Errata for WEYL's Symmetry

by Marius Kempe

p. 20: "The net result is that in all physics nothing has shown up indicating an intrinsic difference of left and right."

Four years after the book was published, physicists discovered a fundamental left-right asymmetry, of which there is a wonderful discussion in Feynman's *Lectures on Physics*, ch. 52. More recent experiments seem to indicate that charge, reflection and time symmetry all go together.

p. 25: "It seems difficult to devise physical laws in which they are not intrinsically alike; but the negative counterpart of the positively charged proton still remains to be discovered."

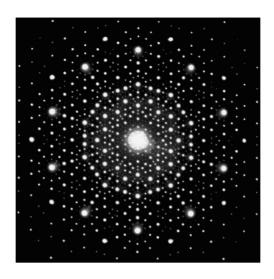
This was discovered even sooner, three years after the book's publication, by Segrè & Chamberlain, and is now called the 'antiproton'. For a description of its physical properties, see §52-8 of Feynman's *Lectures*.

p. 63: "While pentagonal symmetry is frequent in the organic world, one does not find it among the most perfectly symmetrical creations of inorganic nature, among the crystals."

Crystallographers have since discovered crystals with, amongst other properties, five-fold symmetry of sorts, called 'quasicrystals'. The figure on the next page shows a Laue diagram of an aluminium-nickel-cobalt quasicrystal (compare the figures on p. 124 of the book).

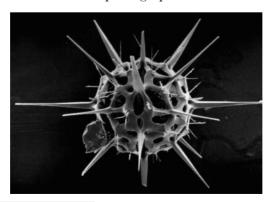
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¹ First described by Shechtman et al. in *Physical Review Letters* (1984), 53 (20): p. 1951-1954.



p. 75: "Here is a page from Haeckel's Challenger monograph showing the skeletons of several Radiolarians."

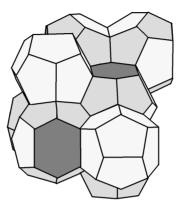
D'Arcy Thompson wrote: "As to Haeckel, I wouldn't trust him round the corner, and I have the gravest doubt whether his pentagonal dodecahedron and various others ever existed outside his fertile fancy. I believe I may safely say that no type-specimens of these exist in the British Museum, or anywhere else." Here is a photograph of a radiolarian.



¹ From a letter quoted in H. S. M. Coxeter's review of *Symmetry (American Mathematical Monthly* (1953), 60 (2): p. 136-139)

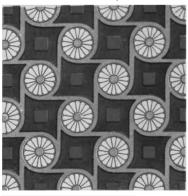
p. 93: "I am inclined to believe that Lord Kelvin's configuration gives the absolute minimum; but so far as I know, this has never been proved."

In fact, a better structure was discovered by Weaire & Phelan in 1994. It is not known whether it is optimal.



p. 103: "Examples for all 17 groups of symmetry are found among the decorative patterns of antiquity, in particular among the Egyptian ornaments."

Only 12 of the 17 symmetry groups can be found in extant ancient Egyptian decorations, with the five groups that preserve hexagonal lattices missing¹. Two example patterns are shown below (and see figure 65 in the book).





¹ See B. Grünbaum in *The Mathematical Intelligencer* (1984), 6 (4), p. 47-53.

p. 104: "Strangely enough the proof was carried out only as late as 1924 by George Pólya, now teaching at Stanford."

Actually the proof was first given by E. S. Fedorov in 1891¹, though Pólya discovered it independently. Moreover, Pólya did not actually give a proof in his 1924 article, but merely stated the result.

p. 104: "Galois' ideas, which for several decades remained a book with seven seals but later exerted a more and more profound influence upon the whole development of mathematics, are contained in a farewell letter written to a friend on the eve of his death, which he met in a silly duel at the age of twenty-one."

This letter was actually only a sketch of various results Galois had obtained. His most important work was a memoir² on the solvability of algebraic equations which he had been submitting versions of to the *Académie des Sciences* every year for three years previous to his death, and which was finally published by Liouville in his journal in 1846. Also, Galois died at twenty.

IMAGE SOURCES

Quasicrystal – Abe et al., *Nature Materials* (2004), 3: p. 759-767 Radiolarian – *Encyclopedia of Life* (eol.org) Weaire-Phelan structure – Tom Ruen, *Wikipedia*

Egyptian patterns – Owen Jones, *The Grammar of Ornament*

¹ 'Symmetry in the plane', Proceedings of the Imperial St. Petersburg Mineralogical Society, 28 (2), p. 345-390.

 $^{^2}$ Translated into English in Appendix 1 of H. M. Edwards, $\textit{Galois Theory}\ (1984)$