# housing regularization cv

March 13, 2025

# 1 Regularization example IN3050

### 1.1 Preamble and dataset

```
[1]: import numpy as np
import matplotlib.pyplot as plt
import sklearn
```

```
[2]: from sklearn.linear_model import LinearRegression from sklearn.linear_model import Ridge
```

In the previous years, IN3050 used the Boston dataset for this notebook, but this dataset is now removed from scikit-learn due to an ethical problem. That's why in 2024, we switched to the California housing dataset.

```
[3]: from sklearn.datasets import fetch_california_housing housing = fetch_california_housing() print(housing.DESCR)
```

.. \_california\_housing\_dataset:

```
California Housing dataset
```

\*\*Data Set Characteristics:\*\*

:Number of Instances: 20640

:Number of Attributes: 8 numeric, predictive attributes and the target

:Attribute Information:

```
    MedInc median income in block group
    HouseAge median house age in block group
    AveRooms average number of rooms per household
    AveBedrms average number of bedrooms per household
    Population block group population
    AveOccup average number of household members
```

Latitude block group latitudeLongitude block group longitude

:Missing Attribute Values: None

This dataset was obtained from the StatLib repository. https://www.dcc.fc.up.pt/~ltorgo/Regression/cal\_housing.html

The target variable is the median house value for California districts, expressed in hundreds of thousands of dollars (\$100,000).

This dataset was derived from the 1990 U.S. census, using one row per census block group. A block group is the smallest geographical unit for which the U.S. Census Bureau publishes sample data (a block group typically has a population of 600 to 3,000 people).

A household is a group of people residing within a home. Since the average number of rooms and bedrooms in this dataset are provided per household, these columns may take surprisingly large values for block groups with few households and many empty houses, such as vacation resorts.

It can be downloaded/loaded using the
:func:`sklearn.datasets.fetch\_california\_housing` function.

- .. rubric:: References
- Pace, R. Kelley and Ronald Barry, Sparse Spatial Autoregressions, Statistics and Probability Letters, 33 (1997) 291-297
- [4]: X = housing.data
  t = housing.target
- [5]: X.shape, t.shape
- [5]: ((20640, 8), (20640,))
- [6]: X[0, :]
- [6]: array([ 8.3252 , 41. , 6.98412698, 1.02380952, 322. , 2.555555556, 37.88 , -122.23 ])
- [7]: t[:10]
- [7]: array([4.526, 3.585, 3.521, 3.413, 3.422, 2.697, 2.992, 2.414, 2.267, 2.611])
- [8]: from sklearn.model\_selection import train\_test\_split
  X\_train, X\_val, t\_train, t\_val = train\_test\_split(X, t, random\_state=2025)

```
[9]: # train_test_split?
[10]: X_train.shape, X_val.shape, t_train.shape, t_val.shape
[10]: ((15480, 8), (5160, 8), (15480,), (5160,))
     1.2 Linear Regression
[11]: # LinearRegression?
[12]: lr =LinearRegression()
      lr.fit(X_train, t_train)
[12]: LinearRegression()
[14]: round(lr.score(X_train, t_train), 4)
[14]: 0.6075
[15]: round(lr.score(X_val, t_val), 4)
[15]: 0.6021
[16]: # lr.score?
[17]: from sklearn.metrics import mean_squared_error as sk_mse
[18]: round(sk_mse(lr.predict(X_val), t_val), 4)
[18]: 0.5347
     1.2.1 So far
     Similar results for train and val. No overfitting. But can we do better?
     1.3 Polynomial features
     We add second order polynomial features
[19]: from sklearn.preprocessing import PolynomialFeatures
[20]: poly = PolynomialFeatures(degree=2, include_bias=False)
      X_poly_train = poly.fit_transform(X_train)
```

[21]: (15480, 44)

[21]: X\_poly\_train.shape

X\_poly\_val = poly.transform(X\_val)

### Comment

- 8 original features
- 8 squares of original features
- 28 polynomial combinations of the features with degree  $\leq 2$

```
[22]: lr_poly = LinearRegression()
lr_poly.fit(X_poly_train, t_train)
```

[22]: LinearRegression()

```
[23]: round(lr_poly.score(X_poly_train, t_train), 4)
```

[23]: 0.6872

```
[24]: round(lr_poly.score(X_poly_val, t_val), 4)
```

[24]: -5.398

```
[25]: round(sk_mse(lr_poly.predict(X_poly_train), t_train), 4)
```

[25]: 0.4152

```
[26]: round(sk_mse(lr_poly.predict(X_poly_val), t_val), 4)
```

[26]: 8.5982

### 1.3.1 So far

Large improvement on *train*. The oposite on *val*. Large difference between *train* and *val*. Overfitting!

### 1.4 Ridge regularization

```
[27]: ridge_poly = Ridge()
ridge_poly.fit(X_poly_train, t_train)
```

/home/andrei/my\_python/lib/python3.12/sitepackages/sklearn/linear\_model/\_ridge.py:215: LinAlgWarning: Ill-conditioned matrix (rcond=2.53832e-19): result may not be accurate. return linalg.solve(A, Xy, assume\_a="pos", overwrite\_a=True).T

[27]: Ridge()

Scikit-learn complains that the data contains values which are too high or too low, leading to possibly inaccurate results. We will deal with this soon.

```
[28]: # Ridge?
```

```
[29]: 2.5762
[30]: round(sk_mse(ridge_poly.predict(X_poly_train), t_train), 4)
[30]: 0.4204
[31]: round(ridge_poly.score(X_poly_train, t_train), 4)
[31]: 0.6833
[32]: round(ridge_poly.score(X_poly_val, t_val), 4)
[32]: -0.917
```

# 1.4.1 So far

Best score on val so far. Still much better on train. Is the regularization optimal? And can we do anything with the warnings?

# 1.5 Tuning regularization

An instance of parameter tuning

```
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=7.23758e-21): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=7.26104e-21): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=7.47217e-21): result may not be accurate.
 return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=9.58809e-21): result may not be accurate.
 return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
```

```
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=3.08655e-20): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=2.53832e-19): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=2.81128e-18): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     Alpha: 0.00000, train score: 0.687, val score: -5.398, train mse: 0.4152,
     val_mse: 8.5982
     Alpha: 0.00010, train score: 0.687, val score: -5.399, train mse: 0.4152,
     val_mse: 8.5992
     Alpha: 0.00100, train score: 0.687, val score: -5.404, train mse: 0.4152,
     val mse: 8.6069
     Alpha: 0.01000, train_score: 0.687, val_score: -5.415, train_mse: 0.4153,
     val mse: 8.6210
     Alpha: 0.10000, train_score: 0.686, val_score:-4.564, train_mse: 0.4164,
     val mse: 7.4776
     Alpha: 1.00000, train_score: 0.683, val_score:-0.917, train_mse: 0.4204,
     val mse: 2.5762
     Alpha: 10.00000, train_score: 0.675, val_score: 0.303, train_mse: 0.4310,
     val mse: 0.9372
     Alpha: 100.00000, train_score: 0.671, val_score:0.371, train_mse: 0.4369,
     val_mse: 0.8451
     Alpha: 1000.00000, train_score: 0.668, val_score:0.386, train_mse: 0.4412,
     val_mse: 0.8246
     Alpha: 10000.00000, train score: 0.665, val score: 0.466, train mse: 0.4450,
     val_mse: 0.7181
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear model/ ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=2.87186e-17): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
[35]: for a in range(10):
          a = .5 + 0.1 * a
          ridge poly = Ridge(alpha=a)
          ridge_poly.fit(X_poly_train, t_train)
          train_score = ridge_poly.score(X_poly_train, t_train)
          val_score = ridge_poly.score(X_poly_val, t_val)
          train_mse = sk_mse(ridge_poly.predict(X_poly_train), t_train)
          val_mse = sk_mse(ridge_poly.predict(X_poly_val), t_val)
```

/home/andrei/my\_python/lib/python3.12/site-

```
⇔{val_score:.3f}, train_mse: {train_mse:.4f}, val_mse: {val_mse:.4f}")
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=1.2775e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=1.52583e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=1.77625e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=2.02857e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
Alpha: 0.5000, train_score: 0.685, val_score:-2.025, train_mse: 0.4185, val_mse:
4.0658
Alpha: 0.6000, train_score: 0.684, val_score:-1.704, train_mse: 0.4189, val_mse:
3.6343
Alpha: 0.7000, train_score: 0.684, val_score:-1.447, train_mse: 0.4193, val_mse:
3.2878
Alpha: 0.8000, train_score: 0.684, val_score:-1.236, train_mse: 0.4197, val_mse:
3.0053
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=2.28264e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=2.53832e-19): result may not be accurate.
 return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=2.79549e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=3.05402e-19): result may not be accurate.
  return linalg.solve(A, Xy, assume a="pos", overwrite_a=True).T
/home/andrei/my_python/lib/python3.12/site-
packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
matrix (rcond=3.75664e-19): result may not be accurate.
 return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
```

print(f"Alpha: {a:.4f}, train\_score: {train\_score:.3f}, val\_score:

```
Alpha: 0.9000, train_score: 0.684, val_score:-1.062, train_mse: 0.4200, val_mse: 2.7716

Alpha: 1.0000, train_score: 0.683, val_score:-0.917, train_mse: 0.4204, val_mse: 2.5762

Alpha: 1.1000, train_score: 0.683, val_score:-0.794, train_mse: 0.4207, val_mse: 2.4109

Alpha: 1.2000, train_score: 0.683, val_score:-0.689, train_mse: 0.4210, val_mse: 2.2698

Alpha: 1.3000, train_score: 0.683, val_score:-0.599, train_mse: 0.4213, val_mse: 2.1483

Alpha: 1.4000, train_score: 0.682, val_score:-0.520, train_mse: 0.4216, val_mse: 2.0429

/home/andrei/my_python/lib/python3.12/site-packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned matrix (rcond=4.0218e-19): result may not be accurate.

return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
```

### 1.5.1 So far

The regularization factor of 10.0 seems optimal. It gives 0.303 on val. Further increasing it ruins train performance.

# 1.6 Scaling

Let's make our data nice and beautiful and get rid of the warnings

```
[36]: from sklearn.pipeline import make_pipeline from sklearn.preprocessing import StandardScaler
```

```
Alpha: 0.0000, train_score: 0.687, val_score:-5.398, train_mse: 0.4152, val_mse: 8.5982
Alpha: 0.0000, train_score: 0.687, val_score:-5.393, train_mse: 0.4152, val_mse: 8.5916
Alpha: 0.0000, train_score: 0.687, val_score:-5.349, train_mse: 0.4152, val_mse: 8.5320
Alpha: 0.0001, train_score: 0.687, val_score:-4.931, train_mse: 0.4152, val_mse: 7.9710
```

```
Alpha: 0.0010, train_score: 0.687, val_score:-2.420, train_mse: 0.4155, val_mse:
     4.5959
     Alpha: 0.0100, train score: 0.685, val score: 0.310, train mse: 0.4182, val mse:
     Alpha: 0.1000, train score: 0.680, val score: 0.523, train mse: 0.4251, val mse:
     0.6417
     Alpha: 1.0000, train score: 0.669, val score: 0.413, train mse: 0.4396, val mse:
     0.7890
     Alpha: 10.0000, train score: 0.662, val score: 0.336, train mse: 0.4485, val mse:
     0.8926
     Alpha: 100.0000, train score: 0.648, val score: 0.524, train mse: 0.4676,
     val_mse: 0.6403
     Alpha: 1000.0000, train_score: 0.616, val_score:0.573, train_mse: 0.5095,
     val mse: 0.5738
     Alpha: 10000.0000, train_score: 0.540, val_score:0.527, train_mse: 0.6100,
     val_mse: 0.6353
[39]: for a in [0.1, 0.2, 0.5, 0.8, 1.0, 1.5, 2, 5]:
          ridge_poly = make_pipeline(StandardScaler(with_mean=False), Ridge(alpha=a))
          ridge_poly.fit(X_poly_train, t_train)
          train_score = ridge_poly.score(X_poly_train, t_train)
          val_score = ridge_poly.score(X_poly_val, t_val)
          train_mse = sk_mse(ridge_poly.predict(X_poly_train), t_train)
          val_mse = sk_mse(ridge_poly.predict(X_poly_val), t_val)
          print(f"Alpha: {a:.4f}, train_score: {train_score:.3f}, val_score:
       ⇔{val score:.3f}, train mse: {train mse:.4f}, val mse:.4f}")
     Alpha: 0.1000, train_score: 0.680, val_score:0.523, train_mse: 0.4251, val_mse:
     0.6417
     Alpha: 0.2000, train_score: 0.677, val_score:0.511, train_mse: 0.4289, val_mse:
     0.6570
     Alpha: 0.5000, train score: 0.672, val score: 0.464, train mse: 0.4352, val mse:
     0.7198
     Alpha: 0.8000, train_score: 0.670, val_score: 0.430, train_mse: 0.4383, val_mse:
     0.7662
     Alpha: 1.0000, train_score: 0.669, val_score:0.413, train_mse: 0.4396, val_mse:
     Alpha: 1.5000, train_score: 0.667, val_score:0.384, train_mse: 0.4417, val_mse:
     0.8283
     Alpha: 2.0000, train_score: 0.666, val_score:0.365, train_mse: 0.4430, val_mse:
     0.8529
     Alpha: 5.0000, train score: 0.664, val score: 0.330, train mse: 0.4463, val mse:
     0.9006
[40]: for b in range(11):
          a = 0.1 * b
          ridge_poly = make pipeline(StandardScaler(with mean=False), Ridge(alpha=a))
          ridge_poly.fit(X_poly_train, t_train)
```

```
train_score = ridge_poly.score(X_poly_train, t_train)
    val_score = ridge_poly.score(X_poly_val, t_val)
    train_mse = sk_mse(ridge_poly.predict(X_poly_train), t_train)
    val_mse = sk_mse(ridge_poly.predict(X_poly_val), t_val)
    print(f"Alpha: {a:.4f}, train_score: {train_score:.3f}, val_score:
  ⇔{val_score:.3f}, train_mse: {train_mse:.4f}, val_mse: {val_mse:.4f}")
Alpha: 0.0000, train score: 0.687, val score: -5.398, train mse: 0.4152, val mse:
8.5982
Alpha: 0.1000, train score: 0.680, val score: 0.523, train mse: 0.4251, val mse:
0.6417
Alpha: 0.2000, train score: 0.677, val score: 0.511, train mse: 0.4289, val mse:
0.6570
Alpha: 0.3000, train score: 0.675, val score: 0.495, train mse: 0.4316, val mse:
0.6787
Alpha: 0.4000, train score: 0.673, val score: 0.479, train mse: 0.4336, val mse:
0.7002
Alpha: 0.5000, train_score: 0.672, val_score: 0.464, train_mse: 0.4352, val_mse:
0.7198
Alpha: 0.6000, train_score: 0.671, val_score: 0.451, train_mse: 0.4364, val_mse:
0.7372
Alpha: 0.7000, train_score: 0.670, val_score: 0.440, train_mse: 0.4374, val_mse:
0.7526
Alpha: 0.8000, train_score: 0.670, val_score: 0.430, train_mse: 0.4383, val_mse:
0.7662
Alpha: 0.9000, train_score: 0.669, val_score: 0.421, train_mse: 0.4390, val_mse:
```

### 1.6.1 So far

0.7890

The optimal score on val (not the best, but without losing the performance on train) is 0.523, achieved with

Alpha: 1.0000, train\_score: 0.669, val\_score: 0.413, train\_mse: 0.4396, val\_mse:

- polynomial features
- regularization
- $\alpha = 0.1$
- scaling

# 1.7 Cross-validation experiments

```
[41]: from sklearn.model_selection import cross_val_score

[42]: cvs = cross_val_score(LinearRegression(), X_train, t_train, cv=4)
    print(cvs)
    print(f"Mean score: {np.sum(cvs)/len(cvs):.4f}")
    print(f"Standard deviation: {np.std(cvs):.4f}")
```

[0.32557717 0.60987968 0.6149328 0.60697934]

Mean score: 0.5393

Standard deviation: 0.1235

#### 1.7.1 Observations

- Some variation in results.
- The conclusions from using one dev-set are less firm
- Hopefully, the mean is a better measure than the individual experiments
- This set seems too small
- (Each training set is slightly smaller than earlier, 75%)

```
[43]: cvs = cross_val_score(Ridge(), X_train, t_train, cv=4)
    print(cvs)
    print(f"Mean score: {np.sum(cvs)/len(cvs):.4f}")
    print(f"Standard deviation: {np.std(cvs):.4f}")
```

[0.3254875 0.609871 0.61501522 0.60696442]

Mean score: 0.5393

Standard deviation: 0.1235

## 1.7.2 With polynomial features

```
[44]: cvs = cross_val_score(Ridge(), X_poly_train, t_train, cv=4)
      print(cvs)
      print(f"Mean score: {np.sum(cvs)/len(cvs):.4f}")
      print(f"Standard deviation: {np.std(cvs):.4f}")
     [0.58055433 0.67157301 0.69887793 0.50789235]
     Mean score: 0.6147
     Standard deviation: 0.0757
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=3.14658e-19): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=3.07154e-19): result may not be accurate.
       return linalg.solve(A, Xy, assume a="pos", overwrite_a=True).T
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=3.10294e-19): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=1.33003e-18): result may not be accurate.
```

return linalg.solve(A, Xy, assume\_a="pos", overwrite\_a=True).T

### 1.7.3 Observations

• The variation is lower than without polynomial features

What if we remove regularization (alpha) from Linear regression?

```
[46]: cvs = cross_val_score(Ridge(alpha=0), X_poly_train, t_train, cv=4)
      print(cvs)
      print(f"Mean score: {np.sum(cvs)/len(cvs):.4f}")
      print(f"Standard deviation: {np.std(cvs):.4f}")
     [-4.86395083 0.67589949 0.63670265 0.5131771 ]
     Mean score: -0.7595
     Standard deviation: 2.3704
     /home/andrei/my python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=5.79581e-21): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=5.60619e-21): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=4.91813e-21): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
     /home/andrei/my_python/lib/python3.12/site-
     packages/sklearn/linear_model/_ridge.py:215: LinAlgWarning: Ill-conditioned
     matrix (rcond=2.49242e-20): result may not be accurate.
       return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
```

# 1.7.4 Observations

- Large variation, one split is really unlucky
- The mean is of course not impressive
- The polynomial features need regularization.

### 1.7.5 With normalization

```
Alpha: 0.0000, train_score: 0.687, val_score:-5.398, train_mse: 0.4152, val_mse:
     8.5982
     Alpha: 0.1000, train score: 0.680, val score: 0.523, train mse: 0.4251, val mse:
     0.6417
     Alpha: 0.2000, train score: 0.677, val score: 0.511, train mse: 0.4289, val mse:
     0.6570
     Alpha: 0.3000, train score: 0.675, val score: 0.495, train mse: 0.4316, val mse:
     0.6787
     Alpha: 0.4000, train_score: 0.673, val_score:0.479, train_mse: 0.4336, val_mse:
     0.7002
     Alpha: 0.5000, train score: 0.672, val score: 0.464, train mse: 0.4352, val mse:
     0.7198
     Alpha: 0.6000, train_score: 0.671, val_score:0.451, train_mse: 0.4364, val_mse:
     0.7372
     Alpha: 0.7000, train_score: 0.670, val_score:0.440, train_mse: 0.4374, val_mse:
     0.7526
     Alpha: 0.8000, train_score: 0.670, val_score:0.430, train_mse: 0.4383, val_mse:
     0.7662
     Alpha: 0.9000, train_score: 0.669, val_score: 0.421, train_mse: 0.4390, val_mse:
     0.7783
     Alpha: 1.0000, train_score: 0.669, val_score:0.413, train_mse: 0.4396, val_mse:
     0.7890
[52]: cvs = cross val score(
          make_pipeline(StandardScaler(with_mean=False),
                        Ridge(alpha=0.1)),
          X_poly_train, t_train, cv=4)
      print(cvs)
      print(f"Mean score: {np.sum(cvs)/len(cvs):.4f}")
      print(f"Standard deviation: {np.std(cvs):.4f}")
     [-4.20902366 0.66736505 0.69693884 0.58101288]
     Mean score: -0.5659
     Standard deviation: 2.1038
```