ABM Prediction

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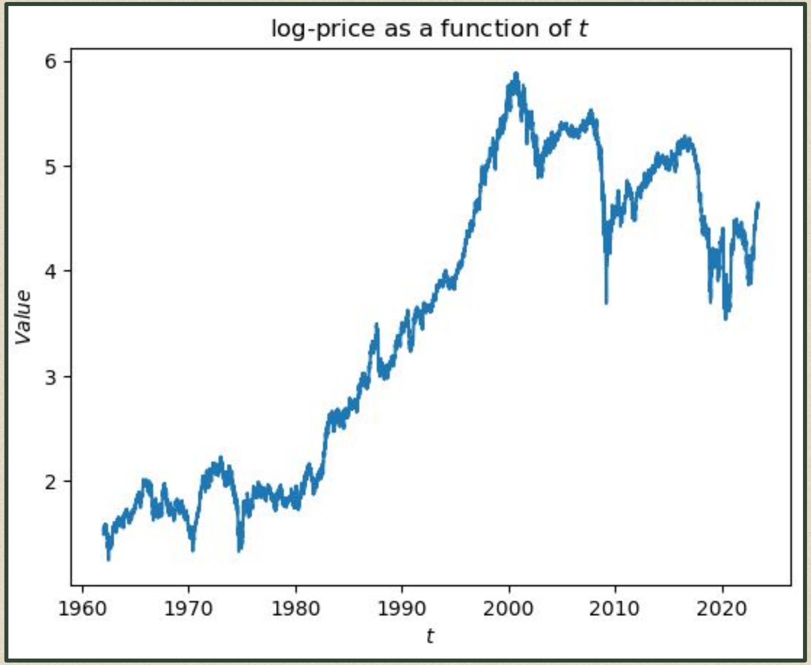
What are the data?



Daily stock prices of General Electrics (from 1962 to 2023)

What is the model?

Majewski et al. type of model

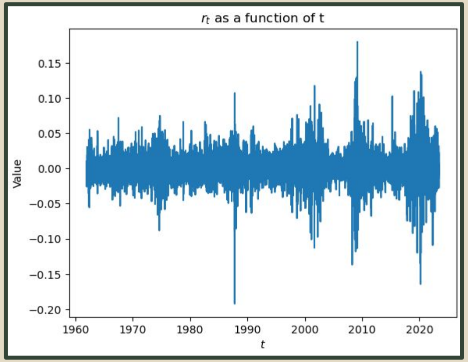
$$\begin{cases} p_{t+1} = p_t + \kappa_1 \left(v_t - p_t \right) + \beta \tanh\left(\gamma\, m_t \right) + \varepsilon_t \\ m_t = (1 - \alpha)\, m_{t-1} + \alpha \left(p_t - p_{t-1} \right) \\ v_t = (1 - \lambda)\, v_{t-1} + \lambda p_t \end{cases}$$

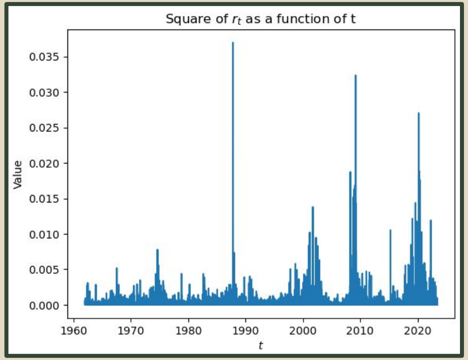
Where $\varepsilon_t \sim \mathcal{N}(0, \sigma(\varepsilon)^2)$

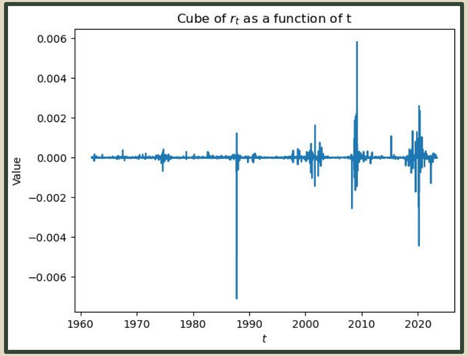
We need to estimate the parameters $\xi = (\kappa_1, \beta, \gamma, \alpha, \lambda, \sigma(\varepsilon))$

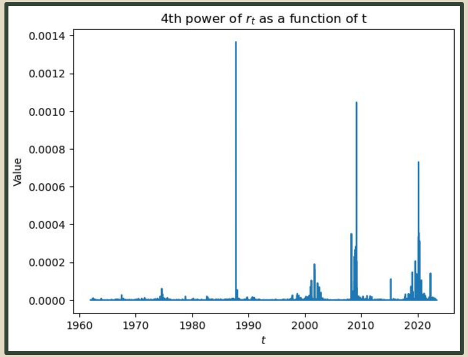
How to find the optimal parameters ?

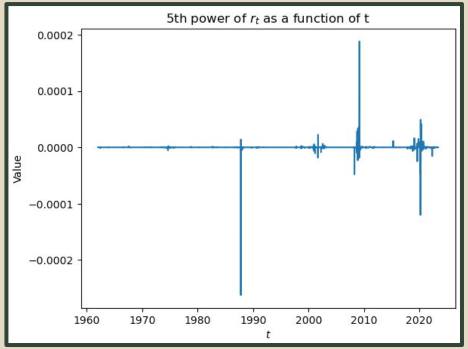
Consider $r, r^2, r^3, r^4, r^5, r^6$ the first six moments of data

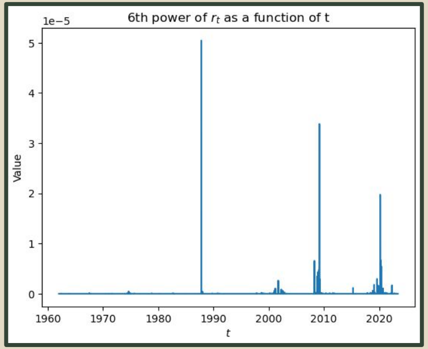












How to find the optimal parameters?

- Consider the function $f : (\kappa_1, \beta, \gamma, \alpha, \lambda, \sigma(\varepsilon)) \mapsto r_{ABM}, r^2_{ABM}, r^3_{ABM}, r^4_{ABM}, r^5_{ABM}, r^6_{ABM}$

  The function is stochastic !

- Consider the vector :

$$X = (\mathbb{E}(r) - \mathbb{E}(r_{ABM}), \mathbb{E}(r^2) - \mathbb{E}(r^2_{ABM}), \ldots, \mathbb{E}(r^6) - \mathbb{E}(r^6_{ABM}), ACF_r(1) - ACF_{r_{ABM}}(1), ACF_r(2) - ACF_{r_{ABM}}(2), ACF_r(3) - ACF_{r_{ABM}}(3))$$

- Minimize 100 times the function: $g : (\kappa_1, \beta, \gamma, \alpha, \lambda, \sigma(\varepsilon)) \mapsto \sum_{i=1}^{9} \sum_{j=1}^{9} X_j W^{(0)}_{i,j} X_i$

  with $W^{(0)} = I_9$

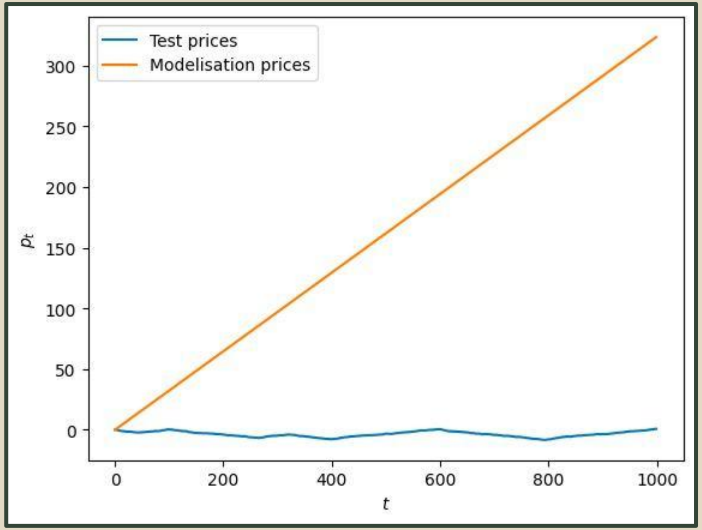
- To minimize, we use the function scipy.optimize.minimize with Powell method

- Calculate the mean of 100 minimisations : we call it $\xi^{(0)}$

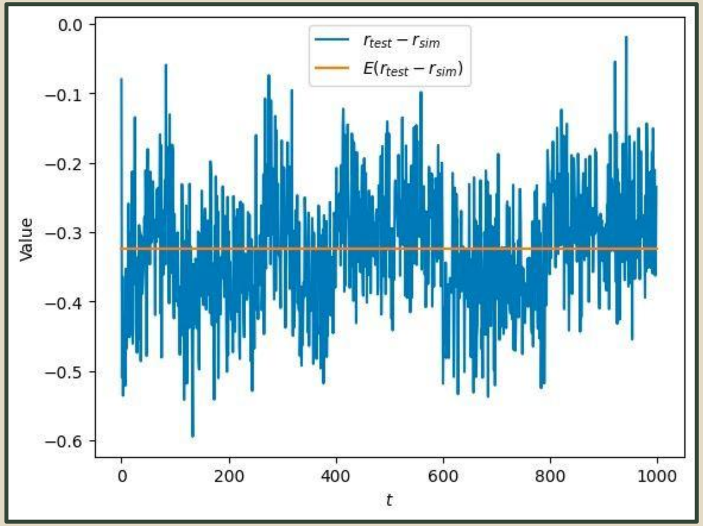
How to find the optimal parameters?

- For each minimized parameters array, we simulate 100 times the data, and calculate the 6 first powers of the returns. This creates an array of size 6x100x15000
- We calculate the mean on the third axis : we now have an array of size 6x100.
- We calculate the ACF, and take lags 1, 2, 3 : we get an array of size 3x100.
- We calculate the covariance matrix with the concatenation. The matrix is of size 9x9.
- We take the inverse of this matrix: $W^{(1)}$
- **With this new matrix, we calculate $\xi^{(1)}$ and we continue until the difference between two vectors is small enough.**

Results



Test prices
Modelisation prices
300
250
200
150
100
50
0
p_t
0
200
400
600
800
1000
t



$r_{test} - r_{sim}$
$E(r_{test} - r_{sim})$
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
Value
0
200
400
600
800
1000
t

Results

$$\xi_{test} = (\kappa = 0.08, \beta = 0.1, \gamma = 50, \alpha = 0.1, \lambda = 0.05, \sigma = 0.075)$$

$$\xi_{mode} = (\kappa = 0.12, \beta = 0.3, \gamma = 54, \alpha = 0.25, \lambda = 0.02, \sigma = 0.05)$$

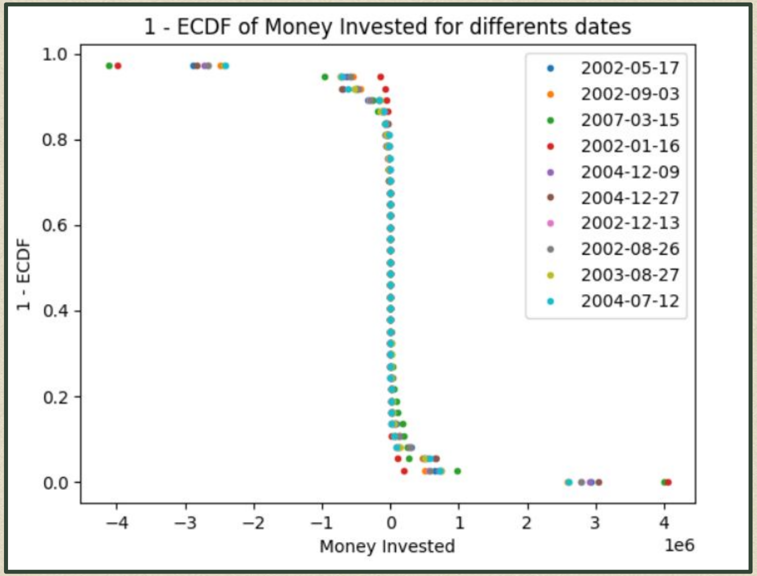
Because the calculus of m_t and v_t do not depend on noise, with the true data, we can predict the future values of m_t and v_t .

Investor behavior

We aim to classify investors in **two categories:**

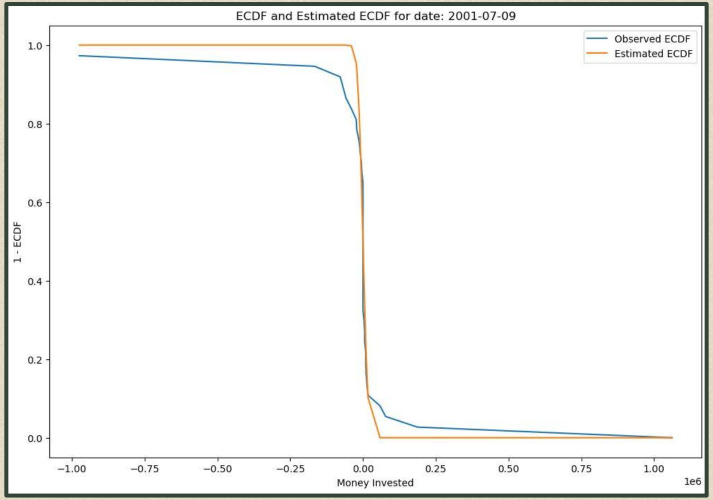
- trend followers
- contrarians

To do so, we plot $1 - ECDF(m)$ where each point represents an investor.



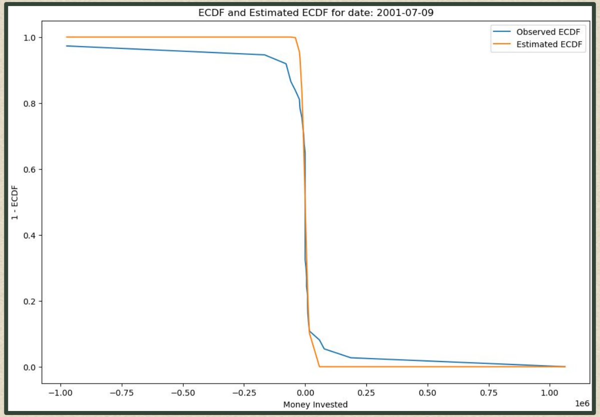
Investor behavior

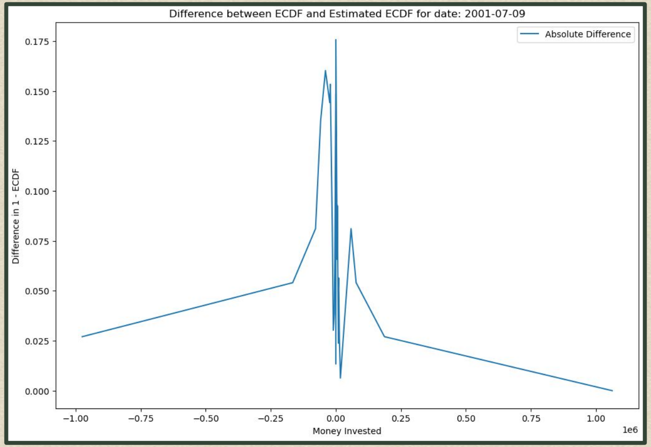
We then plot the 1-ECDF of the money invested:



Investor behavior

We then plot the 1-ECDF of the number of changes of invested volume:






```
message: Optimization terminated successfully.
success: True
 status: 0
    fun: 0.5452154714687333
      x: [ 1.380e+04]
    nit: 2
  direc: [[ 7.749e+02]]
   nfev: 45
```

We begin by try to fit our data with a **centered normal law** but approximation is not satisfying.

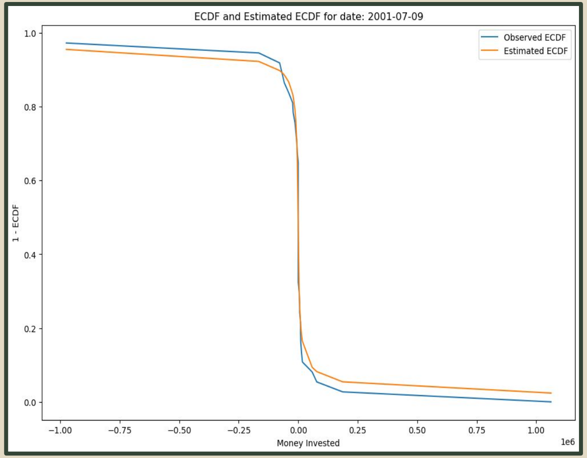
Investor behavior

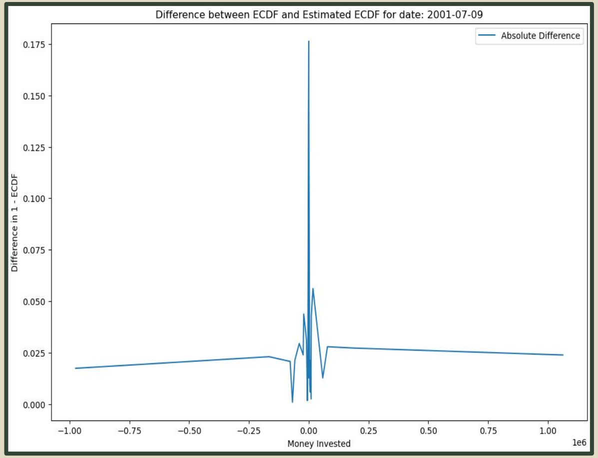
We will search our distribution in those which can be written under the form

$$\frac{\mathcal{T}(k) - \mu}{\sigma}$$

Investor behavior

And it's clearly better !






```
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      x: [ 4.733e-01 -3.099e+02  4.496e+03]
    nit: 14
  direc: [[ 1.358e-03 -9.011e+00  8.230e+01]
          [ 0.000e+00  1.000e+00  0.000e+00]
          [-1.375e-03 -4.388e+00  3.319e+01]]
   nfev: 582
```
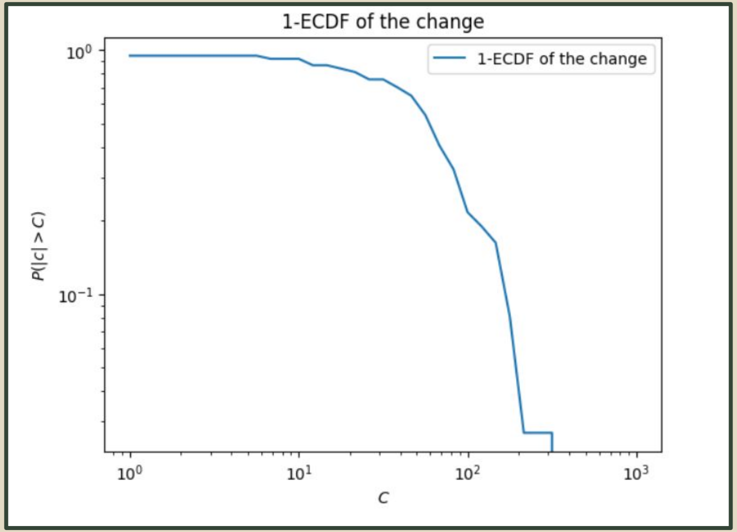

Investor behavior

We plot the 1-ECDF of the number of change



Investor behaviour

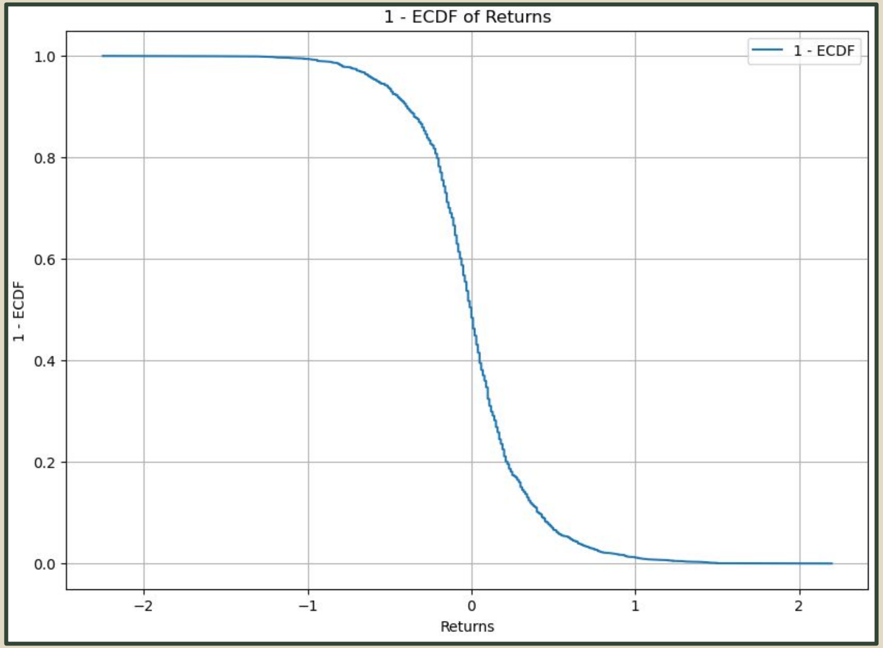
The CDF can be written under the form :

$$\alpha - e^{(x-\beta)}$$

or heavy-tailed with y = 4.35

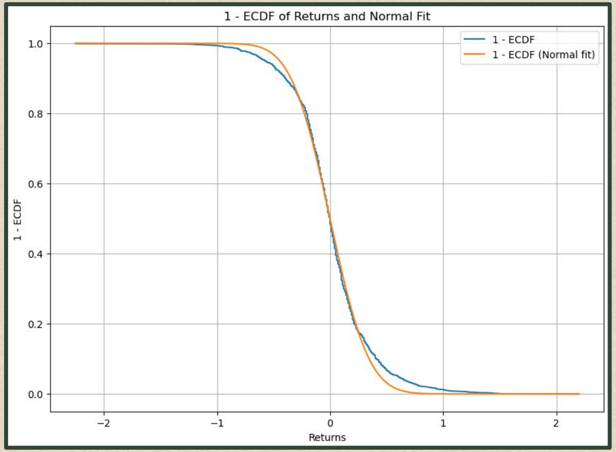
Investor behaviour

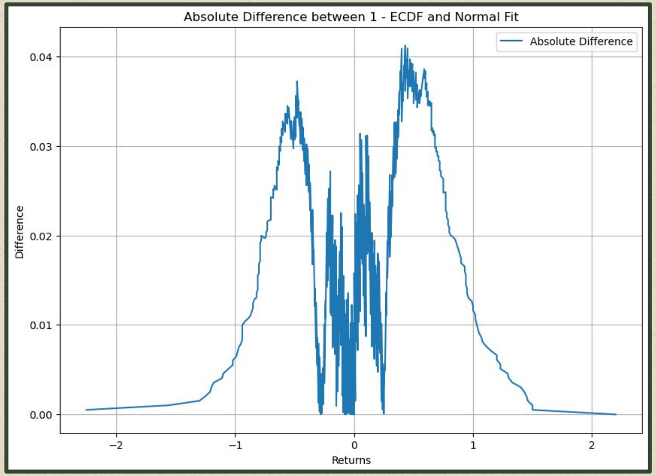
Bonus : we plot the 1-ECDF of the returns



Investor behaviour

We found a normal law !






```
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 success: True
  status: 0
     fun: 0.8719760579593364
       x: [ 2.698e-01]
     nit: 2
   direc: [[ 1.000e+00]]
    nfev: 28
[0.26983099]
```

Investor behavior

We then divide the investors between
mean-reverters, trend followers and neutral.

We calculate correlation between change of inventory and the same-day return for each investor.

If it is above 0, they are **trend followers**,
Below 0 they are **mean-reverters**,
Close to 0 we classify them as **neutral**.

```
investor_id
744      mean-reverting
2955            neutral
2963     mean-reverting
4066     mean-reverting
6018     mean-reverting
6198     mean-reverting
6447     mean-reverting
7319     mean-reverting
7936            neutral
9176     mean-reverting
9185     mean-reverting
9190     mean-reverting
9191     mean-reverting
10510    mean-reverting
10649           neutral
10710           neutral
11513           neutral
11872           neutral
14561           neutral
14774           neutral
15724    mean-reverting
15740    mean-reverting
15814           neutral
15936    mean-reverting
16433    mean-reverting
17837    mean-reverting
18159    mean-reverting
18169    mean-reverting
18178           neutral
18195           neutral
18228           neutral
18232           neutral
18317           neutral
18321    mean-reverting
18322           neutral
18333           neutral
18397    mean-reverting
dtype: object
mean-reverting    21
neutral           16
```

Investor behavior

We compute the probability of an investor changing from a class to another.

second_period	mean-reverting	neutral	trend-following
first_period			
mean-reverting	0.739130	0.217391	0.043478
neutral	0.571429	0.357143	0.071429

We finally determine which investors try to predict future price moves by **calculating the correlation between the change of inventory and future returns**.

neutral	26
mean-reverting	8
trend-following	3

Prediction with an ABM

- Take a model
- Compute the parameters of Majewski *et al.* that fits the most the model
- Compute the volatility and m_t with the last prices
- Predict the next value and repeat the process

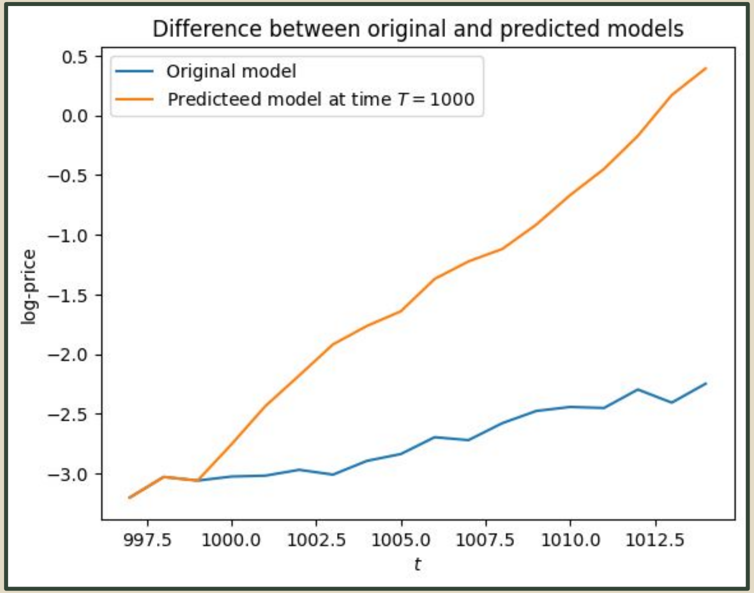
Also for a non stationary signal :

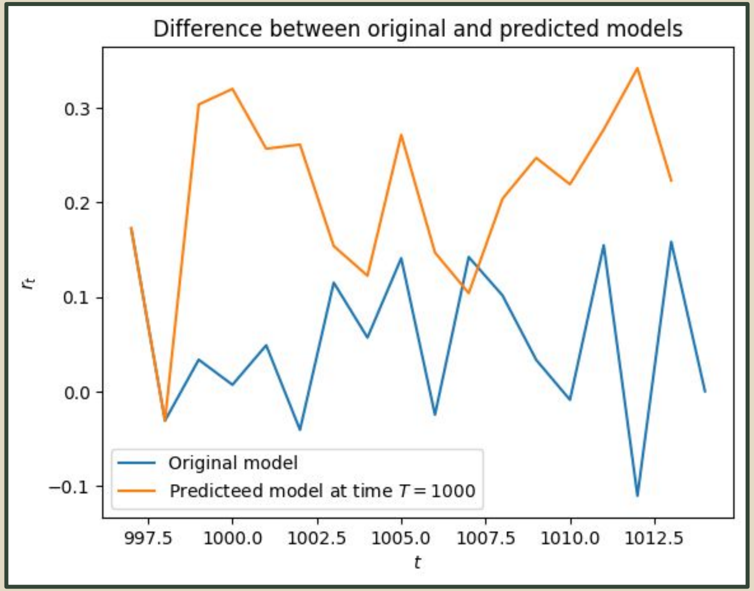
- Predict the n next values and repeat the process

Prediction with an ABM

Results for a model from *Majewski et al* type of model with the 1000 last values :

Parameters given : $\xi_{\text{test}} = (0.08, 0.1, 50, 0.1, 0.05, 0.075)$





Prediction with an ABM

Results :

	True for real model	False for real model
True for predicted model	11	4
False for predicted model	0	0

Precision = 0.7333333333333333

Hit ratio = 1.0

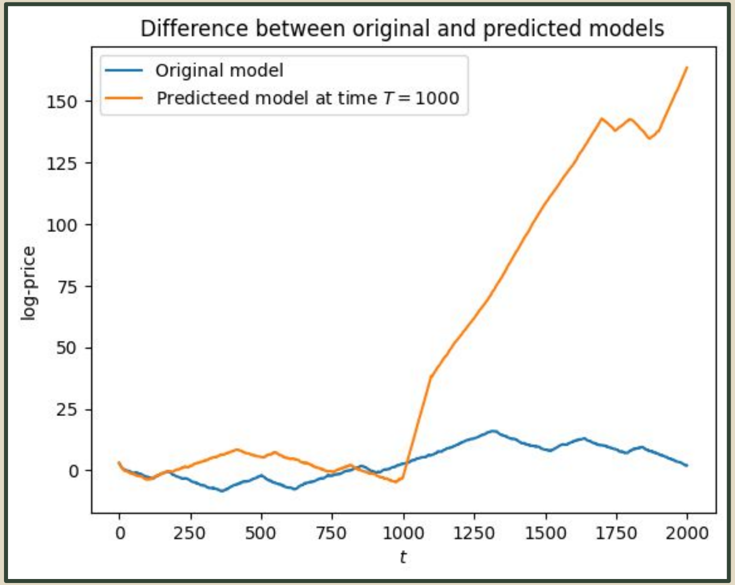
- Good precision and hit ratio
- Not enough prices estimated to compute a safe precision
- Took 9 minutes to compute 15 future values with only 4 iterations max for the GMM method

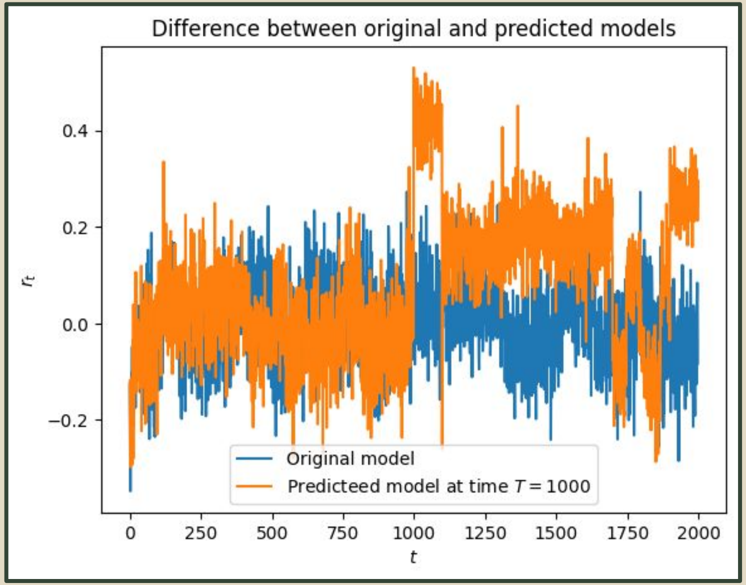
Can you we predict more parameters and do it faster ?

Prediction with an ABM

Results for a model from *Majewski et al* type of model with the 50 last values every 100 values :

Parameters given : $\xi_{\text{test}} = (0.08, 0.1, 50, 0.1, 0.05, 0.075)$





Prediction with an ABM

Results :

```
                              True for real model  False for real model
True for predicted model                      449                   428
False for predicted model                      53                    69
Precision = 0.5119726339794755
Hit ratio = 0.8667953667953668
```

- Took only 2 minutes to predict 1000 prices
- Less good precision but good balance between time and precision

Prediction with an ABM

Results for a model with stocks prices of General Electrics estimated with the last 7000 values :

	True for real model	False for real model
True for predicted model	3	2
False for predicted model	0	0

```
Precision = 0.6
Hit ratio = 1.0
```

- Took 18 minutes to predict only 5 prices
- Precision of 60% → good

Prediction with an ABM

What could have been done to improve the prediction ?

- More estimations of the precision with the prices of the stock exchange to have an idea of the precision for many values
- Take more time to compute with more iterations for the GMM method to see if it improves the precision
- Test on more data of different type (with crypto for example)
- Test if we can earn money with this method 💲

In conclusion

How easy is it to predict the market prices ?

- Financial markets are very complex : more parameters to catch market dynamic ?
- A huge amount of time and computational resources, as well as sophisticated models
- It is easier to understand better market movements, for example by studying investors behaviors

Thanks for your attention !
Tasks distribution:

- Pierre : part 1 & end of part 2
- Paul : part 1 & statistics analysis
- Mohamed : part 1 & beginning of part 2 & beginning of part 3
- Marius : part 1 & end of part 2 & beginning of part 3
- Edward : part 1 & beginning of part 2 & part 3