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1 Unification

1. $MGU = \{l/A, v/B, u/m\}$
2. $MGU = \{m/l, l/S\}$
3. $MGU = \{m/W(l,l), l/A, l/B\}$
   This however is impossible, since we can't map the variable l to two different constants A and B.
4. $MGU = \{m/A, v/B, l/P(l)\}$
   This however does not pass the occur check, since the variable itself occurs in the term it is mapped to.
5. $\{l/P(A,m), P \neq H\}$
   P and H are not the same function, therefore a mapping is impossible.

2 Substitution and Skolemization

1. If we choose x as men, y as women and P(x,y) as x loves y. Then $\forall x \exists y P(x,y) \not\Rightarrow \exists q P(q,q)$, since there can be a woman or a man that does not love themself even if every man loves a woman.
   In formal termes we can make a existential instantiation such that $\forall x \exists P(x,y)$ is performed to $\forall x P(x, SK_0(x))$. Now we apply the universal instantiation with $x = \{A, B\}$, such that $SK_0(A) = B$ and $SK_0(B) = A$. Since $P(A, SK_0(A)) \wedge P(B, (SK_0(B)) \not\Rightarrow P(A,A) \vee P(B,B)$ the implication is not valid.
2. We start by inferring $\exists P(q,q)$ from $\forall x \exists y P(x,y)$. We follow with an existential instantiation: $\forall x \exists y P(x,y)$ is transferred to $\forall x P(x, SK_0(x))$.
   With $x = \{x_1, x_2\}$ we can do universal instantiation such that $P(x_1, SK_0(x_1)) \wedge P(x_2, SK_0(x_2))$.
   With the broken engine we can infer $P(q_1, q_1) \wedge P(q_2, q_2)$.
   Because $q \in \{q_1, q_2\}$ we know $P(q_1, q_1) \wedge P(q_2, q_2)$ implies $\forall q P(q,q)$ and $\exists q P(q,q)$.
3. firstly we transform all mathematical to logical functions:
   - $|x| \rightarrow Abs(x)$
   - $x > y \rightarrow gt(x,y)$

   1 $\forall \epsilon gt(\epsilon, 0) \exists n_0 \forall n gt(n, n_0) \Rightarrow gt(\epsilon, Abs(a_n))$

   Ex. inst. $\forall \epsilon gt(\epsilon, 0) \forall n gt(n, SK_0(\epsilon)) \Rightarrow gt(\epsilon, Abs(a_n))$

   - $\forall \epsilon \forall n gt(\epsilon, 0) \wedge gt(n, SK_0(\epsilon)) \Rightarrow gt(\epsilon, Abs(a_n))$
   - define $e \in \mathbb{R}$ and $m \in \mathbb{N}$
   - $gt(e, 0) \wedge gt(m, SK_0(e)) \Rightarrow GT(e, Abs(a_n))$

   This can now be transformed to CNF:

$$
\begin{aligned}
gt(e,0) \wedge gt(m, SK_0(e)) \Rightarrow GT(e, Abs(a_n)) &\equiv \neg(gt(e,0) \wedge gt(m, SK_0(e))) \vee GT(e, Abs(a_n)) \\
&\equiv \neg gt(e,0) \vee \neg gt(m, SK_0(e))) \vee GT(e, Abs(a_n))
\end{aligned}
$$

   which is in CNF

3 Resolution in First-Order Logic

We have a statement: $A(x) \wedge B(f(x), y)$ and a KB and want to find out through resolution whether the statement is constistent with the KB. Therefore we add the negation $\neg(A(x) \wedge B(f(x), y)) \equiv \neg A(x) \vee \neg B(f(x), y)$ to the KB:

1 $B(f(x_1), x_2) \vee \neg A(x_2) \vee \neg C(x_1)$

2 $\neg D(y_1, y_2) \vee A(f(y_1))$

3 $C(f(G))$

4 $D(G, z_1)$

5 $\neg A(x) \vee \neg B(f(x), y)$

(2,4) $A(f(G))$ with $\theta = \{y_1/G, y_2/z_1\}$

(5,(2,4)) $\neg B(f(f(G)), y_3)$ with $\theta = \{x_3/f(G)\}$

6 = (1,(5,(2,4))) $\neg A(y_3) \vee \neg C(f(G))$ with $\theta = \{x_1/f(G), x_2/x_3\}$

7 =(3,(1,(5,(2,4)))) $\neg A(y_3)$ with $\theta = identity$

6,7 $\square$