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Übungsblatt Nr. 12

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1 Resolution

From the last assignment we know that there are 3 Variables: Z, N, E and we have 5 statements that need to be fulfilled: $Z \vee N \vee E, \neg Z \Rightarrow \neg N, E \Rightarrow N, E \Rightarrow \neg Z, Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$
These can be transformed to CNF such that:

1. $Z \vee N \vee E$
2. $\neg Z \Rightarrow \neg N \equiv Z \vee \neg N$
3. $E \Rightarrow N \equiv \neg E \vee N$
4. $E \Rightarrow \neg Z \equiv \neg E \vee \neg Z$
5. $Z \Rightarrow (Z \wedge N) \vee (Z \wedge E) \equiv \neg Z \vee ((Z \wedge N) \vee (Z \wedge E)) \equiv \neg Z \vee N \vee E$

To make the proof by contradiction, we want to yield $KB \wedge \neg\alpha = false$, where $\alpha = Z \wedge N$ since this is the result from the last homework.
$\neg\alpha$ is therefore $\neg(Z \wedge N) \equiv \neg Z \vee \neg N(6)$.
Combination of 2 and 6 yields: $\neg N$(2,6)
Combination of 3 and 6 yields: $\neg Z \vee \neg E$(3,6)
Combination of 5 and 6 yields: $E \vee \neg Z$(5,6)
Combination of (2,6) and 3 yields: $\neg E$ ((2,6),3)
Combination of (3,6) and (5,6) yields: $\neg Z$ ((3,6),(5,6))
Combination of (2,6), ((2,6),3), ((3,6), (5,6)) and 1 yields: $false$

2 More Resolution

We want to prove $KB \wedge \neg\alpha = false$ with $\alpha = (C \wedge \neg D) \vee E$ from the knowledge base:

1. $A \vee B$
2. $B \vee C \vee E$
3. $\neg B \vee C$
4. $\neg B \vee \neg D$
5. $\neg A \vee \neg D$

Therefore $\neg\alpha = \neg((C \wedge \neg D) \vee E) \equiv \neg(C \wedge \neg D) \wedge \neg E \equiv (\neg C \vee D) \wedge \neg E$
This essentially means, we add two clauses: $(\neg C \vee D)(6)$ and $\neg E(7)$.
Combination of 1 and 4 yields: $A \vee \neg D$(1,4)
Combination of 2 and 7 yields: $(B \vee C)$ (2,7)
Combination of (2,7) and 3 yields: C ((2,7),3)
Combination of (1,4) and 5 yields: $\neg D$((1,4),5)
Combination of ((2,7),3) and 6 yields: D (((2,7),3), 6)
Combination of ((1,4),5) and (((2,7),3),6) yields: $false$

3 Clauses

a)

Assuming $S = True$ for University students and $S = False$ for high school students.

The second option $(S \wedge E) \Leftrightarrow A$ translates to:
Only University students S can present the Exam E if they have submitted at least 5 assignments A.

This is the correct representation of what the professor said, because it excludes high school students from presenting the exam, unlike (i).
(iii) allows all students to present the exam regardless of the amount of submitted papers.

b)

1.

$$\begin{aligned} S \Rightarrow (E \Leftrightarrow A) &\equiv \neg S \vee ((E \Rightarrow A) \wedge (A \Rightarrow E)) \\ &\equiv \neg S \vee ((\neg E \vee A) \wedge (\neg A \vee E)) \\ &\equiv (\neg A \vee E \vee \neg S) \wedge (A \vee \neg E \vee \neg S) \end{aligned}$$

This is a horn clause, since in each clause only one literal is positive.

2.

$$\begin{aligned} (S \wedge E) \Leftrightarrow A &\equiv ((S \wedge E) \Rightarrow A) \wedge (A \Rightarrow (S \wedge E)) \\ &\equiv (\neg(S \wedge E) \vee A) \wedge (\neg A \vee (S \wedge E)) \\ &\equiv (\neg S \vee \neg E \vee A) \wedge (\neg A \vee S) \wedge (\neg A \vee E) \end{aligned}$$

This is a horn clause, since in each clause only one literal is positive.

3.

$$\begin{aligned} S \Rightarrow (A \Rightarrow E) &\equiv (\neg S \vee (\neg A \vee E)) \\ &\equiv (\neg S \vee \neg A \vee E) \end{aligned}$$

This is a horn clause, since only the E is positive.

4 First Order logic

(a) $Occupation(James, Student) \wedge Occupation(James, o)$

(b) $\neg Customer(James, Occupation(p, Architect))$

(c) $\exists Occupation(p, Architect) : \forall Customer(Occupation(p1, Scientiest), p)$

(d) $Boss(Occupation(p, Architect), Casey)$

(e) $\forall p \in Physicists : Occupation(p, Scientist)$

(f) $(Occupation(casey, Physicist) \wedge \neg Occupation(Casey, Architect)) \vee (\neg Occupation(casey, Physicist) \wedge Occupation(Casey, Architect))$

(g) $\neg \exists Boss(Occupation(p1, architect), Occupation(p2, physicist))$