Assignment 1

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1 5-card Poker Hands

Consider the domain of dealing 5-card poker hands from a standard deck of 52 cards, under the assumption that the dealer is fair.

1.1 How many atomic events are there in the joint probability distributions?

Given that the focus is the different 5-card hands, and **not** the order in which we draw the cards, we are interested in the different combinations:
$\binom{52}{5} = \underline{2598960}$

1.2 What is the probability of each atomic event?

The exercise says that the dealer is fair (or at least to assume that he is). Therefore, the probability of each hand (atomic event) should be equal. Given the number of different combinations above, we have the following probability:
$\frac{1}{2598960} = \underline{3.85 \times 10^{-7}}$

1.3 What is the probability of being dealt a royal straight flush? Four of a kind?

Royal Straight Flush
A Royal Straight Flush is a combination of the cards A, K, Q, J, 10, where all is of the same kind. Therefore, there are only 4 possible Royal Straight Flushes. Since we have the same probability of all atomic events, we get the following probability for a Royal Straight Flush:
$4 \times \frac{1}{2598960} = \underline{\frac{1}{649740}}$

Four Of A Kind
When looking at four of a kind, we not only need to look at the four cards, that need to be of the same kind, but also the fifth card that needs to be different. We therefore get:
$\binom{13}{1}\binom{12}{1}\binom{4}{1} = \frac{13!}{1!(13-1)!}\frac{12!}{1!(12-1)!}\frac{4!}{1!(4-1)!}) = 13 \times 12 \times 4 = 624$
$\frac{624}{2598960} = \underline{\frac{1}{4165}}$

2 Bayesian Network Construction

Construct Bayesian Networks that represents the following factorized distribution:

1. $p(A, B, C, D) = p(A|B, C)p(B|D)p(C|D)p(D)$
2. $p(X_1, ..., X_n) = p(X_1) \prod_{i=2}^{i=n} p(X_i|X_{i-1})$
3. $p(RainToday, RainYesterday, FloorWet, UseUmbrellaToday, CloudSky) =$
$p(RainYesterday) \times p(CloudSky) \times$
$p(RainToday|RainYesterday, CloudSky) \times$
$p(FloorWet|RainToday, RainYesterday) \times$
$p(UseUmbrellaToday|RainToday, CloudSky)$

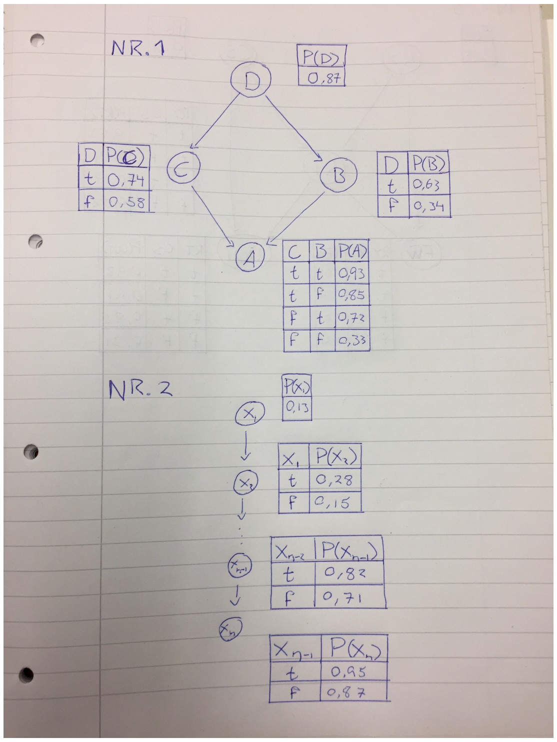

Figure 1: Bayesian Networks for distribution 1 and 2

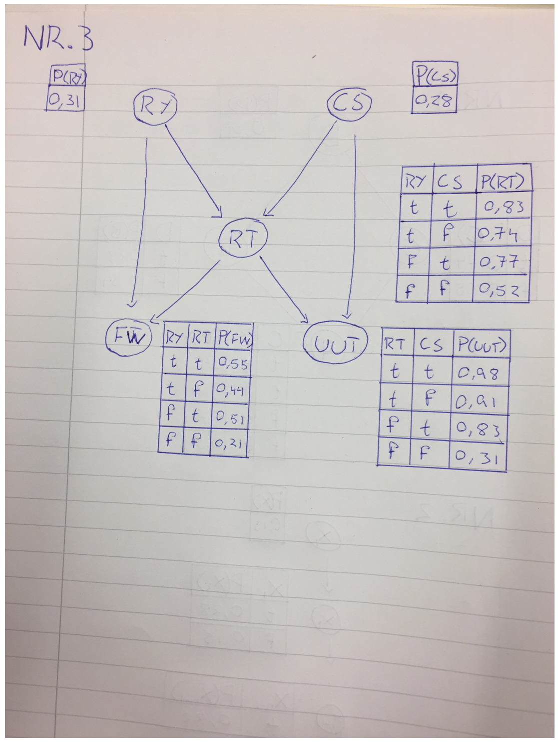

Figure 2: Bayesian Networks for distribution 3

Independence and conditional independence provide the tools for simplifying conditional probability tables. This is to make them much smaller than joint distribution tables which, in the worst case has $O(d^n)$, where d is the largest arity, specifying all relations.

Two events are *independent* iff $P(A|B) = P(A)$ or vise versa. Two events are *conditionally independent* iff for a third event, the following equation holds: $P(A|B, C) = P(A|B)$ (or the other way around).

In a Bayesian network, a node is conditionally independent of its other predecessors, given its parents. An example of this is Network nr. 2, where each node is solely dependent on its single parent, and none of the prior nodes.

As mentioned, the two forms of independence can help with the construction of CSPs because they become much, much, smaller than a full joint distribution table of the same domain.

3 Bayesian Network Application

You are are confronted with three doors A, B, and C. Behind exactly one of the doors there is $10000. The money is yours if you choose the correct door. After you have made your first choice of door but still not opened it, an official comes in. He works according to some rules:

1. He starts by opening a door. He knows where the prize is, and he is not allowed to open that door. Furthermore, he cannot open the door you have chosen. Hence, he opens the door with nothing behind.
2. Now there are two closed doors, one of which contains the prize. The official will ask you if you want to alter your choice (i.e., to trade your door for the other one that is not open).

Should you do that?

I chose to solve this task using the recommended tool GeNIe2.0. I had some initial problems understanding the program, but after the tutorial was shared on blackboard things became much easier. The network file can be found attached, called *Oving1_Network*.xdsl

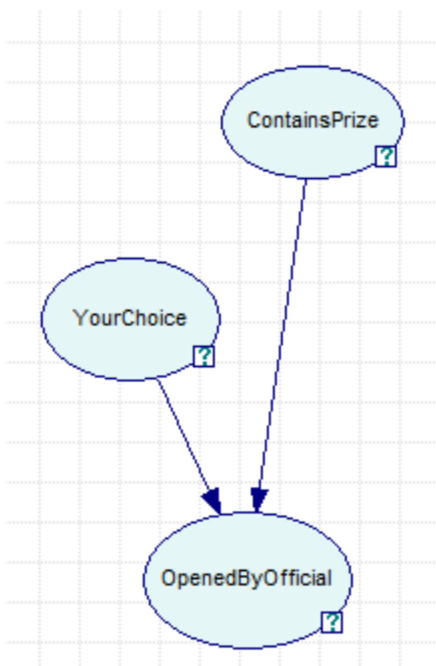

Figure 3: A Bayesian Network illustrating the problem

A	0.33333333
B	0.33333333
C	0.33333333

Figure 4: Conditional Probability table for YourChoice.

A	0.33333333
B	0.33333333
C	0.33333333

Figure 5: Conditional probability table for ContainsPrize.

YourChoice	A			B			C		
ContainsPrize	A	B	C	A	B	C	A	B	C
A	0	0	0	0	0.5	1	0	1	0.5
B	0.5	0	1	0	0	0	1	0	0.5
C	0.5	1	0	1	0.5	0	0	0	0

Figure 6: Conditional probability table for OpenedByOfficial.

Figure 3 illustrates the Bayesian network of the problem. Showing the connection between the different door statuses. Here we see that both *YourChoice* and *ContainsPrize* are not dependent of any of the other nodes. On the other hand, *OpenenedByOfficial* is dependent of both the other nodes. This is logical given the problem description.

Figure 4, 5 and 6 show the conditional probability tables for *YourChoice*, *ContainsPrize* and *OpenedByOfficial*.

The two first tables aren't that interesting. Both *YourChoice* and *ContainsPrize* are not dependent of any other node, and since we are assuming the game is "fair" the probability is equal for any of the choices and potential prize doors. The conditional probability table for *OpenedByOfficial* is more interesting. Here we see that the choice the Official makes is dependent of which door is chosen by the contestant and which door contains the prize. If we look at the figure when Door A is chosen, we see that there are different probabilities for the official opening different doors based on where the prize actually is. The Official will never open door A, since this breaks the rules of the game. If the prize is behind door A, i.e. *YourChoice* was right, we see that there is a 50/50 chance for the Official picking any of the remaining doors, since it does not really matter. If the prize is behind any of the other doors, we see that with the Official will open the third door with 100% probability.

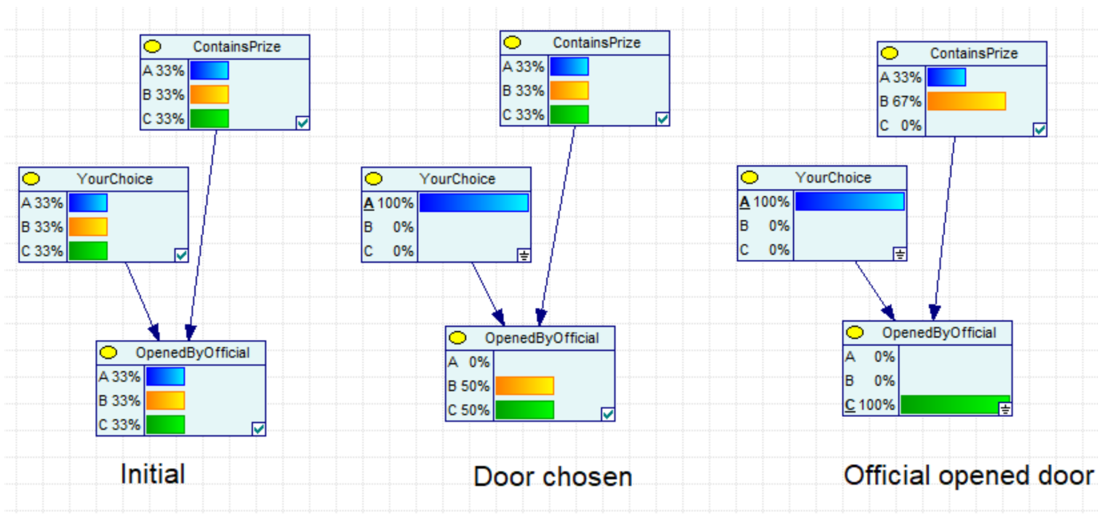

Figure 7: The different probabilities as choices are made.

Figure 7 illustrates how the probabilities change as choices as made.

1. To begin with there is an equal probability for picking any of the different doors for all of the nodes, $\frac{1}{3}$.

2. After *YourChoice* has been made, in this case A, We see that the probability of the choice being right has not changed. However, the probability of which door the official will open has changed. The values are a bit misleading because it may indicate that the right door is A, since then it would not matter which door the Official opens (with respect to the CPT above). However, since we do not know which door is the right door, we end up with a 50/50 chance for B or C, only using the knowledge of *YourChoice*.

3. After the Offical has made his choice (in this case C). We see that the probability of the chosen door being correct is still $\frac{1}{3}$, while the remaining door has a probability of $\frac{2}{3}$.

This may be a bit hard to grasp at first. One might, intuitively, think that once there are only two doors left, the odds will be changed to 50/50. This, however, is not the case. The reason for this is that our choice was made back when there was a $\frac{1}{3}$ chance for all of the doors. So the chosen door has a $\frac{1}{3}$ of a chance while the two remaining have a combined $\frac{2}{3}$ of a chance. When removing one of the two other doors, the final door, not chosen by anyone will have a probability of $\frac{2}{3}$ by itself. Therefore, one should always say "Yes" when given the chance to switch doors.