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1 Propositional logic

The entailment connection: \models is true if and only if $True \models True$

a) Wrong by definition since $True \not\models False$

b) There are cases where $(A \vee B)$ is true but $(A \Leftrightarrow B)$ is not, i.e. $A = 1, B = 0$

A	B	$A \Leftrightarrow B$	$A \wedge \neg B$
0	0	1	0
0	1	0	0
1	0	0	1
1	1	1	0

There are cases in which $A \Leftrightarrow B$ is true but $A \wedge \neg B$ is not. It is therefore not entailed.

d) Yes, since $(A \Rightarrow B) \equiv (\neg A \vee B)$, the entailment is therefore $True \models True$

A	B	C	$(B \Rightarrow A) \vee C$	$(B \Rightarrow C) \wedge A$	$(A \Rightarrow C) \wedge C$	e
0	0	0	1	0	0	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	1	1	0	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Is satisfiable for e.g. $A = B = C = True$

f) No, since there are cases in which $(\neg C \vee D \vee E)$ is true, but $(\neg D \vee \neg E)$ is false, i.e. $C = 0, D = 1, E = 1$. If now Both $A = 0$ and $B = 0$, then there is $True \models False$ which is wrong.

g) $(A \wedge B) \wedge \neg(A \Rightarrow B) \equiv (A \wedge B) \wedge \neg(\neg A \vee B) \equiv (A \wedge B) \wedge (A \wedge \neg B) \equiv A \wedge B \wedge \neg B$. This is a contradiction, meaning it is not satisfiable for any given input.

A	B	C	$(A \wedge B) \Rightarrow C$	$(A \Rightarrow B) \vee (A \vee B)$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	1
1	1	1	1	1

In every case in which the first term is true, the second is as well making this a true entailment.

	A	B	C	$(C \vee (A \wedge B))$	$(A \Rightarrow B)$	$(B \Rightarrow C)$	$(\neg(A \Rightarrow B) \vee (B \Rightarrow C))$
	0	0	0	0	1	1	1
	0	0	1	1	1	1	1
	0	1	0	0	1	0	0
i)	0	1	1	1	1	1	1
	1	0	0	0	0	1	1
	1	0	1	1	0	1	1
	1	1	0	1	1	0	1
	1	1	1	1	1	1	1

In every case in which the first term is true, the second is as well making this a true entailment.

	A	B	$(A \Leftrightarrow B)$	$A \vee \neg B$	j
	0	0	1	1	1
j)	0	1	0	0	0
	1	0	0	1	0
	1	1	1	1	1

Is satisfiable for e.g. $A = B = \text{True}$

2 Normal forms

a)

$$\begin{aligned} A \Rightarrow (B \vee C) &\equiv \neg A \vee (B \vee C) \\ &\equiv (\neg A \vee B \vee C) \end{aligned}$$

b)

$$\begin{aligned} \neg A \Leftrightarrow (\neg B \wedge C) &\equiv (A \Rightarrow (\neg B \wedge C)) \wedge ((\neg B \wedge C) \Rightarrow A) \\ &\equiv (\neg \neg A \vee (\neg B \wedge C)) \wedge (\neg(\neg B \wedge C) \vee A) \\ &\equiv (A \vee (\neg B \wedge C)) \wedge ((B \wedge \neg C) \vee A) \\ &\equiv A \wedge ((\neg B \wedge C) \vee (B \wedge \neg C)) \\ &\equiv A \wedge (\neg B \vee B) \wedge (C \vee B) \wedge (\neg B \vee \neg C) \wedge (C \vee \neg C) \\ &\equiv A \wedge (C \vee B) \wedge (\neg B \vee \neg C) \end{aligned}$$

c)

$$\begin{aligned} (A \wedge B) \Rightarrow C &\equiv \neg(A \wedge B) \vee C \\ &\equiv \neg A \vee \neg B \vee C \\ &\equiv (\neg A \vee \neg B \vee C) \end{aligned}$$

d)

$$\begin{aligned} \neg A \vee (C \wedge B) \Rightarrow B &\equiv \neg(\neg A \vee (C \wedge B)) \vee B \\ &\equiv (A \wedge \neg(C \wedge B)) \vee B \\ &\equiv (A \wedge (\neg C \vee \neg B)) \vee B \\ &\equiv (B \vee A) \wedge (B \vee (\neg C \vee \neg B)) \\ &\equiv (B \vee A) \wedge (B \vee \neg C \vee \neg B) \\ &\equiv (B \vee A) \wedge \text{True} \\ &\equiv (B \vee A) \end{aligned}$$

e)

$$\begin{aligned}
(\neg B \wedge (A \vee C)) \vee (A \wedge (B \vee C)) &\equiv ((\neg B \wedge A) \vee (\neg B \wedge C)) \vee ((A \wedge B) \vee (A \wedge C)) \\
&\equiv (\neg B \wedge A) \vee (\neg B \wedge C) \vee (A \wedge B) \vee (A \wedge C) \\
&\equiv (\neg B \wedge A) \vee (A \wedge B) \vee (\neg B \wedge C) \vee (A \wedge C) \\
&\equiv (A \wedge (B \vee \neg B)) \vee (C \wedge (A \vee \neg B)) \\
&\equiv A \vee (C \wedge (A \vee \neg B)) \\
&\equiv (A \vee C) \wedge (A \vee (A \vee \neg B)) \\
&\equiv (A \vee C) \wedge (A \vee A \vee \neg B) \\
&\equiv (A \vee C) \wedge (A \vee \neg B)
\end{aligned}$$

3 The hacking case

a)

1. $Z \vee N \vee E$
2. $Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$
3. $\neg Z \Rightarrow \neg N$
4. $E \Rightarrow N$
5. $E \Rightarrow \neg Z$

b)

The additional assumption that we make here is that $1 \wedge 2 \wedge 3 \wedge 4 \wedge 5$. That implies that as soon as one statement is not fulfilled we can already exclude the row from the realm of possibilities.

Z	N	E	$Z \vee N \vee E$	$\neg Z \Rightarrow \neg N$	$E \Rightarrow N$	$E \Rightarrow \neg Z$	$Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$	All
0	0	0	0	-	-	-	-	-
0	0	1	1	1	0	-	-	-
0	1	0	1	0	-	-	-	-
0	1	1	1	0	-	-	-	-
1	0	0	1	1	1	1	0	-
1	0	1	1	1	0	-	-	-
1	1	0	1	1	1	1	1	1
1	1	1	1	1	1	0	-	-

We can therefore conclude that Zack and Naomi are the hackers.