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The Concept of Construction and the Representation of Curves in Seventeenth-Century Mathematics

HENK J. M. BOS

Example 1. An exponential curve. My topic is best introduced by an example. I take it from the correspondence between Leibniz and Huygens in 1690–1691. Leibniz wrote about his new differential and integral calculus. Huygens was very skeptical and proposed problems for Leibniz to solve. In the course of this exchange Leibniz came to use an exponential equation to represent a curve. This was entirely new; the only curve equations used until then were algebraic ones. Huygens was even more skeptical about this novelty: he thought that Leibniz boasted, using fancy but empty symbolism. So Leibniz explained further. He took as an example the curve representing the relation between the time t and the velocity v of a body falling in a medium with resistance proportional to v^2. That curve, he said, was given by the following exponential equation:

$$b^t = \frac{1+v}{1-v}. \tag{1}$$

Huygens was still puzzled. He wrote:

> I must confess that the nature of that sort of supertranscendental lines, in which the unknowns enter the exponent, seems to me so obscure that I would not think about introducing them into geometry unless you could indicate some notable usefulness of them [**11**, Vol. 9, p. 537].

And somewhat later he wrote:

> I beg you to tell me whether you can represent the form of that curve by marking points on it or by whatever method [**11**, Vol. 9, p. 570].

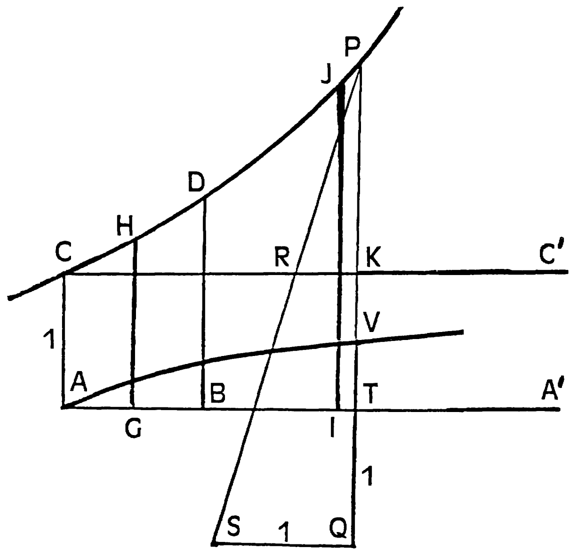

FIGURE 1

Leibniz's answer was affirmative. The equation, he wrote, implies the construction of points on the curve, and he gave the following

Construction [**11**, Vol. 10, pp. 14–15]. Draw (see Figure 1) parallel lines AA' and CC' with distance $AC = 1$. Take B on AA' with $AB = 1$. Take BD of arbitrary length b, perpendicular to AA'. Draw the *Logarithmica* through C and D with axis AA'. The *Logarithmica* is the curve with equation $y = b^x$. It was known to Huygens; not, of course, by its equation, but as the curve with the property that for every sequence of equidistant points on the axis, the corresponding ordinates are in geometrical progression. Hence if G is the middle of AB, then $GH = \sqrt{AC \cdot BD}$, which is constructible by ruler and compass. Again if $AB = BI$, then $IJ = (BD)^2/AC$, which is constructible as well. Thus this property implies a method to construct arbitrarily many points on the curve (by successive halving and doubling of segments on the axis and constructing the corresponding ordinates). It is to this pointwise construction of the curve that Leibniz refers in his explanation to Huygens. With the *Logarithmica* thus constructed, take P arbitrary on that curve, draw the ordinate PT, intersecting CC' in K, prolong to Q with $TQ = 1$. Take $QS = 1$ horizontally to the left, and connect S and P. SP intersects CC' in R. Take V on TP such that $TV = KR$. Then V is on the required curve. To find more points repeat this construction from other points P on the *Logarithmica*. □

Clearly, this is a rather complicated procedure to represent a curve. More surprising is the fact that for Huygens this method of marking points on the curve was much more enlightening than Leibniz's exponential equation. Indeed he wrote back:

> I have looked at your construction of the exponential curve which is very good. Still I do not see that this expression $b^t = \frac{1+v}{1-v}$ is a great help for that: I knew the curve already for a long time [**11**, Vol. 10, pp. 20–21].

Huygens's reaction shows that for him the exponential equation was not a sufficient representation of the curve; he only could understand, and indeed recognize, the curve when a construction of it was given. For him the canonical way of *giving* (and *understanding*) a curve was by a *construction procedure* for making points on it. The example, then, is about different views on the proper way of representing curves.

The representation of curves. I use the term "representation of a curve" as a technical term to denote a description of a curve that is sufficiently informative to consider the curve *known.*

In the seventeenth century, mathematicians were often confronted with the problem of how to represent curves, because they came upon many problems in which it was required to find hitherto unknown curves. Many of these problems were so-called "inverse tangent problems," equivalent to first-order differential equations and often arising from mechanical problems. Solving such problems required a convincing representation of the curve sought. As the analytical methods (analytic geometry, the calculus) were still very new, representation of a curve by its equation was often not considered sufficient (especially in the case of transcendental curves), and more geometrical ways of representation were required.

The representation of curves was an *informal practice*, without fixed criteria of adequacy. There was, at that time, no universally accepted definition of the concept of curve on which a formally determined way of representing curves could be based (nor, apparently, was a need for such a definition felt). Because it was an informal practice, it was subject to much debate; opinions about the proper representation of curves differed among mathematicians; and they changed over the period. These differences of opinion and the ensuing debates are interesting because they reveal much about the changing conceptions and aims within the mathematics of that period. In particular they reveal the complex process of the replacement of geometrical ways of thinking by analytical ones.

Geometrical construction. The example of the exponential curve illustrates that in the seventeenth century the representation of curves often relied on procedures of geometrical construction. At the beginning of the century this concept of construction had been central in a debate occurring within what may be called the *early modern tradition of geometrical problem solving.* The century between 1550 and 1650 was the time in which the classical Greek mathematics was taken up, understood, and elaborated. In particular, the early modern

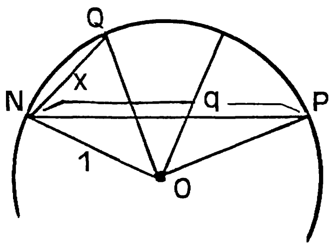

FIGURE 2

mathematicians took over the Greek interest in geometrical problems and their solution by construction.

In this practice they were confronted with two questions of method. The first was: *What means of construction should be used if problems cannot be constructed by ruler and compass*? Many problems (the classical ones as foremost cases) could not be constructed by ruler and compass. Obviously, they had to be solved, but by what means? More sophisticated instruments than ruler and compass? More complicated curves than straight lines and circles? Or should one adopt new postulates in addition to the Euclidean ones that are the basis of ruler and compass constructions? All these possibilities were considered and debated by early modern geometers.

The second methodological issue was *the search for analytic methods*. From the classical Greek geometrical works as they were known, about 1600 mathematicians inferred that the ancients had had a special method, called *analysis*, for finding proofs of theorems and constructions of problems, but that they had kept that method secret, or at least that works about the method had been lost. So the early modern geometers set themselves the task of recreating or creating such analytic methods.

Example 2. Trisection. Rather than discussing geometrical construction and the related methodological questions abstractly, I shall illustrate them by an example. It is taken from Descartes's *Géométrie* (1637) [**9**], and it concerns a classical problem, the trisection of the angle.

Let $\angle NOP$ (see Figure 2) be given, so that the chord $NP = q$ within the circle (radius 1) is known. It is required to construct $\angle NOQ = \frac{1}{3}\angle NOP$.

Descartes proceeded in two steps. He called x the chord NQ of the required angle, and he derived an equation for x. He found, by applying elementary Euclidean geometry:

$$x^3 - 3x + q = 0. \tag{2}$$

The second step was to geometrically construct a root x of the equation. Descartes gave the following

Construction [**9**, pp. 396–397]. With respect to perpendicular axes (see Figure 3) through O, draw a parabola with vertical axis, vertex in O, and passing through the point U, with coordinates of length 1. Take D on the vertical axis

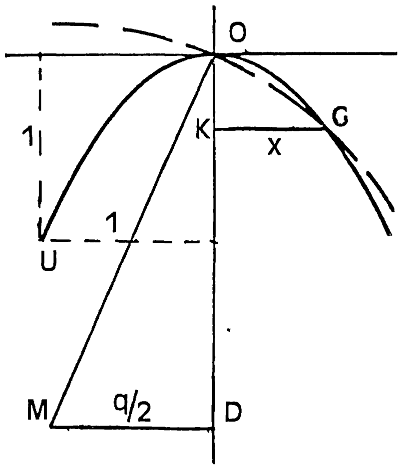

FIGURE 3

below O such that $OD = 2$. Take $DM = q/2$ horizontal to the left. Draw a circle with center M and radius MO; the circle intersects the parabola in O and in three other points, of which G is the one nearest to the vertex. Draw GK horizontally with K on the vertical axis. Then GK is the required root x; taking $NQ = GK$ in Figure 2 gives the required trisection. [The remaining roots occur as the ordinates of the other points of intersection of the circle and the parabola.] □

According to Descartes, this kind of construction was the canonical solution of an equation if it arose in a geometrical context. An algebraical solution (by a Cardano-type formula) would not be sufficient; the problem was geometrical and hence the solution had to be geometrical too. The example illustrates Descartes's particular answer to the methodological questions outlined above: Construction beyond ruler and compass was to be effectuated by the intersection of higher curves (here the parabola and the circle); the analytical method was algebra.

When is a problem solved? At this point the two examples enable me to state, somewhat slogan-like, the central theme of my research. It concerns the questions: When was a problem considered solved? When was an object considered known? In other words, what were the *criteria* for adequate solution and representation in seventeenth-century mathematics?

Such criteria evidently played a role in the mathematical practice of the period (as in fact in any period). They were not formalized, and they were controversial. Studying these criteria, the debates about them, and the changes they underwent often brings to light ways of mathematical thinking that were common and self-evident at the time but are very unfamiliar to us.

The criteria of adequacy have been little studied before by historians of mathematics. The reason for that neglect of an important part of seventeenth-century mathematics is that these criteria concern contemporary practice, whereas historical research has often concentrated on the origin of modern ideas. Also the criteria concern the mathematical *material*, the *objects* (like curves), and the *problems* (construction problems or inverse tangent problems), whereas historical research has tended to concentrate on the *theories* and the *methods* (analytic geometry, calculus) that were developed to deal with those objects and problems.

I have found a study of these criteria of adequacy very revealing and rewarding. In the remainder of this lecture I would like to mention some results of the investigations around the theme outlined above, and give some examples.

Descartes's *Géométrie*. Let me begin with Descartes's *Géométrie* of 1637 (cf. [7]). This was without doubt the most influential book in seventeenth-century mathematics; for one thing, it marked the beginning of analytic geometry. Through it, Descartes's particular choices (mentioned above) with respect to the methodological issues in geometry, his criteria of adequacy, became paradigmatic for mathematicians after him. These choices largely determined the structure of the book and the conception of geometry behind it, as for instance the restriction of geometry to algebraic relationships which Descartes advocated very strongly. His methodological choices explain in particular what may be called Descartes's program for geometry:

Given a geometrical problem, one calls x one of the line segments that have to be constructed. One then derives an equation

$$H(x) = 0 \tag{3}$$

for x, where H is a polynomial. Then, to determine x, the geometer's task is to find *acceptable, simple* curves $\mathcal{F}$ and $\mathcal{G}$, such that the roots of $H(x) = 0$ are equal to the ordinates of intersection points of $\mathcal{F}$ and $\mathcal{G}$. These curves are then the *constructing curves* by which the problem is solved.

In Descartes's view of geometry, these curves should be algebraic. So, if we write $F(x,y) = 0$ and $G(x,y) = 0$ for the equations of these curves, the requirements are that $H(x)$ is a factor of the resultant of F and G:

$$\mathrm{Res}(F,G) = A(x) \cdot H(x), \tag{4}$$

and that $\mathcal{F}$ and $\mathcal{G}$ are in some sense *acceptable* and *simple.* The procedure to find such F and G for given H was called the "construction of the equation."

Descartes treated the construction of equations in general for equations $H(x) = 0$ of degree 2–6. He showed that equations of degree 1 and 2 can be constructed by circles and straight lines, equations of degree 3 and 4 by the intersection of a conic and a circle (in fact, he showed that one fixed parabola is enough), and equations of degree 5 and 6 by the intersection of a circle and a special third-degree curve, the later so-called "Cartesian Parabola." Descartes did not proceed to higher degrees; he simply stated at the end of his book that it would be easy to go on. So he left a program for his successors: to work out a theory of constructing equations.

A forgotten theory. Around 1650, the *Construction of Equations* (cf. [**8**]) was generally considered a sensible subject, a natural and legitimate interpretation of the program of finding exact constructions for geometrical problems of any degree of complexity. The theory attracted considerable attention; many books and articles about it appeared and mathematicians of first rank contributed to it, such as Descartes, Fermat, Newton, l'Hôpital, Riccati, Cramer, Euler, Lagrange. Descartes's opinion that the constructing curves should be algebraic was generally (though not universally) accepted, but there was much debate on the requirement that the curves be "simplest possible." Should the equation be simple? Or the shape of the curve? Or the movement by which it can be traced? Descartes had given little guidance here; he had only stated, without further argument, that a curve is simpler in as much as its degree is lower.[1]

The debates about these questions show how mathematicians struggled to formulate and fix the motivation and the aims of the theory. They often felt strongly about it; witness the legislative, almost moralistic overtones in the debate. Some quotations may illustrate this. Here, for instance, is Fermat:

> Certainly it is an offense against the more pure geometry if one assumes too complicated curves of higher degrees for the solution of some problem, not taking the simpler and more proper ones; for it has often been declared already, both by Pappus and by more recent mathematicians, that it is a considerable error in geometry to solve a problem by means that are not proper to it [**10**, Vol. 1, p. 121].

And Newton:

> Yet it is not its equation but its description which produces a geometrical curve.... It is not the simplicity of its equation but the ease of its description which primarily indicates that a line is to be admitted into the construction of problems.... Either, then, we are, with the ancients, to exclude from geometry all lines except the straight line and circle and may be the conics, or we are to admit them all according to the simplicity of their description [**14**, Vol. 5, pp. 425–427].

Many similar statements occur in the literature on the construction of equations. They use remarkable metaphors: geometry is seen as a lawful territory

[1] Around 1700, mathematicians had come to the following consensus about the degrees of the "best possible" constructing curves for an equation: If the degree of H is n, then constructing curves $F(x,y)=0$ and $G(x,y)=0$ can be found with degrees that are integer approximations of $\sqrt{n}$. The consensus was based on experience. Newton and l'Hôpital gave proofs, but these were incorrect. Euler and others accepted the result without questioning the proof. In modern terms the question is this: Given $H[X]$, a polynomial of degree $n=k\cdot l$; are there polynomials $F[X,Y]$ and $G[X,Y]$ with degrees k and l, such that $H=\mathrm{Res}(F,G)$? It seems that this question is still open. I would be very thankful to any colleague who can give me more definite information about it.

that has to be protected and from which certain practices have to be excluded, or it is seen as a person, who can be offended and whose purity, one would almost say whose chastity, has to be defended. The issue was: to shape the proper rules of the subject and thereby to secure its status as a meaningful and sensible subject. The metaphors indicate that mathematicians felt strongly about it. Still, despite the strong words, the debates remained inconclusive; the questions about the aims of the field, and its proper procedures, could not be answered. After some time the debate died and so did the theory itself; after 1750 it quickly fell into oblivion.

The phenomenon of a theory that starts off as an evidently sensible enterprise and later dies amidst inconclusive discussions on its aims and motivations is a most interesting one. Why did the subject die? The answer turns out to be the following: The construction of equations originated as a sensible procedure within geometry. Purely algebraically, however, it does not make much sense. If a problem consists of a polynomial in one unknown, why should two polynomials in two unknowns constitute a solution? As the theory progressed, the techniques to find constructing curves became more and more algebraic. But the geometrical meaning of the subject—exact construction—and the geometrical criteria of adequacy—simplicity of the curves—refused translation into algebra in a natural way. The subject had a tendency to become algebraic, but its aims, criteria, and meaning proved untranslatable into algebra—it succumbed to this internal contradiction.

In the case of the construction of equations, we can follow in detail a process of development and decline of a mathematical field, a process whose causes were in the sphere of motivation, sense, and meaning. Such processes are little studied, although they are of evident interest for understanding the development of mathematics. The case also provides an informative example (or counterexample) with respect to theories about the historical development of scientific "research programs" as proposed by I. Lakatos and other recent philosophers of science.

Example 3. The *Elastica* and the *Paracentric Isochrone*. I now return to the representation of curves, about which methodological questions were raised remarkably similar to the ones discussed in connection with the construction of equations. Again, I can best illustrate these questions by an example. The example concerns two curves, the *Elastica* and the *Paracentric Isochrone*. In 1694 Jakob Bernoulli published in the *Acta Eruditorum* an article [**2**] about the form of elastic beams under tension. The beams (cf. Figure 4) are fixed vertically at the one end; a weight is attached such that the other end is bent horizontally. Bernoulli considered arbitrary relations between extension and force, but he devoted special attention to the case in which Hooke's law—extension proportional to force—applies. The *Elastica* is the form of the beam in that case.

Bernoulli derived the differential equation of the *Elastica*:

$$dy = \frac{x^2\,dx}{\sqrt{a^4 - x^4}}, \tag{5}$$

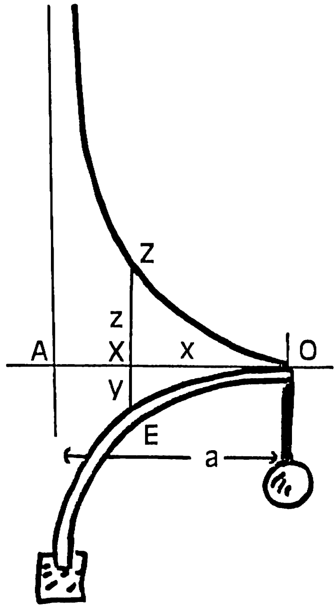

FIGURE 4

where a is the horizontal distance between the two ends of the beam. He represented the solution curve by means of the following

Construction. Take $OA = a$ along a horizontal X-axis (see Figure 4, positive values are taken to the left). Construct above the axis the curve with ordinates z satisfying the equation

$$z = \frac{ax^2}{\sqrt{a^4 - x^4}}. \tag{6}$$

[Bernoulli assumes his readers to be familiar with the construction of algebraic curves.] For any abscissa $OX = x$, determine y such that ay is equal to the area OXZ ($XZ = z(x)$). Take $XE = y$ vertically downwards. Then E is on the *Elastica.* More points on the curve are found by repeating the construction for other values of x. □

The construction is the geometrical equivalent of the analytical formula

$$ay = \int_0^x \frac{ax^2\,dx}{\sqrt{a^4 - x^4}}. \tag{7}$$

Bernoulli could have written the solution of (5) in such an analytical form; it is important to note that he did not do so, but chose to represent the curve by this geometrical construction. It is a so-called "construction by quadrature," assuming (without explanation) that it is possible to determine a rectangle (ay) equal to an area under a given curve. Construction by quadrature was a common way to represent transcendental curves in the seventeenth century, but it was not considered the most desirable kind of representation.

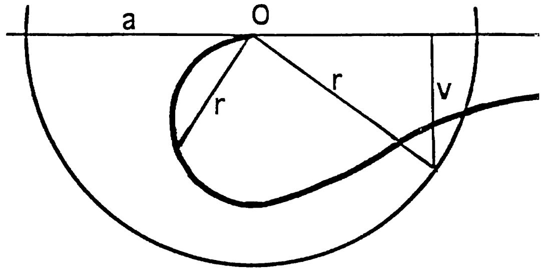

FIGURE 5

Bernoulli further calculated the differential of the arclength $s = OE$ of the *Elastica*:

$$ds = \frac{a^2\,dx}{\sqrt{a^4 - x^4}}. \tag{8}$$

This formula provided the link between the *Elastica* and the *Paracentric Isochrone.* The *Paracentric Isochrone* (see Figure 5) is the curve through a point O with the property that, in a vertical plane, a body moving under influence of gravity along the curve, recedes uniformly from O. That is, if $r(t)$ is the distance of the body to O and t the time, then $r(t)::t$.

Leibniz had challenged mathematicians to determine this curve. In an article [**3**] published together with the one on elastic beams, Bernoulli gave his solution. He derived the differential equation:

$$\frac{d(ar)}{2\sqrt{ar}} = \frac{a^2\,du}{\sqrt{a^4 - u^4}} \tag{9}$$

(with a depending on the initial velocity, r and v as in Figure 5, and $u^2 = av$). Bernoulli could now give a construction "by quadratures," the geometric equivalent of writing

$$\sqrt{ar} = \int_0^u \frac{a^2\,du}{\sqrt{a^4 - u^4}}. \tag{10}$$

Significantly, he did not do so. He recognized the right-hand differential in (9) as the arclength differential of the *Elastica*, and he concluded that this enabled him to give a construction "by rectification." It is as follows:

Construction. Assume an *Elastica RQO* given (see Figure 6). Draw a circle around 0 with radius $OB = a$. Take E arbitrary on OB and draw EQ vertically with Q on the *Elastica.* Take U on the circle such that $UV = OE^2/a$. Take W on OU such that $OW = (\text{arc } OQ)^2/a$. (Here it is assumed that the rectification of the *Elastica* can be performed.) Then W is on the *Paracentric Isochrone.* Repeat the construction for other points E to get arbitrarily many points on the required curve. □

The *best* representations. The remarkable thing about Bernoulli's construction is that according to him this was the *best* way of representing the solution of Leibniz's problem, better than the construction by quadratures which

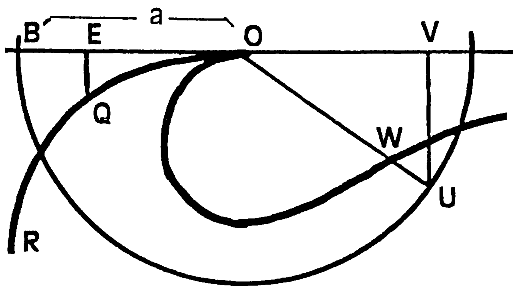

FIGURE 6

is implied in formulas (9)–(10). And this was not merely a curious idiosyncrasy of one mathematician. Shortly afterwards three further articles appeared, by Leibniz [**13**], Johann Bernoulli [**6**] (Jakob's brother), and Jakob himself [**4**], each containing reductions of the integral in (10) to an arclength of a curve. Indeed, while searching for a comparatively simple algebraic curve to reduce the integral, Jakob and Johann independently found the same curve. It was the *Lemniscate*, whose origin, therefore, lies in a preference for rectifications over quadratures in the representation of transcendental curves.

In the course of this exchange of solutions Jakob Bernoulli came to formulate explicitly [**4**, p. 608] his view on the proper representation of transcendental curves. He wrote that one should *at least* give a construction by quadrature of an algebraic curve. It was *better* to give a construction by rectification of an algebraic curve, or a "pointwise construction" (such as Leibniz's construction of the *Logarithmica*, see above). The *best* way to represent a curve, however, was a construction by curves "given in nature" (as the *Elastica* or, e.g., the *Catenary*). Bernoulli preferred rectifications over quadratures because, as he said, measuring length is easier than measuring area. He gave top preference to curves "given in nature" because if these can be found, all laborious construction of algebraic curves and their quadratures or rectifications could be avoided. These views of Jakob solicited several reactions, which I shall not further discuss; I use his statements here primarily to show that there was a debate and to illustrate its nature.

The debate shows striking similarities with the discussions about the construction of equations. In both cases analytical representation was seen as insufficient: a problem was considered solved only when a geometrical construction was given. The crucial point was the interpretation of "simplicity"; the constructing curves were considered better inasmuch as they are "simpler"; rectifications were preferred over quadratures because they were considered "simpler" to effectuate; construction by curves "given in nature" was advocated by Bernoulli because it provided "simpler," easier constructions. There were legislative overtones in both debates; Johann Bernoulli, for instance, uses terminology like "to sin against the laws of geometry" [**6**, p. 121]. And finally in both cases the debate remained inconclusive. With hindsight we can understand this; the relevant theories (equations, differential equations) became more and more analytical, but the concepts of geometrical simplicity could not be convincingly translated and formalized into analytical terms. The discussions were resolved by forgetting the problems.

Although these issues of construction and representation of curves were later forgotten, at the time they had a decisive influence on the development of mathematics. Analytic geometry originated in the context of geometrical construction by the intersection of curves. The first techniques for solving differential equations were elaborated with the aim of finding appropriate geometrical representations of the solution curves of inverse tangent problems. And, for instance, the early studies on elliptic integrals by Jakob Bernoulli, Fagnani, and others were a result of the effort to interpret integrals as arclengths.

Conclusion. I hope I have shown that the question of the *criteria of adequacy of representation and solution* provides an intriguing and fruitful way of looking at the mathematics of the seventeenth century. It provides new insights on three different levels.

On the *technical* level, an awareness of these issues leads to a better understanding of the terminology and the mental images of seventeenth-century mathematical practice. Curves were studied intensively in that period, but most of them (in particular the transcendental ones) could not be represented by equations. An understanding of the alternative ways of representation, of the reasons behind them, and of the mental images of mathematical objects which they presuppose is essential for understanding the texts of the period.

On the level of the *development* of mathematics, the approach helps in understanding certain directions and tendencies in seventeenth-century mathematical research, which would otherwise merely seem peculiar or superfluous, such as the interest in the geometrical construction of roots of equations or in representing integrals as arclengths.

Finally, on a more *general* level, a study of the criteria of adequacy is useful in understanding the processes of change in mathematics caused by the introduction of radically new methods (such as analysis in the seventeenth century) and the process of *habituation* to new ways of mathematical thinking. These processes operate both on the level of technique and on the level of motivation, meaning, and sense of the mathematical enterprise. They are not special to the seventeenth century, they belong to the mathematics of all times. They have received little attention until now; the research on which I am reporting may be of interest as an experiment in how these processes can be studied.

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MATHEMATICAL INSTITUTE, STATE UNIVERSITY OF UTRECHT, UTRECHT, THE NETHERLANDS