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Übungsblatt Nr. 12

(Abgabetermin 31.01.2017)

1 Resolution

From the last assignment we know that there are 3 Variables: Z, N, E and we have 5 statements that need to be fulfilled: $Z \vee N \vee E, \neg Z \Rightarrow \neg N, E \Rightarrow N, E \Rightarrow \neg Z, Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$
These can be transformed to CNF such that:

1. $Z \vee N \vee E$
2. $\neg Z \Rightarrow \neg N \equiv Z \vee \neg N$
3. $E \Rightarrow N \equiv \neg E \vee N$
4. $E \Rightarrow \neg Z \equiv \neg E \vee \neg Z$
5. $Z \Rightarrow (Z \vee N) \wedge (Z \vee E) \equiv \neg Z \vee Z \wedge (N \vee E) \equiv N \vee E$

To make the proof by contradiction, we want to yield $KB \wedge \neg\alpha = false$, where $\alpha = Z \wedge N$ since this is the result from the last homework. $\neg\alpha$ is therefore $\neg(Z \wedge N) \equiv \neg Z \vee \neg N(6)$.
Combination of 2 and 6 yields: $\neg N$(2,6)
Combination of 3 and 6 yields: $\neg Z \vee \neg E$(3,6)
Combination of 5 and 6 yields: $E \vee \neg Z$(5,6)
Combination of (2,6) and 3 yields: $\neg E$ ((2,6),3)
Combination of (3,6) and (5,6) yields: $\neg Z$ ((3,6),(5,6))
Combination of (2,6), ((2,6),3), ((3,6), (5,6)) and 1 yields: $false$

2 More Resolution

We want to prove $KB \wedge \neg\alpha = false$ with $\alpha = (C \wedge \neg D) \vee E$ from the knowledge base:

1. $A \vee B$
2. $B \vee C \vee E$
3. $\neg B \vee C$
4. $\neg B \vee \neg D$
5. $\neg A \vee \neg D$

Therefore $\neg\alpha = \neg((C \wedge \neg D) \vee E) \equiv \neg(C \wedge \neg D) \wedge \neg E \equiv (\neg C \vee D) \wedge \neg E$
This essentially means, we add two clauses: $(\neg C \vee D)(6)$ and $\neg E(7)$.
Combination of 1 and 4 yields: $A \vee \neg D$(1,4)
Combination of 2 and 7 yields: $(B \vee C)$ (2,7)
Combination of (2,7) and 3 yields: C ((2,7),3)
Combination of (1,4) and 5 yields: $\neg D$((1,4),5)
Combination of ((2,7),3) and 6 yields: D (((2,7),3), 6)
Combination of ((1,4),5) and (((2,7),3),6) yields: $false$

3 Clauses

a)

1. is a correct representation, since it only allows the exam to be written, if and only if at least five assignments have been submitted.

2. is also correct

3. is incorrect, since it is still fulfilled if A is false

b)

1. $S \Rightarrow (E \Leftrightarrow A) \equiv \neg S \vee ((E \Rightarrow A) \wedge (A \Rightarrow E)) \equiv \neg S \vee ((\neg E \vee A) \wedge (\neg A \vee E)) \equiv (\neg A \vee E \vee \neg S) \wedge (A \vee \neg E \vee \neg S)$ This is a horn clause, since in each clause only one literal is positive.

2. $(S \wedge E) \Leftrightarrow A \equiv ((S \wedge E) \Rightarrow A) \wedge (A \Rightarrow (S \wedge E)) \equiv (\neg(S \wedge E) \vee A) \wedge (\neg A \vee (S \wedge E)) \equiv (\neg S \vee \neg E \vee A) \wedge (\neg A \vee S) \wedge (\neg A \vee E)$ This is a horn clause, since in each clause only one literal is positive.

3. $S \Rightarrow (A \Rightarrow E) \equiv (\neg S \vee (\neg A \vee E)) \equiv (\neg S \vee \neg A \vee E)$. This is a horn clause, since only the E is positive.

4 First Order logic

1. $Occupation(James, student) \wedge Occupation(James, o)$

2. $\neg Customer(James, Occupation(p, Architect))$

3. $\exists occupation(p, architect) : \forall customer(occupation(p1, scientiest), p)$

4. $Boss(Occupation(p, Architect), Casey)$

5. $\forall p \in Physicists : Occupation(p, scientist)$

6. $occupation(casey, physisist) \vee occupation(casey, architect)$

7. $\neg \exists boss(occupation(p1, architect), occupation(p2, physicist))$