Asset Purchases, Limited Asset Markets Participation and Inequality

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EPFL - Swiss Finance Institute

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Conference in Honor of Ramon Marimon

This paper studies the interaction of financial frictions with unconventional monetary policy and its implications for inequality and the macroeconomy

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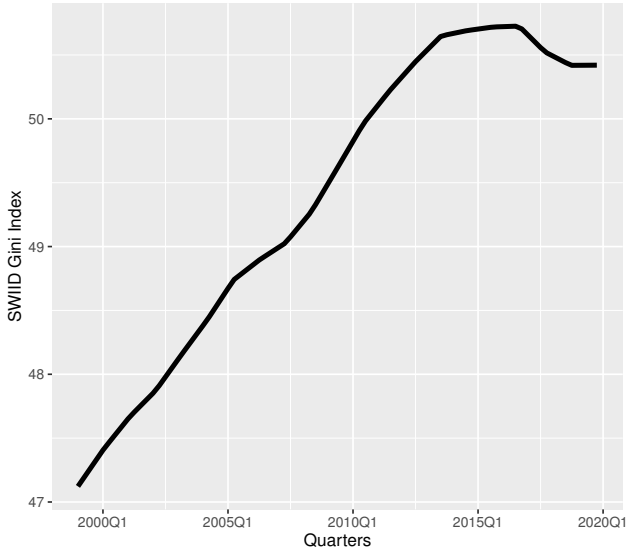
- Creates direct and general equilibrium effects:
  - Direct effects: Increase asset prices, reduce long term rates
  - Indirect effects: Wages increase, economic activity is stimulated, unemployment drops
- Prior consensus: QE increases inequality between those that *do have* financial assets and those who *do not*

Income Inequality Index for the the Euro Area



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(3) Normative exercise: QE can be contractionary and increase inequality when considering a subset of Euro Area members with low asset markets participation + flexible wages

Related Literature

- **Monetary Policy and Inequality in the EA:** Lenza and Slacalek (2018), Slacalek, Tristani, and Violante (2020), Ampudia et. al (2018) Hohberger, Priftis and Vogel (2019)
  → Show the effects of QE on 1) consumption and income inequality, 2) inequality conditional on asset markets participation
- **Financial frictions:** Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2014)
- **TANK:** Galí et al. (2007), Debortoli and Galí (2018), Bilbiie (2008)
  → Combine a TANK model with financial frictions and QE
- **Proxy SVARs:** Gertler & Karadi (2015), Mertens & Ravn (2011), Stock and Watson (2012)
  → Use of Altavila et al. (2019) to provide QE shock aggregate responses for the EA

Quantitative Easing and Inequality: SVAR Evidence

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- External Instrument SVAR approach [Mertens and Ravn (2013), Gertler & Karadi (2015)]

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- QE factor by Euro Area Monetary Policy Event Study Database (EA-MPD); Altavilla et al. (2019)

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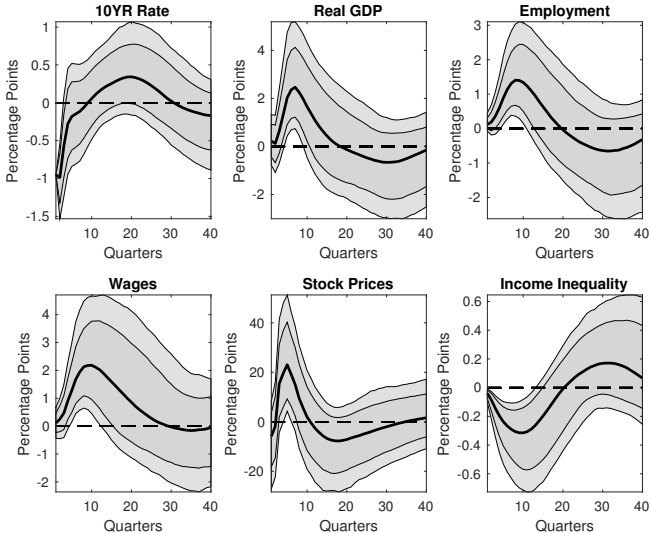
- Identify the coefficients in s with an external instrument
- QE factor by Euro Area Monetary Policy Event Study Database (EA-MPD); Altavilla et al. (2019)
- Document the price changes 10 minutes before and after the ECB MP meeting and estimate by PCA the factors that yield from the monetary policy changes

Impulse Responses to a QE Shock



The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles

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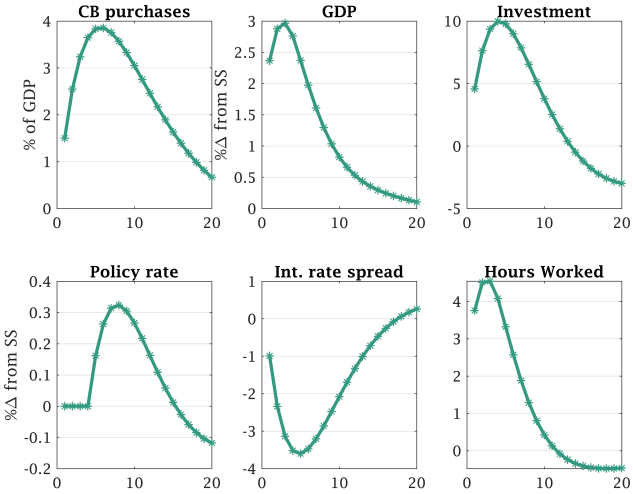
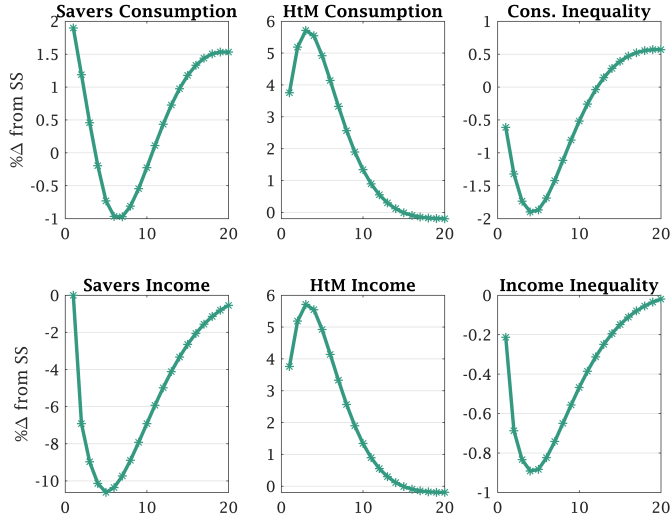
- Bond purchases by an unconstrained central bank by issuing and giving reserves to the banks
  - QE loosens banks constraint and stimulate the supply of loans

More on the model

Impulse Responses to Central Bank Bond Purchases



Consumption and Income Inequality Responses



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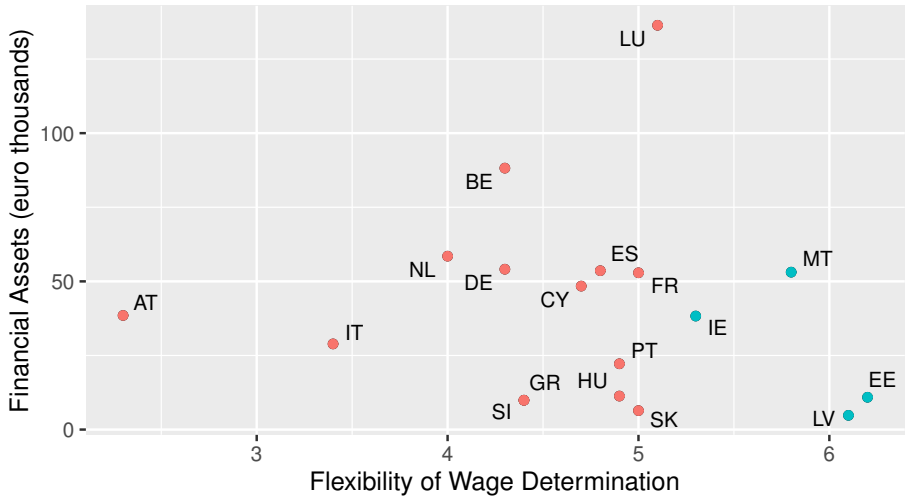
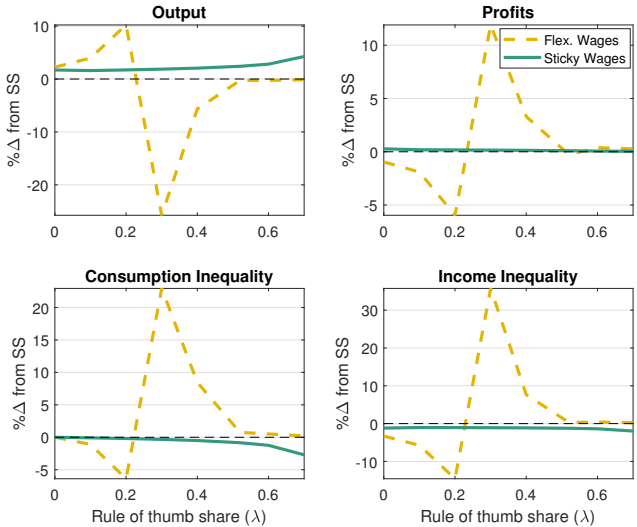
- Extend the work for conventional MP by Bilbiie (2008)
- There is a reversal point in the sign of the monetary policy impact
- Depends on the level of asset market participation and wage flexibility

Financial Assets and Wage Determination



Impact Effects Conditional on Asset Market Participation: QE Shock



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Conclusion

- QE increases aggregate demand and is redistributive by reducing consumption and income inequality in the EA
- I show this in an external instrument SVAR and a DSGE model with heterogeneity and financial frictions
- In economies with low financial inclusion and flexible wages, QE might have inverse effects than those expected.
- Cyclicality of profits plays a crucial role to the sign of the effects. With flexible wages, profits are countercyclical and inequality can increase after a QE shock.

Appendix

Households

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- Optimizers (o)

$$P_t C_t^o + D_t^o + q_t B_t^o + Q_t S_t^o + T_t^o + \underbrace{q_t[\frac{1}{2}(B_t^o - \bar{B}^o)^2] + Q_t[\frac{1}{2}(S_t^o - \bar{S}^o)^2]}_{\text{holding costs}}$$
$$= P_t W_t L_t^o + \Pi_t + R_{d,t} D_{t-1}^o + R_{b,t} B_{t-1}^o + R_{k,t} S_{t-1}^o$$

Back

Households

- Households demand for shares

$$S_t^o = \bar{S}^o + \mathbb{E}_t \, \Lambda_{t,t+1}(R_{k,t+1} - R_{t+1})$$

- Households demand for bonds

$$B_t^o = \bar{B}^o + \mathbb{E}_t \, \Lambda_{t,t+1}(R_{b,t+1} - R_{t+1})$$

Financial Intermediaries

- Bank's balance sheet:

$$\underbrace{Q_t S_{j,t} + q_t B_{j,t} + M^B_{j,t}}_{\text{Assets}} = N_{j,t} + \underbrace{D_{j,t}}_{\text{Liabilities}}$$

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- Bank's evolution of net worth at period $t+1$:

$$N_{j,t+1} = \underbrace{R_{k,t} Q_t S^B_{j,t} + R_{b,t} q_t b^B_{j,t} + R_t M_{j,t}}_{\text{interest gains}} - \underbrace{R_t D_{j,t}}_{\text{interest losses}}$$

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- Easier for the bank to divert loans rather than bonds. Cannot divert reserves $\omega = 0$

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- Easier credit conditions stimulate aggregate demand, ↑ asset prices, ↓ spreads, ↑ bank's NW

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- Households prefer to hold less bonds due to the lower excess returns

Monetary Policy - Government

- The government budget constraint

$$G - T_t + \bar{B}(R_{b,t} - 1) + \underbrace{q_{t-1}B^G_{t-1} + Q_{t-1}S^G_{t-1}}_{\text{Asset Purchases}} = N^G_t + M_t$$

where the government's net worth evolution is

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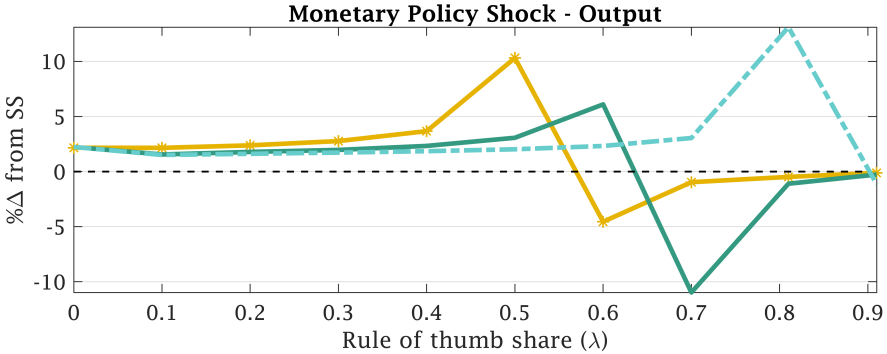

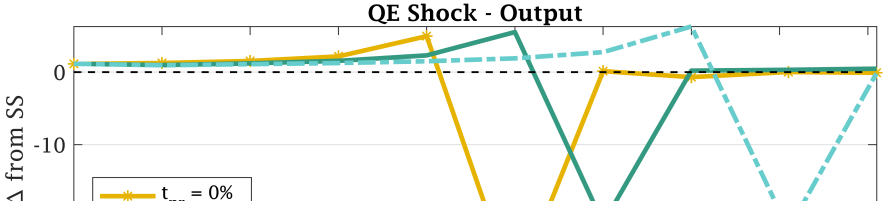

Financial Intermediaries: Solution II

$$\text{Credit spread: } R_{k,t+1} - R_{t+1} = \overbrace{\frac{\lambda_t}{1+\lambda_t}}^{\text{Lagrange multiplier}} \theta$$

$$\text{Bond spread: } R_{b,t+1} - R_{t+1} = \overbrace{\frac{\lambda_t}{1+\lambda_t}}^{\text{Lagrange multiplier}} \Delta\theta$$

$$R_{b,t} = \Delta R_{b,t} + (1-\Delta) R_t$$

$$R_{k,t} = \frac{[Z_t + (1-\delta) Q_t]}{Q_{t-1}}$$

Back

Appendix: Capital Goods Producers

- **Capital goods producers** produce new capital in order to sell it to the goods producers subject to investment adjustment costs.

$$\max_{I_\tau} E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_t I_t - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}} \right) \right] I_\tau \right\}$$

$$Q_t = 1 + \left(\chi \frac{I_\tau}{I_{\tau-1}} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right) + \frac{\chi}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 - \chi \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right) \right)$$

Intermediate Good Firms

- Production Function

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

- Capital evolves according to the law of motion of capital

$$K_{t+1} = I_t + (1 - \delta) K_t.$$

Price Setting

- Intermediate firms are not freely able to change prices each period
- There is a fixed probability $(1-\gamma)$ that a firm can adjust its price.

From the law of large numbers, the following relation for the evolution of the price level emerges:

$$P_t = [(1-\gamma)(P_t^*)^{1-\epsilon} + \gamma(\Pi_{t-1}P_{t-1})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

where P_t^* represents the price chosen by firms resetting prices at time t.

Wage Setting: Perfectly Competitive Labour Markets

- Households choose optimally their labour supply taking wages as given

$$u^j_{c,t} W_t = \chi (L^j_t)^\epsilon. \tag{1}$$

Wage Setting: Sticky Wages

- Wage decisions are delegated to a continuum of labour unions
- The problem of the union is to maximize its objective function:

$$\lambda \left[u^r_{c,t} W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] + (1-\lambda) \left[u^o_{c,t} W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right]$$

.

- subject to a labour demand schedule

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t} \right)^{-\epsilon_w} L_t$$

where ϵ_w is the elasticity of substitution between labour inputs.

- In each period, a union faces a constant probability $1 - \xi_w$ of being able to re-optimize the nominal wage.

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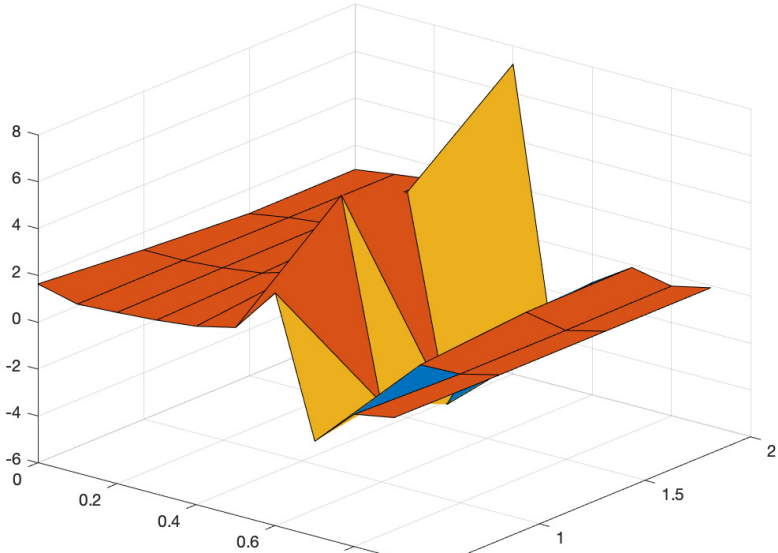
Wage Setting: Sticky Wages

- Wage decisions are delegated to a continuum of labour unions
- Hours are determined by firms taking the wages set by unions as given
- Households supply the hours required by the firms given the wage set by unions
- Probability $1 - \xi_\omega$ that the wage for each particular labour service $W_{h,t}$ is set optimally

The union buys homogeneous labour at nominal price $W_{h,t}$, repackages it by adding a mark-up and chooses the optimal wage W_t^* to maximize the objective function. The FOC is:

$$\left(\frac{\lambda}{u_{c,t}^r u_{l,t}^r} + \frac{1-\lambda}{u_{c,t}^o u_{l,t}^o} \right) W_t = \mu^W$$

Robustness to Inverse Frisch Elasticity: MP



Robustness to Inverse Frisch Elasticity: QE