T_1 (Cantor)

($\mathbb{R}$ este o mulțime ne-numărabilă)

T_2 (Cantor)

~~Pt~~ $(\forall)A$ mulțime

$\cancel{(\exists)}$ $f: A \to \mathcal{P}(A)$ $\xrightarrow{\text{surjectivă}}$

părțile lui A

În particular $A \not\sim \mathcal{P}(A)$

Relații $\boxed{\text{Seminar 2}}$

O relație este un triplet $\alpha = (A, B, \rho)$ unde

$\begin{cases} A, B = \text{mulțimi } (\neq \emptyset) \\ \rho \subseteq A \times B = \{(a,b) \mid a \in A, b \in B\} \end{cases}$

O funcție $f = (A, B, \Gamma_f)$ este o relație (între A și B) cu prop. că $(\forall)\, a \in A, \exists!\, b \in B$ a.î. $(a,b) \in \Gamma_f$

Γ_f = graficul funcției f: $\Gamma_f = \{(a, f(a)) \mid a \in A\}$ $\left(\overset{\shortparallel}{G_f}\right)$

Notație

$$\begin{array}{ccc} f: A & \to & B \\ \downarrow & & \downarrow \\ \forall a & & \exists!\, f(a) \end{array}$$

Compunerea relațiilor

$\alpha = (A, B, \rho)$, $\beta = (B, C, \rho')$

$\beta \circ \alpha := (A, C, \rho' \circ \rho) := \{(a, c) \mid \exists\, b \in B$ a.î. $(a,b) \in \rho, (b,c) \in \rho'\}$

Notație: $(a,b) \in \rho \iff a \rho b$

$(a,b) \notin \rho \iff a \not\rho b$

!Ex

1) $(\forall)\ \alpha = (A,B,\rho)$, $\beta = (B,C,\tau)$, $\gamma = (C,D,\varepsilon)$

$(\gamma \circ \beta) \circ \alpha = \gamma \circ (\beta \circ \alpha)$

2) Dacă $\Delta_A = \{(a,a) \mid a \in A\}$ și $1_A = (A, A, \Delta_A)$,

- $\alpha \circ 1_A = \alpha\ (\forall)\alpha = (A,B,\rho)$

  $1_B \circ \alpha = \alpha$

$A = \{2,4,6,8\}$, $B = \{1,3,5,7\}$

$\rho = \{(x,y) \mid x \geq 6 \lor y \leq 1\} \subseteq A \times B$

$\rho = \{(x,y) \mid x \geq 6 \lor y = 1\} = \{(x,y) \mid x \in \{6,8\} \lor y = 1\}$

$= \{(6,1), (6,3), (6,5), (6,7), (8,1), (8,3), (8,5), (8,7),$
$(2,1), (4,1)\} \subseteq A \times B$

- $A = B = \mathbb{N}$, $\rho = \{(3,5), (5,3), (3,3), (5,5)\}$

  $\tau = \{(x,y) \mid x \leq y\} \subseteq \mathbb{N} \times \mathbb{N}$

  $\gamma = \{(x,y) \mid y - x = 12\} \subseteq \mathbb{N} \times \mathbb{N}$

a) $\rho^{-1}, \tau^{-1}, \gamma^{-1}$

$\rho^{-1} = \{(5,3), (3,5), (3,3), (5,5)\} = \rho$

$\tau^{-1} = \{(x,y) \mid (y,x) \in \tau\} = \{(x,y) \mid y \leq x\}$

$\gamma^{-1} = \{(x,y) \mid (y,x) \in \tau\} = \{(x,y) \mid x - y = 12\}$

$\rho \circ \nabla = \{(a,c) \mid a \in \mathbb{N}, c \in \mathbb{N}, (\exists) b \in \mathbb{N}$ a.î. $(a,b) \in \nabla, (b,c) \in \cancel{\ } \rho\}$

$= \{(a,c) \mid a \leq b, (b,c) \in \{(3,5), (5,3), (3,3), (5,$

$= \{(0,5), (1,5), (2,5), (3,5), (0,3), (1,3), (2,3), (3,3), (4,3), (5,3), (4,5), (5,5)\}$

$\nabla \circ \rho = \{(a,c) \mid a \in \mathbb{N}, c \in \mathbb{N}, \exists b \in \mathbb{N},$ a.î. $(a,b) \in \rho, (b,c) \in \nabla\}$

$= \{(a,c)\ \exists b$ a.î. $(a,b) \in \rho, b \leq c\}$

$= \{(3,c) \mid c \geq 5\} \cup \{(3,3), (3,4)\} \cup \{(5,3) \mid c \geq 3\}$

$\rho \circ \gamma = \{(a,c) \mid a \in \mathbb{N}, c \in \mathbb{N}, (\exists) b \in \mathbb{N}$ a.î. $(a,b) \in \gamma, (b,c) \in \rho\}$

$= \{(a,c) \mid \cancel{\ }, \exists b$ a.î. $b - a = 12, (b,c) \in \rho\}$

$= \emptyset$

$\gamma \circ \rho = \{(a,c) \mid \exists b$ a.î. $(a,b) \in \rho \wedge (b,c) \in \gamma\}$

$= \{(a,c) \mid \exists b$ a.î. $(a,b) \in \rho \wedge c - b = 12\}$

$= \{(3,17), (3,15), (5,15), (5,17)\}$

Def $\alpha=(A,B,\rho)$

$\alpha^{-1}=(B,A,\rho^{-1})$ cu $\rho^{-1}=\{(b,a) \mid (a,b)\in\rho\}$

$\boxed{\sigma\circ\delta,\ \delta\circ\sigma \quad \text{Temă}}$

Exc:

$\rho=\{(2a,3b) \mid a,b\in\mathbb{Z}\}$

$\rho^n=?$, $n\in\mathbb{Z}$

$\rho^2=\rho\circ\rho=\{(x,z) \mid \exists y\in\mathbb{Z}$ a.î. $(x,y)\in\rho \wedge (y,z)\in\rho\}$

$=\{(x,z) \mid \exists a,b\in\mathbb{Z}$ a.î. $x=2a, y=3b \wedge (3b,z)\in\rho)\}$

$=\{(2a,z) \mid a\in\mathbb{Z}, \exists b\in\mathbb{Z}$ a.î. $(3b,z)\in\rho\}$

$=\{(2a,3b') \mid a,b'\in\mathbb{Z}\}=\rho \Rightarrow \rho^n=\rho$

$(\forall)\, n\in\mathbb{N}^*$

$\sigma=\rho^{-1}=\{(3a,2b) \mid a,b\in\mathbb{Z}\}$

$\sigma^2=\sigma\circ\sigma=\{(x,z) \mid x,z\in\mathbb{Z}, \exists y\in\mathbb{Z}$ a.î. $(x,y)\in\sigma,\ (y,z)\in\sigma$

$=\{(3a,2b) \mid a,b\in\mathbb{Z}$ a.î. $\exists y\in\mathbb{Z}\ (y,z)\in\sigma$

$=\{(3a,2b') \mid a,b'\in\mathbb{Z}\}=\rho\ (\forall)\, n\in\mathbb{N}$

$\Rightarrow \rho^n=\begin{cases}\sigma, & n\in\mathbb{Z}\setminus\mathbb{N}\\ \rho, & n\in\mathbb{N}^*\\ 1_{\mathbb{Z}}, & n=0\end{cases}$

$$(\alpha \circ \beta)^{-1} = \beta^{-1} \circ \alpha^{-1} = \underbrace{\alpha^{-1} \circ \dots \circ \alpha^{-1}}_{n \text{ ori}}$$

$$(\alpha^{n})^{-1} = (\alpha^{-1})^{n}$$

$$\rho = \{(a, a+3) \mid a \in \mathbb{Z}\}$$

$$\rho^{n} = ?$$

$$\rho^{2} = \rho \circ \rho = \{(a, c) \mid \exists b \in \mathbb{Z} \text{ a.î } (a, b) \in \rho, (b, c) \in \rho\} \quad c, a \in \mathbb{Z}$$

$$= \{(a, c) \mid b = a+3,\ c = b+3\}$$

$$= \{(a, c) \mid c = a+6\}$$

$$= \{(a, a+6) \mid a \in \mathbb{Z}\}$$

$$\rho^{n} = \{(a, a+3n)\}$$

$$n \to n+1$$

$$\rho^{n} \circ \rho = \{(a, c) \mid a, c \in \mathbb{Z} \mid \exists b \in \mathbb{Z} \text{ a.î } (a, b) \in \rho, (b, c) \in \rho^{n}\}$$

$$= \{(a, c) \mid b = a+3,\ c = b+3n\}$$

$$= \{(b-3, b+3n) \mid b \in \mathbb{Z}\}$$

$$= \{(a, a+3(n+1)) \mid a \in \mathbb{Z}\}$$

$$\rho^{-1} = \{(a+3, a) \mid a \in \mathbb{Z}\} = \{(a, a-3) \mid a \in \mathbb{Z}\}$$

$$\rho^{-n} = \{(a, a-3n) \mid a \in \mathbb{Z}\} \ (\forall) n \in \mathbb{N}^{*}$$

$\rho \cup \rho^{-1} = \{(a, b) \mid 3 \nmid (a-b)\} \subseteq \mathbb{Z} \times \mathbb{Z}$

$(\rho \cup \rho^{-1})^n = ?$ Tema