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| 1 | 2 | 3 | $\Sigma$ |
|---|---|---|---|
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# Übungsblatt Nr. 11

(Abgabetermin 24.01.2017)

## 1 Propositional logic

The entailment connection: $\vDash$ is true if and only if $True \vDash True$

a) Wrong by definition since $True \nvDash False$

b) There are cases where $(A \vee B)$ is true but $(A \Leftrightarrow B)$ is not, i.e. $A = 1, B = 0$

c)

| A | B | $A \Leftrightarrow B$ | $A \wedge \neg B$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |

There are cases in which $A \Leftrightarrow B$ is true but $A \wedge \neg B$ is not. It is therefore not entailed.

d) Yes, since $(A \Rightarrow B) \equiv (\neg A \vee B)$, the entailment is therefore $True \vDash True$

e) Yes, since only one upper level term must be satisfiable since it is connected with disjunctions. $(A \Rightarrow B)$ is satisfiable, therefore the whole sentence is.

f) No, since there are cases in which $(\neg C \vee D \vee E)$ is true, but $(\neg D \vee \neg E)$ is false, i.e. $C = 0, D = 1, E = 1$. If now Both $A = 0$ and $B = 0$, then there is $True \vDash False$ which is wrong.

g) $(A \wedge B) \wedge \neg(A \Rightarrow B) \equiv (A \wedge B) \wedge \neg(\neg A \vee B) \equiv (A \wedge B) \wedge (A \wedge \neg B) \equiv A \wedge B \wedge \neg B$. This is a contradiction, meaning it is not satisfiable for any given input.

h)

| A | B | C | $(A \wedge B) \Rightarrow C$ | $(A \Rightarrow B) \vee (A \vee B)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

In every case in which the first term is true, the second is as well making this a true entailment.

i)

| A | B | C | $(C \vee (A \wedge B))$ | $(A \Rightarrow B)$ | $(B \Rightarrow C)$ | $(\neg(A \Rightarrow B) \vee (B \Rightarrow C))$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

In every case in which the first term is true, the second is as well making this a true entailment.

j) $(A \Leftrightarrow B) \wedge (A \vee \neg B) \equiv (\neg A \wedge B) \wedge (\neg B \wedge A) \wedge (A \vee \neg B) \equiv (\neg A \wedge B) \wedge (\neg B \wedge A)$. The equivalence entails the following term. The sentence is therefore satisfiable for all cases in which the equivalence is, i.e. $A = 1, B = 1$.

# 2 Normal forms

a)

$$\begin{aligned} A \Rightarrow (B \vee C) &\equiv \neg A \vee (B \vee C) \\ &\equiv (\neg A \vee B \vee C) \end{aligned}$$

b)

$$\begin{aligned} \neg A \Leftrightarrow (\neg B \wedge C) &\equiv (A \Rightarrow (\neg B \wedge C)) \wedge ((\neg B \wedge C) \Rightarrow A) \\ &\equiv (\neg\neg A \vee (\neg B \wedge C)) \wedge (\neg(\neg B \wedge C) \vee A) \\ &\equiv (A \vee (\neg B \wedge C)) \wedge ((B \wedge \neg C) \vee A) \\ &\equiv A \wedge ((\neg B \wedge C) \vee (B \wedge \neg C)) \\ &\equiv A \wedge (\neg B \vee B) \wedge (C \vee B) \wedge (\neg B \vee \neg C) \wedge (C \vee \neg C) \\ &\equiv A \wedge (C \vee B) \wedge (\neg B \vee \neg C) \end{aligned}$$

c)

$$\begin{aligned} (A \wedge B) \Rightarrow C &\equiv \neg(A \wedge B) \vee C \\ &\equiv \neg A \vee \neg B \vee C \\ &\equiv (\neg A \vee \neg B \vee C) \end{aligned}$$

d)

$$\begin{aligned} \neg A \vee (C \wedge B) \Rightarrow B &\equiv \neg(\neg A \vee (C \wedge b)) \vee B \\ &\equiv (A \wedge \neg(C \wedge B)) \vee B \\ &\equiv (A \wedge (\neg C \vee \neg B)) \vee B \\ &\equiv (B \vee A) \wedge (B \vee (\neg C \vee \neg B)) \\ &\equiv (B \vee A) \wedge (B \vee \neg C \vee \neg B) \\ &\equiv (B \vee A) \wedge True \\ &\equiv (B \vee A) \end{aligned}$$

e)

$$\begin{aligned} (\neg B \wedge (A \vee C)) \vee (A \wedge (B \vee C)) &\equiv ((\neg B \wedge A) \vee (\neg B \wedge C)) \vee ((A \wedge B) \vee (A \wedge C)) \\ &\equiv (\neg B \wedge A) \vee (\neg B \wedge C) \vee (A \wedge B) \vee (A \wedge C) \\ &\equiv (\neg B \wedge A) \vee (A \wedge B) \vee (\neg B \wedge C) \vee (A \wedge C) \\ &\equiv (A \wedge (B \vee \neg B)) \vee (C \wedge (A \vee \neg B)) \\ &\equiv A \vee (C \wedge (A \vee \neg B)) \\ &\equiv (A \vee C) \wedge (A \vee (A \vee \neg B)) \\ &\equiv (A \vee C) \wedge (A \vee A \vee \neg B) \\ &\equiv (A \vee C) \wedge (A \vee \neg B) \end{aligned}$$

# 3 The hacking case

## a)

1. $Z \vee N \vee E$
2. $Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$
3. $\neg Z \Rightarrow \neg N$
4. $E \Rightarrow N$
5. $E \Rightarrow \neg Z$

### b)

The additional assumption that we make here is that $1 \wedge 2 \wedge 3 \wedge 4 \wedge 5$. That implies that as soon as one statement is not fulfilled we can already exclude the row from the realm of possibilities.

| Z | N | E | $Z \vee N \vee E$ | $\neg Z \Rightarrow \neg N$ | $E \Rightarrow N$ | $E \Rightarrow \neg Z$ | $Z \Rightarrow (Z \wedge N) \vee (Z \wedge E)$ | All |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | - | - | - | - | - |
| 0 | 0 | 1 | 1 | 1 | 0 | - | - | - |
| 0 | 1 | 0 | 1 | 0 | - | - | - | - |
| 0 | 1 | 1 | 1 | 0 | - | - | - | - |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | - |
| 1 | 0 | 1 | 1 | 1 | 0 | - | - | - |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | - | - |

We can therefore conclude that Zack and Naomi are the hackers.