Übungsblatt Nr. 11

(Abgabetermin 24.01.2017)

1 Propositional logic

The entailment connection: \vDash is true if and only if $True \vDash True$

- a) Wrong by definition since $True \not\vDash False$
- b) There are cases where $(A \vee B)$ is true but $(A \Leftrightarrow B)$ is not, i.e. A = 1, B = 0

	Α	В	$A \Leftrightarrow B$	$A \wedge \neg B$
	0	0	1	0
c)	0	1	0	1
	1	0	0	0
	1	1	1	0

There are cases in which $A \Leftrightarrow B$ is true but $A \land \neg B$ is not. It is therefore not entailed.

- d) Yes, since $(A \Rightarrow B) \equiv (\neg A \lor B)$, the entailment is therefore $True \models True$
- e) Yes, since only one upper level term must be satisfiable since it is connected with disjunctions. $(A \Rightarrow B)$ is satisfiable, therefore the whole sentence is.
- f) No, since there are cases in which $(\neg C \lor D \lor E)$ is true, but $(\neg D \lor \neg E)$ is false, i.e. C = 0, D = 1, E = 1. If now Both A = 0 and B = 0, then there is $True \models False$ which is wrong.
- g) $(A \wedge B) \wedge \neg (A \Rightarrow B) \equiv (A \wedge B) \wedge \neg (\neg A \vee B) \equiv (A \wedge B) \wedge (A \wedge \neg B) \equiv A \wedge B \wedge \neg B$. This is a contradiction, meaning it is not satisfiable for any given input.

	A	В	$^{\rm C}$	$ (A \land B) \Rightarrow C$	$(A \Rightarrow B) \lor (A \lor B)$
	0	0	0	1	1
	0	0	1	1	1
	0	1	0	1	1
h)	0	1	1	1	1
	1	0	0	1	1
	1	0	1	1	1
	1	1	0	0	1
	1	1	1	1	1

In every case in which the first term is true, the second is as well making this a true entailment.

	A	В	С	$(C \vee (A \wedge B))$	$(A \Rightarrow B)$	$(B \Rightarrow C)$	$(\neg(A \Rightarrow B) \lor (B \Rightarrow C))$
	0	0	0	0	1	1	1
	0	0	1	1	1	1	1
	0	1	0	0	1	0	0
i)	0	1	1	1	1	1	1
	1	0	0	0	0	1	1
	1	0	1	1	0	1	1
	1	1	0	1	1	0	1
	1	1	1	1	1	1	1

In every case in which the first term is true, the second is as well making this a true entailment.

j) $(A \Leftrightarrow B) \land (A \lor \neg B) \equiv (\neg A \land B) \land (\neg B \land A) \land (A \lor \neg B) \equiv (\neg A \land B) \land (\neg B \land A)$. The equivalence entails the following term. The sentence is therefore satisfiable for all cases in which the equivalence is, i.e. A = 1, B = 1.

2 Normal forms

a)

$$A \Rightarrow (B \lor C) \equiv \neg A \lor (B \lor C)$$
$$\equiv (\neg A \lor B \lor C)$$

b)

$$\neg A \Leftrightarrow (\neg B \land C) \equiv (A \Rightarrow (\neg B \land C)) \land ((\neg B \land C) \Rightarrow A)$$

$$\equiv (\neg \neg A \lor (\neg B \land C)) \land (\neg (\neg B \land C) \lor A)$$

$$\equiv (A \lor (\neg B \land C)) \land ((B \land \neg C) \lor A)$$

$$\equiv A \land ((\neg B \land C) \lor (B \land \neg C))$$

$$\equiv A \land (\neg B \lor B) \land (C \lor B) \land (\neg B \lor \neg C) \land (C \lor \neg C)$$

$$\equiv A \land (C \lor B) \land (\neg B \lor \neg C)$$

c)

$$(A \land B) \Rightarrow C \equiv \neg (A \land B) \lor C$$
$$\equiv \neg A \lor \neg B \lor C$$
$$\equiv (\neg A \lor \neg B \lor C)$$

d)

$$\neg A \lor (C \land B) \Rightarrow B \equiv \neg(\neg A \lor (C \land b)) \lor B$$

$$\equiv (A \land \neg(C \land B)) \lor B$$

$$\equiv (A \land (\neg C \lor \neg B)) \lor B$$

$$\equiv (B \lor A) \land (B \lor (\neg C \lor \neg B))$$

$$\equiv (B \lor A) \land (B \lor \neg C \lor \neg B)$$

$$\equiv (B \lor A) \land True$$

$$\equiv (B \lor A)$$

e)

$$(\neg B \land (A \lor C)) \lor (A \land (B \lor C)) \equiv ((\neg B \land A) \lor (\neg B \land C)) \lor ((A \land B) \lor (A \land C))$$

$$\equiv (\neg B \land A) \lor (\neg B \land C) \lor (A \land B) \lor (A \land C)$$

$$\equiv (\neg B \land A) \lor (A \land B) \lor (\neg B \land C) \lor (A \land C)$$

$$\equiv (A \land (B \lor \neg B)) \lor (C \land (A \lor \neg B))$$

$$\equiv A \lor (C \land (A \lor \neg B))$$

$$\equiv (A \lor C) \land (A \lor (A \lor \neg B))$$

$$\equiv (A \lor C) \land (A \lor A \lor \neg B)$$

$$\equiv (A \lor C) \land (A \lor A \lor \neg B)$$

3 The hacking case

a)

- 1. $Z \vee N \vee E$
- 2. $Z \Rightarrow (Z \land N) \lor (Z \land E)$
- 3. $\neg Z \Rightarrow \neg N$
- 4. $E \Rightarrow N$
- 5. $E \Rightarrow \neg Z$

b)

The additional assumption that we make here is that $1 \land 2 \land 3 \land 4 \land 5$. That implies that as soon as one statement is not fulfilled we can already exclude the row from the realm of possibilities.

\mathbf{Z}	N	E	$Z \lor N \lor E$	$\neg Z \Rightarrow \neg N$	$E \stackrel{\circ}{\Rightarrow} N$	$E \Rightarrow \neg Z$	$Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$	All
0	0	0	0	-	-	-	-	-
0	0	1	1	1	0	-	-	-
0	1	0	1	0	-	_	-	_
0	1	1	1	0	-	_	-	-
1	0	0	1	1	1	1	0	_
1	0	1	1	1	0	_	-	-
1	1	0	1	1	1	1	1	1
1	1	1	1	1	1	0	-	-

We can therefore conclude that Zack and Naomi are the hackers.