Finding the optimal number of terms for e^x

In [165]:
```
import matplotlib.pyplot as plt
import math
import numpy as np
```

The approximation takes the form of:

$e(x) = \ldots + \frac{x^i}{i!} + \ldots$ for $i \in 1, \ldots, n$ where n equals the number of terms for the approximation.

Upper bounds of n

The nominator and denominator are stored as single-precision floating point units. That means they can reach the maximal value

In [207]:
```
single_float_limit = 3.4 * 10 ** 38
```

The denonimator enforces an upper bound of 34 for n.

In [173]:
```
math.factorial(34) < single_float_limit
```

Out[173]:
```
True
```

In [170]:
```
math.factorial(35) < single_float_limit
```

Out[170]:
```
False
```

In [191]:
```
max_n(0), max_n(13), max_n(14)
```

Out[191]:
```
(34, 34, 33)
```

The enumerator bounds n only on x > 13. Determine for each x the maximal number of n such that $x^n < 3.4 * 10^{38}$

In [174]:
```
def max_n(x):
    n = 34
    while(x ** n > single_float_limit):
        n = n - 1
    return n
```

Upper bounds of x

In [175]:
```
math.exp(100)
```

Out[175]:
```
2.6881171418161356e+43
```

In [209]:
```
x = 100
while math.exp(x) > single_float_limit:
    x = x - 1

x , math.exp(x)
```

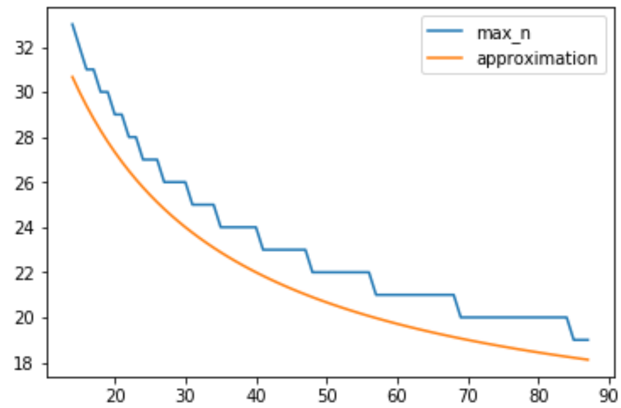
Out[209]: (88, 1.6516362549940018e+38)

Approximation

In [210]:
```
def approx(x):
    return 400/(x + 10) + 14

t = range(14, 88)
plt.plot(t, [max_n(x) for x in t], label='max_n')
plt.plot(t, [approx(x) for x in t], label='approximation')
plt.legend()
```

Out[210]: <matplotlib.legend.Legend at 0x120a2e630>



Test: the approximation should always be lower equal the actual value

In [205]:
```
correct = True
for x in range(14, 88):
    if approx(x) > max_n(x):
        correct = False
correct
```

Out[205]: True

Therefore the optimal number of terms can be approximated by the following:

$x < 14 \rightarrow n = 34$
$x >= 14 \rightarrow n = \frac{400}{x+10} + 14$