Deliverable 2

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1 Models and Entailment in Propositional Logic

1.1 For each statement below, determine whether the statement is true or false by building the complete model table.

a $\neg A \land \neg B \models \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \land \neg B$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

From the table we see that the statement is **true** (See right to left on the last two columns).

b $\neg A \lor \neg B \models \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \lor \neg B$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

From the table we see that the statement is **false** (See right to left on the last two columns).

c $\neg A \land B \models A \lor B$

A	B	$\neg A$	$\neg A \land B$	$A \lor B$
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	0	0	1

From the table we see that the statement is **true**.

d $A \to B \models A \leftrightarrow B$

A	B	$A \to B$	$B \to A$	$A \leftrightarrow B$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

From the table we see that the statement is **false**.

e $(A \to B) \leftrightarrow C \models A \lor \neg B \lor C$

A	B	C	$A \to B$	$(A \to B) \leftrightarrow C$	$A \lor \neg B \lor C$
0	0	0	1	0	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	0	1
1	1	0	1	0	1
1	1	1	1	1	1

From the table we see that the statement is **true**.

f $(\neg A \to \neg B) \land (A \land \neg B)$ **is satisfiable**

A	B	$\neg A$	$\neg B$	$\neg A \to \neg B (P)$	$A \land \neg B (Q)$	$P \land Q$
0	0	1	1	1	0	0
0	1	1	0	0	0	0
1	0	0	1	1	1	1
1	1	0	0	1	0	0

From the table we see that the statement **is satisfiable**.

g $(\neg A \leftrightarrow \neg B) \land (A \land \neg B)$ **is satisfiable**

A	B	$\neg A$	$\neg B$	$\neg A \leftrightarrow \neg B (P)$	$A \land \neg B (Q)$	$P \land Q$
0	0	1	1	1	0	0
0	1	1	0	0	0	0
1	0	0	1	0	1	0
1	1	0	0	1	0	0

From the table we see that the statement **is not satisfiable**.

1.2 **Consider a logical knowledge base with 100 variables, $A_1, A_2, ..., A_{100}$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it.**
Feel free to express your answer as a fraction of Q (without writing out the whole number) or to use other symbols to represent large numbers.
Example: The logical sentence A_1 will be satisfied by $\frac{1}{2}Q = \frac{1}{2}2^{100} = 2^{99}$ models.

a $\neg A_{38} \land \neg A_{49}$ **valid models:** $\frac{1}{4}Q$

b $A_{27} \land \neg A_{46} \land A_{57}$**valid models:** $\frac{1}{8}Q$

c $A_{27} \land (A_{46} \lor \neg A_{57}$**valid models:** $\frac{3}{8}Q$

d $\neg A_{85} \rightarrow \neg A_{91}$ **valid models:** $\frac{3}{4}Q$

e $(\neg A_{14} \leftrightarrow \neg A_{19}) \land (A_{21} \rightarrow A_{22})$ **valid models:** $\frac{3}{8}Q$

f $A_{41} \land \neg A_{59} \land A_{64} \land \neg A_{85} \land A_{87} \land A_{90}$ **valid models:** $\frac{1}{64}Q$

1.3 **Consider the Wumpus world in Figure 7.2 / 6.2 . Suppose the agent starts in room in the bottom right corner, i.e. [4, 1]. The agent then moves north/upward twice, visiting the rooms [4, 2] and [4, 3]. The agent perceives a breeze in [4, 1] and [4, 3], but not in [4, 2]. From these percepts, we are interested in determining the possible configuration of pits in the adjacent rooms, i.e. [3, 1], [3, 2], [3, 3] and [4, 4].**
Build the full model table of all possible worlds by constructing a truth table with the variables $P_{3,1}, P_{3,2}, P_{3,3}$ and $P_{4,4}$, each of which specifies whether there is a pit in a particular room. Mark the worlds in which the knowledge base is true, i.e. the pit configurations that are consistent with the perceived breezes. Additionally, mark the worlds in which each of the following sentences is true:

1. $\alpha 1$ = There is no pit in [3,2]
2. $\alpha 2$ = There is a pit in [4,4]
3. $\alpha 3$ = There is no a pit in [4,4]
4. $\alpha 4$ = There is a pit in [3,3] or [4,4]

Nr	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$	$P_{4,4}$
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0
16	1	1	1	1

The character has perceived a breeze in position [4,1] and [4,3] this gives that the table lines 17 to 23 are worlds in which the knowledge base is true.

1. $\alpha 1$: Lines 1 to 4 an lines 9 to 12.
2. $\alpha 2$: Every other line, starting with 2
3. $\alpha 3$: Every other line, starting with 1
4. $\alpha 4$: Every line except 1, 5, 9 and 13.

$KB \models \alpha 1$ and $\alpha 4$

2 Resolution in Propositional Logic

2.1 Convert each of the following sentences to Conjunctive Normal Form (CNF).

a $A \vee (B \wedge C \wedge \neg D)$

$$(A \vee B) \wedge (A \vee C) \wedge (A \vee \neg D)$$

b $\neg(A \rightarrow \neg B) \wedge \neg(C \rightarrow \neg D)$

$$A \wedge B \wedge C \wedge D$$

c $\neg((A \rightarrow B) \wedge (C \rightarrow D))$

$$(A \vee C) \wedge (A \vee \neg D) \wedge (\neg B \vee C) \wedge (\neg B \vee \neg D)$$

d $(A \wedge B) \vee (C \rightarrow D)$

$$(A \vee \neg C \vee D) \wedge (B \vee \neg C \vee D)$$

e $A \leftrightarrow (B \rightarrow \neg C)$

$$(\neg A \vee \neg B \vee \neg C) \wedge (A \vee B) \wedge (A \vee C)$$

2.2 Consider the following Knowledge Base (KB):

- $(D \wedge E) \rightarrow C$
- $\neg A \rightarrow \neg B$
- $\neg C \wedge E$
- $\neg D \rightarrow B$

Use resolution to show that $KB \models A$

1:	$\neg C \wedge E$	KB
2:	$(D \wedge E) \rightarrow C$	KB
3:	$\neg(D \wedge E) \vee C$	2
3:	$\neg D$	1 + 3
4:	$\neg D \rightarrow B$	KB
5:	B	3 + 4
6:	$\neg A \rightarrow \neg B$	KB
7:	$A \vee \neg B$	6
8:	A	5 + 7

2.3 Do exercise 7.17 (assuming this is a typo, and that 7.18 is the correct exercise) / 6.18 from the textbook (Consider the following sentence. . .), but with the following sentence instead of the one in the textbook:

$$(\neg Party \rightarrow \neg(Food \lor Drinks)) \rightarrow (Food \rightarrow Party)$$

$Party = p$, $Food = f$ **and** $Drinks = d$

a Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable

p	f	d	$\neg p$	$\neg(f \lor d)$	$\neg p \rightarrow \neg(f \lor d)$	$f \rightarrow p$	$(\neg p \rightarrow \neg(f \lor d)) \rightarrow (f \rightarrow p)$
0	0	0	1	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	1	1	1
1	0	1	0	0	1	1	1
1	1	0	0	0	1	1	1
1	1	1	0	0	1	1	1

We see from the truth table that the sentence is valid.

b Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

Left side:

$$\neg p \rightarrow \neg(f \lor d)$$
$$\neg\neg p \lor \neg(f \lor d)$$
$$p \lor (\neg f \land \neg d)$$
$$\underline{(p \lor \neg f) \land (p \lor \neg d)}$$

Right side:

$$f \rightarrow p$$
$$\neg f \lor p$$
$$\underline{p \lor \neg f}$$

c Prove your answer to (a) using resolution

1:
2:
3:
4:

3 Representations in First-Order Logic

3.1 Consider a vocabulary with the following symbols:

$Occupation(p, o)$: Predicate. Person p has occupation o.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of Person $p2$
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of $p2$
Doctor, Surgeon , Lawyer, Actor: Constants denoting occupations.
Emily, Joe: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a Emily is either a surgeon or a lawyer

$Occupation(Emily, Surgeon) \vee Occupation(Emily, Lawyer)$

b Joe is an actor, but he also holds another job.

$Occupation(Joe, Actor) \wedge \exists_{o \neq Actor}(Occupation(Joe, o)$

c All surgeons are doctors

$\forall_p(Occupation(p, Surgeon) \Rightarrow Occupation(p, Doctor))$

d Joe does not have a lawyer (i.e., is not a customer of any lawyer).

$\neg\exists_p(Occupation(p, Lawyer) \wedge Customer(Joe, p))$

e Emily has a boss who is a lawyer.

$\exists_p(Boss(p, Emily) \wedge Occupation(p, Lawyer)$

f There exists a lawyer all of whose customers are doctors.

$\exists_{p_1}(Occupation(p_1, Lawyer) \wedge \forall_{p_2}(Customer(p_2, p_1) \Rightarrow Occupation(p_2, Doctor)))$

g Every surgeon has a lawyer.

$\forall_{p_1}(Occupation(p_1, Surgeon) \Rightarrow \exists_{p_2}(Occupation(p_2, Lawyer) \wedge Customer(p_1, p_2)))$

3.2 Consider a first-order logical knowledge base that describes worlds containing movies, actors, directors and characters. The vocabulary contains the following symbols:

- $PlayedInMovie(a, m)$: predicate. Actor/person a played in the movie m.
- $PlayedCharacter(a, c)$: predicate. Actor/person a played character c.
- $CharacterInMovie(c, m)$: predicate. Character c is in the movie m.
- $Directed(p, m)$: person p directed movie m.
- $Male(p)$: p is male.
- $Female(p)$: p is female.
- Constants related to the name of the movie, person or character with obvious meaning (to simplify you may use the surname or abbreviation).

Express the following statements in First-Order Logic:

a The character Batman was played by Christian Bale, George Clooney and Val Kilmer.

$PC(cb, bm) \wedge PC(gc, bm) \wedge PCh(vk, bm)$

b The character Batman was played by male actors.

$\forall_p (PC(p, bm) \Rightarrow Male(p))$

c The character Batwoman was played by female actresses.

$\forall_p (PC(p, bw) \Rightarrow Female(p))$

d Heath Ledger and Christian Bale did not play the same characters.

$PC(hl, c_1) \Rightarrow \neg PC(cb, c_1) \wedge PC(cb, c_2) \Rightarrow \neg PC(hl, c_2)$

e In all Batman movies directed by Christopher Nolan, Christian Bale played the character Batman (*tip*: note that in this case Batman is a character of the movie, not the name of the movie).

$\forall_m (D(cn, m) \wedge CIM(bm, m) \Rightarrow PC(cb, bm))$

f The Joker and Batman are characters that appear together in some movies.

$\exists_m (CIM(tj, m) \wedge CIM(bm, m))$

g Kevin Costner directed and starred in the same movie.

$\exists_m (D(kc, m) \wedge PIM(kc, m))$

h George Clooney and Tarantino never played in the same movie and Tarantino never directed a film that George Clooney played.

$\neg\exists_m ((PIM(t, m) \wedge PIM(gc, m)) \vee (D(t, m) \wedge PIM(gc, m)))$

i Uma Thurman is female actress who played a character in *some* movies directed by Tarantino.

$F(ut) \wedge \exists_m (D(t, m) \wedge PIM(ut, m))$

3.3 Arithmetic assertions can be written using FOL. Use the predicates ($<, \leq, \neq, =$), usual arithmetic operations as function symbol ($+, -, \times, /$), biconditionals to create new predicates, and integer number constants to express the following statements in FOL:

a **An integer number x is divisible by y if there is some integer z less than x such that $x = zy$ (in other words, define the predicate $Divisible(x, y)$).**
$Divisible(x, y) \rightarrow \exists z(z < x \wedge x = y \times z)$

b **A number is even if and only if it is divisible by 2 (define the predicate $Even(x)$).**
$Even(x) \leftrightarrow Divisible(x, 2)$

c **An odd number is not divisible by 2 (define the predicate $Odd(x)$).**
$Odd(x) \rightarrow \neg Divisible(x, 2)$

d **The result of summing an even number with 1 is an odd number (define the predicate $Odd(x)$).**
$Odd(x) \rightarrow (Even(y) + 1 = x)$

e **A prime number is divisible only by itself (define the predicate $Prime(x)$).**
$Prime(x) \rightarrow Divisible(x, x) \wedge \forall_{n \neq x} \neg Divisible(x, n)$

f **There is only one even prime number.**
$\forall_x (Prime(x) \wedge Even(x)) \rightarrow \neg \exists_{n \neq x} (Prime(n) \wedge Even(n))$

g **Every integer number is equal to a product of prime numbers. (you can use $\prod_{i=1}^{k} p_k$ to express a product of numbers, or use ... to express a repeating pattern, like $p_1, ..., p_n$, meaning p_1, p_2, p_3 until p_n).**
$\forall_x \exists_p (Even(x) \rightarrow x = \prod_{i=1}^{k} p_i)$ **where p is a prime**

4 Resolution in First-Order Logic

4.1 Find the unifier (Θ) if possible for each pair of atomic sentences. Here, $Owner(x, y)$, $Horse(x)$ and $Rides(x, y)$ are predicates, while $FastestHorse(x)$ is a function that maps a person to the name of their fastest horse:

a $Horse(x)$ . . . $Horse(Rocky)$
Answer: $\Theta = \{x/Rocky\}$

b $Owner(Leo, Rocky)$ . . . $Owner(x, y)$
Answer: $\Theta = \{x/Leo, y/Rocky\}$

c $Owner(Leo, x)$ . . . $Owner(y, Rocky)$
Answer: $\Theta = \{x/Rocky, y/Leo\}$

d $Owner(Leo, x)$ . . . $Rides(Leo, Rocky)$
Answer: $\Theta = \{x/Rocky\}$

e $Owner(x, FastestHorse(x))$ . . . $Owner(Leo, Rocky)$
Answer: $\Theta = \{x/Leo\}$

f $Owner(Leo, y)$ . . . $Owner(x, FastestHorse(x))$
Answer: $\Theta = \{x/Leo, y/FastestHorse(Leo)\}$

g $Rides(Leo, FastestHorse(x))$ . . . $Rides(y, FastestHorse(Marvin))$
Answer: $\Theta = \{y/Leo, FastestHorse(x)/FastestHorse(Marvin)\}$

4.2 Using the predicates $Philosopher(x)$, $StudentOf(y,x)$, $Write(x,z)$, $Read(y,z)$ and $Book(z)$ perform skolemization with the following expressions:

a $\exists x \exists y : Philosopher(x) \wedge StudentOf(y,x)$
Answer: $Philosopher(C_1) \wedge StudentOf(C_2, C_1)$

b $\forall y,x : Philosopher(x) \wedge StudentOf(y,x) \Rightarrow [\exists z : Book(z) \wedge Write(x,z) \wedge Read(y,z)]$
Answer: $Philosopher(x) \wedge StudentOf(y,x) \Rightarrow [Book(f(x)) \wedge Write(x, f(x)) \wedge Read(y, f(x))]$

4.3 Use resolution to prove SuperActor(Tarantino) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, Θ. We are using in this case the same predicates of Exercise 3.1 (movies, actors, etc)

- $\forall x : SuperActor(x) \leftrightarrow [\exists m : PlayedInMovie(x,m) \wedge Directed(x,m)]$
- $\forall m : Directed(Tarantino, m) \leftrightarrow PlayedInMovie(UmaTherman, m)$
- $\exists m : PlayedInMovie(UmaTherman, m) \wedge PlayedInMovie(Tarantino, m)$

a Show all the steps in the proof (or the diagram).

CNF:

- $\forall x : [\neg SuperActor(x) \vee PlayedInMovie(x,m)] \wedge$
  $[\neg SuperActor(x) \vee Directed(x,m)] \wedge$
  $[\neg PlayedInMovie(x,m) \vee \neg Directed(x,m) \vee SuperActor(x)]$
- $\forall m : [\neg Directed(Tarantino, m) \vee PlayedInMovie(UmaTherman, m)] \wedge$
  $[\neg PlayedInMovie(UmaTherman, m) \vee Directed(Tarantino, m)]$
- $\exists m : PlayedInMovie(UmaTherman, m) \wedge PlayedInMovie(Tarantino, m)$

Skolemization:

- $[\neg SuperActor(x) \vee PlayedInMovie(x, f(x))] \wedge$
  $[\neg SuperActor(x) \vee Directed(x, (fx))] \wedge$
  $[\neg PlayedInMovie(x, f(x)) \vee \neg Directed(x, f(x)) \vee SuperActor(x)]$
- $[\neg Directed(Tarantino, C) \vee PlayedInMovie(UmaTherman, C)] \wedge$
  $[\neg PlayedInMovie(UmaTherman, C) \vee Directed(Tarantino, C)]$
- $PlayedInMovie(UmaTherman, C) \wedge PlayedInMovie(Tarantino, C)$

We see from the original sentences that since one sentence is about all movies ($\forall m$:) and the other is about a single instance of a movie ($\exists$) we can use the same constant C for both cases.

1:	$PIM(ut, C) \land PIM(t, C)$	KB
2:	$[\neg D(t, C) \lor PIM(ut, C)] \land [\neg PIM(ut, C) \lor D(t, C)]$	KB
3:	$[D(t, C)]$	1 + 2
4:	$[\neg SA(x) \lor PIM(x, f(x))] \land$ $[\neg SA(x) \lor D(x, (fx))] \land$ $[\neg PIM(x, f(x)) \lor \neg D(x, f(x)) \lor SA(x)]$	KB
5:	$\Theta = [x/t, f(x)/C]$	Unification
6:	$SA(t)$	1 + 3 + 4 + 5

b Translate the information given in FOL into English (or Norwegian) and describe in high level the reasoning you could apply in English to have the same result (in other words, describe a proof of the result in natural language).

We know that there exists a movie where both Uma Therman and Tarantino play a character in the movie. Also, we know that every movie Tarantino has directed, Uma Therman has played in, and vise versa. Since every movie Uma Therman has played in, Tarantino directed, and we know that there is a movie where both played in it, we know that Tarantino is a super actor, since there must exist a movie wehre he both directed it and played in it.