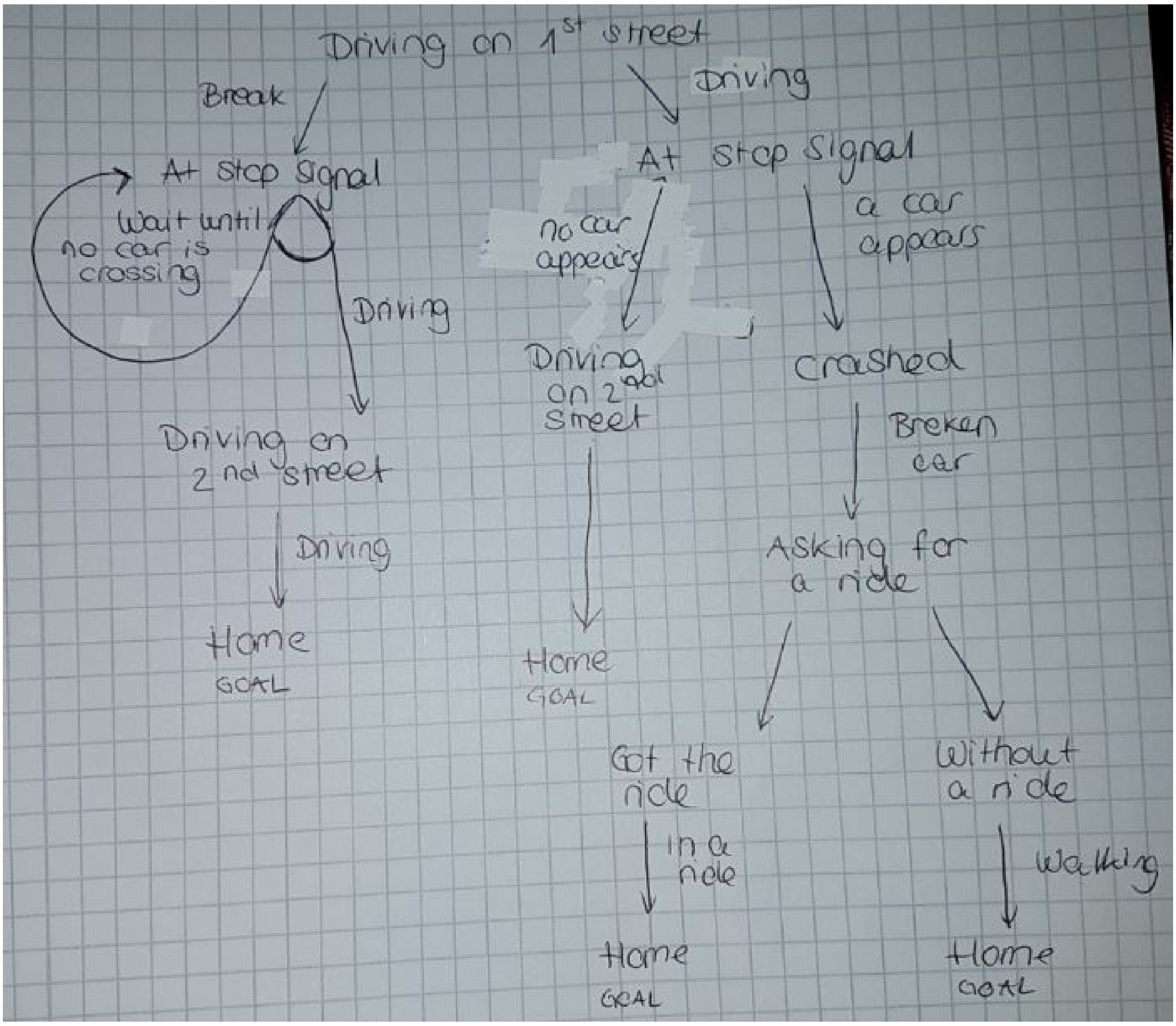
Assignment Nr. 4

(Abgabetermin 23.11.2016)

1) AND-OR Trees

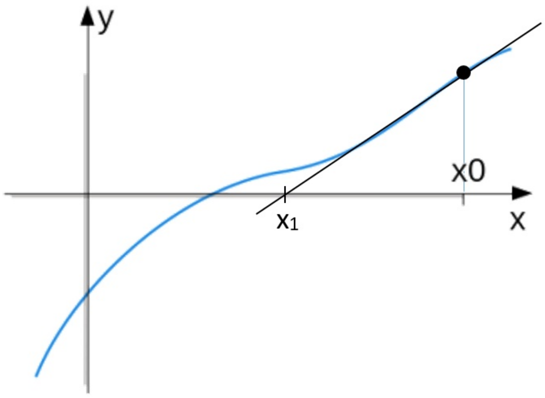


New states as on picture.

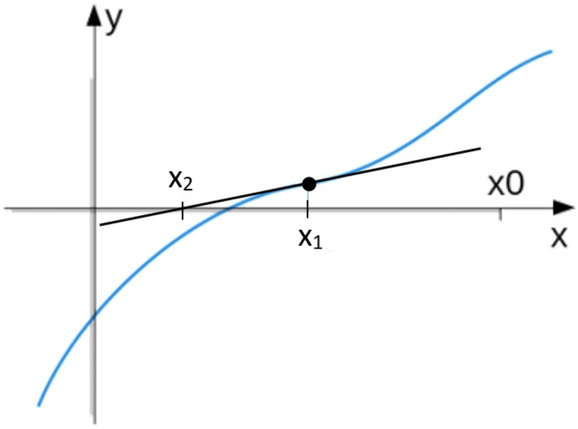
2) Newton's Method

a)

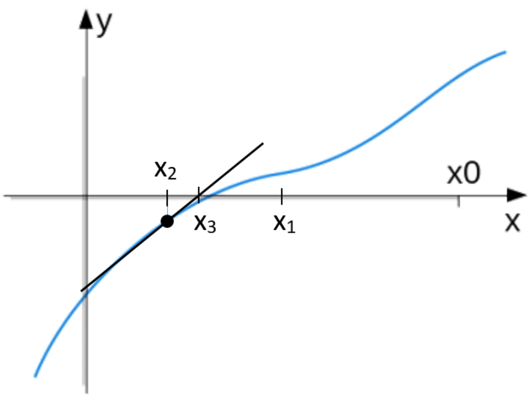
The tangent of x_0 crossing the x-axis gives the next point x_1.



The tangent of x_1 crossing the x-axis gives the next point x_2.



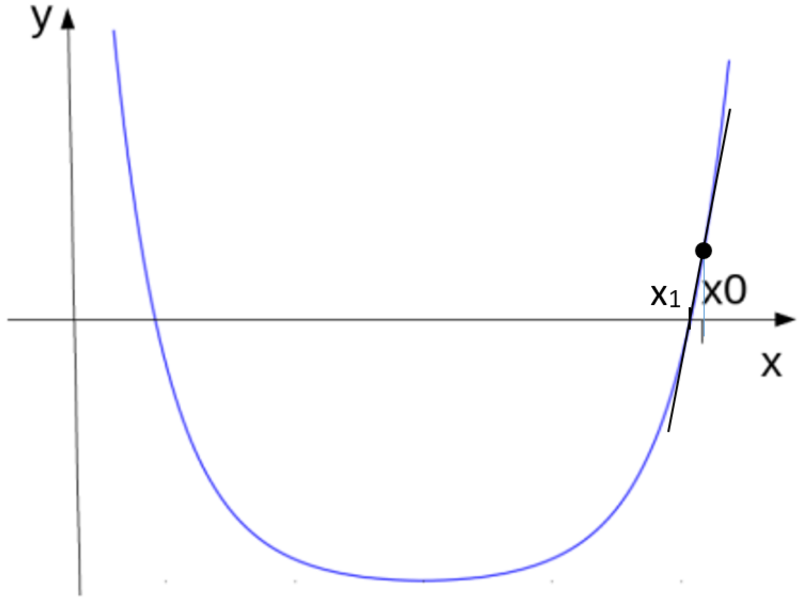
The tangent of x_1 crossing the x-axis gives the next point x_2.



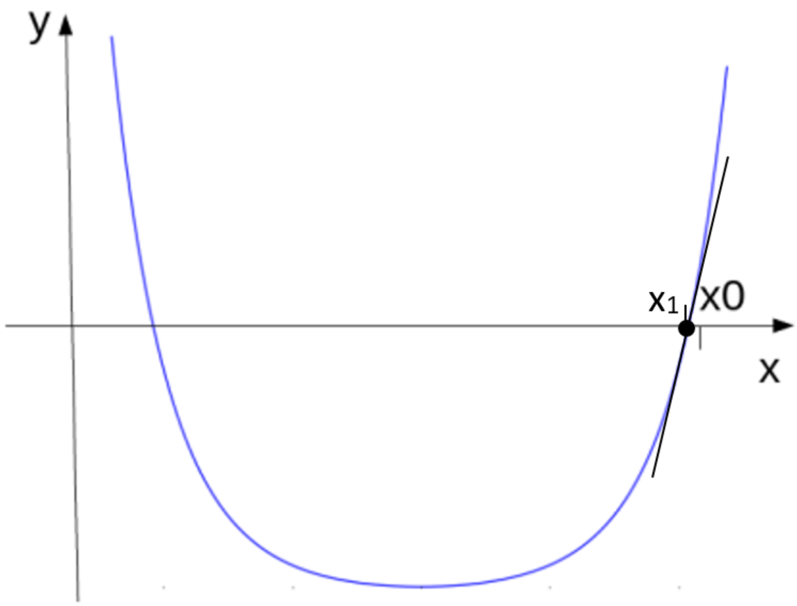
The root is approximated fairly close with these 3 steps.

b)

The tangent of x_0 crossing the x-axis gives the next point x_1.

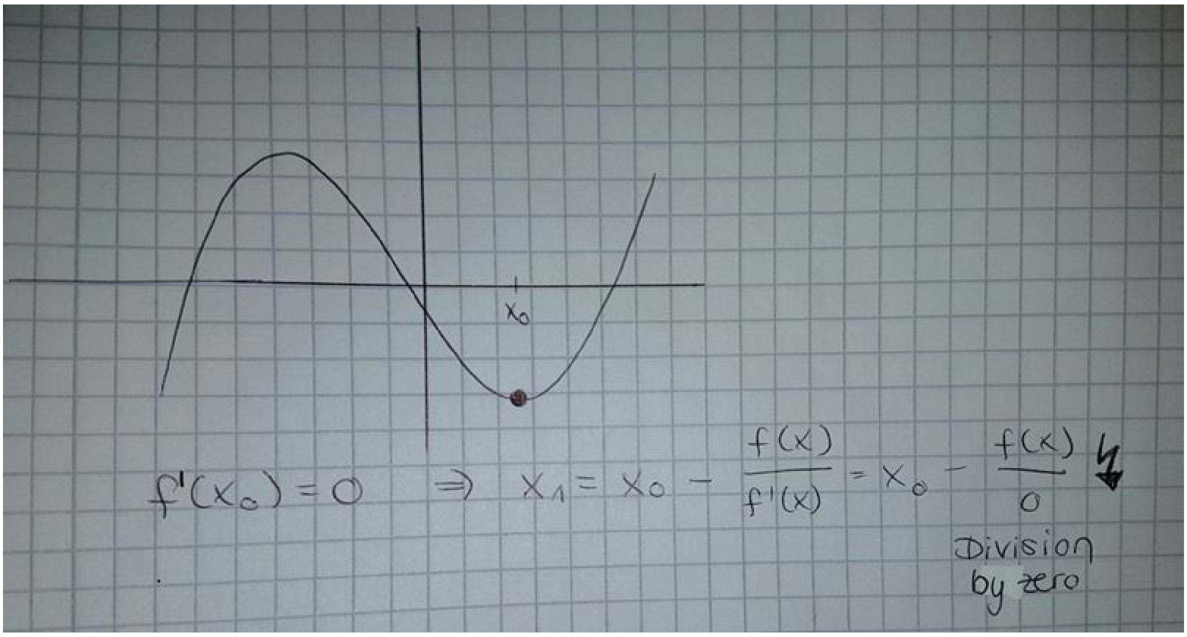


The tangent of x_0 is already very close to the root. The tangent of x_1 crossing the x-axis gives the next point x_2.



By further drawing of steps, the accuracy of digits will rise, since the tangent ist so close to the root or on the picture appears to cross the root, no further drawings would make sense, since they would look almost the same, since my lines are not accurate enough.

c)



If the derivative in the starting point x_0 equals 0, no further steps can be made.

d)

First two steps of the Newton's method for:

$$f(x_1,x_2)x_1^2 - x_1^3 + \mathbf{e}^{x_1} - \mathbf{e}^{-x_2^2} - 2\cdot\mathbf{e}^{-(x_2-7)^2}$$

Gradient:

$$Grad(f) = \Big(2x_1 - 3x_1^2 + \mathbf{e}^{x_1}, 2x_2\cdot\mathbf{e}^{-x_2^2} + 4\cdot\mathbf{e}^{-(x_2-7)^2}\cdot(x_2-7)\Big)$$

Hessian Matrix:

$$\begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} 2 - 6x_1 + \mathbf{e}^{x_1} & 0 \\ 0 & \mathbf{e}^{-x_2^2}\cdot(2-4x_2^2) - 4\cdot\mathbf{e}^{-(x_2-7)^2}\cdot(97 - 28x_2 + 2x_2^2) \end{pmatrix}$$

First Step: $a = (x_1, x_2) = (-1, 0.1)$

$$Grad(f)(a) = \Big(2\cdot(-1) - 3(-1)^2 + \mathbf{e}^{-1}, 2\cdot(0.1)\cdot\mathbf{e}^{-(0.1)^2} + 4\cdot\mathbf{e}^{-(0.1-7)^2}\cdot(0.1-7)\Big) = \Big(\frac{1}{\mathbf{e}} - 5, 0.198\Big)$$

Hessian Matrix:

$$\begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} 2 - 6(-1) + \mathbf{e}^{(-1)} & 0 \\ 0 & \mathbf{e}^{-(0.1)^2}\cdot(2-4(0.1)^2) - 4\cdot\mathbf{e}^{-((0.1)-7)^2}\cdot(97 - 28(0.1) + 2(0.1)^2) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{e} + 8 & 0 \\ 0 & \end{pmatrix}$$

And so on