1 Greedy best-first search

a)

node	options and its heuristic	option with best heuristic
a	b: 17, d: 12	d: 12
d	g: 10 ,a: 17	g: 10
g	e: 8, i: 4, g:10	i: 4
i	h: 5, l: 0, i: 4	l: 0
l	done	

The path we get from our heuristic therefore is: $a \to d \to g \to i \to l$
The optimal path is either: $a \to d \to g \to i \to l$ or $a \to d \to g \to i \to h \to j \to k \to l$ as can be found out by a test with dijkstra.
since the path that was found out is equivalent to one of the optimal paths we found the optimal solution.

b)

node	options and its heuristic	option with best heuristic
c	b:14, e: 10	e:10
e	g: 9, f: 5, c: 13	f:5
f	k: 0, h: 3, e: 10	k:0
k	done	

The path we get from our heuristic therefore is: $c \to e \to f \to k$
This is not the optimal path since the path $c \to e \to f \to h \to j \to k$ is shorter (since $14 < 19$). We did not find the optimal solution.

c)

node	options and its heuristic	option with best heuristic
a	b: 13, d: 3	d: 3
d	g: 8, a: 14	g: 8
g	e: 6, i: 4, d: 3	d: 3
d	g: 8, a: 14	g: 8
g	e: 6, i: 4, d: 3	d: 3
$\vdots$	$\vdots$	$\vdots$

We are not even able to get a path with this heuristic, since the heuristic is always jumping between d and g and everytime the greedy choice would be to stay in this loop. No path is obviously not the optimal path since $a \to d \to g \to i \to h \to j$ is shorter (since $14 < \infty$). We did not find the optimal solution.

2 Pathfinding with A*

Shortest path between a and h

Expanded Node	Frontier
Start with $a = 0$	$c = 9, d = 10, b = 11$
a, c	$d = 10, b = 11, f = 16$
a, c, d	$b = 11, e = 15, f = 15$
a, c, d, b	$e = 15, f = 15$
a, c, d, b, e	$f = 15, h = 18$
a, c, d, b, e, f	$h = 18, g = 19$
a, c, d, b, e, f, g	$g = 19$

Two shortest paths were found. One of which is not optimal: (a, d, e, h) leads to a distance from a to h of 11 units, whereas (a, d, f, h) leads to a distance of 10, which is optimal.

a

There exists another shortest path with a length of 10: (a, c, f, h)

b

An admissible heuristic never overestimates the distance to the goal, whereas a consistent heuristic only has to fulfill the following property for each node n and its sucessor-nodes n':

$$h(n) \leq c(n, a, n') + h(n')$$

c

Let $h(d) = 6$. Now h is no longer consistent:

$$h(d) \leq c(d, 3, e) + h(e)$$
$$\Leftrightarrow 6 \leq 3 + 2 = 5 \lightning$$

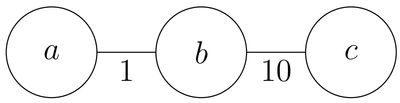
And no longer admissible, because the true distance between d and h is 5.

Alternatively let $h(d) = 3$. Now h is no longer consistent:

$$
\begin{aligned}
&h(a) \leq c(a, 5, d) + h(d) \\
\Leftrightarrow\ &9 \leq 5 + 3 = 8 \ \lightning
\end{aligned}
$$

But h is still admissible, because the true distance between a and d is 5.

d



Let $h(a) = 11, h(b) = 9, h(c) = 0$
Inconsistent:

$$
\begin{aligned}
&h(a) \leq c(a, 1, b) + h(b) \\
\Leftrightarrow\ &11 \leq 1 + 9 = 10 \ \lightning
\end{aligned}
$$

Admissible:

$$
\begin{aligned}
\text{True distance from } c \text{ to } a &: 11 \\
\text{Heuristic} &: 11 \\
11 \leq 11 &\Rightarrow h \text{ is admissible}
\end{aligned}
$$

3 A* in lisp