Seminar 1

daniel.bulacu@unibuc.ro

1. $A = \{3n-2 \mid n \in \mathbb{N}\}$ $\quad A \cap B = \emptyset$?

$B = \{1003 - 2m \mid m \in \mathbb{N}\}$

$x \in A \cap B \Leftrightarrow x \in A, x \in B \Leftrightarrow \exists n, m \in \mathbb{N}$ a.î

$x = 3n-2 = 1003 - 2m$

$3n - 2 = 1003 - 2m$

$\boxed{3n + 2m = 1005}$

Obs: $\left.\begin{matrix} 3 \mid 3n \\ 3 \mid 1005 \end{matrix}\right\} \Rightarrow 3 \mid 2m \Rightarrow 3 \mid m$

$m = 3m'; \ m' \in \mathbb{Z}$

$3n + 6m' = 1005 \;/:3 \neq 0$

$n + 2m' = 335 \Leftrightarrow 0 \leq m' \leq 167$

$n = 335 - 2m' =$

$A \cap B = \{3n-2 \quad \{1003 - 6m' \mid 0 \leq m' \leq 167\}$

□ 2 $ax + by = c$, $a, b, c \in \mathbb{Z}^*$

!! - ec are sol în $\mathbb{Z} \times \mathbb{Z} \Leftrightarrow (a,b) \mid c$

"$\Rightarrow$" Fie $(x_0, y_0) \in \mathbb{Z} \times \mathbb{Z}$ soluție.

$ax_0 + by_0 = c$

$(a,b) \mid a$ și $(a,b) \mid b \Rightarrow (a,b) \mid ax_0 + by_0 = c$

($''u''$)

"$\Leftarrow$" $(a,b) \mid c$

Notăm $d = (a,b)$ // cmmdc $= 1$

$\Rightarrow a = da'$

$b = db'$, $(a', b') = 1$

$d \mid c$

$c = dc'$, $c' \in \mathbb{Z}$

$da' + db' = dc' \;/: d \neq 0$

$a' + b' = c'$

$(a', b') = 1 \Leftrightarrow \exists\, u, v \in \mathbb{Z}$ a.î. $ua' + vb' = 1 / \cdot c'$

$a'uc' + b'vc' = c' /: d$

$auc' + bvc' = c \Rightarrow (uc', vc')$ sol ec.

$281x - 133y = 3$

$281 : 133 = 2 \text{ r } 15$

$133 : 15 = 8 \text{ r } 13$

$15 : 13 = 1 \text{ r } 2$

$13 : 2 = 6 \text{ r } 1$

$2 : (1) = 2 \text{ r } 0$

$15 = 281 - 2 \cdot 133$

$13 = 133 - 15 \cdot 8 = 133 - (281 - 2 \cdot 133) \cdot 8$

$= -8 \cdot 281 + 17 \cdot 133$

$2 = 15 - 13 = 281 - 2 \cdot 133 + 8 \cdot 281 - 17 \cdot 133$

$= 9 \cdot 281 - 19 \cdot 133$

$1 = 13 - 2 \cdot 6 = -8 \cdot 281 + 17 \cdot 133 - 6(9 \cdot 281 - 19 \cdot 133)$

$= (-8 \cdot 281) + 17 \cdot 133 (-54 \cdot 281) + 114 \cdot 133$

$= -62 \cdot 281 + 131 \cdot 133$

$-62 \cdot 281 + 131 \cdot 133 = 1 \mid \cdot 3$

$-186 \cdot 281 + 393 \cdot 133 = 3 \Rightarrow \begin{cases} x = -186 \\ y = -393 \end{cases}$

~~$= 1 \cdot 281 + 131 \cdot 133 = 1 \mid \cdot 3$~~

~~$-186 \cdot 281 + 393 \cdot 133 = 3$~~ $281 \cdot (-62) + 133 \cdot (-97)$

$\Rightarrow x = -186$

$y = -393$

□ $ax + by = c, \quad a, b, c \in \mathbb{Z}^*$

$d = (a, b) \mid c$

$a = d a'$

$b = d b'$

$(a', b') = 1$

$c = d c'$

$\Rightarrow ax + by = c \Leftrightarrow a'x + b'y = c'$

Fie (x_0, y_0) sol particulară

Dacă (x, y) este altă sol $\Rightarrow a'x_0 + b'y_0 =$

$= a'x + b'y$

$a'(x_0 - x) = b'(y - y_0) \Rightarrow a' \mid b'(y - y_0)$

$(a', b') = 1$

$\Rightarrow a' \mid (y - y_0) \Rightarrow \exists t \in \mathbb{Z}$ a.î

$y - y_0 = a' \cdot t$

$\Rightarrow (x_0 - x) = b' \cdot t \Rightarrow (x, y) = (x_0 - b't, y_0 +$

$t \in \mathbb{Z}$

$281x - 133y = 3$

$\begin{cases} x = -186 + 133t \\ y = -395 + 281t \end{cases}, t \in \mathbb{Z}$ (nu generală)

de ce

$A = \{3n-2 \mid n \in \mathbb{Z}\}$

$B = \{1003-2m \mid m \in \mathbb{Z}\}$

$A \cap B = ?$

$x \in A \cap B \Leftrightarrow x = 3n-2 = 1003-2m$, $n, m \in \mathbb{Z}$

$3n-2 = 1003-2m \Leftrightarrow 3n+2m = 1005$

$1 = 3-2 \mid \cdot 1005$

$3 \cdot 1005 - 2 \cdot 1005 = 1005$

$x_0 = 1005$

$y_0 = -1005$

$n = 1005 - 2t$, $t \in \mathbb{Z}$

$m = -1005 + 3t$

$A \cap B = \{3(1003-2t)-2 \mid t \in \mathbb{Z}\}$

$= \{-6t + 3013 \mid t \in \mathbb{Z}\}$

$= \{6t + 1 \mid t \in \mathbb{Z}\}$

□ $M = \{ x \in \mathbb{Q} \mid x = \frac{n^2+3}{n^2+n},\ n \in \{1, \dots, 50\} \subset \mathbb{N} \}$

$card(M) = ?$ ✓

Fie $n, m \in \{1, 2, \dots 50\}$, $n \neq m$

$$\frac{n^2+3}{n^2+n} = \frac{m^2+3}{m^2+m}$$

$(n^2+3)(m^2+m) = (n^2+n)(m^2+3)$

$n^2m^2 + 3n^2 + 3m^2 + 9 = n^2m^2 + mn^2 + nm^2 + 3n$

~~$n^2m^2 + 3n^2 + 3m = 3m^2 + nm^2 + 3n$~~

$n^2m - nm^2 + 3(n^2 - m^2) + 3(m - n) = 0$

$(n-m)[nm + 3(m+n) - 3] = 0 \mid :(n-m) \neq 0$

$nm - 3(m+n) - 3 = 0$

$m(n-3) - 3(n-3) = 12$

$(m-3)(n-3) = 12$

I $m-3 = 1, \quad n-3 = 12$

II $m-3 = -1, \quad n-3 = -12$

III

$card(M) = 47$

$\Rightarrow n = 15$

$m = 3 - 2 = (4, 15) \quad (3, 3)$

$m = 4 \quad (5, 9)$

$(6, 7)$

$M = \{ x \in \mathbb{Q} \mid x = \frac{n^2+3}{n^2+n},\ n \in \{1,2,\dots,50\} \subset \mathbb{N} \}$

Card(M)=?

Fie $n, m \in \{1,2,3,\dots 50\}$, $n \neq m$ a.î $\frac{n^2+3}{n^2+n} = \frac{m^2+3}{m^2+m}$

$(n^2+3)(m^2+m) = (n^2+n)(m^2+3)$

~~$n^2m^2 + 3n^2 + 9 + 3m^2 = n^2m^2 + nm^2 + n^2m + mn$~~

~~$n^2m - nm^2 + 3(n^2 - m^2) + 3(m-n) = 0$~~

~~$(n-m)[nm$~~

$n^2m^2 + n^2m + 3m^2 + 3n = n^2m^2 + nm^2 + 3m^2 + 3n$ $+mn$

$n^2m + 3m^2 + 3n = nm^2 + 3m^2 + 3n$

~~mnm~~

$n^2m - nm^2 + 3(n^2 - m^2) + 3(m-n) = 0$

$(n-m)[nm - 3(m+n) - 3] = 0 \ / : (n-m) \neq 0$

$nm - 3(m+n) - 3 = 0$

$m(n-3) - 3(n-3) = 12$

$(m-3)(n-3) = 12$

I $n-3 = 1 \Rightarrow n = 4,\ m = 15 \in \mathbb{N}$ $(4,15)$

II $n-3 = -1 \Rightarrow n = 2,$ ~~$m = 11 = 15$~~ $m = -9 \notin \mathbb{N}$

III $n-3 = -2 \Rightarrow n = 1,\ m-3 = -6 \Rightarrow m = -3 \notin \mathbb{N}$

IV $n-3 = 2 \Rightarrow n = 5,\ m-3 = 6 \Rightarrow m = 9 \in \mathbb{N}$ $(5,9)$

V $n-3 = 3 \Rightarrow n = 6,\ m = 7$ $(6,7)$

...

Card(M) = 47