Introduction to security and cryptography Lecture 3: Hash functions, Message Authenticated Codes

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Hash functions

Definition

A hash function H takes as input a bit-string of any finite length and returns a corresponding *digest* of **fixed length**.

$$h : \{0, 1\}^* \rightarrow \{0, 1\}^n$$

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Example

$$h : \{0,1\}^* \to \{0,1\}$$
$$b_1 \dots b_n \mapsto b_1$$

Properties of Cryptographic Hash Functions (1/3)

Pre-image resistance
Given an output y, it is computationally infeasible to compute x
such that

$$h(x) = y$$

Second Pre-image resistance
Given an input x, it is computationally infeasible to compute x'
such that

$$h(x') = h(x)$$

Properties of Cryptographic Hash Functions (3/3)

Collision resistance
It is computationally infeasible to compute x and x' such that

$$h(x) = h(x')$$

Avalanche effect
If we flip one bit of a message x to get x', the hashes $h(x)$ and $h(x')$ will be completely different.

Ex:

- SHA3-256(1) = 67b176705b46206614219f47a05aee7ae6a3edbe850bbbe214c536b989aea4d2
- SHA3-256(2) = b1b1bd1ed240b1496c81ccf19ceccf2af6fd24fac10ae42023628abbe2687310

Note on hash functions

Hash functions are not used for encryption! They are used for *fingerprinting*: from an arbitrary long message or document, we always obtain a digest of small size (a few bits).

Goal of a hash function: two different messages will have different hashes, and from the hash one cannot retreive the original message.

Merkle-Damgård Construction (variant for SHA, MD5)

Let f be a compression fonction

$$f : \{0,1\}^{n+k} \to \{0,1\}^n$$

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Let f be a compression fonction

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1. Break the message x to hash in blocks of size k:

   $x = x_1 x_2 \ldots x_t$
2. Pad x_t with a specific padding[1]
3. Define x_{t+1} as the binary representation of the bit length of x
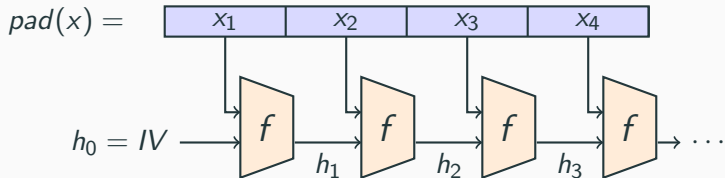4. Iterate over the blocks

$$
\begin{aligned}
h_0 &= 0^n \text{ or any IV} \\
h_i &= f(h_{i-1}||x_i) \\
h(x) &= h_{t+1}
\end{aligned}
$$

[1]Notably, the final size of the last block must be k

Merkle-Damgård Construction

$$pad(x) = \boxed{\begin{array}{c|c|c|c} x_1 & x_2 & x_3 & x_4 \end{array}}$$

$h_0 = IV \longrightarrow f \xrightarrow{h_1} f \xrightarrow{h_2} f \xrightarrow{h_3} f \rightarrow \cdots$

Merkle-Damgård Construction

Examples

- MD5 (1991)
- SHA-1 (1995)

Merkle-Damgård Construction

Theorem
If the compression function f is collision resistant, and if the padding is done correctly, then the obtained hash function h is collision resistant.

Merkle-Damgård weaknesses

- Second preimage attacks are more efficient than bruteforce
- Multicollisions (several texts that have the same hash) are not much harder than simple collisions
- Length extension attacks

MD5 is quite easy to attack. Attacks found in 1996. First collision in 2004. Nowadays, finding a collision takes less than a minute on a laptop. Some people even have fun:
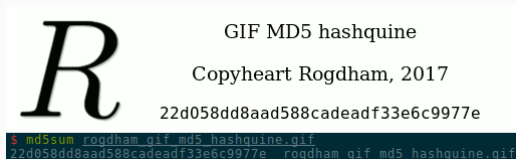
SHA-1 is getting there too. First attack discovered in 2005, first collision in 2017.

Call for a new hash

In 2007, NIST calls for a new family of hash functions (what would become SHA-3).

They should have original architecture, be flexible in the design.

51 participants, some were proven weak quickly, some where eliminated for complicated design etc.

The winner is Keccak, in 2012. It relies on the novel (2006-2007) sponge construction.

Sponge Construction

Let f be a permutation fonction or a transformation

$$f : \{0,1\}^n \rightarrow \{0,1\}^n$$

1. Break the message m to hash in blocks of size r ($r < n$)

$$m = m_0 m_1 \ldots m_t$$

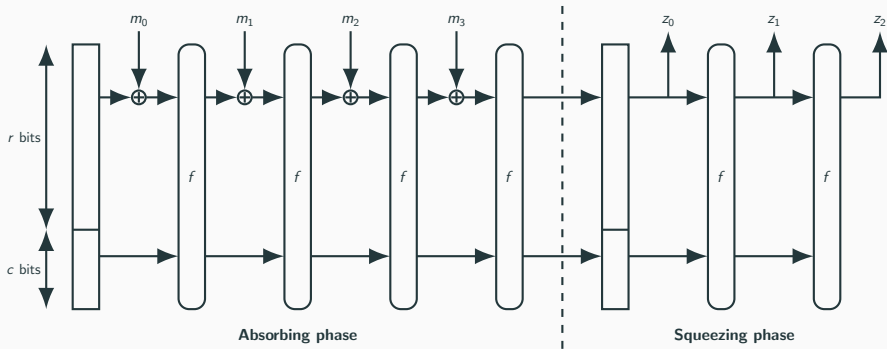2. Pad m_t with zeros as necessary
3. Iterate over the blocks

$$
\begin{aligned}
r_0 &= 0^r \\
c_0 &= 0^{n-r} \\
c_i \| r_i &= f(c_{i-1} \| (r_{i-1} \oplus m_{i-1})) \\
h(m) &= c_t \| r_t
\end{aligned}
$$

Sponge Construction

Example

- SHA-3

Use of Cryptographic Hash Functions

Compute a digest y from a given message m. The digest should be specific to the message.

Examples

- Use y in place of m in a trustworthy way.
- *"Did you get m correctly? Here's y to check."* (file-sharing)
- *"I sign y to prove that I wrote m."*

MAC

MAC (Message Authentication Code)

Idea
A MAC guarantees the integrity of the message and authenticates
the sender via a secret key. The MAC is sent along with the
message. The message is sent in cleartext (not encrypted).

Difference with Hash Functions
We need a *secret key* to verify the integrity of the message. This
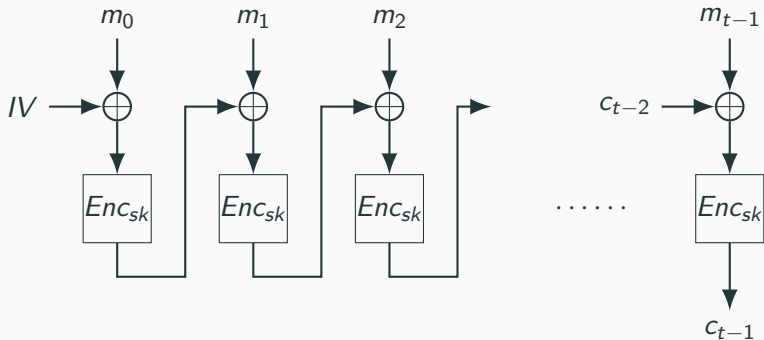guarantees the authenticity of the message.

MAC (Message Authentication Code)

$$\text{Alice } sk \longleftrightarrow \text{Bob } sk$$
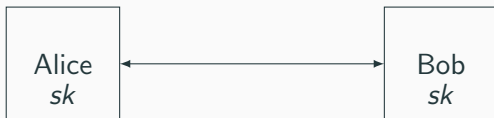
$$m$$
$$\text{MAC}_{sk}(m) = \text{mac} \qquad \text{mac} \stackrel{?}{=} \text{MAC}_{sk}(m)$$
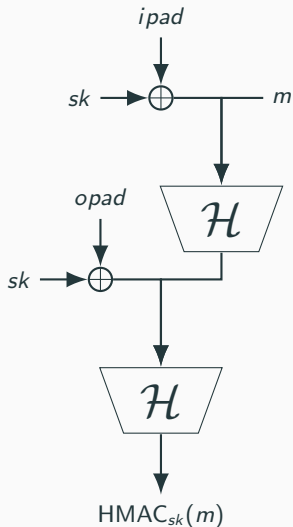
MAC Based on Block Cipher (CBC-MAC)

Then use bits 16 to 64 of c_{t-1} as CBC-MAC (according to NIST standards)

Not secure with variable-length messages.

MAC Based on Hash Function (HMAC)

With $ipad = 0x36363636\ldots36$ and $opad = 0x5c5c5c5c\ldots5c$. Formalized in RFC2014, used in IPSec, TLS...

We want both confidentiality, integrity and authenticity (this is called **authenticated encryption**).

Which one to chose?

- Encrypt-then-MAC
- MAC-then-encrypt (SSL/TLS)
- Encrypt-and-MAC (SSH)

Authenticated encryption

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Which one to chose?

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- Encrypt-and-MAC (SSH)

According to Bellare and Namprempre (2000), Encrypt-then-MAC has best security guarantees.

Authenticated encryption modes

Suppose we have a message m, a MAC function MAC, a secret key sk and a symmetric cipher, with encryption Enc.

- Encrypt-then-MAC: send $Enc_{sk}(m), MAC_{sk}(Enc_{sk}(m))$
- MAC-then-encrypt: send $Enc_{sk}(m, MAC_{sk}(m))$
- Encrypt-and-MAC: send $Enc_{sk}(m), MAC_{sk}(m)$

Things to remember

- Properties of a cryptographic hash function
- MD5, SHA-1, Keccak
- Basic concept of Merkle-Damgård, sponge construction
- MAC, HMAC
- Authenticated encryption, Encrypt-then-MAC

Exercises

1. What is the difference between Keccak and SHA-3 ?
2. What is the difference between hashing and encrypting?
3. Let us consider the hash function H which, for an input m, outputs the 3 last bits of m. Do you think it is secure? Why?
4. Is HMAC a symmetric or an asymmetric scheme?

Thank you for your attention

Any question?