

Homework Assignment 11

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1 Introduction

In this assignment we implemented the RANSAC algorithm to fit a line, solidified understanding of planar homography, and explored multi-view geometric constraints for keypoint matching. We also looked at the configuration of a fundamental matrix and image stitching using SIFT, homography, and RANSAC.

1.1 Contributions

Kayla contributed with SIFT extractions, arranging SIFT features, and SIFT visualizations. Marium implemented RANSAC algorithm, the Direct Linear Transform method to compute homography between two planar images, and used the homography with RANSAC to improve keypoint matching. Both discussed the derivation of fundamental matrix and image stitching and Marium was primarily responsible for implementing them. Both also consulted each other back and forth when either was stuck.

2 A: RANSAC

For this part, we used a dataset of (x, y) coordinates to fit a line model using a RANSAC algorithm. As provided, at least 1/4th of the points were assumed to be close to the good fit. Fig. 1 shows the scatter plot of the data points together with the line fit found with an error of 0.0928. The RANSAC algorithm was implemented as follows:

- we computed k , the number of iterations, using the formula $\lceil \frac{\log(1-p)}{\log(1-w^n)} \rceil$, where p quantifies the probability of success we require from the algorithm, w is the probability of getting an inlier, and n is the number of points needed to fit the model. We set $p = 0.99$, that is we required the algorithm to compute with 99% success rate, $w = 0.6$, that is we wanted the inlier points ratio to be 3/5 of the data points around the line that fit the model, and $n = 2$ since we only need 2 points to fit a line. This computed the value for our $k = 11$ as the number of iterations.
- using the k found, we excluded picked two random points from the dataset and used the homogenous least squares model to find a line instance. For each of these instances, we computed a perpendicular distance for each of the points using a threshold of 0.2. That is, if for any point the distance was within 0.2 of the line distance, we included that point as an inlier. The value for the threshold was chosen by consulting the scatter plot of the data points.
- if the amount of inliers found exceeded 75 (1/4th of our 300 data points), then we assume to have found a good model and fit a line using all the inliers and the two randomly sampled points. The red line shown in Fig. 1 is this line model.

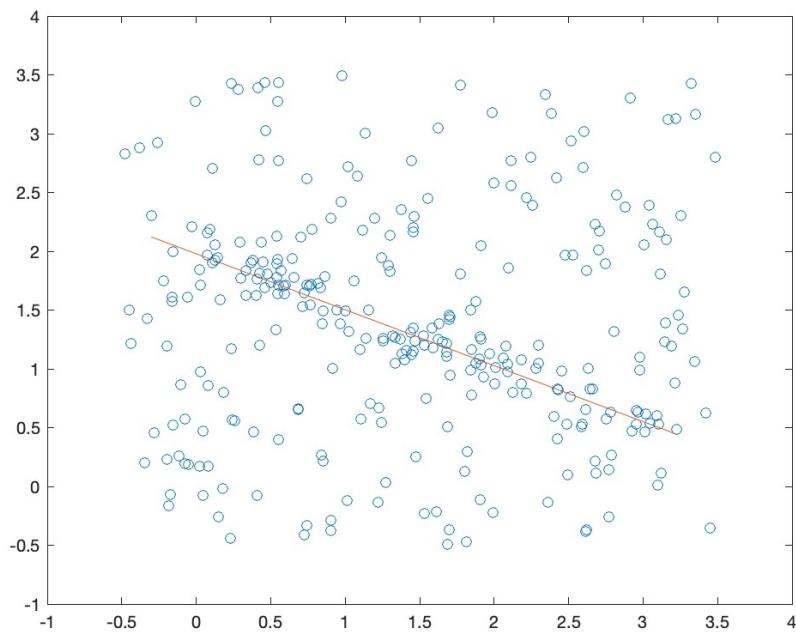


Figure 1: Figure showing the data points of (x,y) coordinates as a scatter plot and a line model fitted using the RANSAC algorithm. This line was found with an error of 0.0928 based on inliers found from the algorithm.

3 B: Homography

For this part, we implemented the Direct Linear Transform (DLT) method and tested it on the synthetic data and the slide/frame image pair data from assignment 9. For the synthetic data, we randomly generated 4, 5, and 6 pairs of (x, y) coordinates, twice: one as "actual" points and one as the "estimate" or the point match. We did this 10 times and yield the RMS errors as shown in Table 1 below. It's a little difficult to tell if the code is working since the errors stay between (0.3, 0.4) (range tested through trial-and-error) for any number of pairs. We conclude that since we only need 4 good matches, the number of pairs is not making much of a difference in the homography found using the DLT method.

Number of Pairs	RMS
4	0.3108
5	0.3235
6	0.3267

Table 1: RMS error of the transformation for 4, 5, and 6 randomly generated pairs of (x, y) -coordinates lying between a $[0, 1] \times [0, 1]$ window.

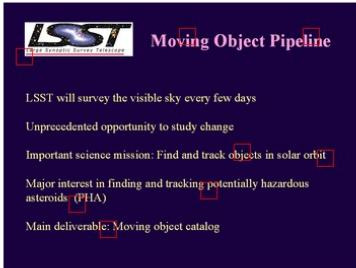
Moving on to the slide/frame image pairs, we collected 8 distinct points using mouse clicks as shown in Fig. 2 for all three slide/frame pairs. Then using 4 of these clicked points, we computed the homography using the DLT method for each slide/frame pair. Using this homography, we estimated the corresponding slide coordinates to the points on the video frame image. The Fig. 3 shows the visualization on the video frame images, displaying the 8 clicked points in red and the 8 mapped locations in yellow. *Please note: previously, we had issue with DLT due to which the estimated (yellow) points were shifted. This issue occurred because of two things: 1) we were rescaling the points to $[0, 1]$, and 2) we forgot to normalize the homogenous coordinates so that the last column had all 1s.*



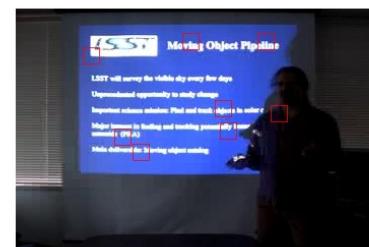
(a) Slide 1



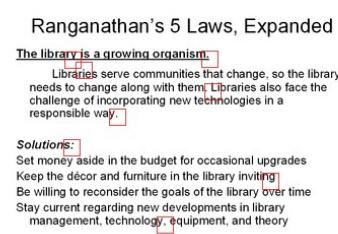
(b) Frame 1



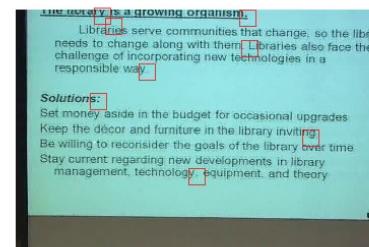
(c) Slide 2



(d) Frame 2



(e) Slide 3

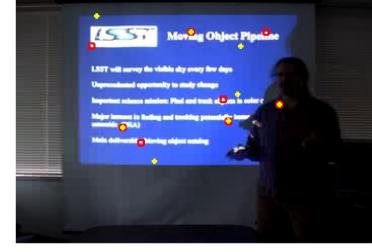


(f) Frame 3

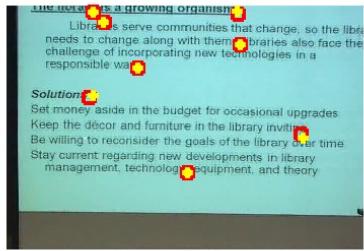
Figure 2: Figure showing collected 8 matching points using manual mouse clicking. The points were chosen by focusing on distinct locations that were least ambiguous.



(a) Estimated points for Frame 1



(b) Estimated points for Frame 2



(c) Estimated points for Frame 3

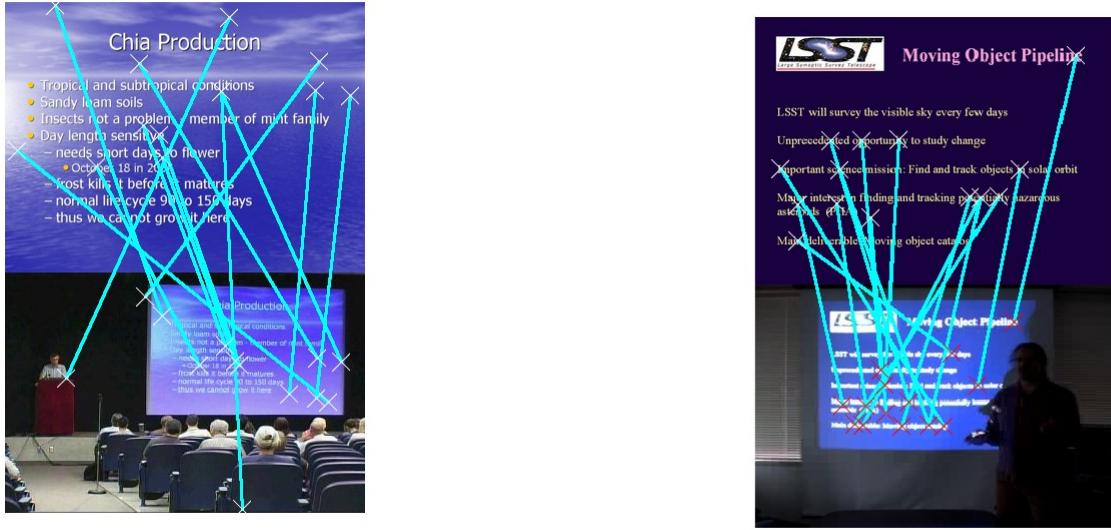
Figure 3: Figure showing the clicked points (in red) against the estimated points computed using homography (in yellow). Note that the subset of points chosen to compute the homography have a visible overlap (that is, yellow marker is perfectly contained in the red square).

4 C: Homography and RANSAC

For this part we again used the slide/frame image pairs from assignment 9 to compute homography for several points (instead of the mouse-clicks chosen in Part B) using the RANSAC algorithm. We achieved some meaningful results by doing the following:

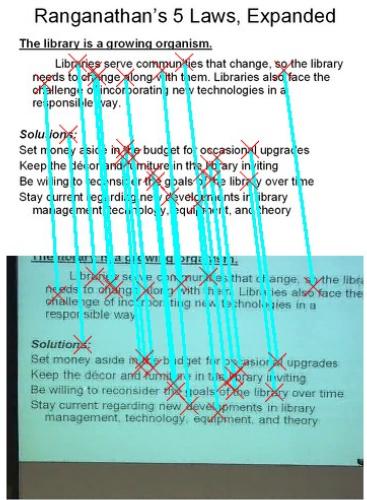
- for each slide/frame pair, we used the *.sift* files from assignment 9 for SIFT features and created a $M \times 11$ matrix holding the (x, y) coordinates, scale, and direction for video frame image, corresponding (x, y) coordinates, scale, and direction for the slide image, their corresponding Euclidean distance using the nearest neighbor between their 128-element feature vector, angles between their corresponding 128-element feature vector, and Chi-squared measure between their 128-element feature vector. M is the number of features extracted returned by the SIFT.
- we sorted our data matrix by the Euclidean distance and used it as our data set for the RANSAC algorithm, which we ran $k = 178$ times, computed with $n = 4$ (number of data points for the model – need 4 for homography), $w = 0.4$ (the inlier ratio), and $p = 0.99$ (the success probability).
- for each slide/pair image, we found the best homography H , using 4 random rows from the matrix as our inliers, their DLT as our initial model, and then used the frame keypoints from the rest of the data to estimate the slide keypoints. If the RMS between the slide coordinates from the original data matrix and the estimated slide keypoints was within a threshold (chosen as 50), then we included that estimate in our inlier set.
- once the size of the inlier set was over some set value (of $N = 150$), we assumed to have found a good model and recomputed our homography using the DLT with the intial inliers and additional inliers that hold the threshold.
- Fig. 4 shows the mapping of keypoints from estimated slide image to the corresponding keypoints on the video frame.

Challenges faced: initially, we had major trouble trying to match the keypoints to reasonable estimated keypoints (going both slide to frame and frame to slide). The estimated keypoints seemed to be in the vicinity of where they should land and by close observation the line often crossed where it supposedly should be... it also looked like the estimated points are bunching up together. On heft debugging, it turned out there was a couple issues: the RANSAC threshold to decide on the inliers needed reconfiguration and the computing estimated keypoints needed normalization so that the last column of homogenous coordinates was always full of 1s.



(a) Slide/Frame 1 - First pair seems to perform the worst; on close inspection, the gradient (going from purplish tone to blue) on the slide is opposite to gradient on the frame. Therefore, multiple keypoints are matched opposite considering the water-like texture/background of the slide/frame.

(b) Slide/Frame 2 - Second pair performed better than the first. There are still some mis-matches, but we were happy to see the perfect matches like the end of word "pipeline", "in" in the third line, and so on.



(c) Slide/Frame 3 - Third pair performed the best.

Figure 4: Figure showing the mapping of keypoints for the provided slide-frame pairs. These slide keypoints were estimated from the frame keypoints using the homography computed with RANSAC algorithm. We picked our keypoints based on the lowest Euclidean distance and were getting similar results using the distance between the matches as the angle between feature vectors.

5 D1: Configuring a fundamental matrix

5.1

Fig. 5 shows the 20 matching image points using mouse-clicking as done in HW3 previously. These coordinates were saved and submitted with the submitted assignment files named *FM_1.txt* and *FM_2.txt*.



(a) 20 keypoints for scene 1



(b) 20 keypoints for scene 2

Figure 5: Figure showing matches across two arbitrary views of the same scene. The image points were chosen carefully such that the points occur in both scenes.

5.2

Using the fact that $(x')^T F x = 0$ everytime, we can compute the fundamental matrix in the following way. Let:

$$\mathbf{X}' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix},$$

Then we have:

$$\begin{aligned} [x' \quad y' \quad 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} &= 0 \\ \implies [x'F_{11} + y'F_{21} + F_{31} \quad x'F_{12} + y'F_{22} + F_{32} \quad x'F_{13} + y'F_{23} + F_{33}] \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} &= 0 \end{aligned}$$

Let \mathbf{F}_i be the row vectors of the matrix F for $i = \{1, 2, 3\}$.

$$\Rightarrow [[x' & y' & 1] \cdot \mathbf{F}_1^T & [x' & y' & 1] \cdot \mathbf{F}_2^T & [x' & y' & 1] \cdot \mathbf{F}_3^T] \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Therefore, we have:

$$[x'x & y'x & x & x'y & y'y & y & x' & y' & 1] \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0,$$

which we can solve using homogenous least squares method for any number of points \mathbf{X} and corresponding \mathbf{X}' .

5.3

Using the derivation in Part 5.2, we find F using the 12 (training) coordinates collected in Part 5.1. We tested the computed F on the other 8 coordinates and yielded an RMS of 0.0249.

5.4

Using the given image-pairs and the clicked point-pairs, for this part we used the derived F to draw the epipolar line on both images for each point in the other image. Fig. 6 shows these lines together with the marked points. On observation, it looks like that the epipolar lines help determine the position of the camera for both scenes! (This question was a lot of fun to tackle.)



(a) Epipolar lines on scene 1 using points from scene 2



(b) Epipolar lines on scene 2 using points from scene 1

Figure 6: Figure showing epipolar lines on both scenes. These were determined by first collecting 20 points by mouse-clicking and then distribution the points into training (of 12 points) and test (of 8 points) sets. Using the training points, we determined the fundamental matrix F using the derivation in 5.2 and used the 8 test points to see how close they are to 0, since $(x')^T F x = 0$ everytime.

6 D2: Image Stitching

For this part we used a pair of images to attempt image stitching using SIFT and homography computed using the RANSAC algorithm. Starting with extracting SIFT features, we used sorted matches (sorted according to the Euclidean distance) to compute homography from the left image to the right image. The keypoint matches yielded are shown in Fig. 7.

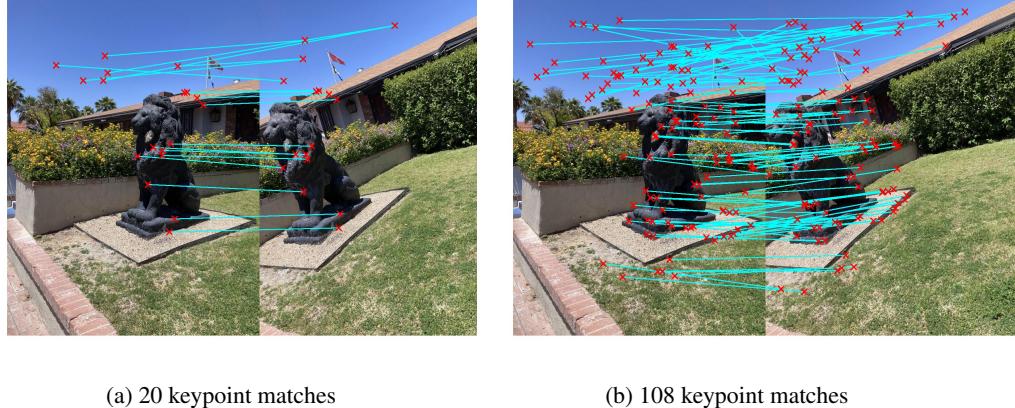


Figure 7: Figure showing keypoint matches across two views of the same scene. These matches were extracted for image stitching.

We then created a white RGB image to hold two images together in a stitch and fixed the position for the left image. Then for each pixel in the right image, we estimated the mapping of the pixel locations in the stitch using the homography computed using the RANSAC algorithm. We faced some trouble getting the stitch correctly. Fig. 8 shows the example of output we received. Conceptually, it made sense of what we were trying to achieve (mapping the points in the second image on top of the first image using the homography – as homography tells us the geometric transformation from one image to another). Most of our “stitched” results looked like they require rotation and translation but from class lectures, we grasped that homography takes care of rotation and translation, so we weren’t sure of what exactly was going awry. It also looked like the second image was being mapped as sheered. I looked into the imwarp function on MATLAB, but wasn’t sure if that was the correct route to take. The other challenge we faced is that while mapping, some points were exploding or splitting from the middle, this can be observed in images (c), (d), and (f) of Fig. 8. The mapping looks funny for most, we believe this happened solely in the step where we computed mapping points using homography found from RANSAC. From results of previous parts of the assignment, we believe that our DLT method and RANSAC algorithm were performing correctly, so it was hard to debug here what’s going wrong.



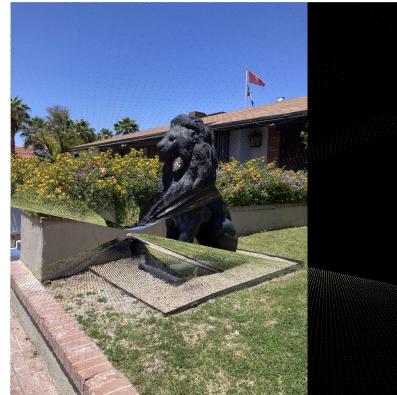
(a)



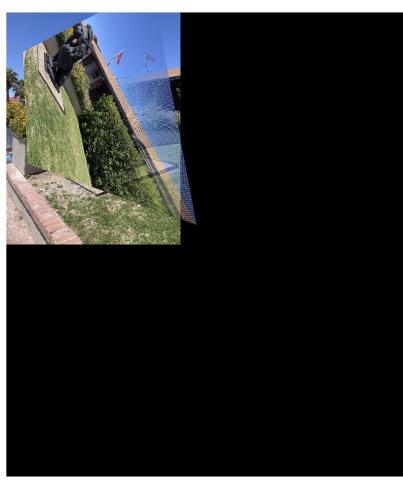
(b)



(c)



(d)



(e)



(f)

Figure 8: Figure showing results from attempted image stitching. Please see text on the previous page for more details regarding these images.