Earnings and Consumption Dynamics: A Nonlinear Panel Data Framework - By Arellano, Blundell and Bonhomme

Marius Grünewald[1]

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[1]European University Institute

Motivation

- New framework for estimating nonlinear earning dynamics

  $\Longrightarrow$ Implications for consumption (variation)

- New method for studying persistence of shocks to income

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- New framework for estimating nonlinear earning dynamics

  $\Longrightarrow$ Implications for consumption (variation)

- New method for studying persistence of shocks to income

Why should we care

- Policy Relevance
  a) Persistence affects income inequality
  b) Insurance and taxation
- Methodology Contribution

Overview

Canconical Model

Canonical Model

Earnings: $y_{it} = \eta_{it} + \epsilon_{it}$

- η_{it} - persistent component, $\eta_{it} = \eta_{i,t-1} + \nu_{it}$
- η_{it} - indep. of conditional quantile τ
- ϵ_{it} - transitory component, i.i.d.
- ν_{it} - independent of $\eta_{i,t-1}$

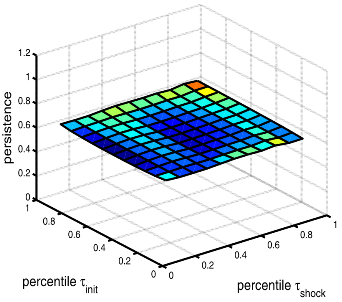
$\implies$ Random walk and independent shock

$\implies$ $\frac{\partial \eta_{it}}{\partial \eta_{i,t-1}} = \rho_t(\eta_{i,t-1}) = 1$

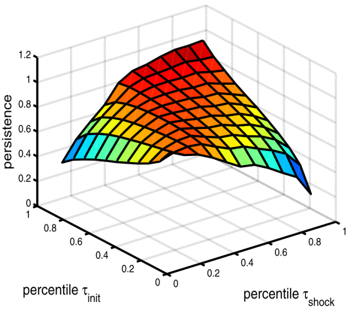
Simulate Life Cycle Model: Given policy functions, get assets and consumption

$\implies$ DONE

(c) Earnings, canonical model



(a) Earnings, PSID data



Are linear models appropriate? Precautionary savings, borrowing constraints?

The Model

Earnings

Earnings: $y_{it} = \eta_{it} + \epsilon_{it}$

Think in Quantiles: Given $\eta_{i,t-1}$ every quantile has different persistence (Nonlinear)

- η_{it} - persistent component, $\eta_{it} = Q_t(\eta_{i,t-1}, u_{it})$, where $Q_t(\eta_{i,t-1}, \tau) = \tau$-th quantile of $\eta_{it}|\eta_{i,t-1}\, \forall\, \tau \in (0,1)$
- $(u_{it}|\eta_{i,t-1}) \sim U(0,1)$
- ϵ_{it} - transitory component, i.i.d.

$\implies \frac{\partial Q_t(\eta_{i,t-1},\tau)}{\partial \eta} = \rho_t(\eta_{i,t-1}, \tau)$, depends on size and sign of u_{it}

On average: $\rho_t(\tau) = \mathbb{E}\left[\frac{\partial Q_t(\eta_{i,t-1},\tau)}{\partial \eta}\right]$

Intuitive: ρ's measure *persistence of earnings history*

Consumption

Distributions known, today's realizations known

(1) Assets: $A_{it} = (1 + r)A_{i,t-1} + Y_{i,t-1} - C_{i,t-1}$

(2) Log earnings: $\ln Y_{it} = \kappa_t + \eta_{it} + \epsilon_{it}$

(3) $V_t(A_{it}, \eta_{it}, \epsilon_{it}) =$
$\max_{C_{it}} u(C_{it}) + \beta \mathbb{E}_t[V_{t+1}(A_{i,t+1}, \eta_{i,t+1}, \epsilon_{i,t+1})]$

Important now: Conditional distribution of $\eta_{i,t+1}$ given $\eta_{i,t}$

$$\begin{aligned} &\text{Solving for } C_{it} \text{ using (1), (2) and (3)} \\ \Longrightarrow \quad & C_{it} = G_t(A_{it}, \eta_{it}, \epsilon_{it}) \end{aligned}$$

But how to get there? Get it directly from the data!

Bringing it to the data

- Log consumption: $g_t(a_{it}, \eta_{it}, \epsilon_{it}, \nu_{it})$, with ν_{it} being unobservables
- Net assets: $a_{it} = h_t(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, \nu_{it})$, with ν_{it} i.i.d. and independent
- Average Consumption: $\mathbb{E}[g_t(a, \eta, \epsilon, \nu_{i,t})]$, given earnings, assets
- Derivative at the average: $\phi_t(a, \eta, \epsilon) = \mathbb{E}\frac{\partial g_t(a,\eta,\epsilon,\nu_{i,t})}{\partial \eta}$

Identification

Earnings

Not the central point of the paper, but...

- Nonlinear Model with latent variable $(\eta_{it}, \epsilon_{it})$
- Goal: Identify joint distributions of η_i's, ϵ_i's from earnings data
- Crucial assumption: The distributions of $(y_{it}|y_{i,t-1})$ and $(\eta_{it}|y_{i,t-1})$ both need to satisfy **completeness**
- Intuition: If we only use $y_{i,t-1}$ to estimate $h(y_{it})$, $\mathbb{E}[h(y_{it})|y_{i,t-1}] = 0$ the only instance where this is optimal is if $h = 0$. Otherwise, we gain information by adding a non-noise variable, i.e. $\eta_{i,t-1}$

$\implies$ $\eta_{i,t}$ serially correlated

Consumption

Identification strategy:

- Note: $f(a_1|y) = \int f(a_1|\eta_1) f(\eta_1|y) d\eta_1$
- Start with first period. Get initial conditional distribution $f(a_1|y) = \mathbb{E}[f(a_1|\eta_{i1})|y_i = y]$ as ϵ_{it} and u_{it} independent.
- Next, $f(c_1|a_1, y) = \mathbb{E}[f(c_1|a_{i1}, \eta_{i1}, y_{i1})|a_{i1} = a_1, y_i = y]$, as ν_{it} independent
- Next, do the same with $t = 2$ and apply Bayesian updating

Estimation

Estimation

(1) Use Quantile regression to jointly estimate

- $Q_t(\cdot, \cdot)$, ϵ_{it}, η_1
- $Q_t(\eta_{i,t-1}, \tau) = \sum_{k=0}^{K} a_k^Q(\tau)\phi_k(\eta_{i,t-1}, age_{it})$
- $Q_{\cdot}(age_{it}, \tau) = \sum_{k=0}^{K} a_k^{\cdot}(\tau)\phi_k(age_{it})$
- In practice: Use Hermite Polynomials (low order)

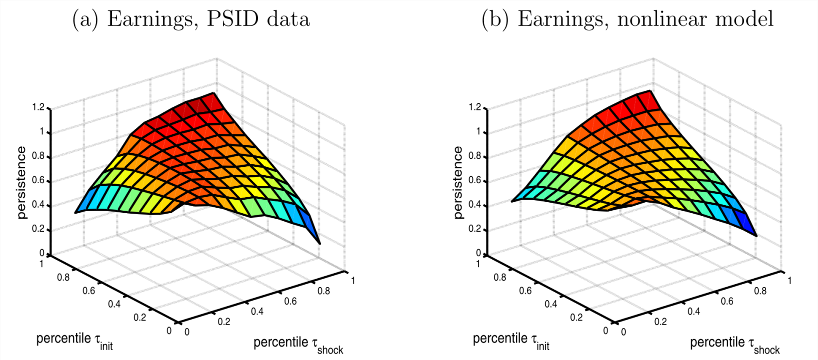
(2) Given these, estimate c and a

- $g_t(a_{it}, \eta_{it}, \epsilon_{it}, \tau) = \sum_{k=1}^{K} b_k^g \tilde{\phi}_k(a_{it}, \eta_{it}, \epsilon_{it}, age_{it}) + b_0^g(\tau)$
- $Q_t(\eta_{i,1}, age_{i1}, \tau) = \sum_{k=0}^{K} b_k^a(\tau)\tilde{\phi}_k(\eta_{i,1}, age_{i1})$
- $h_t(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, \tau) =$
  $\sum_{k=1}^{K} b_k^h \tilde{\phi}_k(a_{i,t-1}, c_{i,t-1}, y_{i,t-1}, \eta_{i,t-1}, age_{it}) + b_0^k(\tau)$

Note: All estimates will depend on age and quantile

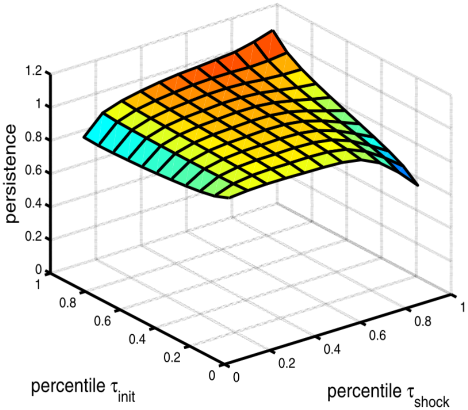
Results

Persistence of Earnings



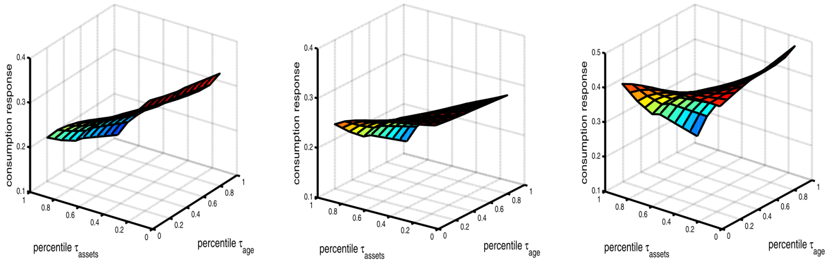
Persistence of η - Simulated Data

(d) Persistent component η_{it}, nonlinear model

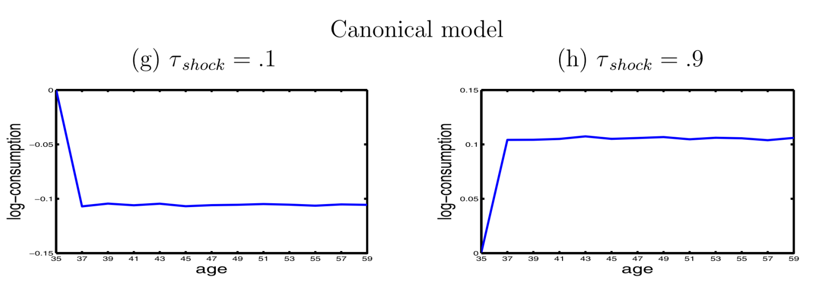


Consumption Responses

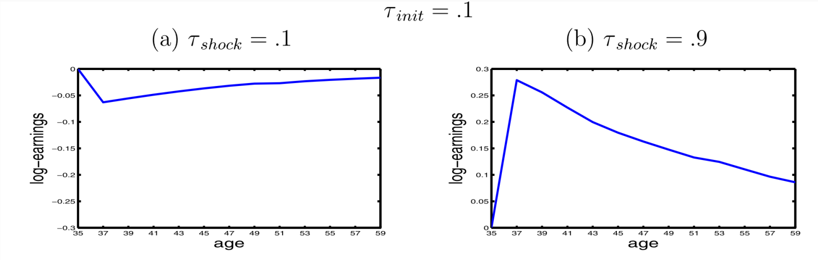
(a) Response to earnings PSID data	(b) Response to earnings Nonlinear model	(c) Response to η_{it} Nonlinear model

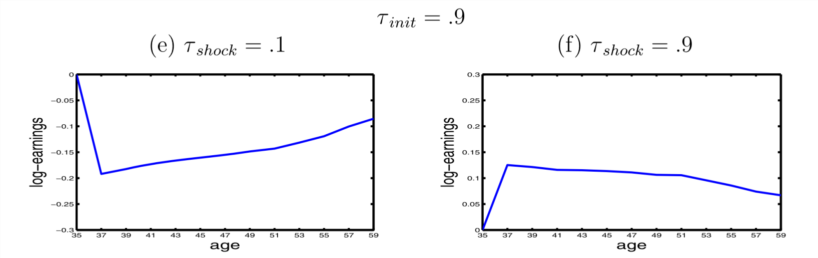


Impulse Response Function - Canonical Model



Impulse Response Function - Nonlinear Model





Conclusion

Take-away

1. We can now model nonlinear persistence
2. Simulation-based quantile regression method and conditions for Nonparametric identification
3. Consumption responses vary
4. Asymmetric persistence pattern, more unusual less persistent