Otto Neugebauer and Richard Courant: On Exporting the Göttingen Approach to the History of Mathematics

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Years Ago *features essays by historians and mathematicians that take us back in time. Whether addressing special topics or general trends, individual mathematicians or "schools" (as in schools of fish), the idea is always the same: to shed new light on the mathematics of the past. Submissions are welcome.*

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> The common belief that we gain "historical perspective" with increasing distance seems to me utterly to misrepresent the actual situation. What we gain is merely confidence in generalizations which we would never dare make if we had access to the real wealth of contemporary evidence.
>
> – Otto Neugebauer, *The Exact Sciences in Antiquity* (Neugebauer 1969, viii)

Otto Neugebauer was one among several distinguished mathematicians who began to take a deep interest in ancient mathematics, astronomy, and related exact sciences soon after the First World War. During the Weimar era, he took up research in this field as a protégé of Richard Courant in Göttingen, a position that placed him at the hub of power at a leading mathematical center. In 1931 he assumed editorial responsibility for a new reviewing journal, *Zentralblatt für die Mathematik und ihre Grenzgebiete*. In 1934 Neugebauer left Göttingen for Copenhagen, where he continued his editorial work as well as his pioneering research and teaching activity. He also continued editing Springer's *Zentralblatt* until Nazi racial policies led to the removal of Jewish colleagues from its board. This paved the way toward the founding of *Mathematical Reviews,* which Neugebauer managed from his new post at Brown University, beginning in 1940. This editorial activity placed him once again at the heart of a major mathematical center, especially after Providence, Rhode Island, became the operational headquarters for the American Mathematical Society. Neugebauer thus put down new roots while maintaining close contacts with fellow refugees, many of whom he knew from his years in Göttingen (for further details, see Swerdlow 1993a).

Unlike many other Europeans, Neugebauer experienced relatively few problems adapting to life in the United States. The moment he arrived on American soil, he began writing in English and soon afterward applied for U.S. citizenship. Apparently he never again returned to Germany and was quite content with his life at Brown, with occasional visits to Princeton's Institute for Advanced Study. Whereas many European émigrés saw the United States as a cultural backwater, Neugebauer clearly sympathized with the freer forms of social interaction he encountered among Americans. A telling anecdote concerns the response he sent to a former German colleague, who had complained to him that he should at least show the courtesy of writing him in his *Muttersprache*. To this, Neugebauer replied:

> As to the last paragraph of your letter, I must remark that the language I use in my letters does not depend on my

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mother but on my secretary. It interests me very much that the so-called German mathematicians now require the editor of an international journal to use their language. During the time I was editor of the *Zentralblatt*, no American mathematician required that I use the English language. I regret, however, that you do not know me personally well enough to know that I would prefer to use exactly the language that I want to use, even if I have to interrupt my relations with German mathematicians (quoted in Swerdlow 1993a, 155).

Philip Davis, who often ate lunch with Neugebauer and his entourage, thought it pointless to talk with him about the nature of mathematical knowledge (Davis 1994, 130). He nevertheless enjoyed Neugebauer's dry sense of humor: "Not unlike Mark Twain, he perceived the human world as consisting largely of fools, knaves, and dupes; and when he was overwhelmed by this perception, he took refuge in his love of animals which was tender and deep" (Davis 1994, 129). For Davis, Neugebauer was a thoroughgoing Platonist, though not really in the philosophical sense of the term; he was far too antiphilosophical in his outlook to be labeled in such a way. Yet he firmly believed in the immutable character of mathematical knowledge, which meant that his field of historical inquiry, the exact sciences, differed from all other forms of human endeavor in one fundamental respect: in this realm there is no sense of historical contingency. After an investigator had cracked the linguistic or hieroglyphic codes that serve to express a culture's scientific knowledge, he or she suddenly held the keys to deciphering ancient sources. And since these pertain to mathematical matters, one could, in principle, argue inductively in order to reconstruct what they contained, namely a fixed and determinable pattern of scientific results. Clearly, this type of puzzle-solving held great fascination for Neugebauer, and he pursued it with considerable success in his research on Mesopotamian astronomy, beginning in the mid-1930s.

Neugebauer's work on Greek mathematics during these politically turbulent times was far scantier. Nevertheless, his views on Greek mathematics formed a central component of his overall vision of the ancient mathematical sciences. Regarding historiography, he adopted a rigorous empirical approach that worked well in some cases, but often led him to make sweeping claims based on little more than hunches. When it came to purely human affairs, on the other hand, Neugebauer professed that he held no *Weltanschauung*. Indeed, he took pains to make this known to those who, like Oskar Becker, mingled ideology with science (see Siegmund-Schultze 2009, 163). Not surprisingly, his general outlook had much to do with the special context in which he experienced mathematics as a young man.

After the First World War, Neugebauer studied physics in Graz with Michael Radaković and mathematics with Roland Weitzenböck. In 1920–1921, these two collaborated in teaching a course on relativity theory and its mathematical foundations, a course that Neugebauer attended with interest. His notes from this and other lecture courses offered at Graz can be found today among his papers housed at Princeton's Institute for Advanced Study (http://library.ias.edu/finding-aids/neugebauer). One also finds among these his handwritten notes from a lecture delivered by Hermann Weyl on his approach to general relativity. From Graz, Neugebauer went on to Munich, where he took courses in physics with Arnold Sommerfeld (boundary-value problems in Maxwell's theory) and with Karl Ferdinand Herzfeld (quantum mechanics of atomic models). Already in 1926, Herzfeld left Germany to take up a visiting professorship at Johns Hopkins, where he remained for the next ten years. One of his doctoral students at Hopkins was John Archibald Wheeler. Presumably it was on Sommerfeld's advice that Neugebauer decided to move on to Göttingen for the summer semester of 1922. In that term he attended a course taught by Hilbert on statistical methods in physics as well as Born's lectures on the kinetic theory of matter. Afterward, he turned to pure mathematics, taking three courses taught by Edmund Landau: analytic number theory, entire transcendental functions, and trigonometric functions. Later, even after he had begun his deep immersion into historical studies of ancient mathematics, he assisted Courant with his elementary courses, but also attended advanced offerings, such as Emmy Noether's lectures on algebraic functions or Gustav Herglotz's on celestial mechanics. These experiences would not only exert a deep influence on Neugebauer's general scientific outlook, they also had profound consequences for his approach to historical research.

As a close ally of Courant, Neugebauer shared a positivist vision of mathematics as an integral part of scientific culture. In particular, both men were deeply influenced by the universalism advocated by Göttingen's two aging sages, Felix Klein and David Hilbert, who broke with an older German tradition in which mathematical research was largely isolated from developments in neighboring disciplines such as astronomy and physics. Hilbert's strong epistemic claims for mathematics had also deeply alienated conservative humanists on the Göttingen faculty, many of whom feared a realignment of traditional disciplinary boundaries (see Rowe 1986). This background should be borne in mind when discussing Neugebauer's later career, both in Copenhagen as well as at Brown. It also helps to account for the reason his revisionist approach to Greek mathematics led to a clash of opinions within the newly emergent community of historians of science in the United States.

Neugebauer's Cornell Lectures

In 1949, when Otto Neugebauer delivered six lectures on ancient sciences at Cornell University, he was the first historian of mathematics to be given the honor of speaking in its distinguished Messenger lecture series. He did not waste this opportunity. Afterward, he went over his notes and gave the text its final, carefully sculpted form that we find today in the six chapters of Neugebauer's *The Exact Sciences in Antiquity*, published in 1951 with high-quality plates by Brown University Press. The text begins by describing a famous work in the history of art:

> When in 1416 Jean de France, Duc de Berry, died, the work on his "Book of the Hours" was suspended. The brothers Limbourg, who were entrusted with the illuminations of this book, left the court, never to complete what is now considered one of the most magnificent of late medieval manuscripts which have come down to us.

A "Book of Hours" is a prayer book which is based on the religious calendar of saints and festivals throughout the year. Consequently we find in the book of the Duc of Berry twelve folios, representing each one of the months. As an example we may consider the illustration for the month of September. As the work of the season, the vintage is shown in the foreground (Plate 1 [reproduced below]). In the background we see the Château de Saumur, depicted with the greatest accuracy of architectural detail. For us, however, it is the semicircular field on top of the picture, where we find numbers and astronomical symbols, which will give us some impression of the scientific background of this calendar. Already a superficial discussion of these representations will demonstrate close relations between the astronomy of the late Middle Ages and antiquity (Neugebauer 1969, 3).

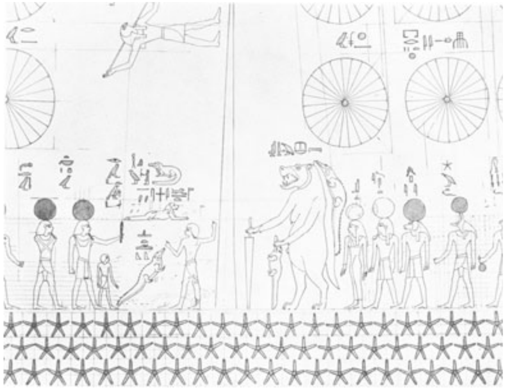
Neugebauer went on to note four different types of writing for the numbers that appear in the *Book of Hours:* Hindu-Arabic as well as Roman numerals, number words (September through December for the seventh to the tenth months of the Roman calendar), and alphabetic numbers, here calculated modulo 19, the system used in connection with the Metonic lunar cycle. Regarding the latter, Neugebauer noted that 19 was called the "golden number" in the late Middle Ages, after a 13th-century scholar wrote that this lunar cycle excels all others "as gold excels all other metals." He then comments as follows about the state of scientific progress in the Latin West when seen against the backdrop of earlier developments: "In the twelfth century this very primitive method [for calculating the date of a new moon] was considered by scholars in Western Europe as a miracle of accuracy, though incomparably better results had been reached by Babylonian and Greek methods since the fourth century B.C. and though these methods were ably handled by contemporary Islamic and Jewish astronomers" (Neugebauer 1969, 8). Clearly, Neugebauer wanted his audience to realize that it was one thing to appreciate a magnificent work of art, and quite another to think of it as a canvas for clues about the state of mathematical and astronomical knowledge in the culture within which it was produced.

"September" from the *Book of Hours* of the Duke of Berry. [Plate 1 from Neugebauer's *The Exact Sciences in Antiquity*, 2nd ed., Brown University Press, 1957.].

For the second edition of *The Exact Sciences in Antiquity*, Neugebauer updated the material and added two technical appendices, but he still hoped to have "avoided … converting my lectures into a textbook" (Neugebauer 1969, ix). Evidently, he valued the less formal form of exposition associated with oral exposition, a hallmark of the Göttingen tradition. Noel Swerdlow made these remarks about the style of presentation: "Neugebauer here allowed himself the freedom to comment on subjects from antiquity to the Renaissance. The expert can learn something from it, and from its notes, every time it is read, and for the general reader it is, in my opinion, the finest book ever written on any aspect of ancient science" (Swerdlow 1993a, 156). High praise, indeed; let's consider another passage that brings out the speculative side of these lectures as well as Neugebauer's famously dry sense of humor. Here he comments ironically on the significance of modern celestial mechanics in suggesting how human understanding has been shaped by accidental circumstances.

The structure of our planetary system is indeed such that Rheticus could say, "the planets show again and again all the phenomena which God desired to be seen from the earth." The investigations of Hill and Poincaré have demonstrated that only slightly different initial conditions would have caused the moon to travel around the earth in a curve [with nodal loops and] … with a speed exceedingly low in the outermost quadratures as compared with the motion at new and full moon. Nobody would have had the idea that the moon could rotate on a circle around the earth and all philosophers would have declared it as a logical necessity that a moon shows six half moons between two full moons. And what could have happened with our concepts of time if we were members of a double-star system (perhaps with some uneven distribution of mass in our little satellite) is something that may be left to the imagination (Neugebauer 1969, 152–153).

Significantly, Neugebauer dedicated this now classic book to "Richard Courant, in Friendship and Gratitude." Elaborating on that dedication in the preface, he wrote that it was Courant who enabled him to pursue graduate studies in ancient mathematics, and he went on to remark: "more than that I owe [to him] the experience of being introduced to



Drawing of the Ceiling in the Tomb of Senmut made by Expedition of the Metropolitan Museum of Art, New York , Plate 10.

Geometrical Problems in the Style of Heron, Greek Papyrus in the Cornell Collection, Plate 12.

Otto Neugebauer: Courant's Right-hand Man.

A cuneiform tablet from the ‘‘Old Babylonian'' period, ca. 1600 B.C.E., Plate 3.

Plimpton 322, the most famous of all Babylonian mathematical tablets, Plate 7a, discussed by Neugebauer in *The Exact Sciences in Antiquity*, pp. 36-40.

modern mathematics and physics as a part of intellectual endeavour, never isolated from each other nor from any other field of our civilization'' (Neugebauer 1969, vii).

Neugebauer's Ties with Courant's Göttingen

Neugebauer was a man who chose his words carefully, and so we may be sure that this public acknowledgment of his debt to Courant was far more than just a friendly gesture. His allusion to physics brings to mind the famous Courant-Hilbert volume from 1924, which gave physicists the tools they needed to handle Schrödinger's equation and related problems in quantum mechanics. Neugebauer witnessed much of this first-hand in Göttingen. Nor should we forget that he had studied physics under Arnold Sommerfeld before he came to Göttingen. Yet, clearly, what Neugebauer had in mind here went far beyond the usual appeal to the unity of mathematical and physical ideas, for he wrote that Courant's vision saw these fields of intellectual endeavor as ''never isolated from each other nor from any other field of our civilization.'' This brief remark comes very close to capturing the essence of Neugebauer's own understanding of what it meant to study the history of mathematics, a topic that deserves closer attention. Regarding Courant's own vision, Neugebauer said the following on the occasion of Courant's 75th birthday:

> … the real core of his work [consisted] in the conscious continuation and ever widening development of the ideas of Riemann, Klein, and Hilbert, and in his insistence on demonstrating the fundamental unity of all mathematical disciplines. One must always remain aware of these basic motives if one wants to do justice to Courant's work and to realize its inner consistency (Neugebauer 1963, 1).

Otto Neugebauer's personal relationship with Richard Courant reflects many of the broader mathematical and scientific interests the two men shared. As director of the Göttingen Mathematics Institute during the Weimar years, Courant was faced with numerous challenges as he struggled to uphold its international scientific reputation. Part of his strategy was conservative in nature. Through his connections with Ferdinand Springer, Courant launched the famed ''yellow series,'' one of several initiatives that enabled Springer to attain a pre-eminent position as a publisher in the fields of mathematics and theoretical physics (Remmert and Schneider 2010).

Richard Courant as Mathematical Entrepreneur.

Not surprisingly, Neugebauer took an active part in preparing some of these volumes, including the Hurwitz-Courant lectures on function theory. He later gave a vivid account of typical scenes in the production of these books:

> A long table in [Carl] Runge's old office was the battleground on which took place what Courant's assistants used to call the ''Proof-Reading-Festivals'' (''Korrekturfeste''). … During this period Courant wrote his first group of famous books, the second edition of the ''Hurwitz-Courant,'' the first volume of the ''Courant-Hilbert,'' and the ''Calculus.'' All of his assistants during these years participated at one or the other time in the preparation of the manuscripts: [Kurt] Friedrichs, [Hans] Levy, [Willy] Feller, [Franz] Rellich, [B. L.] van der Waerden, and others; red ink, glue, and personal temperament were available in abundance. Courant had certainly no easy time in defending his position and reaching a generally accepted solution under the impact of simultaneously uttered and often widely divergent individual opinions about proofs, style, formulations, figures, and many other details. At the end of such a meeting he had to stuff into his briefcase galleys (or even page proofs) which can only be described as Riemann surfaces of high genus and it needed completely unshakeable faith in the correctness of the uniformisation theorems to believe that these proofs would ever be mapped on *schlicht* pages (Neugebauer 1963, 6–7).

Neugebauer also played a role in the preparations for and publication of Klein's influential lectures on the mathematics of the 19th century, in many ways a prototype for later projects directed by Courant. Although called to the front during the years that Klein delivered these lectures, Courant had himself helped prepare the material on one of Klein's central topics, the arithmetization of analysis. Hilbert's Assistent, Alfréd Haar, likewise worked up valuable material on the history of mathematical astronomy during this same informal seminar (these documents are located in Cod. Ms. F. Klein, Niedersächsische Staats- und Universitätsbibliothek Göttingen). Such communal work would later become a hallmark of the Courant-Neugebauer partnership.

Courant was an innovator with a deep belief in the vitality of older traditions. His yellow series looked backward as well as forward; in fact, surprisingly few of its volumes betray a commitment to what came to be identified as modern, abstract mathematics. Far more evident was the way in which Courant and his coeditors built on the tradition of Klein and Hilbert, and with the yellow series he found a way to make local knowledge accessible well beyond the borders of Germany. Neugebauer would ultimately devote himself to the study of the same nexus of mathematical sciences in antiquity. For the history of the ancient exact sciences, Springer's short-lived *Quellen und Studien* series, launched in 1929 and edited by Neugebauer, Julius Stenzel, and Otto Toeplitz, created a new standard for studies in this fast-breaking field.

Yet Neugebauer could hardly have foreseen this explosion of interest in ancient as well as modern mathematics.

Plaque honoring Otto Neugebauer, Mathematical Institute, Göttingen.

Along the way to becoming a historian, he would gain a deep respect for the unity of mathematical knowledge through his interactions with Göttingen mathematicians. When he arrived, Courant was offering a seminar on two topics: algebraic surfaces and algebraic number theory. This blending of topics was, no doubt, unusual. Even more unusual was the quality of the group who attended, which included Courant and his two *Assistenten*, Helmut Kneser and Carl Ludwig Siegel, along with such young talents as Emil Artin and Kurt Friedrichs. Neugebauer was asked to give the opening presentation, and his performance delighted Courant; thus began their lifelong friendship (Reid 1976, 91). Soon thereafter Courant gave Neugebauer various special duties to perform at the hub of operations, located on the third floor of the *Auditorienhaus*. There one found the famous *Lesezimmer*, an impressive collection of mathematical models long cared for by Felix Klein's assistants. Now Neugebauer stood guard while Klein received nearly daily reports through those who were busy helping him prepare his collected works. Neugebauer's new interest in Egyptian mathematics also came to Klein's attention, along with a complaint that he had stuffed all the books on mathematics education tightly together on a high shelf, making them nearly inaccessible. By now Klein was an infirm old man who rarely left his home, which overlooked the botanical garden immediately behind the Auditorienhaus, but he still kept up a busy and tightly organized schedule. Neugebauer remembered how Klein called him over to be gently scolded. When he arrived, Klein greeted him by saying: "There came a new Moses into Egypt and he knew not Pharaoh" (Reid 1976, 100). The young Neugebauer surely realized that watching over the Lesezimmer was no trifling matter.

In his recollections of Courant's role in revitalizing Göttingen mathematics, Neugebauer placed great emphasis on the loyalty he was able to instill among those in his circle (Neugebauer 1963). Peter Lax also wrote in very much the same vein about Courant's later career at New York University (Lax 2003). When Courant himself looked back on his years in Göttingen, his own deep sense of loyalty toward Hilbert mingled with a sense of nostalgia for the past. On one wall in his home north of New York City, hung a portrait of Hilbert, across from that of another Göttingen Ordinarius, Carl Runge, his father-in-law. When I first came to New York in 1981, I rented a room around the corner from Nina Courant, who invited me to visit on several occasions and sometimes introduced me to several others in her circle. She once told me how her husband needed to escape from the turmoil in the city. He loved to return to the quiet of their home in New Rochelle, where a number of other NYU faculty members also lived, among them Kurt Friedrichs. Neugebauer, too, had a strong aversion to big cities. He valued quiet and seclusion to such an extent that when he had the chance to choose his office at Brown he took rooms in the basement, figuring that people weren't likely to disturb him down there. (Swerdlow 1993a, 153)

Neugebauer's Revisionist Approach to Greek Mathematics

Neugebauer saw himself as a "scientific historian," a tradition Noel Swerdlow traced back to Jean Etienne Montucla and the Enlightenment (Swerdlow 1993b). Yet unlike Montucla, whose work was encyclopedic in scope, Neugebauer's special métier was that of a mathematical detective, one who went about combing artifacts for underlying patterns. What he told about the famous *Book of Hours* of the Duc de Berry was merely an example, meant to illustrate a "much more general phenomenon." His larger point was that "[f]or the history of mathematics and astronomy the traditional division of political history into Antiquity and Middle Ages is of no significance" (Neugebauer 1969, 3). He also regarded the stylistic categories used by cultural historians as irrelevant for the history of mathematics, claiming that it was nonsense to think of Euler's work as late baroque. As for mathematical astronomy, its practitioners were steeped in ancient methods until the time of Newton. "One can perfectly well understand the *Principia*," Neugebauer wrote, "without much knowledge of earlier astronomy, but one cannot read a single chapter in Copernicus or Kepler without a thorough knowledge of Ptolemy's *Almagest*" (Neugebauer 1969, 3–4). He made a similar distinction between ancient and modern in the history of mathematics, taking the latter to commence with the creation of analysis by Newton and his contemporaries.

Neugebauer had no patience for those historians who simply wanted to chronicle the great names and works of the past. Thus he loathed the work of the Belgian George Sarton, who had already left Europe during the First World War to settle in the United States, where he did much to promote the history of science as an academic discipline. Sarton saw the field largely as a humanistic endeavor; nevertheless, he had the highest respect for Neugebauer's achievements. Sarton's views emerge clearly from correspondence during September 1933 with Abraham Flexner. At the time, Flexner was contemplating the possibility of founding a school for studies of science and culture at the Institute for Advanced Study. Sarton thought that Neugebauer was just the man for such an enterprise, a point he made by humbly contrasting the nature of their work: "As compared with Neugebauer I am only a dilettante. He works in the *front trenches* while I amuse myself way back in the rear—praising the ones, blaming the others; saying this ought to be done, etc.–& doing very little myself. What Neugebauer does is fundamental, what I do, secondary" (Pyenson 1995, 268).

Neugebauer certainly did view Sarton as a dilettante through and through, which helps to explain why he seldom published in Sarton's *Isis,* the official journal of the History of Science Society. Not that Neugebauer had anything against him personally. When I interviewed Neugebauer in 1982, he made a point of telling me what he thought of Sarton by lumping him together with Moritz Cantor, another encyclopedist of great breadth and little depth whom he regarded as a modern-day Isidor of Seville. Neugebauer's heroes were, for the most part, fellow mathematicians: he admired historians such as the Danish algebraic geometer, H. G. Zeuthen, who tried to use small clues in order to frame a picture of early Greek mathematics, in particular the theory of conic sections as it existed before the time of Apollonius. Zeuthen, incidentally, held a similarly dismissive view of M. Cantor's approach to the history of mathematics (see Lützen and Purkert 1994).

In any case, Sarton's plan to bring Neugebauer to Princeton came to naught, mainly because the IAS saw no way to take on the whole *Zentralblatt* operation, too. Soon afterward, a better option emerged when Harald Bohr arranged a three-year appointment as professor at Copenhagen beginning in January 1934. Neugebauer managed to get most of his property out of Germany, but had to abandon a house with a partially paid mortgage. In Copenhagen, his research was supported in part by the Rockefeller Foundation.

Almost immediately he began preparing a series of lectures on Egyptian and Babylonian mathematics that he would publish in Courant's yellow series as *Vorgriechische Mathematik* (Neugebauer 1934). According to Swerdlow, this volume was "as much a cultural as a technical history of mathematics" and represented "Neugebauer's most thorough and successful union of the two interpretations" (Swerdlow 1993a, 145). More striking still is the unfinished character of this work, which represents the first and final volume in a projected trilogy that remained incomplete. Neugebauer had planned to tackle Greek mathematics proper in the second volume, whereas the third would have dealt with mathematical astronomy, both in the Greek tradition culminating with Ptolemy as well as the largely unknown work of late Babylonian astronomers. Thus, his original aim, as spelled out in the foreword to the first volume, was to achieve a first overview of the ancient mathematical sciences in their entirety, something that had never before been attempted.

Swerdlow has offered compelling reasons to explain why Neugebauer dropped this project, one being that he simply found the rich textual sources for Mesopotamian mathematical astronomy far more important than anything he could ever have written about Greek mathematics. Nevertheless, we can trace a fairly clear picture of the line of argument Neugebauer had in mind from the summary remarks at the conclusion of his *Vorgriechische Mathematik* as well as some of his other publications from the 1930s. Particularly suggestive is an essay entitled "Zur geometrischen Algebra," published in 1936 in *Quellen und Studien* (Neugebauer 1936). Significantly, Neugebauer takes as his motto a famous fragment from the late Pythagorean Archytas of Tarentum, which reads: "It seems that logistic far excels the other arts in regard to wisdom, and in particular in treating more clearly what it wishes than geometry. And where geometry fails, logistic brings about proofs" (Neugebauer 1936, 245).

Much has been written about this passage, in particular about what might be meant by the term "logistic." This notion pops up in Platonic dialogues and quite clearly it has more to do with ancient arithmetic than it does logic. The whole matter was discussed at great length by Jakob Klein in his study "Die griechische Logistik und die Entstehung der Algebra" (Klein 1936), which appeared alongside Neugebauer's article (it was later translated into English by Eva Brann [Klein 1968]). In fact, both scholars were chasing after the same elusive goal, although there the similarity ends. Klein was a classical philologist who later became a master teacher of the "Great Books" curriculum at St. John's College in Annapolis, Maryland. Not surprisingly, he was intent on squeezing as much out of Plato as he possibly could. Thus he distinguished carefully between practical and theoretical logistic, offering a new interpretation of Diophantus's *Arithmetica* that placed it within the latter tradition. Neugebauer had no patience for the nuances of meaning classicists liked to pull out of their texts. Indeed, he had an entirely different agenda. His point was that rigorous axiomatic reasoning in the style of Euclid arose rather late, and that Archytas, a contemporary of Plato, was bearing witness to the primacy of algebraic content over the geometrical form in which the Greeks dressed their mathematics. With that, we can take another step forward toward attaining a closer understanding of Neugebauer's *Weltanschauung*.

Decades earlier, the Danish historian of mathematics H. G. Zeuthen already advanced the idea that the Greeks had found it necessary to geometrize their purely algebraic results after the discovery of incommensurable magnitudes (Zeuthen 1896, 37–39). Neugebauer took up this by now standard interpretation, adopted by Heath and nearly everyone else, but he then went much further, arguing that the algebraic content—found not only in Book II of Euclid but throughout the entire corpus of Apollonius's *Conica*—could be traced back to results and methods of the Babylonians (on Apollonius, see Neugebauer 1932). In "Zur geometrischen Algebra" he wrote:

> The answer to the question what were the origins of the fundamental problem in all of geometrical algebra [meaning the application of areas, as given by Euclid's propositions II.44 and VI.27-29] can today be given completely: they lie, on the one hand, in the demands of the Greeks to secure the general validity of their mathematics in the wake of the emergence of irrational magnitudes, on the other, in the resulting necessity *to translate the results of the pre-Greek "algebraic" algebra as well.*
>
> Once one has formulated the problem in this way, everything else is completely trivial [!] and provides *the smooth connection between Babylonian algebra and the formulations of Euclid* (Neugebauer 1936, 250, my translation, his italics).

The mathematical arguments underlying this claim are, as Neugebauer noted, by no means difficult to follow. Still, he was surely quite aware that this interpretation amounted to a wild leap of historical imagination. For by asserting that the fundamental core of early Greek mathematics was Babylonian—having been transmitted during an earlier epoch and possibly over a lengthy period of time—he was advancing a bold new conjecture based on nothing more than what he discerned to be a common body of knowledge entirely algebraic in nature. What made this claim for transmission so bold was that it, in fact, lacked any solid documentary evidence whatsoever. Summarizing his position, Neugebauer offered these remarks: "Every attempt to connect Greek thought with the pre-Greek meets with intense resistance. The possibility of having to modify the usual picture of the Greeks is always undesirable, despite all shifts of view, ... [and yet] the Greeks stand in the middle and no longer at the beginning" (Neugebauer 1936, 259).

When we try to square this stance with Neugebauer's stated belief that one should be wary of generalizations about the distant past—the position quoted in the motto to this essay—the problems with such an argument only become more acute. Perhaps these evident difficulties help explain the intensely passionate language he used in the concluding parts of this text. The tone in *The Exact Sciences in Antiquity* is far milder, and yet his argumentation remains substantively the same (Neugebauer 1969, 146–151). There is even brief mention of the same quotation from Archytas, and one senses what Swerdlow might have meant when he wrote that Neugebauer soon grew bored with Greek mathematics (Swerdlow 1993a, 146). One could hardly do better than to quote his description of Neugebauer's methodological orientation:

> At once a mathematician and cultural historian, Neugebauer was from the beginning aware of both interpretations and of the contradiction between them. Indeed, a notable tension between the analysis of culturally specific documents, whether the contents of a single clay tablet or scrap of papyrus or an entire Greek treatise, and the continuity and evolution of mathematical methods regardless of ages and cultures, is characteristic of all his work. And it was precisely out of this tension that was born the detailed and technical cross-cultural approach, in no way adequately described as the study of "transmission," that he applied more or less consistently to the history of the exact sciences from the ancient Near East to the European Renaissance.
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> But if the truth be told, on a deeper level Neugebauer was always a mathematician first and foremost, who selected the subjects of his study and passed judgment on them, sometimes quite strongly, according to their mathematical interest (Swerdlow 1993a, 141–142).

Greek Mathematics Reconsidered

By the 1950s, however, a first wave of negative reaction began to swell up. One can well imagine that for many experts on ancient science, Neugebauer's research on Greek mathematics represented part of a fairly large-scale intrusion by mathematicians into a field that was formerly dominated by classicists. Yet it seems this field itself had already begun to fissure in Weimar Germany, which suggests that a quite general reorientation was long underway. At any rate, Neugebauer knew that he had plenty of company. He could thus cite the work of classical scholars such as Eva Sachs and Erich Frank, so-called "hyper-critical" philologists, in defending his arguments for recasting the early history of Greek mathematics. This revisionist approach aimed to debunk the notion of a "Greek miracle" that sprang up during the sixth century from the shores of Ionia. Neugebauer, too, was convinced that most of the sources that reported on the legendary feats of ancient heroes—Thales, Pythagoras, and their intellectual progeny—were just that: legends that had grown with the passing of time. So his watchword remained skepticism with regard to the accomplishments of the early Greeks, whereas Toeplitz, Becker, and others began to analyze extant sources with a critical eye toward their standards of exactness (see Christianidis 2004 for a recent account of older as well as the newer historiography on Greek mathematics).

In 1951 this earlier revisionist work came under strong attack in the pages of George Sarton's *Isis*. Georgio de Santillana, who fled fascist Italy to take up a post at MIT, briefly recounted this background before launching into a scathing critique of Erich Frank's book (Frank 1923). Frank had made a sweeping attempt to deny that the so-called Pythagoreans had played any substantive role in early Greek science. Before proceeding to demolish Frank's argument, however, de Santillana and his coauthor, Walter Pitts took a swipe at the pernicious influence of the mathematical historians who had since entered the field, naming people such as Neugebauer, B. L. van der Waerden, and Kurt Reidemeister (de Santillana and Pitts 1951). This hefty reaction thus came long before Sabetai Unguru mounted an even more sweeping assault on the historiography of ancient Greek mathematics in his 1975 article in *Archive for History of Exact Sciences* (Unguru 1975). These controversies clearly reflect a strong polarization of opinion among experts along sharply disciplinary lines. On a small scale, the picture suggests themes that later became famous in C. P. Snow's essay on the sharp division separating the "Two Cultures" (for more on these debates, see Rowe 1996).

In his frontal assault, Unguru seized upon a general tendency among historians of ancient mathematics, many of whom were retired mathematicians, to make use of transcriptions into familiar modern notation, a methodology heavily employed by Neugebauer and his contemporaries. Indeed, in his 1936 article "Zur geometrischen Algebra," Neugebauer further legitimized such a methodology by arguing that the *content* of much Greek mathematics was algebraic, even though its *form* was geometric (Neugebauer 1936, 245-246). For him, as for other leading mathematicians—B. L. van der Waerden, Hans Freudenthal, and André Weil—who took strong issue with Unguru's views (see the final section of Christianidis 2004), there was nothing problematic about such a viewpoint. After all, mathematicians are constantly trying to find the deeper core of truth behind the symbols they use to express things they struggle to grasp. Unguru nevertheless stuck to his guns, insisting that such claims on behalf of mathematically educated historians were simply fallacious. Legitimate historical inquiry, he has maintained, cannot proceed on the assumption that mathematical content can be separated from the form in which it is expressed.

Today it would appear that most historians of mathematics have come to accept this central tenet. Indeed, at the recent symposium honoring Neugebauer at New York University's Institute for Studies of the Ancient World, Alexander Jones told me that Unguru's position could now be regarded as the accepted orthodoxy. Sabetai Unguru, however, begs to differ; he quickly alerted me to recent work by experts on Babylonian mathematics who, in his view, continue to commit the same kinds of sins he has railed about for so long. As is well known, people don't change their minds, even less their habits of thought, very easily.

Neugebauer rarely took part in controversies such as these, despite the fact that his name was often invoked by others. One must imagine that this had something to do with his background and special place within the discipline. After all, he had many friends and allies within the world of mathematics, most of whom deeply admired his achievements as an historian. Already in 1936, he was invited to deliver a plenary lecture at the International Congress of Mathematicians held in Oslo. Moreover, Neugebauer's own attitude toward his work seems to have contained an element of playfulness. When he came to the end of his Messenger lectures on the exact sciences in antiquity, he offered a simile to describe the historian's craft:

> In the Cloisters of the Metropolitan Museum in New York there hangs a magnificent tapestry which tells the tale of the Unicorn. At the end we see the miraculous animal captured, gracefully resigned to his fate, standing in an enclosure surrounded by a neat little fence. This picture may serve as a simile for what we have attempted here. We have artfully erected from small bits of evidence the fence inside which we hope to have enclosed what may appear as a possible, living creature. Reality, however, may be vastly different from the product of our imagination; perhaps it is vain to hope for anything more than a picture which is pleasing to the constructive mind when we try to restore the past (Neugebauer 1969, 177).

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