that bids will stay secret
- A possible solution?
  - Alice, Bob, Charlie submit **hashes** h(A), h(B), h(C)
  - All hashes received and posted online
  - Then bids A, B, and C submitted and revealed
- Hashes don't reveal bids (one way)
- Can't change bid after hash sent (collision)
- But there is a serious flaw here…

Hashing for Spam Reduction

- Spam reduction
- Before accept email, want proof that sender had to "work" to create email
  - Here, "work" == CPU cycles
- Goal is to limit the amount of email that can be sent
  - This approach will not eliminate spam
  - Instead, make spam more costly to send

Spam Reduction

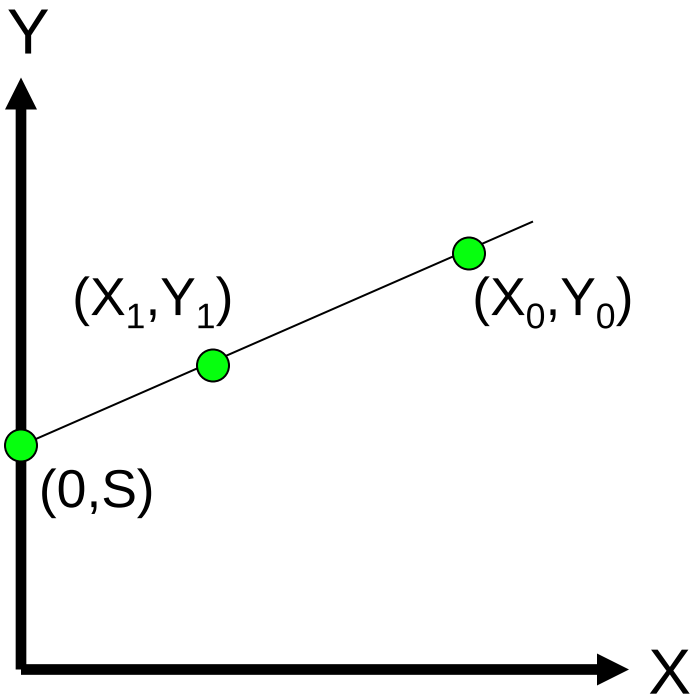
- Let M = complete email message
  R = value to be determined
  T = current time
- Sender must determine **R** so that
  h(M,**R**,T) = (00…0,X), that is,
  initial N bits of hash value are **all zero**
- Sender then sends (M,**R**,T)
- Recipient accepts email, provided that…
  h(M,**R**,T) begins with N zeros

Spam Reduction

- Sender: h(M,R,T) begins with N zeros
- Recipient: verify that h(M,R,T) begins with N zeros
- **Work for sender:** on average **2^N hashes**
- **Work for recipient:** always **1 hash**
- Sender's work increases exponentially in N
- Small work for recipient, regardless of N
- Choose N so that…
  - Work acceptable for normal amounts of email
  - Work is too high for spammers

Secret Sharing

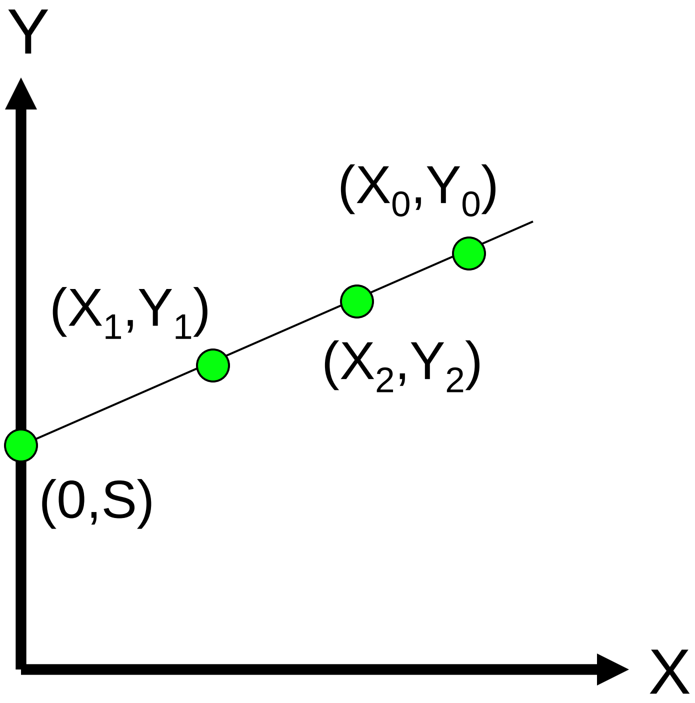
Shamir's Secret Sharing



2 out of 2

- Two points determine a line
- Give (X_0,Y_0) to Alice
- Give (X_1,Y_1) to Bob
- Then Alice and Bob must cooperate to find secret S
- Also works in discrete case
- Easy to make "m out of n" scheme for any $m \leq n$

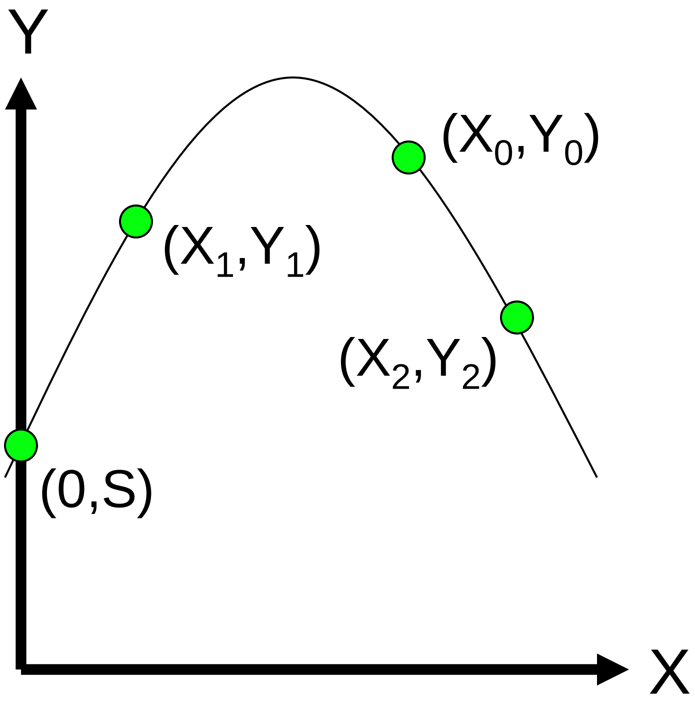
Shamir's Secret Sharing



2 out of 3

- Give (X_0,Y_0) to Alice
- Give (X_1,Y_1) to Bob
- Give (X_2,Y_2) to Charlie
- Then any ***two*** can cooperate to find secret S
- No ***one*** can determine S
- A "2 out of 3" scheme

Shamir's Secret Sharing



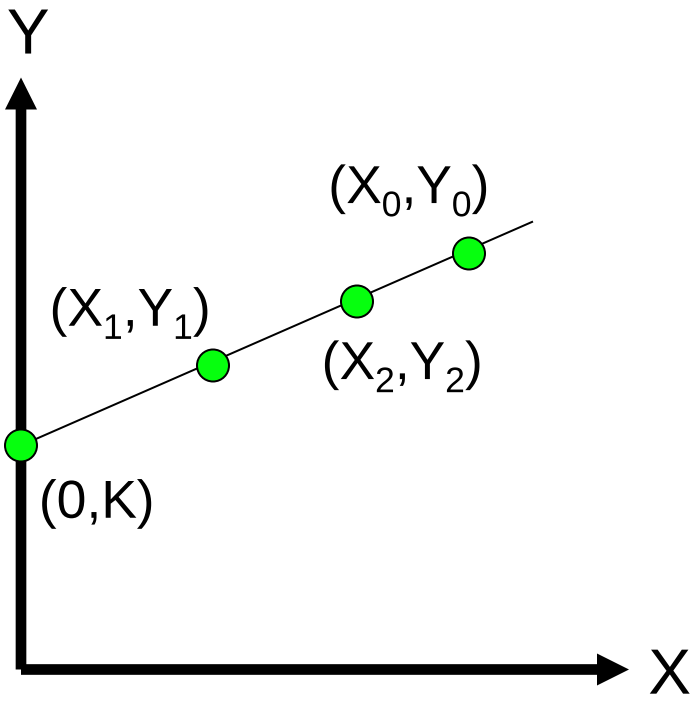
3 out of 3

- Give (X_0,Y_0) to Alice
- Give (X_1,Y_1) to Bob
- Give (X_2,Y_2) to Charlie
- 3 pts determine parabola
- Alice, Bob, **and** Charlie must cooperate to find S
- A "3 out of 3" scheme
- What about "3 out of 4"?

Secret Sharing Use?

- **Key escrow** — suppose it's required that your key be stored somewhere
- Key can be "recovered" with court order
- But you don't trust FBI to store your keys
- We can use secret sharing
    - Say, three different government agencies
    - Two must cooperate to recover the key

Secret Sharing Example



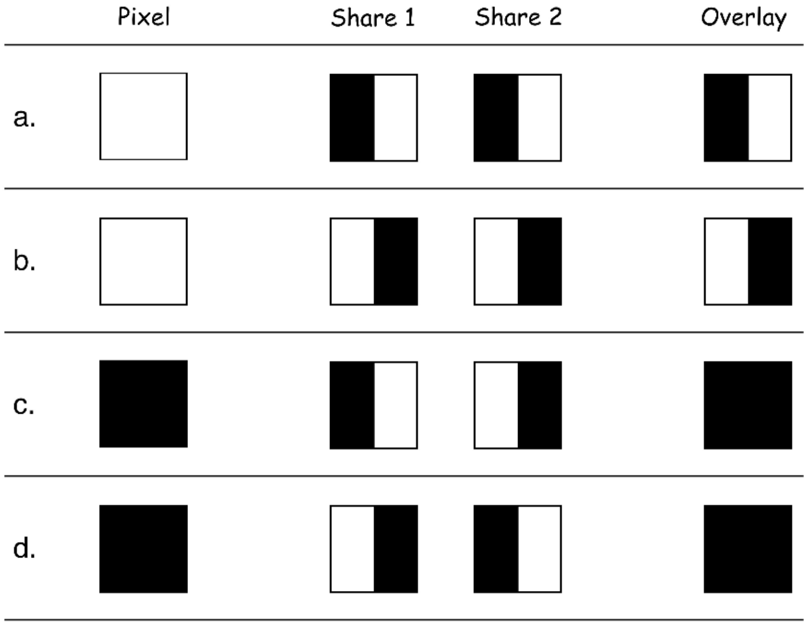
- Your symmetric key is K
- Point (X_0,Y_0) to FBI
- Point (X_1,Y_1) to DoJ
- Point (X_2,Y_2) to DoC
- To recover your key K, two of the three agencies must cooperate
- No one agency can get K

Visual Cryptography

- Another form of secret sharing…
- Alice and Bob "share" an image
- Both must cooperate to reveal the image
- Nobody can learn anything about image from Alice's share or Bob's share
  - That is, both shares are required
- Is this possible?

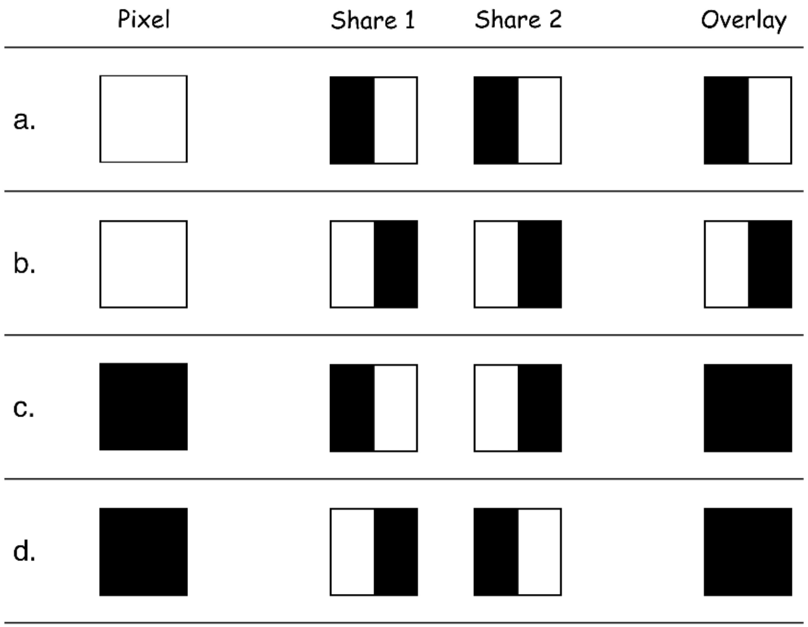
Visual Cryptography

- How to "share" a pixel?
- Suppose image is black and white
- Then each pixel is either black or white
- We split pixels as shown



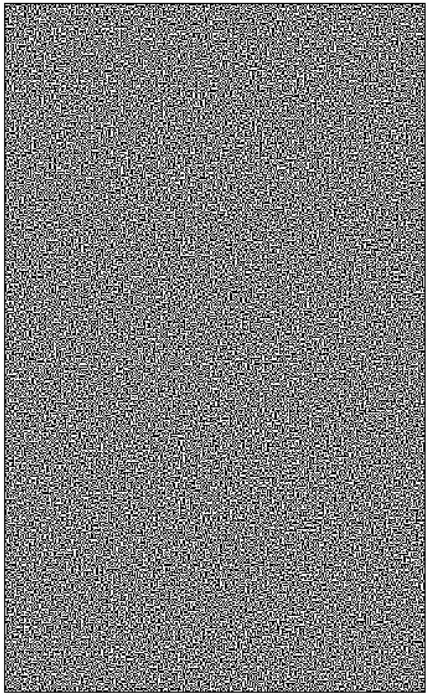
Sharing Black & White Image

- ☐ If pixel is white, randomly choose a or b for Alice's/Bob's shares
- ☐ If pixel is black, randomly choose c or d
- ☐ **No information** in one "share"

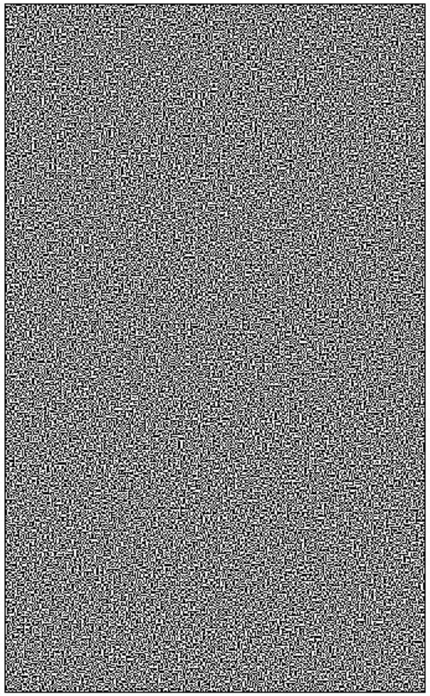


Visual Crypto Example

- Alice's share



- Bob's share



- Overlaid shares

Visual Crypto

- How does visual "crypto" compare to regular crypto?
- In visual crypto, no key…
  - Or, maybe both images are the key?
- With encryption, exhaustive search
  - Except for the one-time pad
- Exhaustive search on visual crypto?
  - No exhaustive search is possible!

Visual Crypto

- Visual crypto — no exhaustive search…
- How does visual crypto compare to crypto?
  - Visual crypto is "information theoretically" secure — also true of secret sharing schemes
  - With regular encryption, goal is to make cryptanalysis computationally infeasible
- Visual crypto an example of **secret sharing**
  - Not really a form of crypto, in the usual sense

Random Numbers in Cryptography

Random Numbers

- Random numbers used to generate **keys**
  - Symmetric keys
  - RSA: Prime numbers
  - Diffie Hellman: secret values
- Random numbers used for nonces
  - Sometimes a sequence is OK
  - But sometimes nonces must be random
- Random numbers also used in simulations, statistics, etc.
  - In such apps, need "statistically" random numbers

Random Numbers

- ***Cryptographic random*** numbers must be statistically random and **unpredictable**
- Suppose server generates symmetric keys
  - Alice: K_A
  - Bob: K_B
  - Charlie: K_C
  - Dave: K_D
- Alice, Bob, and Charlie don't like Dave…
- Alice, Bob, and Charlie, working together, must not be able to determine K_D

Non-random Random Numbers

- Online version of Texas Hold 'em Poker
  - ASF Software, Inc.

- Random numbers used to shuffle the deck
- Program did not produce a random shuffle
- A serious problem, or not?

Card Shuffle

- There are 52! > 2^{225} possible shuffles
- The poker program used "random" 32-bit integer to determine the shuffle
  - So, only 2^{32} distinct shuffles could occur
- Code used Pascal pseudo-random number generator (PRNG): Randomize()
- Seed value for PRNG was function of number of milliseconds since midnight
- Less than 2^{27} milliseconds in a day
  - So, less than 2^{27} possible shuffles

Card Shuffle

- Seed based on milliseconds since midnight
- PRNG re-seeded with each shuffle
- By synchronizing clock with server, number of shuffles that need to be tested $< 2^{18}$
- Could then test all 2^{18} in real time
  - Test each possible shuffle against "up" cards
- Attacker knows **every card** after the first of five rounds of betting!

Poker Example

- Poker program is an extreme example
  - But common PRNGs are predictable
  - Only a question of how many outputs must be observed before determining the sequence
- Crypto random sequences not predictable
  - For example, keystream from RC4 cipher
  - But "seed" (or key) selection is still an issue!
- How to generate initial **random** values?
  - Keys (and, in some cases, seed values)

What is Random?

- True “random” is hard to define
- **Entropy** is a measure of randomness
- Good sources of “true” randomness
  - Radioactive decay — but, radioactive computers are not too popular
  - Hardware devices — many good ones on the market
  - Lava lamp — relies on chaotic behavior

Randomness

- Sources of randomness via software
  - Software is supposed to be deterministic
  - So, must rely on external "random" events
  - Mouse movements, keyboard dynamics, network activity, etc., etc.
- Can get **quality** random bits by such methods
- But **quantity** of bits is very limited
- Bottom line: "The use of pseudo-random processes to generate secret quantities can result in pseudo-security"

Information Hiding

Information Hiding

- Digital Watermarks
  - Example: Add "invisible" info to data
  - Defense against music/software piracy
- Steganography
  - "Secret" communication channel
  - Similar to a **covert channel** (more later)
  - Example: Hide data in an image file

Watermark

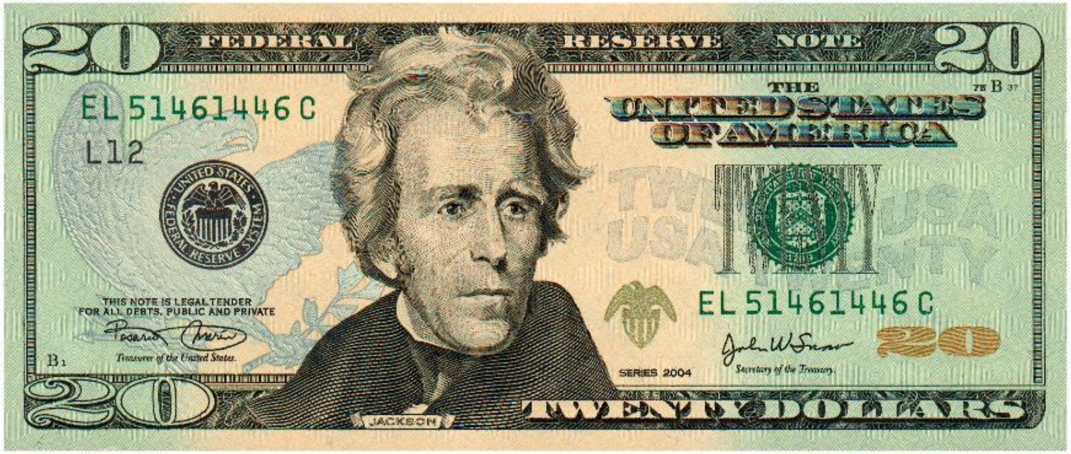
- Add a “mark” to data
- Visibility (or not) of watermarks
  - Invisible — Watermark is not obvious
  - Visible — Such as **TOP SECRET**
- Strength (or not) of watermarks
  - Robust — Readable even if attacked
  - Fragile — Damaged if attacked

Watermark Examples

- Add **robust invisible** mark to digital music
  - If pirated music appears on Internet, can trace it back to original source of the leak
- Add **fragile invisible** mark to audio file
  - If watermark is unreadable, recipient knows that audio has been tampered with (integrity)
- Combinations of several types are sometimes used
  - E.g., visible plus robust invisible watermarks

Watermark Example (1)

- Non-digital watermark: U.S. currency



- Image embedded in paper on rhs
  - Hold bill to light to see embedded info

Watermark Example (2)

- Add **invisible** watermark to photo
- Claim is that 1 inch^2 contains enough info to reconstruct entire photo
- If photo is damaged, watermark can be used to reconstruct it!

Steganography

- According to Herodotus (Greece 440 BC)
  - Shaved slave's head
  - Wrote message on head
  - Let hair grow back
  - Send slave to deliver message
  - Shave slave's head to expose a message warning of Persian invasion
- Historically, steganography used by military more often than cryptography

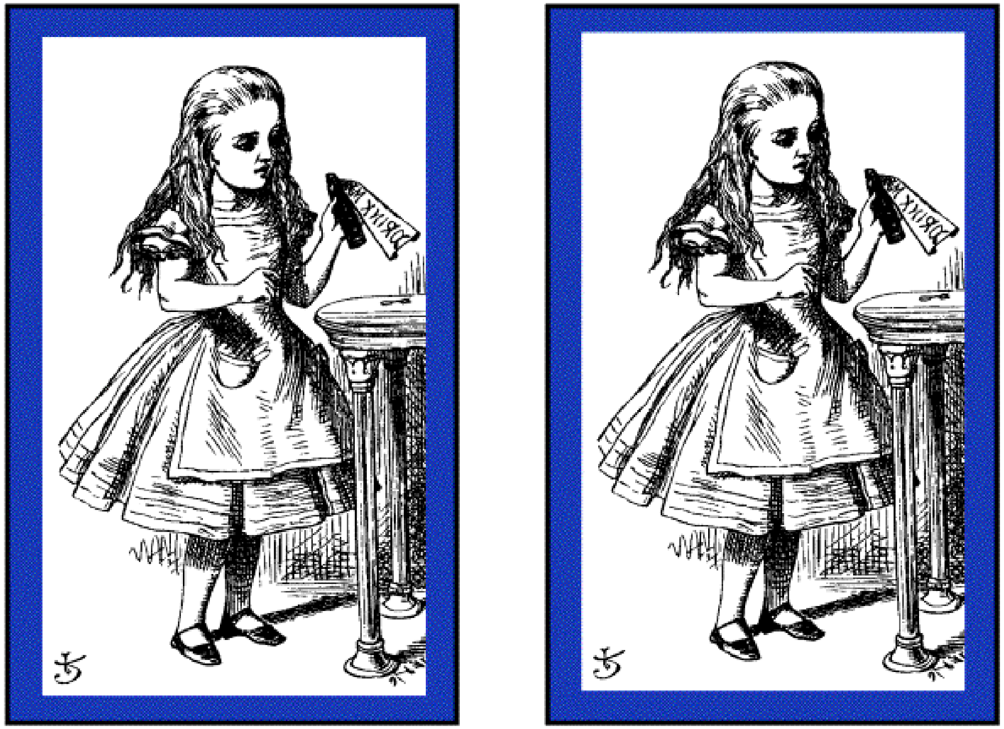
Images and Steganography

- Images use 24 bits for color: RGB
  - 8 bits for red, 8 for green, 8 for blue
- For example
  - **0x7E 0x52 0x90** is this color
  - **0xFE 0x52 0x90** is this color
- While
  - **0xAB 0x33 0xF0** is this color
  - **0xAB 0x33 0xF1** is this color
- Low-order bits don't matter…

Images and Stego

- Given an uncompressed image file…
  - For example, BMP format
- …we can insert information into low-order RGB bits
- Since low-order RGB bits don't matter, changes will be "invisible" to human eye
  - But, computer program can "see" the bits

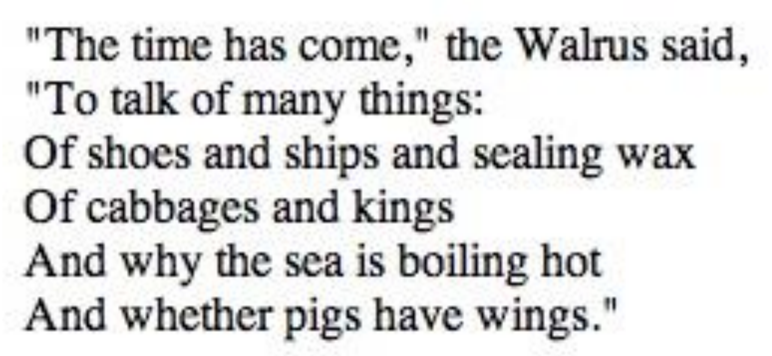
Stego Example 1



- Left side: plain Alice image
- Right side: Alice with entire *Alice in Wonderland* (pdf) "hidden" in the image

Non-Stego Example

- Walrus.html in web browser



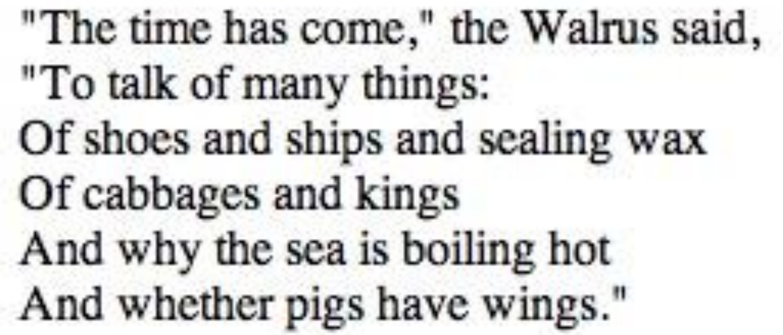
"The time has come," the Walrus said,
"To talk of many things:
Of shoes and ships and sealing wax
Of cabbages and kings
And why the sea is boiling hot
And whether pigs have wings."

- "View source" reveals:

```
<font color=#000000>"The time has come," the Walrus said,</font><br>
<font color=#000000>"To talk of many things: </font><br>
<font color=#000000>Of shoes and ships and sealing wax </font><br>
<font color=#000000>Of cabbages and kings </font><br>
<font color=#000000>And why the sea is boiling hot </font><br>
<font color=#000000>And whether pigs have wings." </font><br>
```

Stego Example 2

- stegoWalrus.html in web browser



"The time has come," the Walrus said,
"To talk of many things:
Of shoes and ships and sealing wax
Of cabbages and kings
And why the sea is boiling hot
And whether pigs have wings."

- "View source" reveals:

```
<font color=#000101>"The time has come," the Walrus said,</font><br>
<font color=#000100>"To talk of many things: </font><br>
<font color=#010000>Of shoes and ships and sealing wax </font><br>
<font color=#010000>Of cabbages and kings </font><br>
<font color=#000000>And why the sea is boiling hot </font><br>
<font color=#010001>And whether pigs have wings." </font><br>
```

- "Hidden" message: **011 010 100 100 000 101**

Steganography

- Some formats (e.g., image files) are more difficult than html for **humans** to read
  - But easy for computer programs to read…
- Easy to hide info in **unimportant bits**
- Easy to damage info in unimportant bits
- To be ***robust***, must use **important bits**
  - But stored info must not damage data
  - Collusion attacks are also a concern
- Robust steganography is tricky!

Information Hiding: The Bottom Line

- Not-so-easy to hide digital information
  - "Obvious" approach is **not** robust
  - **Stirmark:** tool to make most watermarks in images unreadable without damaging the image
  - Stego/watermarking are active research topics
- If information hiding is suspected
  - Attacker may be able to make information/watermark unreadable
  - Attacker may be able to read the information, given the original document (image, audio, etc.)

Chapter 6:
Advanced Cryptanalysis

For there is nothing covered, that shall not be revealed;
neither hid, that shall not be known.
— Luke 12:2

The magic words are squeamish ossifrage
— Solution to RSA challenge problem
posed in 1977 by Ron Rivest, who
estimated that breaking the message
would require 40 quadrillion years.
It was broken in 1994.

Advanced Cryptanalysis

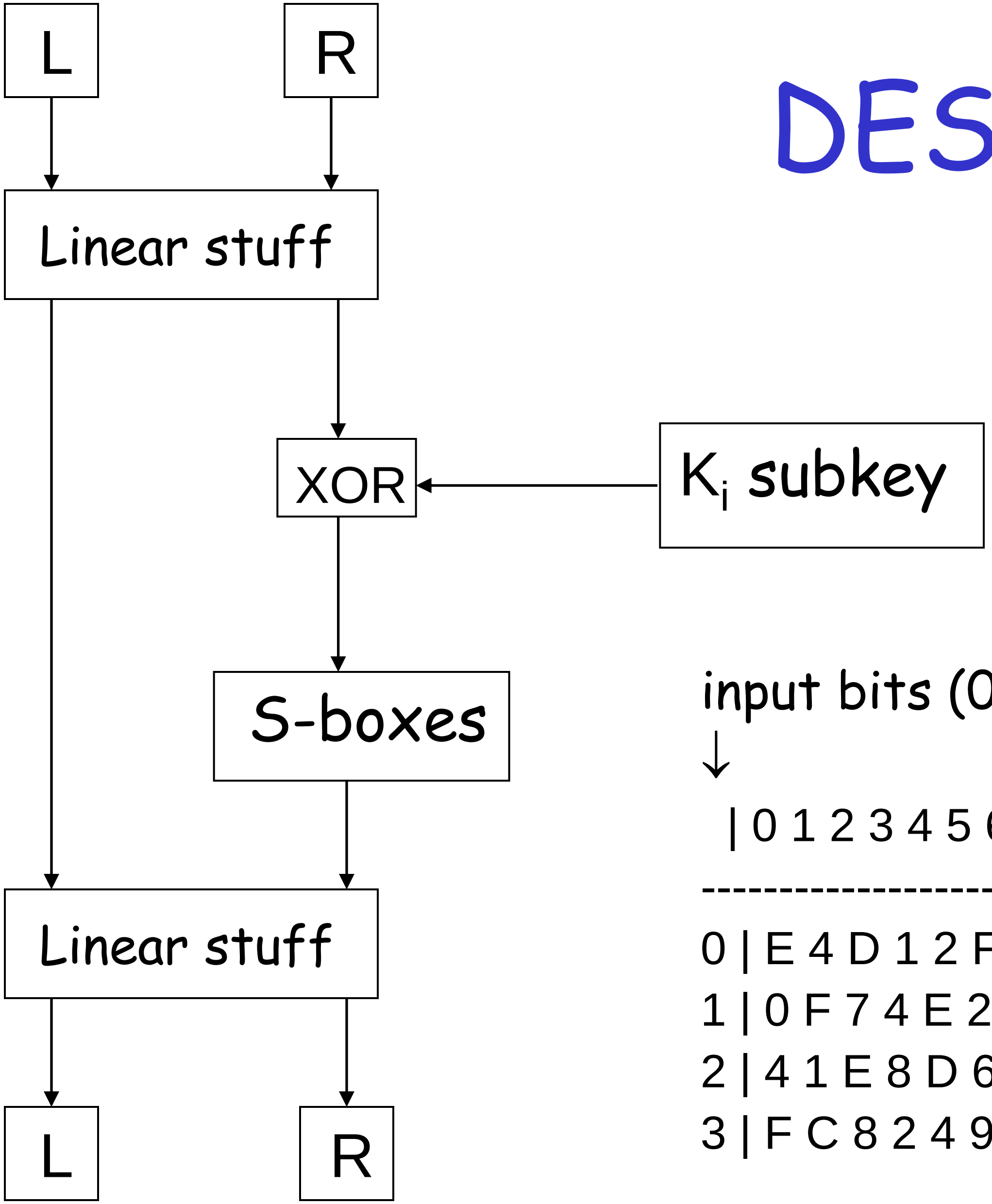
- Modern block cipher cryptanalysis
  - Differential cryptanalysis
  - Linear cryptanalysis
- Side channel attack on RSA
- Lattice reduction attack on knapsack
- Hellman's TMTO attack on DES

Linear and Differential Cryptanalysis

Introduction

- Both linear and differential cryptanalysis developed to attack DES
- Applicable to other block ciphers
- Differential — Biham and Shamir, 1990
  - Apparently known to NSA in 1970s
  - For analyzing ciphers, not a practical attack
  - A chosen plaintext attack
- Linear cryptanalysis — Matsui, 1993
  - Perhaps not know to NSA in 1970s
  - Slightly more feasible than differential
  - A known plaintext attack

DES Overview



- 8 S-boxes
- Each S-box maps 6 bits to 4 bits
- Example: S-box 1

input bits (0,5)
↓
input bits (1,2,3,4)

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	E	4	D	1	2	F	B	8	3	A	6	C	5	9	0	7
1	0	F	7	4	E	2	D	1	A	6	C	B	9	5	3	4
2	4	1	E	8	D	6	2	B	F	C	9	7	3	A	5	0
3	F	C	8	2	4	9	1	7	5	B	3	E	A	0	6	D

Overview of Differential Cryptanalysis

Differential Cryptanalysis

- Recall that all of DES is linear except for the S-boxes
- Differential attack focuses on overcoming this nonlinearity
- Idea is to compare input and output **differences**
- For simplicity, first consider only one round and only one S-box

Differential Cryptanalysis

- Suppose a cipher has 3-bit to 2-bit S-box

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

- Sbox(abc) is element in row a column bc
- Example: Sbox(010) = 11

Differential Cryptanalysis

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

- Suppose $X_1 = 110$, $X_2 = 010$, $K = 011$
- Then $X_1 \oplus K = 101$ and $X_2 \oplus K = 001$
- $\text{Sbox}(X_1 \oplus K) = 10$ and $\text{Sbox}(X_2 \oplus K) = 01$

Differential Cryptanalysis

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

- Suppose
  - **Unknown key:** K
  - **Known inputs:** X = 110, X = 010
  - **Known outputs:** Sbox($X \oplus K$) = 10, Sbox($X \oplus K$) = 01
- Know $X \oplus K \in \{000,101\}$, $X \oplus K \in \{001,110\}$
- Then $K \in \{110,011\} \cap \{011,100\} \Rightarrow K = 011$
- Like a known plaintext attack on S-box

Differential Cryptanalysis

- Attacking one S-box not very useful!
  - And Trudy can't always see input and output
- To make this work we must do 2 things

1. Extend the attack to **one round**
   - Have to deal with all S-boxes
   - Choose input so only one S-box "active"
2. Then extend attack to (almost) **all rounds**
   - Output of one round is input to next round
   - Choose input so output is "good" for next round

Differential Cryptanalysis

- We deal with input and output differences
- Suppose we know inputs X and X
  - For X the input to S-box is $X \oplus K$
  - For X the input to S-box is $X \oplus K$
  - Key K is unknown
  - **Input difference:** $(X \oplus K) \oplus (X \oplus K) = X \oplus X$
- Input difference is independent of key K
- **Output difference:** $Y \oplus Y$ is (almost) input difference to next round
- Goal is to "chain" differences thru rounds

Differential Cryptanalysis

- If we obtain known output difference from known input difference…
  - May be able to chain differences thru rounds
  - It's OK if this only occurs with some probability
- If input difference is 0…
  - …output difference is 0
  - Allows us to make some S-boxes "inactive" with respect to differences

S-box Differential Analysis

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

- Input diff 000 not interesting
- Input diff 010 always gives output diff 01
- More biased, the better (for Trudy)

X ⊕ X	Sbox(X) ⊕ Sbox(X)			
	00	01	10	11
000	8	0	0	0
001	0	0	4	4
010	0	8	0	0
011	0	0	4	4
100	0	0	4	4
101	4	4	0	0
110	0	0	4	4
111	4	4	0	0

Overview of Linear Cryptanalysis

Linear Cryptanalysis

- Like differential cryptanalysis, we target the nonlinear part of the cipher
- But instead of differences, we approximate the nonlinearity with **linear equations**
- For DES-like cipher we need to approximate S-boxes by linear functions
- How well can we do this?

S-box Linear Analysis

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

- Input $x_0x_1x_2$ where x_0 is row and x_1x_2 is column
- Output y_0y_1
- Count of 4 is unbiased
- Count of 0 or 8 is best for Trudy

		output		
		y_0	y_1	$y_0 \oplus y_1$
	0	4	4	4
i	x_0	4	4	4
n	x_1	4	6	2
p	x_2	4	4	4
u	$x_0 \oplus x_1$	4	2	2
t	$x_0 \oplus x_2$	0	4	4
	$x_1 \oplus x_2$	4	6	6
	$x_0 \oplus x_1 \oplus x_2$	4	6	2

Linear Analysis

- For example, $y_1 = x_1$ with prob. 3/4
- And $y_0 = x_0 \oplus x_2 \oplus 1$ with prob. 1
- And $y_0 \oplus y_1 = x_1 \oplus x_2$ with prob. 3/4

	column			
row	00	01	10	11
0	10	01	11	00
1	00	10	01	11

		output		
		y_0	y_1	$y_0 \oplus y_1$
	0	4	4	4
i	x_0	4	4	4
n	x_1	4	6	2
p	x_2	4	4	4
u	$x_0 \oplus x_1$	4	2	2
t	$x_0 \oplus x_2$	0	4	4
	$x_1 \oplus x_2$	4	6	6
	$x_0 \oplus x_1 \oplus x_2$	4	6	2

Linear Cryptanalysis

- Consider a single DES S-box
- Let Y = Sbox(X)
- Suppose $y_3 = x_2 \oplus x_5$ with high probability
  - I.e., a good linear approximation to output y_3
- Can we extend this so that we can solve linear equations for the key?
- As in differential cryptanalysis, we need to "chain" thru multiple rounds

Linear Cryptanalysis of DES

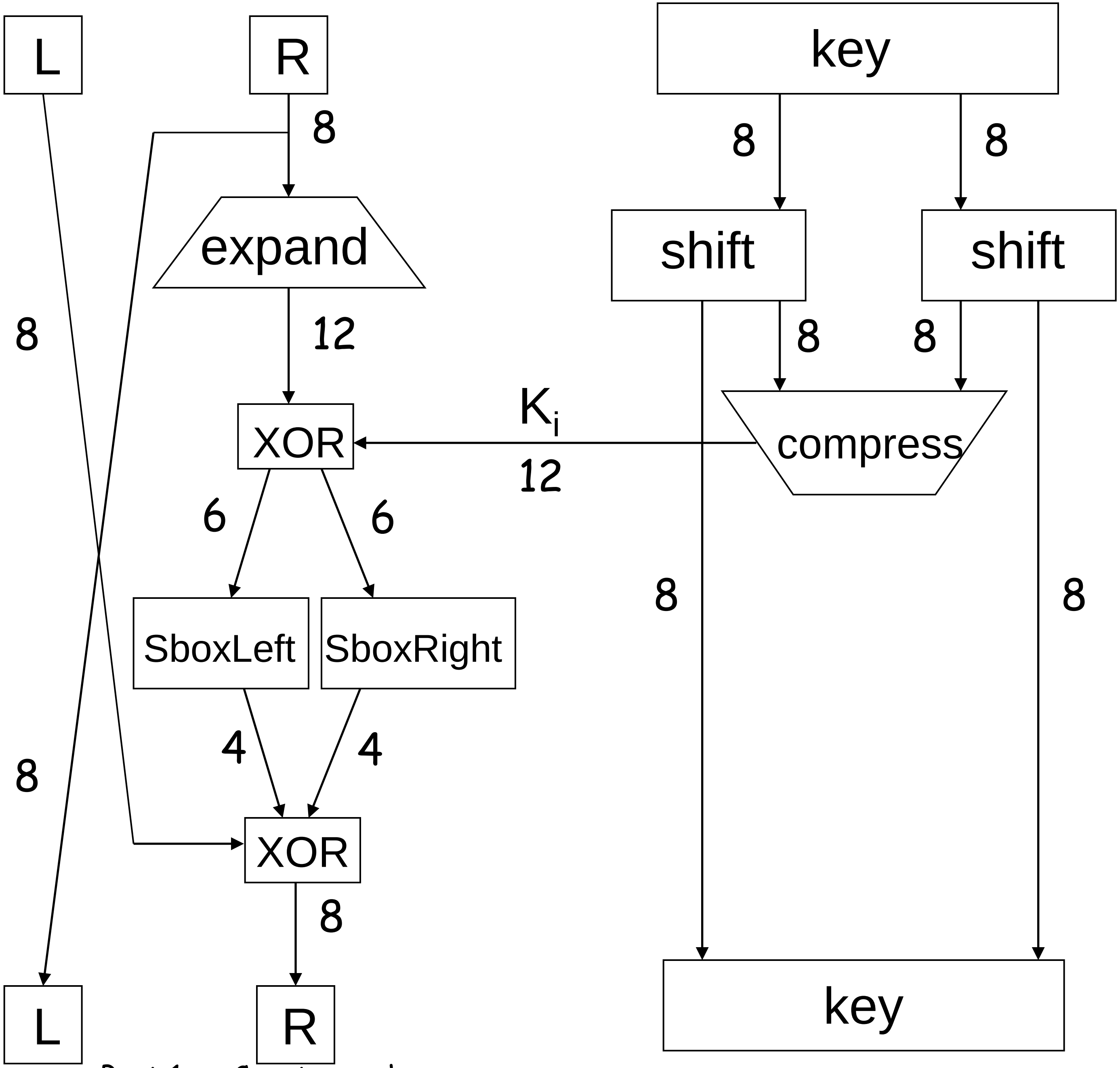
- DES is linear except for S-boxes
- How well can we approximate S-boxes with linear functions?
- DES S-boxes designed so there are no good linear approximations to any one output bit
- But there **are** linear combinations of output bits that can be approximated by linear combinations of input bits

Tiny DES

Tiny DES (TDES)

- A much simplified version of DES
  - 16 bit block
  - 16 bit key
  - 4 rounds
  - 2 S-boxes, each maps 6 bits to 4 bits
  - 12 bit subkey each round
- Plaintext = (L_0, R_0)
- Ciphertext = (L_4, R_4)
- No useless junk

One Round of TDES



TDES Fun Facts

- TDES is a Feistel Cipher
- (L_0,R_0) = plaintext
- For i = 1 to 4

$$L_i = R_{i-1}$$
$$R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$$

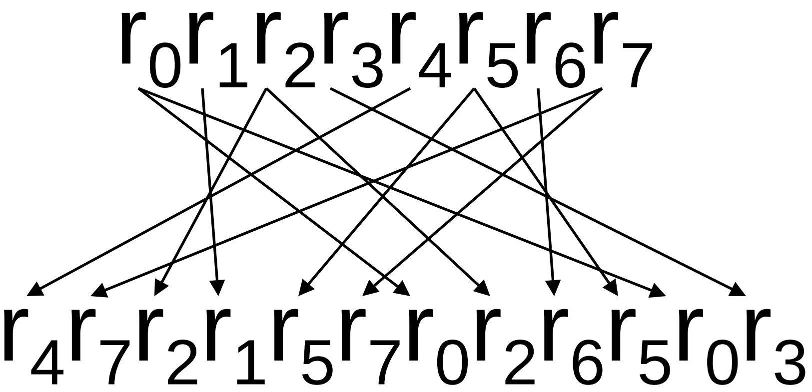
- Ciphertext = (L_4,R_4)
- $F(R_{i-1}, K_i) = \text{Sboxes}(\text{expand}(R_{i-1}) \oplus K_i)$ where $\text{Sboxes}(x_0x_1x_2\ldots x_{11}) = (\text{SboxLeft}(x_0x_1\ldots x_5), \text{SboxRight}(x_6x_7\ldots x_{11}))$

TDES Key Schedule

- Key: K = $k_0k_1k_2k_3k_4k_5k_6k_7k_8k_9k_{10}k_{11}k_{12}k_{13}k_{14}k_{15}$
- Subkey
  - Left: $k_0k_1\ldots k_7$ rotate left 2, select 0,2,3,4,5,7
  - Right: $k_8k_9\ldots k_{15}$ rotate left 1, select 9,10,11,13,14,15
- Subkey $K_1 = k_2k_4k_5k_6k_7k_1k_{10}k_{11}k_{12}k_{14}k_{15}k_8$
- Subkey $K_2 = k_4k_6k_7k_0k_1k_3k_{11}k_{12}k_{13}k_{15}k_8k_9$
- Subkey $K_3 = k_6k_0k_1k_2k_3k_5k_{12}k_{13}k_{14}k_8k_9k_{10}$
- Subkey $K_4 = k_0k_2k_3k_4k_5k_7k_{13}k_{14}k_{15}k_9k_{10}k_{11}$

TDES expansion perm

- Expansion permutation: 8 bits to 12 bits



$r_0r_1r_2r_3r_4r_5r_6r_7$

$r_4r_7r_2r_1r_5r_7r_0r_2r_6r_5r_0r_3$

- We can write this as

  $\text{expand}(r_0r_1r_2r_3r_4r_5r_6r_7) = r_4r_7r_2r_1r_5r_7r_0r_2r_6r_5r_0r_3$

TDES S-boxes

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	C	5	0	A	E	7	2	8	D	4	3	9	6	F	1	B
1	1	C	9	6	3	E	B	2	F	8	4	5	D	A	0	7
2	F	A	E	6	D	8	2	4	1	7	9	0	3	5	B	C
3	0	A	3	C	8	2	1	E	9	7	F	6	B	5	D	4

- Right S-box
- SboxRight

- Left S-box
- SboxLeft

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	6	9	A	3	4	D	7	8	E	1	2	B	5	C	F	0
1	9	E	B	A	4	5	0	7	8	6	3	2	C	D	1	F
2	8	1	C	2	D	3	E	F	0	9	5	A	4	B	6	7
3	9	0	2	5	A	D	6	E	1	8	B	C	3	4	7	F

Differential Cryptanalysis of TDES

TDES

- TDES SboxRight

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	C	5	0	A	E	7	2	8	D	4	3	9	6	F	1	B
1	1	C	9	6	3	E	B	2	F	8	4	5	D	A	0	7
2	F	A	E	6	D	8	2	4	1	7	9	0	3	5	B	C
3	0	A	3	C	8	2	1	E	9	7	F	6	B	5	D	4

- For X and X suppose X ⊕ X = 001000
- Then SboxRight(X) ⊕ SboxRight(X) = 0010 with probability 3/4

Differential Crypt. of TDES

- The game plan…
- Select P and P so that

  $P \oplus P$ = 0000 0000 0000 0010 = 0x0002
- Note that P and P differ in exactly 1 bit
- Let's carefully analyze what happens as these plaintexts are encrypted with TDES

TDES

- If $Y \oplus Y = 001000$ then with probability 3/4 $\text{SboxRight}(Y) \oplus \text{SboxRight}(Y) = 0010$
- $Y \oplus Y = 001000 \Rightarrow (Y \oplus K) \oplus (Y \oplus K) = 001000$
- If $Y \oplus Y = 000000$ then for any S-box, we have $\text{Sbox}(Y) \oplus \text{Sbox}(Y) = 0000$
- Difference of (0000 0010) is expanded by TDES expand perm to diff. (000000 001000)
- **The bottom line:** If $X \oplus X = 00000010$ then $F(X, K) \oplus F(X, K) = 00000010$ with prob. 3/4

TDES

- From the previous slide
  - Suppose $R \oplus R = 0000\ 0010$
  - Suppose K is unknown key
  - Then with probability 3/4

    $F(R,K) \oplus F(R,K) = 0000\ 0010$
- The bottom line? With probability 3/4…
  - Input to next round same as current round
- So we can chain thru multiple rounds

TDES Differential Attack

- Select P and P with $P \oplus P = \text{0x0002}$

$(L_0,R_0) = P$	$(L_0,R_0) = P$	$P \oplus P = \text{0x0002}$
$L_1 = R_0$ $R_1 = L_0 \oplus F(R_0,K_1)$	$L_1 = R_0$ $R_1 = L_0 \oplus F(R_0,K_1)$	With probability 3/4 $(L_1,R_1) \oplus (L_1,R_1) = \text{0x0202}$
$L_2 = R_1$ $R_2 = L_1 \oplus F(R_1,K_2)$	$L_2 = R_1$ $R_2 = L_1 \oplus F(R_1,K_2)$	With probability $(3/4)^2$ $(L_2,R_2) \oplus (L_2,R_2) = \text{0x0200}$
$L_3 = R_2$ $R_3 = L_2 \oplus F(R_2,K_3)$	$L_3 = R_2$ $R_3 = L_2 \oplus F(R_2,K_3)$	With probability $(3/4)^2$ $(L_3,R_3) \oplus (L_3,R_3) = \text{0x0002}$
$L_4 = R_3$ $R_4 = L_3 \oplus F(R_3,K_4)$	$L_4 = R_3$ $R_4 = L_3 \oplus F(R_3,K_4)$	With probability $(3/4)^3$ $(L_4,R_4) \oplus (L_4,R_4) = \text{0x0202}$
$C = (L_4,R_4)$	$C = (L_4,R_4)$	$C \oplus C = \text{0x0202}$

TDES Differential Attack

- Choose P and P with $P \oplus P = 0x0002$
- If $C \oplus C = 0x0202$ then

  $R_4 = L_3 \oplus F(R_3, K_4)$ $R_4 = L_3 \oplus F(R_3, K_4)$

  $R_4 = L_3 \oplus F(L_4, K_4)$ $R_4 = L_3 \oplus F(L_4, K_4)$

  and $(L_3, R_3) \oplus (L_3, R_3) = 0x0002$
- Then $L_3 = L_3$ and $C=(L_4, R_4)$ and $C=(L_4, R_4)$ are both known
- Since $L_3 = R_4 \oplus F(L_4, K_4)$ and $L_3 = R_4 \oplus F(L_4, K_4)$, for correct choice of subkey K_4 we have

  $R_4 \oplus F(L_4, K_4) = R_4 \oplus F(L_4, K_4)$

TDES Differential Attack

- Choose P and P with $P \oplus P = \text{0x0002}$
- If $C \oplus C = (L_4, R_4) \oplus (L_4, R_4) = \text{0x0202}$
- Then for the correct subkey K_4

  $R_4 \oplus F(L_4, K_4) = R_4 \oplus F(L_4, K_4)$

  which we rewrite as

  $R_4 \oplus R_4 = F(L_4, K_4) \oplus F(L_4, K_4)$

  where the only unknown is K_4
- Let $L_4 = l_0l_1l_2l_3l_4l_5l_6l_7$. Then we have

  $0010 = \text{SBoxRight}(\,l_0l_2l_6l_5l_0l_3 \oplus k_{13}k_{14}k_{15}k_9k_{10}k_{11})$

  $\oplus\ \text{SBoxRight}(\,l_0l_2l_6l_5l_0l_3 \oplus k_{13}k_{14}k_{15}k_9k_{10}k_{11})$

TDES Differential Attack

Algorithm to find right 6 bits of subkey K_4

```
count[i] = 0, for i = 0,1,. . .,63
for i = 1 to iterations
    Choose P and P with $P \oplus P$ = 0x0002
    Obtain corresponding C and C
    if $C \oplus C$ = 0x0202
        for K = 0 to 63
            if 0010 == (SBoxRight( $l_0 l_2 l_6 l_5 l_0 l_3 \oplus K$) $\oplus$ SBoxRight( $l_0 l_2 l_6 l_5 l_0 l_3 \oplus K$))
                ++count[K]
            end if
        next K
    end if
next i
```

All K with max count[K] are possible (partial) K_4

TDES Differential Attack

- Experimental results
- Choose 100 pairs P and P with P ⊕ P= 0x0002
- Found 47 of these give C ⊕ C = 0x0202
- Tabulated counts for these 47
  - Max count of 47 for each

    K ∈ {000001,001001,110000,111000}
  - No other count exceeded 39
- Implies that K_4 is one of 4 values, that is,

  $k_{13}k_{14}k_{15}k_9k_{10}k_{11} \in$ {000001,001001,110000,111000}
- Actual key is K=1010 1001 1000 0111

Linear Cryptanalysis of TDES

Linear Approx. of Left S-Box

- TDES left S-box or SboxLeft

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	6	9	A	3	4	D	7	8	E	1	2	B	5	C	F	0
1	9	E	B	A	4	5	0	7	8	6	3	2	C	D	1	F
2	8	1	C	2	D	3	E	F	0	9	5	A	4	B	6	7
3	9	0	2	5	A	D	6	E	1	8	B	C	3	4	7	F

- Notation: $y_0y_1y_2y_3 = \text{SboxLeft}(x_0x_1x_2x_3x_4x_5)$
- For this S-box, $y_1=x_2$ and $y_2=x_3$ both with probability 3/4
- Can we "chain" this thru multiple rounds?

TDES Linear Relations

- Recall that the expansion perm is

  expand($r_0r_1r_2r_3r_4r_5r_6r_7$) = r_4r_7**r_2r_1**$r_5r_7r_0r_2r_6r_5r_0r_3$
- And $y_0y_1y_2y_3$ = SboxLeft($x_0x_1x_2x_3x_4x_5$) with $y_1=x_2$ and $y_2=x_3$ each with probability 3/4
- Also, expand(R_{i-1}) $\oplus$ K_i is input to Sboxes at round i
- Then $y_1=r_2\oplus k_m$ and $y_2=r_1\oplus k_n$ both with prob 3/4
- New right half is $y_0y_1y_2y_3$… plus old left half
- **Bottom line:** New right half bits: $r_1 \leftarrow r_2 \oplus k_m \oplus l_1$ and $r_2 \leftarrow r_1 \oplus k_n \oplus l_2$ both with probability 3/4

Recall TDES Subkeys

- Key: $K = k_0k_1k_2k_3k_4k_5k_6k_7k_8k_9k_{10}k_{11}k_{12}k_{13}k_{14}k_{15}$
- Subkey $K_1 = k_2k_4\mathbf{k_5k_6}k_7k_1k_{10}k_{11}k_{12}k_{14}k_{15}k_8$
- Subkey $K_2 = k_4k_6\mathbf{k_7k_0}k_1k_3k_{11}k_{12}k_{13}k_{15}k_8k_9$
- Subkey $K_3 = k_6k_0\mathbf{k_1k_2}k_3k_5k_{12}k_{13}k_{14}k_8k_9k_{10}$
- Subkey $K_4 = k_0k_2k_3k_4k_5k_7k_{13}k_{14}k_{15}k_9k_{10}k_{11}$

TDES Linear Cryptanalysis

- Known $P=p_0p_1p_2\ldots p_{15}$ and $C=c_0c_1c_2\ldots c_{15}$

$(L_0,R_0) = (p_0\ldots p_7,p_8\ldots p_{15})$	**Bit 1, Bit 2** (numbering from 0)	**probability**
$L_1 = R_0$	p_9, p_{10}	1
$R_1 = L_0 \oplus F(R_0,K_1)$	$p_1 \oplus p_{10} \oplus k_5, p_2 \oplus p_9 \oplus k_6$	3/4
$L_2 = R_1$	$p_1 \oplus p_{10} \oplus k_5, p_2 \oplus p_9 \oplus k_6$	3/4
$R_2 = L_1 \oplus F(R_1,K_2)$	$p_2 \oplus k_6 \oplus k_7, p_1 \oplus k_5 \oplus k_0$	$(3/4)^2$
$L_3 = R_2$	$p_2 \oplus k_6 \oplus k_7, p_1 \oplus k_5 \oplus k_0$	$(3/4)^2$
$R_3 = L_2 \oplus F(R_2,K_3)$	$p_{10} \oplus k_0 \oplus k_1, p_9 \oplus k_7 \oplus k_2$	$(3/4)^3$
$L_4 = R_3$	$p_{10} \oplus k_0 \oplus k_1, p_9 \oplus k_7 \oplus k_2$	$(3/4)^3$
$R_4 = L_3 \oplus F(R_3,K_4)$		
	$k_0 \oplus k_1 = c_1 \oplus p_{10}$ $k_7 \oplus k_2 = c_2 \oplus p_9$ $(3/4)^3$	$(3/4)^3$
$C = (L_4,R_4)$		

TDES Linear Cryptanalysis

- Experimental results
- Use 100 known plaintexts, get ciphertexts.
  - Let $P=p_0p_1p_2\ldots p_{15}$ and let $C=c_0c_1c_2\ldots c_{15}$
- Resulting counts
  - $c_1 \oplus p_{10} = 0$ occurs 38 times
  - $c_1 \oplus p_{10} = 1$ occurs 62 times
  - $c_2 \oplus p_9 = 0$ occurs 62 times
  - $c_2 \oplus p_9 = 1$ occurs 38 times
- Conclusions
  - Since $k_0 \oplus k_1 = c_1 \oplus p_{10}$ we have $k_0 \oplus k_1 = 1$
  - Since $k_7 \oplus k_2 = c_2 \oplus p_9$ we have $k_7 \oplus k_2 = 0$
- Actual key is K = 1010 0011 0101 0110

To Build a Better Block Cipher…

- How can cryptographers make linear and differential attacks more difficult?
  1. **More rounds** — success probabilities diminish with each round
  2. **Better confusion** (S-boxes) — reduce success probability on each round
  3. **Better diffusion** (permutations) — more difficult to chain thru multiple rounds
- Limited mixing and limited nonlinearity, means that more rounds required: TEA
- Strong mixing and nonlinearity, then fewer (but more complex) rounds: AES

Side Channel Attack on RSA

Side Channel Attacks

- Sometimes possible to recover key without directly attacking the crypto algorithm
- A **side channel** consists of "incidental info"
- Side channels can arise due to
  - The way that a computation is performed
  - Media used, power consumed, emanations, etc.
- Induced faults can also reveal information
- Side channel may reveal a crypto key
- Paul Kocher one of the first in this field

Types of Side Channels

- Emanations security (EMSEC)
  - Electromagnetic field (EMF) from computer screen can allow screen image to be reconstructed at a distance
  - Smartcards have been attacked via EMF emanations
- Differential power analysis (DPA)
  - Smartcard power usage depends on the computation
- Differential fault analysis (DFA)
  - Key stored on smartcard in GSM system could be read using a flashbulb to induce faults
- Timing analysis
  - Different computations take different time
  - RSA keys recovered over a network (openSSL)!

The Scenario

- Alice's public key: (N,e)
- Alice's private key: d
- Trudy wants to find d
- Trudy can send any message M to Alice and Alice will respond with M^d mod N
  - That is, Alice signs M and sends result to Trudy
- Trudy can precisely time Alice's computation of M^d mod N

Timing Attack on RSA

- Consider $M^d \bmod N$
- We want to find **private key** d, where $d = d_0 d_1 \ldots d_n$
- Spse repeated squaring used for $M^d \bmod N$
- Suppose, for efficiency

```
mod(x,N)
if x >= N
    x = x % N
end if
return x
```

Repeated Squaring

```
x = M
for j = 1 to n
    x = mod(x^2,N)
    if d_j == 1 then
        x = mod(x*M,N)
    end if
next j
return x
```

Timing Attack

- If $d_j = 0$ then
  - $x = \mathrm{mod}(x^2,N)$
- If $d_j = 1$ then
  - $x = \mathrm{mod}(x^2,N)$
  - $x = \mathrm{mod}(x*M,N)$
- Computation time differs in each case
- Can attacker take advantage of this?

Repeated Squaring

```
x = M
for j = 1 to n
    x = mod(x^2,N)
    if d_j == 1 then
        x = mod(x*M,N)
    end if
next j
return x
```

mod(x,N)

```
if x >= N
    x = x % N
end if
return x
```

Timing Attack

- Choose M with $M^3 < N$
- Choose M with $M^2 < N < M^3$
- Let x = M and x = M
- Consider j = 1
  - x = mod(x^2,N) does no "%"
  - x = mod($x*M$,N) does no "%"
  - x = mod(x^2,N) does no "%"
  - x = mod($x*M$,N) does "%" **only if** d_1=1
- If d_1 = 1 then j = 1 step takes longer for M than for M
- But more than one round…

Repeated Squaring

```
x = M
for j = 1 to n
    x = mod($x^2$,N)
    if $d_j$ == 1 then
        x = mod($x*M$,N)
    end if
next j
return x
```

mod(x,N)

```
if x >= N
    x = x % N
end if
return x
```

Timing Attack on RSA

- An example of a chosen plaintext attack
- Choose $M_0, M_1, \ldots, M_{m-1}$ with
  - $M_i^3 < N$ for $i=0,1,\ldots,m-1$
- Let t_i be time to compute $M_i^d \bmod N$
  - $t = (t_0 + t_1 + \ldots + t_{m-1}) / m$
- Choose $M_0, M_1, \ldots, M_{m-1}$ with
  - $M_i^2 < N < M_i^3$ for $i=0,1,\ldots,m-1$
- Let t_i be time to compute $M_i^d \bmod N$
  - $t = (t_0 + t_1 + \ldots + t_{m-1}) / m$
- If $t > t$ then $d_1 = 1$ otherwise $d_1 = 0$
- Once d_1 is known, find d_2 then d_3 then …

Side Channel Attacks

- If crypto is secure Trudy looks for shortcut
- What is good crypto?
  - More than mathematical analysis of algorithms
  - Many other issues (such as side channels) must be considered
  - See Schneier's article
- Lesson: **Attacker's don't play by the rules!**

Knapsack Lattice Reduction Attack

Lattice?

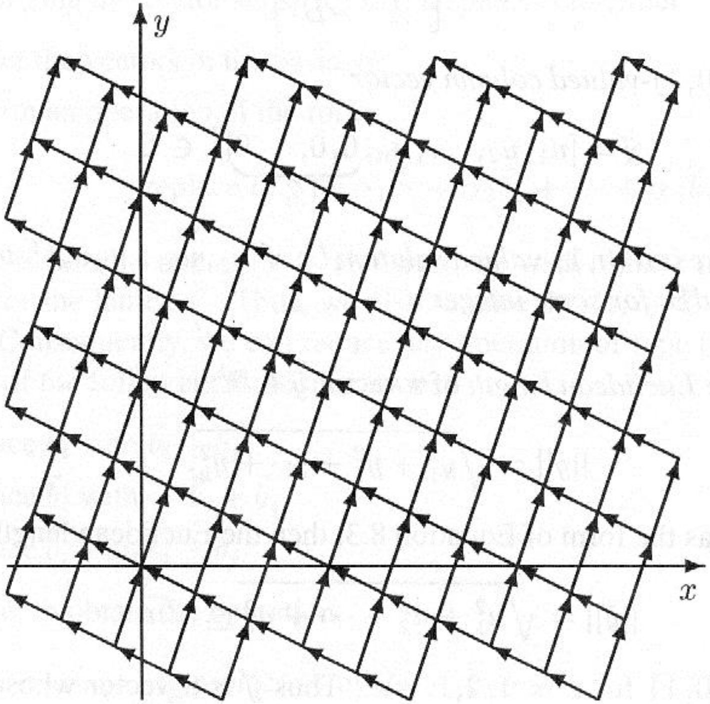
- Many problems can be solved by finding a "short" vector in a **lattice**
- Let $b_1, b_2, \ldots, b_n$ be vectors in $\Re^m$
- All $\alpha_1 b_1 + \alpha_2 b_2 + \ldots + \alpha_n b_n$, each α_i is an integer is a discrete set of points

What is a Lattice?

- Suppose $b_1=[1,3]^T$ and $b_2=[-2,1]^T$
- Then any point in the plane can be written as $\alpha_1 b_1+\alpha_2 b_2$ for some $\alpha_1,\alpha_2 \in \Re$
    - Since b_1 and b_2 are linearly independent
- We say the plane $\Re^2$ is spanned by (b_1,b_2)
- If α_1,α_2 are restricted to **integers**, the resulting span is a **lattice**
- Then a lattice is a discrete set of points

Lattice Example

- Suppose $b_1=[1,3]^T$ and $b_2=[-2,1]^T$
- The lattice spanned by (b_1,b_2) is pictured to the right



Exact Cover

- **Exact cover** — given a set S and a collection of subsets of S, find a collection of these subsets with each element of S is in exactly one subset
- Exact cover is can be solved by finding a "short" vector in a lattice

Exact Cover Example

- Set S = {0,1,2,3,4,5,6}
- Spse m = 7 elements and n = 13 subsets

Subset:	0	1	2	3	4	5	6	7	8	9	10	11	12
Elements:	013	015	024	025	036	124	126	135	146	1	256	345	346

- Find a collection of these subsets with each element of S in exactly one subset
- Could try all 2^{13} possibilities
- If problem is too big, try **heuristic search**
- Many different heuristic search techniques

Exact Cover Solution

- Exact cover in matrix form
  - Set S = {0,1,2,3,4,5,6}
  - Spse m = 7 elements and n = 13 subsets

Subset:	0	1	2	3	4	5	6	7	8	9	10	11	12
Elements:	013	015	024	025	036	124	126	135	146	1	256	345	346

subsets

elements

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \\ u_{11} \\ u_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

m x n

m x 1

n x 1

Solve: AU = B
where $u_i \in \{0,1\}$

Solution:
$U = [0001000001001]^T$

Example

- We can restate AU = B as MV = W where

$$\begin{bmatrix} I_{n\times n} & 0_{n\times 1} \\ A_{m\times n} & -B_{m\times 1} \end{bmatrix} \begin{bmatrix} U_{n\times 1} \\ 1_{1\times 1} \end{bmatrix} = \begin{bmatrix} U_{n\times 1} \\ 0_{m\times 1} \end{bmatrix} \Longleftrightarrow AU = B$$

Matrix M Vector V Vector W

- The desired solution is U
  - Columns of M are **linearly independent**
- Let $c_0,c_1,c_2,\ldots,c_n$ be the columns of M
- Let $v_0,v_1,v_2,\ldots,v_n$ be the elements of V
- Then $W = v_0c_0 + v_1c_1 + \ldots + v_nc_n$

Example

- Let L be the lattice spanned by $c_0, c_1, c_2, \ldots, c_n$ (c_i are the columns of M)
- Recall MV = W
  - Where $W = [U,0]^T$ and we want to find U
  - But if we find W, we've also solved it!
- Note W is in lattice L since all v_i are integers and $W = v_0 c_0 + v_1 c_1 + \ldots + v_n c_n$

Facts

- $W = [u_0,u_1,\ldots,u_{n-1},0,0,\ldots,0] \in L$, each $u_i \in \{0,1\}$
- The length of a vector $Y \in \Re^N$ is

  $$||Y|| = \text{sqrt}(y_0^2+y_1^2+\ldots+y_{N-1}^2)$$

- Then the length of W is

  $$||W|| = \text{sqrt}(u_0^2+u_1^2+\ldots+u_{n-1}^2) \leq \text{sqrt}(n)$$

- So W is a very **short** vector in L where
  - First n entries of W all 0 or 1
  - Last m elements of W are all 0
- Can we use these facts to find U?

Lattice Reduction

- If we can find a short vector in L, with first n entries all 0 or 1 and last m entries all 0…
  - Then we *might* have found solution U
- **LLL** lattice reduction algorithm will efficiently find short vectors in a lattice
- About 30 lines of pseudo-code specify LLL
- No guarantee LLL will find desired vector
- But probability of success is often good

Knapsack Example

- What does lattice reduction have to do with the knapsack cryptosystem?
- Suppose we have
  - Superincreasing knapsack

    S = [2,3,7,14,30,57,120,251]
  - Suppose $m = 41$, $n = 491 \Rightarrow m^{-1} = 12 \bmod n$
  - Public knapsack: $t_i = 41 \cdot s_i \bmod 491$

    T = [82,123,287,83,248,373,10,471]
- **Public key:** T **Private key:** (S, m^{-1}, n)

Knapsack Example

- **Public key:** T **Private key:** (S, m^{-1}, n)

  S = [2,3,7,14,30,57,120,251]

  T = [82,123,287,83,248,373,10,471]

  n = 491, $m^{-1} = 12$
- Example: 10010110 is encrypted as

  82+83+373+10 = 548
- Then receiver computes

  $548 \cdot 12 = 193 \bmod 491$

  and uses S to solve for 10010110

Knapsack LLL Attack

- Attacker knows public key

  T = [82,123,287,83,248,373,10,471]
- Attacker knows ciphertext: 548
- Attacker wants to find $u_i \in \{0,1\}$ s.t.

$$82u_0+123u_1+287u_2+83u_3+248u_4+373u_5+10u_6+471u_7=548$$

- This can be written as a matrix equation (dot product): $T \cdot U = 548$

Knapsack LLL Attack

- Attacker knows: T = [82,123,287,83,248,373,10,471]
- Wants to solve: $T \cdot U = 548$ where each $u_i \in \{0,1\}$
  - Same form as AU = B on previous slides!
  - We can rewrite problem as MV = W where

$$M = \begin{bmatrix} I_{8\times 8} & 0_{8\times 1} \\ T_{1\times 8} & -C_{1\times 1} \end{bmatrix} = \left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline 82 & 123 & 287 & 83 & 248 & 373 & 10 & 471 & -548 \end{array}\right]$$

- LLL gives us short vectors in the lattice spanned by the columns of M

LLL Result

- LLL finds short vectors in lattice of M
- Matrix M′ is result of applying LLL to M

$$M' = \left[\begin{array}{cccccccc|c} -1 & -1 & 0 & \overset{*}{1} & 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 2 \\ 1 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ \hline 1 & -1 & 1 & 0 & 0 & 1 & -1 & 2 & 0 \end{array}\right]$$

- Column marked with "*" has the right form
- Possible solution: $U = [1,0,0,1,0,1,1,0]^T$
- Easy to verify this is actually the plaintext

Bottom Line

- Lattice reduction is a surprising method of attack on knapsack
- A cryptosystem is only secure as long as nobody has found an attack
- Lesson: **Advances in mathematics can break cryptosystems!**

Hellman's TMTO Attack

Popcnt

- Before we consider Hellman's attack, consider a simple **T**ime-**M**emory **T**rade**O**ff
- "Population count" or popcnt
  - Let x be a 32-bit integer
  - Define $\text{popcnt}(x)$ = number of 1's in binary expansion of x
  - How to compute $\text{popcnt}(x)$ efficiently?

Simple Popcnt

- Most obvious thing to do is

```
popcnt($x$) // assuming $x$ is 32-bit value
    $t = 0$
    for $i = 0$ to 31
        $t = t + ((x >> i) \& 1)$
    next $i$
    return $t$
end popcnt
```

- But is it the most efficient?

More Efficient Popcnt

- Precompute popcnt for all 256 bytes
- Store precomputed values in a table
- Given x, lookup its bytes in this table
  - Sum these values to find popcnt(x)
- Note that precomputation is done once
- Each popcnt now requires 4 steps, not 32

More Efficient Popcnt

```
Initialize: table[$i$] = popcnt($i$) for $i$ = 0,1,…,255

popcnt($x$) // assuming $x$ is 32-bit value
    $p$ = table[ $x$ & 0xff ]
            + table[ ($x$ >> 8) & 0xff ]
            + table[ ($x$ >> 16) & 0xff ]
            + table[ ($x$ >> 24) & 0xff ]
    return $p$
end popcnt
```

TMTO Basics

- A precomputation
  - One-time work
  - Results stored in a table
- Precomputation results used to make each subsequent computation faster
- Balancing "memory" and "time"
- In general, larger precomputation requires more initial work and larger "memory" but each subsequent computation is less "time"

Block Cipher Notation

- Consider a block cipher

  $$C = E(P, K)$$

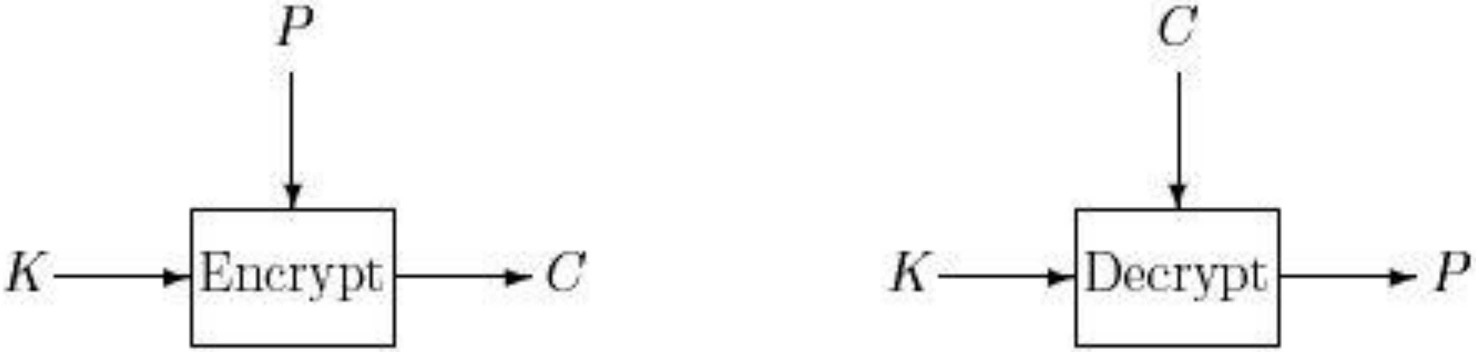
  where

  P is plaintext block of size n

  C is ciphertext block of size n

  K is key of size k

Block Cipher as Black Box



- For TMTO, treat block cipher as black box
- Details of crypto algorithm not important

Hellman's TMTO Attack

- **Chosen plaintext attack:** choose P and obtain C, where $C = E(P, K)$
- Want to find the key K
- Two "obvious" approaches
  1. Exhaustive key search
     - "Memory" is 0, but "time" of 2^{k-1} for each attack
  2. Pre-compute $C = E(P, K)$ for all possible K
     - Then given C, can simply look up key K in the table
     - "Memory" of 2^k but "time" of 0 for each attack
- TMTO lies between 1. and 2.

Chain of Encryptions

- Assume block and key lengths equal: $n = k$
- Then a **chain** of encryptions is

$$SP = K_0 \ = \text{Starting Point}$$

$$K_1 = E(P, SP)$$

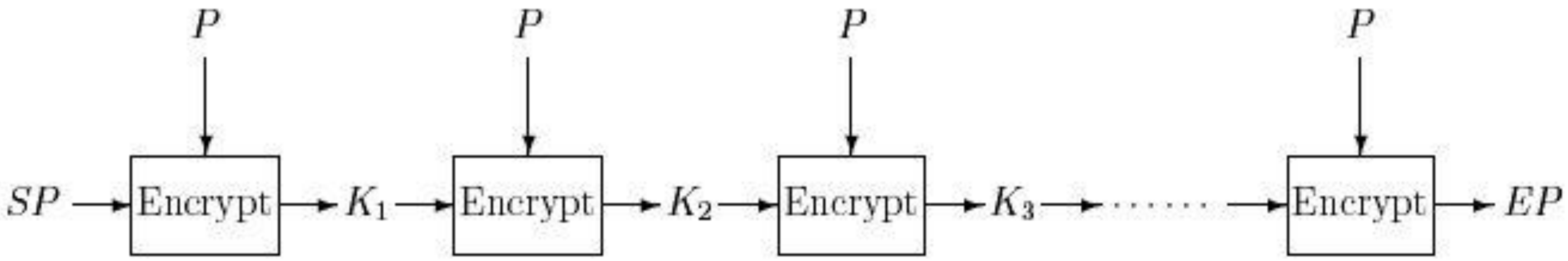
$$K_2 = E(P, K_1)$$

$$\vdots$$

$$\vdots$$

$$EP = K_t = E(P, K_{t-1}) = \text{End Point}$$

Encryption Chain



- Ciphertext used as **key** at next iteration
- Same (chosen) **plaintext** at each iteration

Pre-computation

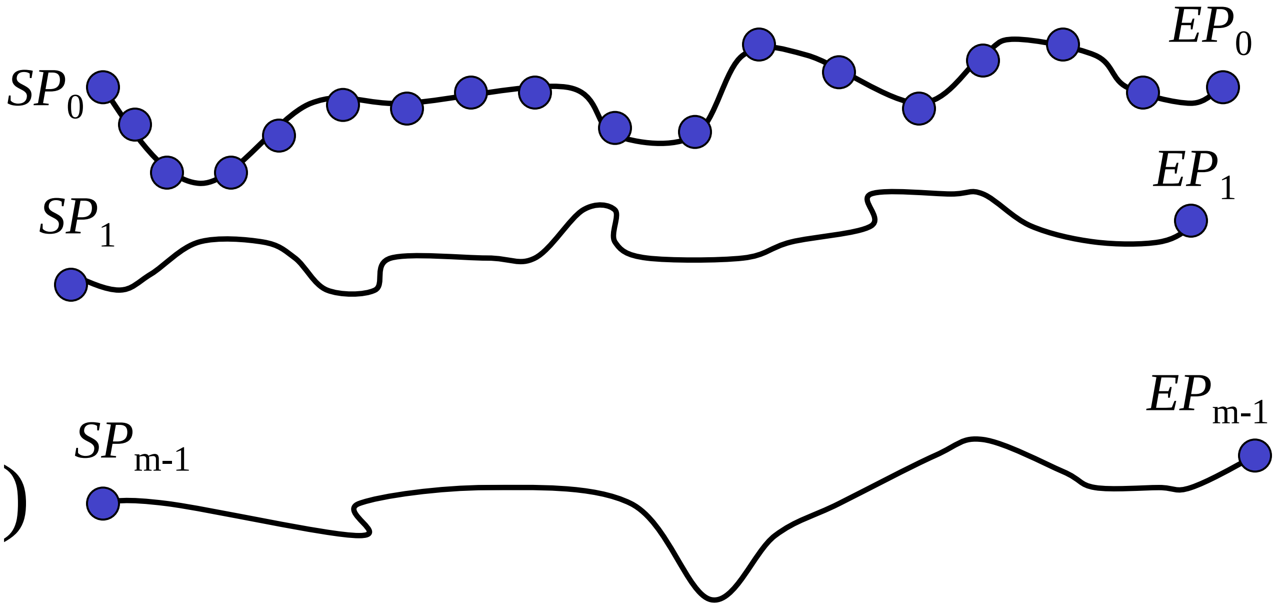
- Pre-compute m encryption chains, each of length $t+1$
- Save only the start and end points

(SP_0, EP_0)

(SP_1, EP_1)

:

(SP_{m-1}, EP_{m-1})



TMTO Attack

- **Memory:** Pre-compute encryption chains and save (SP_i, EP_i) for $i = 0,1,\ldots,m-1$
  - This is one-time work
- Then to attack a particular unknown key K
  - For the same chosen P used to find chains, we know C where $C = E(P, K)$ and K is unknown key
  - **Time:** Compute the chain (maximum of t steps)

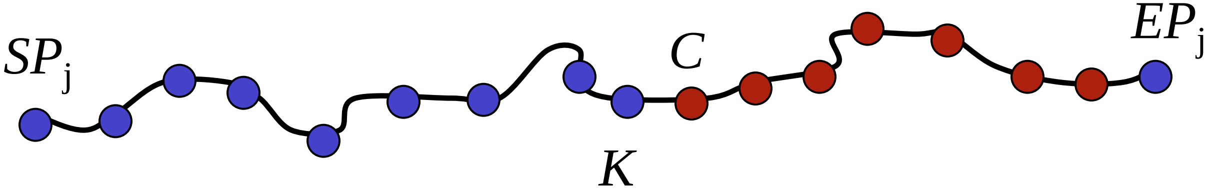
$$X_0 = C,\ X_1 = E(P, X_0),\ X_2 = E(P, X_1),\ldots$$

TMTO Attack

- Consider the computed chain

$$X_0 = C,\ X_1 = E(P, X_0),\ X_2 = E(P, X_1),\ldots$$

- Suppose for some i we find $X_i = EP_j$



- Since $C = E(P, K)$ key K before C in chain!

TMTO Attack

- To summarize, we compute chain

  $X_0 = C, X_1 = E(P, X_0), X_2 = E(P, X_1), \ldots$
- If for some i we find $X_i = EP_j$
- Then reconstruct chain from SP_j

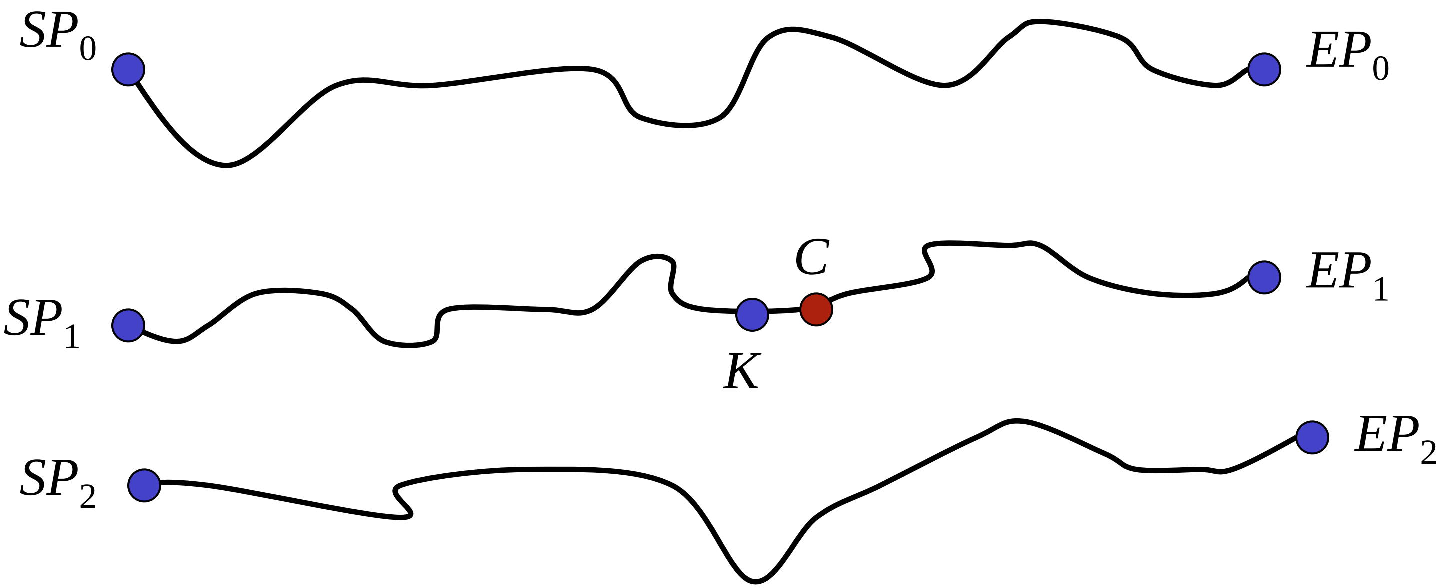
  $Y_0 = SP_j, Y_1 = E(P,Y_0), Y_2 = E(P,Y_1), \ldots$
- Find $C = Y_{t-i} = E(P, Y_{t-i-1})$ (always?)
- Then $K = Y_{t-i-1}$ (always?)

Trudy's Perfect World

- Suppose block cipher has $k = 56$
  - That is, the key length is 56 bits
- Suppose we find $m = 2^{28}$ chains, each of length $t = 2^{28}$ and no chains overlap
- **Memory:** 2^{28} pairs (SP_j, EP_i)
- **Time:** about 2^{28} (per attack)
  - Start at C, find some EP_j in about 2^{27} steps
  - Find K with about 2^{27} more steps
- Attack never fails

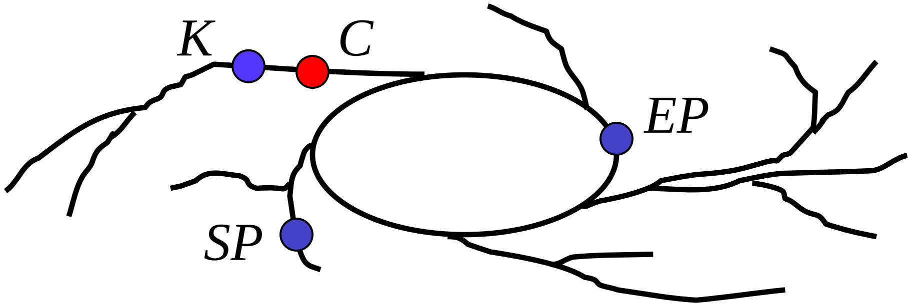
Trudy's Perfect World

- No chains overlap
- Any ciphertext C is in some chain



The Real World

- Chains are not so well-behaved!
- Chains can **cycle** and **merge**



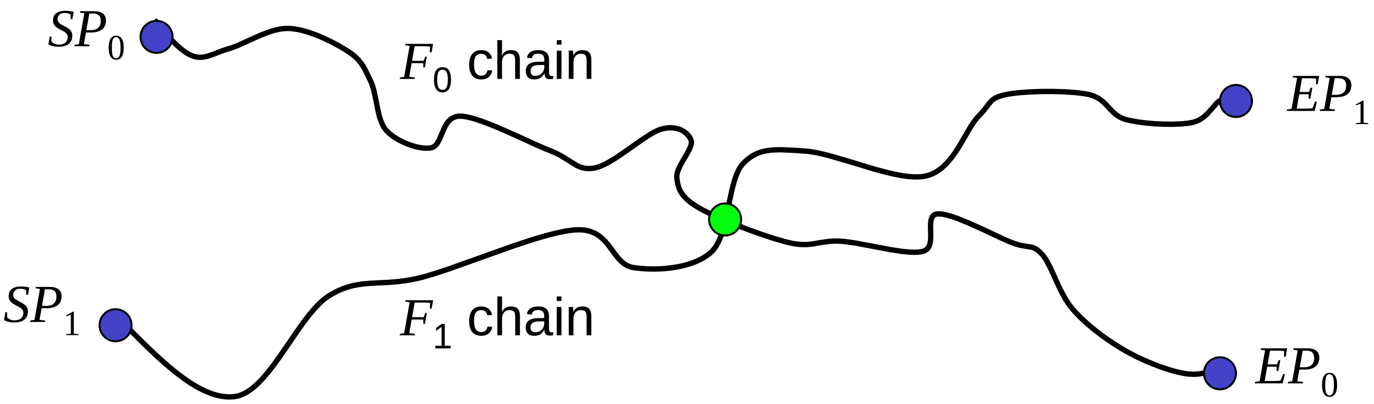
- Chain from C goes to EP
- Chain from SP to EP does not contain K
- Is this Trudy's nightmare?

Real-World TMTO Issues

- Merging, cycles, false alarms, etc.
- Pre-computation is lots of work
  - Must attack many times to make it worthwhile
- Success is not assured
  - Probability depends on initial work
- What if block size not equal key length?
  - This is easy to deal with
- What is the probability of success?
  - This is not so easy to compute

To Reduce Merging

- Compute chain as $F(E(P, K_{i-1}))$ where F permutes the bits
- Chains computed using different functions can intersect, but they will **not** merge



Hellman's TMTO in Practice

- Let
  - m = random starting points for each F
  - t = encryptions in each chain
  - r = number of "random" functions F
- Then mtr = total precomputed chain elements
- Pre-computation is $O(mtr)$ work
- Each TMTO attack requires
  - $O(mr)$ "memory" and $O(tr)$ "time"
- If we choose $m = t = r = 2^{k/3}$ then
  - Probability of success is at least 0.55

TMTO: The Bottom Line

- Attack is feasible against DES
- Pre-computation is about 2^{56} work
- Each attack requires about
  - 2^{37} "memory"
  - 2^{37} "time"
- Attack is not particular to DES
- No fancy math is required!
- Lesson: **Clever algorithms can break crypto!**

Crypto Summary

- Terminology
- Symmetric key crypto
  - Stream ciphers
    - A5/1 and RC4
  - Block ciphers
    - DES, AES, TEA
    - Modes of operation
    - Integrity

Crypto Summary

- Public key crypto
  - Knapsack
  - RSA
  - Diffie-Hellman
  - ECC
  - Non-repudiation
  - PKI, etc.

Crypto Summary

- Hashing
  - Birthday problem
  - Tiger hash
  - HMAC
- Secret sharing
- Random numbers

Crypto Summary

- Information hiding
  - Steganography
  - Watermarking
- Cryptanalysis
  - Linear and differential cryptanalysis
  - RSA timing attack
  - Knapsack attack
  - Hellman's TMTO

Coming Attractions…

- Access Control
  - Authentication -- who goes there?
  - Authorization -- can you do that?
- We'll see some crypto in next chapter
- We'll see **lots** of crypto in protocol chapters