Optimal Delayed Taxation in the Presence of Financial Frictions

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Abstract

In the presence of financial frictions, the timing of cash flows matters. We apply this insight to optimal income taxation by proposing a new policy: delayed taxation. Introducing a delay between the accrual and payment of income taxes provides two sources of welfare gains when some agents are borrowing constrained. First, it improves consumption smoothing by allowing constrained agents to borrow at a lower rate. Second, it reduces the present value tax rate from the perspective of constrained agents, thereby reducing the distortionary effects of income taxation. In a dynamic optimal tax model, we characterize the conditions under which marginally delayed taxation is welfare enhancing. We decompose the welfare gains and contrast them with those obtained by implementing age-dependent taxation or offering subsidized loans. We then characterize optimal delayed taxation in a simple calibrated model. This exercise reveals substantial welfare gains from delayed taxation. When limiting the amount the government can borrow to finance a given reform, delayed taxation significantly outperforms age-dependent taxation and offering subsidized loans. Finally, we empirically test the hypothesis that delayed taxation reduces income tax distortions in the context of young workers in Norway, where a kinked income-contingent student debt conversion scheme replicates an income tax with delayed payments. Bunching analyses reveal elasticities that are an order of magnitude lower than those we find for a regular income tax threshold. Consistent with our theory, proxies for being more constrained are associated with lower sensitivities to the de facto delayed tax, but not to the regular tax. Taken together, our results underscore the potential for delayed taxation to be a powerful new component of optimal tax policy.

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1 Introduction

Standard models of optimal taxation trade off welfare gains from redistribution against efficiency losses from distorting economic behavior. In incomplete markets, workers are typically unable to insure themselves against idiosyncratic productivity shocks. This implies that tax policy can generate welfare gains by redistributing resources within agents across states of the world, which is the focus of a large theoretical literature (Eaton and Rosen 1980, Varian 1980, Heathcote, Storesletten, and Violante 2017). However, few optimal tax frameworks consider redistribution within agents purely over time. This focus is without loss if capital markets allow agents to save and borrow freely. In practice, however, many agents face imperfect credit markets in which it is either very costly or impossible to borrow against future earnings. This consideration opens new avenues for welfare gains from tax policy. In particular, it implies that the *timing* of taxes matters (Heathcote, 2005). One way to alter the timing is through age-dependent taxation (Lozachmeur 2006, Blomquist and Micheletto 2008, Weinzierl 2011, Bastani et al. 2013, Heathcote et al. 2020). Younger workers are more likely to face binding credit constraints, and thus welfare gains can be achieved by taxing younger workers less.

Our paper proposes a new tax policy that more directly addresses credit market imperfections. *Delayed taxation* allows workers to delay the payment but not the accrual of their taxes. Introducing a wedge between the timing of tax payment and tax accrual produces welfare gains by both improving intertemporal consumption smoothing and reducing the distortionary effects of income taxation. To illustrate, consider a simple model with two groups of workers subject to an income tax. One group has flat or declining wage histories and thus wants to save. The other group has an upward wage trajectory and wants to borrow against future earnings. However, they are constrained in the sense that the interest rate on borrowing is higher than the interest rate on saving. The government allows workers to delay their tax payments at an interest rate equal to the rate on savings. In this environment, net savers are indifferent to delaying their tax payments. Constrained workers, however, strictly prefer to delay their taxes. Since the interest rate on the tax is lower than their marginal borrowing rate, delayed taxation implies a lower effective tax rate in present value terms. Thus, delayed taxation has two sources of welfare gains. The first source is improved intertemporal consumption smoothing among constrained workers, since delaying their taxes helps smooth their consumption intertemporally. The second source comes from reduced distortions. Constrained workers now choose their labor supply according to a lower effective tax rate, which increases the amount of tax revenue available for redistribution or the provision of public goods.

By shifting the timing of tax *payments*, we tie effective present value tax rates to an individual's marginal borrowing rate. This essentially allows the government to impose different tax rates on constrained and unconstrained workers without conditioning the tax schedule on taxpayer characteristics (i.e., "tagging"). This self-selection mechanism is an attractive feature relative to age-based taxation. Government-provided subsidized loans share this self-selection feature, but do not produce welfare gains by reducing income tax distortions.

Our study provides an in-depth examination of delayed taxation through a blend of theoretical analysis, numerical simulations, and empirical evidence. In the first part of our paper, we introduce delayed taxation into a dynamic optimal tax model with linear taxation. We consider heterogeneous workers who differ in their labor market productivity, which allows us to study the welfare gains from delayed taxation in the presence of distributional considerations. We show how the optimal delayed tax policy (i.e., the fraction of taxes that can be delayed) depends on standard behavioral elasticities of labor supply with respect to marginal tax rates and on the magnitude of financial frictions, measured by the difference between the interest rates faced by net borrowers and net savers. In other words, assessing the potential welfare gains from delayed taxation does not require any new types of behavioral elasticities.

To understand the welfare effects of delayed taxation, it is easiest to consider the case where an existing tax system is marginally complemented by delayed taxation. In this case, we provide a simple decomposition into four effects: (i) welfare gains from increased intertemporal consumption smoothing, (ii) a positive fiscal externality due to a positive substitution effect on the labor supply of young workers, which is (iii) partially offset by a negative intertemporal substitution effect on the tax revenue of older workers, and (iv) a negative income effect on the present value of tax revenue due to relaxed financial constraints.[1] If the government simply offered loans to financially constrained agents, the positive effects on labor supply would be absent. If the government were to engage in age-based taxation by offering tax breaks to young workers, this would affect all workers, not just those who are financially constrained. In addition, there would be a mechanical loss of tax revenue with age-based taxation.

In the second part of our paper, we provide numerical solutions by calibrating the optimal tax model to the Norwegian economy. The Norwegian register data, which cover the entire population, allow us to compute realized earnings trajectories for a large sample of young workers. We consider all workers between the ages of 20 and 30 in 1990, for whom we can compute a measure of effective wages in both 1990 and 2011. This allows for considerable variation in initial earnings and wage trajectories, and thus in the extent to which workers may wish to borrow against future earnings. We then consider the optimal tax schedule that the government would impose on these workers to maximize welfare in the presence of financial frictions. We model financial frictions as a wedge between the interest rate on borrowing and the interest rate on saving, which (as the wedge grows large) nests the standard "no-borrowing" constraint (see, e.g., Heathcote 2005).

We consider four different tax reforms: (i) introducing age-independent linear taxation, (ii) allowing age-dependent marginal tax rates, (iii) allowing both age-dependent marginal tax rates and lump-sum transfers, and (iv) delaying taxation. We also examine the welfare effects of allowing the government to lend directly to households at the government interest rate. In the benchmark age-independent tax system, the government imposes a high marginal tax rate in part to redistribute across workers and in part to provide lump-sum transfers that help smooth intertemporal consumption.

We find that all policies offer substantial welfare gains in the presence of financial frictions. We focus on the welfare implications of the different reforms when the borrowing rate is 10%

[1] With labor supply fixed, delaying a dollar of taxes increases the present-value wealth of constrained workers because of the high marginal interest rate they face.

and the interest rate on savings is 3%. In this environment, delayed taxation increases money-metric welfare by almost 9%, more than twice as much as the welfare gains from age-dependent taxation. Delayed taxation even rivals the welfare gains from a fully age-dependent tax and transfer system in which lump-sum transfers can also be age-dependent. This is a strong result in the sense that fully age-dependent taxes and transfers involve a larger set of policy parameters. The favorable targeting properties of delayed taxation imply that it can achieve higher welfare gains with fewer tax parameters. We further calculate that the MVPF (Hendren and Sprung-Keyser 2022, Finkelstein and Hendren 2020) of delayed taxation is 3. This implies that the money-metric welfare effects of marginally delaying taxation are three times higher than its fiscal cost. When the borrowing rate is lower (e.g., 5%), the MVPF is infinite, indicating that delayed taxation can be implemented as a Pareto-improving reform.

Both age-dependent taxation and delayed taxation are policies that help consumers smooth intertemporal consumption. Behind the scenes, the government must increase its own borrowing to achieve these welfare gains. In the case of delayed and age-dependent taxation, the government borrows to offset the effects of the delayed tax revenue. In practice, the government's ability to borrow may be limited by political (e.g., debt ceilings) or financial (e.g., credit ratings) constraints. Therefore, we investigate what welfare gains can be achieved when the government's ability to borrow is constrained. We first consider the case where the government cannot increase its borrowing at all. Interestingly, this still allows for meaningful welfare gains of about 0.5% of benchmark GDP under both age-dependent and delayed taxation. Relaxing the government's borrowing constraint increases the potential welfare gains, but more so for delayed taxation. If the government is allowed to increase borrowing by 10% of GDP, the welfare gains from age-dependent taxation are about 3.5% and the gains from delayed taxation are 4.2%.

Since delayed taxation can be structured as an income-contingent subsidized loan program, a natural point of comparison is a loan program in which the government offers loans at the government interest rate up to a uniform loan limit. When government borrowing is restricted, we find that this lending policy produces welfare gains of about 2.8% of GDP, which is only two-thirds that of delayed taxation. The smaller gains from government lending are consistent with our theoretical results. By offering (uniform) loans, the government does not take advantage of the opportunity to reduce income tax distortions.

In robustness tests, we verify that our main results hold when the benchmark is changed to a nonlinear progressive tax as in Heathcote et al. (2020). The welfare gains from delayed taxation are in fact larger when tax progressivity is more flexible. We further verify that delayed taxation yields substantial welfare gains (exceeding those of age-dependent taxation) when our model is calibrated to the US PSID data used by Weinzierl (2011). Our qualitative results are also robust to varying both the extent of financial frictions faced by workers and the constraints on government borrowing.

In the third and final part of the paper, we empirically test whether delayed taxation actually reduces income tax distortions when taxpayers are financially constrained. Conducting such a test is challenging because few settings allow for substantial variation in the timing of tax payments.

Taxes are typically paid either immediately (through withholding) or one year later when tax returns are due. We overcome this challenge by studying the effects of a student debt conversion scheme in Norway, which constitutes a *de-facto* delayed tax.

Norwegian students receive an annual loan of approximately $13,000 from the government to pay for consumer expenses. If students remain in good standing, half of the loan payment is usually forgiven at the end of the year. However, if the student earns above a threshold of about $17,000, each additional dollar of earnings reduces the amount forgiven by 50 cents. In other words, additional earnings increase the student's debt. Thus, this program creates a large jump in the effective marginal tax rate, where marginal taxes can be financed with the same generous terms as subsidized student loans. This quasi-experimental setting is well suited to examine how financial frictions can make delayed taxation less distortionary. First, students are almost by definition highly constrained. Only a few years later, they face significantly higher incomes against which it is difficult to borrow. The large increase in the effective tax rate at the earnings threshold is also more than significant enough for any student to be aware of: At the threshold, the marginal after-tax wage falls from 75 cents to 25 cents once we account for the increase in student debt. Despite this drastic reduction in the marginal (effective) wage, students are astoundingly irresponsive. While there is clear visual evidence of bunching, indicating that students do respond, these responses pale in comparison to the effective after-tax wage reduction that occurs. Our bunching analysis yields an implied elasticity of labor earnings to after-tax wages of only 0.016.

To shed light on the observed non-bunching behavior at the delayed tax threshold, we examine how students' characteristics covary with their position relative to the debt conversion threshold. These analyses suggest that non-bunchers (and their parents) have significantly lower liquid assets, but not lower future earnings. This is exactly what we would expect to see if irresponsiveness to the threshold is driven by financially constrained agents. We also find no evidence that the educational attainment of students' parents changes in a manner consistent with these characteristics driving differences in bunching behavior. Building on these analyses, we examine heterogeneity in bunching by the ex ante financial situation of students and their parents. Students with liquidity below the median (and their parents as well) have an implied labor earnings elasticity less than half as large as those above the median. In our modeling framework, this heterogeneity can be rationalized by the fact that less liquid students optimize to a 10 percentage point higher marginal borrowing rate.

We also examine the bunching behavior of students at a regular tax threshold. This allows us to compare implied labor supply elasticities under a de facto delayed tax and a regular tax scheme, but in a similar sample of individuals.[2] The regular tax threshold occurs around $6,000, where the marginal income tax rate jumps from 0 to 25 percent. Using the same techniques as before, we estimate an implied labor supply elasticity of 0.13. This is about eight times larger than the

[2]Ideally, this will control for unobservable factors that influence labor supply optimization. An alternative would be to compare our elasticity under delayed taxation with elasticities from other research. However, this raises the concern that differences in labor market or financial frictions, or differences in structural elasticities, are driving the differences in elasticities.

elasticity implied by the delayed tax threshold. Under some simplifying assumptions about the exact elasticities measured by our bunching framework, these large elasticity differences can be rationalized by the fact that students face an average marginal borrowing rate of more than 20 percent and thus are considerably less responsive to the de facto delayed tax scheme created by the student loan program. We argue that it is unlikely that this difference can be explained by the fact that the kink occurs at different income levels. Using a regression-based approach that controls for differences in observables such as occupation codes and age, we find a qualitatively similar difference in elasticity.

Our theoretical framework implies that the sensitivity to delayed but not regular taxation is moderated by the severity of borrowing constraints. This is strongly supported by the data. Those who do not respond to the delayed tax incentive (i.e., non-bunchers) are more likely to borrow unsecured and have less liquid assets. We do not find this pattern in the regular tax sample.

The main empirical finding that delaying the payment of a tax reduces distortions is not surprising. In the absence of strong debt aversion and in the presence of borrowing-constrained agents, this is what we would expect from economic theory. Rather, the contribution is to test the applicability of life-cycle model reasoning in modeling the labor supply decisions of constrained workers, which is necessary to assess the potential of delayed taxation as a new policy tool.

Literature. Our paper relates to the optimal tax literature that allows tax rates to depend on taxpayer characteristics, i.e., tagging (Akerlof, 1978). Delayed taxation causes financially constrained taxpayers to see a reduction in effective (present value) tax rates. As taxpayers age and borrowing constraints are no longer binding, effective tax rates equal the higher nominal rate. In this sense, delayed taxation has a strong element of age-dependent taxation (Lozachmeur 2006; Weinzierl 2011;Gervais 2012; Bastani, Blomquist, and Micheletto 2013; Heathcote, Storesletten, and Violante 2020). The key differences are that (i) delayed taxation does not necessarily require the government to condition tax rates on taxpayer characteristics (which is likely to be controversial), and (ii) it does not rely on using age as a proxy for liquidity constraints. Instead, delayed taxation allows constrained borrowers to self-select into the system, thereby introducing a voluntary type of history dependence.

More broadly, this paper contributes to the literature on dynamic optimal taxation (see, for example, Ndiaye 2020, Yu 2021, and the surveys in Golosov and Tsyvinski 2015 and Stantcheva 2020). Most closely related are studies that consider changing the timing of tax payments (e.g., Lockwood 2020) or incorporating financial frictions (e.g., Andreoni 1992, Dávila and Hébert 2019). The central contribution of this paper is to propose and study the welfare implications of the simple idea that —in the presence of credit market imperfections—changing the timing of income tax payments can offer substantial welfare gains, in large part by reducing the distortionary effects of income taxation. We strengthen this contribution by providing quasi-experimental evidence that delayed taxation does indeed reduce income tax distortions. To our knowledge, this idea has not been explored before, either theoretically or empirically.

On the empirical front, this paper contributes to the growing literature studying bunching at tax thresholds (see, e.g., Saez 2010; Bastani and Selin 2014; Seim 2017; Søgaard 2019; and the review by Kleven 2016), loan term thresholds (see, e.g., Bachas, Kim, and Yannelis 2021; Bäckman, van Santen et al. 2020; DeFusco and Paciorek 2017; DeFusco, Johnson, and Mondragon 2020; de Silva 2023; Le Barbanchon 2020; and Best, Cloyne, Ilzetzki, and Kleven 2018). Our contribution is to study bunching at a threshold where the *payment* of marginally accrued taxes is substantially delayed. This adds an intertemporal dimension to bunching behavior that is not present in studies that consider sensitivity to taxation. This paper also relates to the emerging literature on the effects of debt on labor supply (see, e.g., Zator 2019; Bruze, Hilsløv, and Maibom 2024; Bernstein 2021; Doornik et al. 2021; Brown and Matsa 2020; Donaldson et al. 2019). We also contribute to research that considers how different tax instruments may affect behavioral elasticities. For example, Kostøl and Myhre (2020) consider how labor supply elasticities are affected by providing more information on kinks and notches, and for the price elasticity of giving, Fack and Landais (2016) consider the effect of changing documentation requirements, and Ring and Thoresen (2021) consider the effect of wealth taxation.

This paper is organized as follows. Section 2 studies delayed taxation in a dynamic optimal tax framework. Section 3 provides numerical solutions to the optimal delayed tax problem. Section 4 discusses whether there are existing tax regimes that are similar to delayed taxation. Section 5 uses a de facto delayed tax system in Norway to test some of the behavioral implications of our theoretical framework. Section 6 briefly discusses implementation issues and unmodeled trade-offs associated with the introduction of delayed taxation.

2 The welfare gains of delayed taxation

We consider an economy consisting of heterogeneous agents who live and work for two periods and differ in their exogenous lifetime labor productivity profiles (w_1^i, w_2^i), where w_t^i denotes the market productivity of agent i in period $t = 1, 2$. In each period, an agent earns an income of $y_t^i = w_t^i \ell_t^i$, where ℓ_t^i is the labor supply. For tractability reasons, we focus on a dynamic extension of the linear (progressive) taxation framework (Sheshinski, 1972). This setting allows us to study the welfare gains from delayed taxation in the presence of distributional considerations. The structure of the tax and transfer schedules are given by

$$T_1(y_1) = -G_1 + \delta \tau_1 y_1,$$
$$T_2(y_1, y_2) = -G_2 + \tau_2 y_2 + (1 + r)(1 - \delta)\tau_1 y_1,$$

where $\tau_t \geq 0$ denotes the nominal (statutory) marginal tax rate in period t and $G_t \geq 0$ denotes the lump-sum transfer.[3] $\delta \in [0, 1]$ is the fraction of taxes accrued in period 2 that must be paid in period 1 while the fraction $1 - \delta$ must be paid in period 2. The government charges an interest on the delayed tax payment equal to r.

Agents can save between periods, and the saving technology of private agents is given by the

[3]We realistically rule out lump-sum taxes. On the infeasibility of lump-sum taxes, see Smith (1991) for a discussion of Margaret Thatcher's disastrous attempt to introduce a poll tax in the United Kingdom between 1989 and 1990.

function $R(s)$, which is the amount by which disposable income in the second period is increased if the individual saves (or borrows) the amount s.[4]

Consumption c_t^i is given by the per-period budget constraints,

$$c_1^i = w_1^i \ell_1 [1 - \delta \tau_1] + x_1^i + G_1 - s^i, \tag{1}$$
$$c_2^i = w_2^i \ell_2 [1 - \tau_2] + x_2^i + G_2 - (1 + r)[1 - \delta] \tau_1 w_1^i \ell_1^i + R(s^i), \tag{2}$$

where x_t^i is exogenous income. Individual preferences are represented by the utility function:

$$u_1(c_1^i) - v(\ell_1^i) + \beta [u_2(c_2^i) - v(\ell_2^i)], \tag{3}$$

where u is increasing, twice differentiable and strictly concave, and v is increasing, twice differentiable, and strictly convex.

The individual's problem. The problem solved by an individual i with a lifetime wage profile of (w_1^i, w_2^i) is to choose ℓ_1^i, ℓ_2^i, s^i in order to maximize (3) subject to constraints (1) and (2). The first-order conditions are:

$$(\ell_1) : u_1'(c_1^i) w_1^i [1 - \delta \tau_1] - \beta (1 + r) u_2'(c_2^i) [1 - \delta] \tau_1 w_1^i = v'(\ell_1^i), \tag{4}$$
$$(\ell_2) : u_2'(c_2^i) w_2^i [1 - \tau_2] = v'(\ell_2), \tag{5}$$
$$(s) : u_1'(c_1^i) = \beta u_2'(c_2^i) R'(s^i). \tag{6}$$

The first order condition for ℓ_1 (equation 4) can be written:

$$u_1'(c_1^i) w_1^i \left(1 - \tau_1 \Big[\delta + [1 - \delta] \theta \Big]\right) = v'(\ell_1^i) \quad \text{where} \quad \theta^i = \frac{\beta (1 + r) u'(c_2^i)}{u'(c_1^i)}. \tag{7}$$

Thus, the labor supply in period 1 depends on the extent to which taxes are delayed (as reflected by δ) and the wedge θ between the marginal utility of consumption in period 1 and the discounted marginal utility of consumption in period 2. When agents are free to save and borrow at the government interest rate of r, the Euler equation implies that $\theta^i = 1$, which makes the first-order condition independent of δ. However, in the presence of financial frictions, we have that $\theta^i < 1$, which implies that when $0 < \delta < 1$, the effective marginal tax rate faced by agents is lower than in an economy without delayed taxation.

For the second period, the first-order condition for ℓ_2 (equation 5) is the same as in a standard model without delayed taxation. Second-period labor supply is only affected indirectly through c_2.

As can be seen from the simplified FOC for ℓ_1 (7), the effects of delayed taxation depend on the degree of consumption smoothing. Assuming an interior solution for s^i such that the Euler

[4] We exclude taxes on capital income. Including a proportional tax on capital income would not affect the qualitative nature of our results.

equation (6) holds, we may substitute (6) into (7) to obtain

$$u_1'(c_1^i)w_1^i \left(1 - \underbrace{\tau_1 \left[1-(1-\delta)\Delta_r^i\right]}_{\text{Effective Period-1 Tax Rate}} \right) = v'(\ell_1^i). \tag{8}$$

where

$$\Delta_r^i = 1 - \frac{1+r}{R'(s^i)} \tag{9}$$

is the interest wedge faced by net borrowers. Equation (8) illustrates that delaying taxes (increasing $1-\delta$) reduces the distortion on period 1 labor supply at a rate determined by the difference in interest rates between government and private agents, Δ_r^i, which captures the "strength" of the financial frictions faced by agent i. Note that this is a testable implication that we turn to in Section 5.[5]

We define the effective tax rate and after-tax wages as

$$\tilde{\tau}_1^i = \tau_1 \left[\delta + [1-\delta]\frac{1+r}{R'(s)}\right] = \tau_1 \left[1-(1-\delta)\Delta_r^i\right], \tag{10}$$

$$\tilde{w}_1^i = w_1^i \left(1-\tilde{\tau}_1^i\right) \quad \text{and} \quad \tilde{w}_2^i = w_2^i(1-\tau_2), \tag{11}$$

where (10) defines the effective tax rate for period 1, and (11) defines the "net" wage rate relevant to the labor supply decision in periods 1 and 2, respectively. The definition of the effective tax rate emphasizes the fact that the effective tax rate is equal to the nominal tax rate if there is no delayed taxation ($1-\delta=0$) or no financial frictions ($\Delta_r^i = 0$).

Note that period-1 and period-2 labor supply can be related by substituting (6) into (8) and then substituting in (5):

$$\beta R'(s^i)\frac{\tilde{w}_1^i}{\tilde{w}_2^i} = \frac{v'(\ell_1^i)}{v'(\ell_2^i)}. \tag{12}$$

Intuitively, borrowing constrained individuals smooth their consumption by increasing their labor supply in period 1, making it less elastic to taxes.

The extent of financial frictions is determined by the savings technology, $R(s)$. In some parts of our analysis, we consider a piece-wise linear saving technology

$$R(s) = \begin{cases} (1+r)s & \text{if } s \geq 0, \\ (1+r_b)s & \text{if } s < 0, \end{cases} \tag{13}$$

where $r_b - r > 0$ reflects the credit penalty faced by financially constrained agents. An interior solution $s<0$ is optimal if $r_b - r$ is sufficiently close to zero and the wage profile is sufficiently

[5]Firstly, (compensated) behavioral responses to a delayed tax are smaller than to a regular tax when agents face financial frictions. Secondly, behavioral responses to a delayed tax are mitigated by the severity of an individuals' borrowing constraints.

steep. Note that (13) can be seen as a generalization of the general "no-borrowing constraint" in macro (obtained when $r_b \to \infty$).

The government's problem. Assuming that the government is utilitarian (for notational simplicity, all our results generalize to the case where the government weights the welfare of different individuals differently), and denoting by π^i the proportion of agents of type i in the population, the government's problem is:

$$\max_{G_1, G_2, \tau_1, \tau_2, \delta} \sum_i \pi^i V^i, \tag{14}$$

subject to:

$$\sum_i \pi^i \left(\tau_1 w_1^i \ell_1^i + \frac{\tau_2 w_2^i \ell_2^i}{1+r} \right) \geq G_1 + \frac{G_2}{1+r} + M, \tag{15}$$

$$0 \leq \delta \leq 1, \tag{16}$$

where M is an exogenous revenue requirement that is not refunded to agents. Note that δ does not enter (15) because the government is indifferent between receiving tax revenue in period 1 or period 2. This is because the government charges an interest rate of r on delayed taxes, which is the same interest rate the government faces (see discussion of Assumption 1 below).

Before proceeding, two comments are in order regarding the government optimization problem. First, dynamic models of optimal taxation typically assume that the budget constraint is satisfied in expectation, as we have formulated our government budget constraint (15). This implicitly assumes that the government is free to borrow if, for example, it runs a deficit in period 1. In reality, however, governments often face constraints on the amount of borrowing they can undertake. In our numerical simulations, we go beyond the standard model by shedding light on the consequences of imposing constraints on the amount of such implicit borrowing (see section 3.4).

Second, both age-dependent and delayed taxation have in common that they allow significant redistribution from period 2 to period 1 (achieved through lower tax payments in period 1). This creates incentives to move abroad or to engage in other forms of tax evasion. Assumption 1 makes it clear that we assume perfect enforcement of both age-dependent taxation and delayed taxation.

Assumption 1 (No Default Assumption) *Workers cannot default on taxes or government loans, nor can they evade taxes or move abroad. In other words, there's perfect enforcement in all dimensions.*

The purpose of this assumption is to construct a simple benchmark economy in which we can compare welfare effects of different tax policies. For delayed taxation, the implication is that δ does not enter the government budget constraint. For age-dependent taxation, the implication is that age-dependent taxation (with high second-period tax rates) does not cause tax evasion or outmigration in the second period. One question is why workers face high interest rates in private lending markets at the same time as the government views lending to workers as risk free. There are several possible foundations for this. For example, the government may have superior collection technology, for example through income-tax withholding or withholding government transfers. A case in point is the United States, where the interest rate on delinquent taxes is set at the federal short-term rate plus 3 percent. This rate is significantly lower than interest rates on unsecured consumer credit. In addition, several of our simulations have the feature that second-period transfers exceed the amount of delayed taxes for all workers, implying that delayed taxes may simply be withheld from government transfers. Another factor is that private credit markets may be monopolistic, causing private lenders to set actuarially high interest rates.

Central to our discussion is the introduction of delayed taxation as an innovative policy instrument that may be more politically feasible than age-dependent taxation. We argue that concerns about default apply equally to both delayed and age-dependent taxation, which is why our theoretical and quantitative analyses provide a horse-race between these policies in addition to a subsidized lending scheme.

Assumption 2 (Private Credit Markets) *The interest rate on borrowing is exogenous and any profits from lending are irrelevant for welfare.*

Similar to Lozachmeur (2006) and Weinzierl (2011), we assume exogenous financial frictions and interest rates, and that financial intermediaries are irrelevant for welfare.[6] These assumptions keep the analyses tractable and focused on the key mechanisms. One may think of this as there being a monopolist foreign lender. Any profits they make are not taxed and thus whether their profits decrease due to delayed or age-dependent taxation reducing private borrowing is irrelevant for welfare.

Characterizing optimal tax policy. Now we want to characterize the optimal solution to the government's problem. For this purpose, let λ denote the Lagrange multiplier associated with the government budget constraint (15). The Lagrangian is

$$W = \sum_i \pi^i V^i - \lambda \left(- \sum_i \pi^i \left(\tau_1 w_1^i \ell_1^i + \frac{\tau_2 w_2^i \ell_2^i}{1+r} \right) + G_1 + \frac{G_2}{1+r} + M \right).$$

We first characterize the optimal age-independent tax system, given by the solution to the

[6] Weinzierl (2011) considers two main cases. In the first case, there is (exogenously) neither borrowing nor saving. In the second case, individuals may save or borrow at an (exogenous) interest rate of 5%. Lozachmeur (2006) imposes a no-borrowing constraint but lets individuals save at an interest rate of zero.

above optimization problem, assuming $\tau_1 = \tau_2$, $G_1 = G_2$. For this purpose, we define:

$$g_1^i = \frac{u_1'(\cdot)}{\lambda}, \tag{17}$$

$$g_2^i = \beta(1+r)\frac{u_2'(\cdot)}{\lambda}, \tag{18}$$

$$\varepsilon_{ts}^i = \frac{1-\tau_s}{y_t^i}\frac{dy_t^i}{d(1-\tau_s)}, \tag{19}$$

$$\varepsilon_t^i = \frac{1-\tau}{y_t^i}\frac{dy_t^i}{d(1-\tau)}, \tag{20}$$

$$\rho^i = \frac{d}{dG_1}\left(\tau_1 y_1^i + \frac{\tau_2 y_2^i}{1+r}\right) \le 0, \tag{21}$$

$$\eta^i = \frac{d}{dG}\left(\tau_1 y_1^i + \frac{\tau_2 y_2^i}{1+r}\right) \le 0, \tag{22}$$

where (17)–(18) defines the social value of giving an additional dollar to an agent of type i in period $s = 1, 2$ (in money metric terms) and (19) is the elasticity of period t income with respect to the period s net-of-tax rate. This elasticity captures labor supply adjustments to within-period tax changes as well as across-period tax changes (intertemporal labor substitution effects). Equation (20) is the elasticity of period t income with respect to a change in $1 - \tau$ (a change in the net-of-tax rate in both periods). Equations (21) and (22) define income effect parameters that represent the reduction in present value taxes caused by an increase in lump-sum transfers or unearned income in period 1 and both periods, respectively.

Proposition 1 (Benchmark Linear Tax Scheme) *Consider the government's optimization program (14) with $\tau_1 = \tau_2 = \tau$ and $G_1 = G_2 = G$, and some fixed value of $1 - \delta$.*

(i) The optimal marginal tax rate τ satisfies

$$\sum_i \pi_i y_1^i \left[\delta g_1^i + (1-\delta) g_2^i\right] + \frac{1}{1+r}\sum_i \pi_i y_2^i \left[g_2^i\right] \tag{23}$$

$$= \sum_i \pi_i y_1^i \left[1 + \frac{\tau}{1-\tau}\varepsilon_1^i\right] + \frac{1}{1+r}\sum_i \pi_i y_2^i \left[1 + \frac{\tau}{1-\tau}\varepsilon_2^i\right]. \tag{24}$$

(ii) The optimal per-period transfer G satisfies

$$\sum_i \pi^i \left(g_1^i + \frac{1}{1+r} g_2^i\right) = \sum_i \pi^i \left(1 + \frac{1}{1+r} - \eta^i\right). \tag{25}$$

Proof. See Appendix A.1. ■

The formulations in equations (23) and (25) reflect a dynamic extension of the seminal work on optimal taxation by Atkinson and Stiglitz (1980), among others. The determination of the optimal linear (progressive) tax rate involves balancing equity, as reflected on the left-hand side of the equation (23), with efficiency, as reflected on the right-hand side of the same equation. Similarly, the optimal level of the transfer depends on a balance between equity, shown on the LHS of equation (25), and costs, shown on the RHS of equation (25). These costs are the direct costs of providing the transfer, adjusted for the resulting negative impact on the tax base due to income effects.[7]

We then turn to an age-dependent tax system, allowing $\tau_1 \neq \tau_2$, and an age-dependent transfer system, allowing $G_1 \neq G_2$, while considering a fixed amount of delayed taxation $1 - \delta$.

Proposition 2 (Optimal Age-Dependent Taxation) *Consider the optimization program (14), with some fixed value of $1-\delta$.*

(i) The optimal marginal tax rates (τ_1, τ_2) satisfy:

$$\sum_i \pi_i y_1^i \left[\delta g_1^i + [1-\delta] g_2^i\right] = \sum_i \pi_i y_1^i \left[1 - \frac{\tau_1}{1-\tau_1}\left(\varepsilon_{11}^i + \frac{1}{1+r}\frac{\tau_2}{\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{21}^i\right)\right], \tag{26}$$

$$\sum_i \pi_i y_2^i \left[g_2^i\right] = \sum_i \pi_i y_2^i \left[1 - \frac{\tau_2}{1-\tau_2}\left(\varepsilon_{22}^i + (1+r)\frac{\tau_1}{\tau_2}\frac{y_1^i}{y_2^i}\varepsilon_{12}^i\right)\right]. \tag{27}$$

(i) The optimal transfers G_1 and G_2 satisfy:

$$\sum_i \pi^i g_1^i = \sum_i \pi^i \left[1 - \rho^i\right], \tag{28}$$

$$\sum_i \pi^i g_2^i = \sum_i \pi^i \left[1 - \rho^i \frac{1+r}{R'(s^i)}\right]. \tag{29}$$

Proof. See Appendix A.2. ■

The weighted average of the social weights on the LHS of (26) reflects that the burden of a marginal increase in τ_1 is borne partly in period 1 and partly in period 2 when $1-\delta > 0$. If there is either no delayed taxation ($\delta = 1$) or no agents are financially constrained ($R'(s^i) = 1+r$ for all i), then the LHS simplifies to $\sum_i \pi_i y_1^i g_1^i$.[8] The RHS of (26) and (27) reflect that a marginal change in a tax rate in one period affects labor supply in *both* periods. The intertemporal substitution of labor supply in response to the age-dependent tax rates τ_1 and τ_2 is an important feature of our framework.

Equations (28) and (29) require that G_t, for $t = 1, 2$, are set so that the average social value of giving everyone an additional dollar in period t ($\sum_i \pi^i g_t^i$) is exactly equal to the resource cost of an additional dollar ($\sum_i \pi^i = 1$) minus the loss of tax revenue due to fiscal externalities (individuals reduce their labor supply when transfers are increased). Note again that $g_2^i = \frac{1+r}{R'(s^i)} g_1^i$. If $R'(s) = 1+r$ when $s > 0$ and $R'(s) > 1+r$ when $s < 0$, then if at least one agent borrows, we have $\sum_i \pi^i g_1^i > \sum_i \pi g_2^i$, working towards $G_1 > G_2$. Note that since $\frac{1+r}{R'(s^i)} \leq 1$, the negative income effects on tax revenue are generally less severe for period 2 labor supply than they are for period 1 labor supply. This is because financially constrained agents discount future cash flows at a higher rate.

We now turn to the optimal amount of delayed taxation, $1-\delta$. To set the stage for our

[7] In Marginal Value of Public Funds (MVPF) language, the optimality conditions can be stated as $MVPF_\tau = MVPF_G = 1$.

[8] Note that $g_2^i = \frac{1+r}{R'(s^i)} g_1^i$ by virtue of (6), thus the value of δ is irrelevant if $R'(s^i) = 1+r$ for all i.

next proposition, we derive a lemma showing that *marginally* increasing $1-\delta$ has effects on ℓ_t, $t=1,2$, that are proportional to the effect of marginally changing $1-\tau_1$. This result allows us to characterize optimal delayed taxation in terms of standard labor supply elasticities and relate it to age-dependent taxation.

Lemma 1 *When the marginal interest rate, $R'(s^i)$, is well defined and $R''(s^i)=0$ for all i,*[9] *then*

$$\frac{d\ell_1^i}{d(1-\delta)} = \tau_1 \left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right] \frac{d\ell_1^i}{d(1-\tau_1)}, \tag{30}$$

and

$$\frac{d\ell_2^i}{d(1-\delta)} = \tau_1 \left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right] \frac{d\ell_2^i}{d(1-\tau_1)}. \tag{31}$$

Proof. See Appendix A.3. ■

Proposition 3 characterizes the optimal amount of delayed taxation $1-\delta$. To better convey the effects of delayed taxation, we express this proposition in terms of compensated tax elasticities (indicated by the superscript c).

Proposition 3 (Optimal Delayed Taxation) *Consider the optimization program (14), given some fixed values of τ_1, τ_2, G_1, G_2 (not necessarily optimal or age-dependent). Assuming an interior solution for δ, the optimal share of delayed taxation, $1-\delta$, satisfies:*

$$\tau_1 \cdot \sum_i \pi_i y_1^i \left(g_1^i - g_2^i\right) = -\sum_i \pi_i y_1^i \left[\tau_1 \left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right] \left(\frac{\tau_1}{1-\tau_1}\varepsilon_{11}^{i,c} + \frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{21}^{i,c} + \rho^i\right)\right], \tag{32}$$

Proof. See Appendix A.4. ■

The LHS is the welfare effect of increased consumption smoothing, and the RHS captures the fiscal externalities of marginally delayed taxation. If there were no financial frictions, that is, $\Delta_r^i = 0$ for all i , then both the left and right sides of (32) would equal zero. In other words, if no one is financially constrained, it does not matter whether the government delays taxation. There are no consumption-smoothing benefits, since agents can already borrow freely at the government rate, and on the right-hand side there are no fiscal externalities, since the present value of the delayed tax from the perspective of an unconstrained agent is equal to the nominal tax. However, in the presence of at least one borrowing-constrained agent, that is, an agent who borrows at some $R'(s) = 1 + r_b > 1 + r$, the left-hand side of (32) is positive and equal to the marginal welfare gains from improved consumption smoothing. The right-hand side is also nonzero due to fiscal externalities caused by constrained agents who now face a lower effective tax rate in period 1.

[9] If $R(s)$ is piecewise linear with a higher interest rate when $s < 0$, the assumption is satisfied by $s^i \neq 0$ for all i.

2.1 Welfare effects of marginal reforms

An interesting question is under what conditions the introduction of delayed taxation increases welfare in an economy without delayed taxation. We start with our benchmark economy, which is characterized by a tax system with $\tau_1 = \tau_2 = \tau$ and no delayed taxation $1 - \delta = 0$. We then consider a marginal delay in taxation (i.e., a marginal increase in $1 - \delta$). Note that this reform has no mechanical cost to the government. To illustrate the economic forces at play, consider the expression for the marginal money-metric welfare effect, $\frac{1}{\lambda}\frac{dW}{d(1-\delta)}\big|_{\delta=1}$, based on equation (A24) from the proof of Proposition 3 in Appendix A.4:

$$\tau \sum_i \pi^i \Delta_r^i \Bigg[\underbrace{y_1^i \cdot \frac{1}{\lambda} \cdot u'(c_1^i)}_{\substack{\text{[i] Welfare gains} \\ \text{from intertemporal} \\ \text{consumption smoothing}}} + \underbrace{\frac{\tau}{1-\tau} y_1^i \varepsilon_{11}^{i,c}}_{\substack{\text{[ii] Increase in period-1} \\ \text{tax revenues} \\ \text{(substitution)}}} + \underbrace{\frac{1}{1+r}\frac{\tau}{1-\tau} y_2^i \varepsilon_{21}^{i,c}}_{\substack{\text{[iii] Decrease in period-2} \\ \text{tax revenues} \\ \text{(intertemp. substitution)}}} + \underbrace{y_1^i \rho^i}_{\substack{\text{[iv] Decrease} \\ \text{in PV taxes} \\ \text{(income effect)}}} \Bigg], \tag{33}$$

which includes [i] positive welfare effects from increasing intertemporal consumption smoothing, [ii] a positive fiscal externality from increasing tax revenues in period 1 through a substitution effect, [iii] a partially offsetting negative intertemporal substitution effect on tax revenues due to a decrease in labor supply in period 2, and [iv] a negative income effect on present value tax revenues. Note that the entire money-metric welfare effect comes from net borrowers since $\Delta_r^i = 0$ when $s^i > 0$.

Lemma 2 below formally presents the welfare effects of three marginal reforms: (i) delaying taxation, (ii) offering a uniform loan, and (iii) lowering the marginal tax rate in period 1. This will allow us to later establish Proposition 4, which relates the welfare effects of delayed taxation to loans and age-dependent taxation.

Lemma 2 *Assume that $R(s)$ is piecewise linear around $s = 0$, $s^i \neq 0$ for all i, and consider an initial benchmark economy without delayed taxation ($\delta = 1$) and age-independent taxation. The money-metric welfare effect of*

(i) a marginal introduction of delayed taxation is:

$$\frac{1}{\lambda}\frac{dW}{d(1-\delta)}\Big|_{\delta=1} = \tau \Delta_r \left(\sum_{i:s^i<0} \pi^i y_1^i g_1^i + \sum_{i:s^i<0} \pi^i \mathcal{X}^i + \sum_{i:s^i<0} \pi^i y_1^i \rho^i \right). \tag{34}$$

(ii) the government offering a marginal loan, $dx > 0$, at an interest rate of r:

$$\frac{1}{\lambda}\frac{dW}{dx}\Big|_{\delta=1} = \sum_i \pi^i \left(g_1^i - g_2^i \right) + \sum_i \pi^i \Delta_r^i \rho^i = \Delta_r \sum_{i:s^i<0} \pi^i (g_1^i + \rho^i). \tag{35}$$

(iii) a marginal increase in $1 - \tau_1$, while keeping $1 - \tau_2$ fixed at $1 - \tau$:

$$\frac{1}{\lambda}\frac{dW}{d(1-\tau_1)}\Big|_{\delta=1} = \sum_i \pi^i y_1^i g_1^i + \sum_i \pi^i y_1^i \rho^i + \sum_i \pi^i \mathcal{X}^i - \sum_i \pi^i y_1^i, \tag{36}$$

where $\Delta_r = 1 - \frac{1+r}{1+r_b} > 0$ *and* $\mathcal{X}^i = \frac{\tau}{1-\tau}\left(y_1^i \varepsilon_{11}^{i,c} + \frac{1}{1+r} y_2^i \varepsilon_{21}^{i,c}\right)$ *denotes the (income-weighted) substitution effects of the tax change for individual* i.

Proof. See Appendix A.5 ■

Using Lemma 2, we can now establish Proposition 4.

Proposition 4 (Decomposing Delayed Taxation) *Assume that $R(s)$ is piecewise linear around $s = 0$, $s^i \neq 0$ for all i, and consider an initial benchmark economy with neither delayed taxation nor age-dependent taxation. Then, the marginal welfare effect of introducing delayed taxation can be written in terms of either a uniform loan or an age-dependent tax change as follows:*

$$\frac{1}{\lambda}\frac{dW}{d(1-\delta)}\Big|_{\delta=1} = \underbrace{\tau \bar{y}_1 \frac{1}{\lambda}\frac{dW}{dx}}_{\substack{\text{Lending} \\ \text{effect} \\ (49\%)}} + \tau\Delta_r\Big[\underbrace{\sum_{i:s^i<0} \pi^i \mathcal{X}^i}_{\substack{\text{Less} \\ \text{distortions} \\ (72\%)}} + \underbrace{\sum_{i:s^i<0} \pi^i (y_1^i - \bar{y}_1) g_1^i}_{\substack{\text{Loan} \\ \text{targeting} \\ (-23\%)}} + \underbrace{\sum_{i:s^i<0} \pi^i (y_1^i - \bar{y}_1)\rho^i}_{\substack{\text{Income effect} \\ \text{targeting} \\ (2\%)}}\Big] \tag{37}$$

$$= \tau\Delta_r\Big[\underbrace{\frac{1}{\lambda}\frac{dW}{d(1-\tau_1)}}_{\substack{\text{Age-dependent} \\ \text{tax decrease} \\ (47\%)}} + \underbrace{\sum_{i} \pi^i y_1^i}_{\substack{\text{Mechanical} \\ \text{effect} \\ (98\%)}} - \underbrace{\sum_{i:s^i>0} \pi^i \mathcal{X}^i}_{\substack{\text{Savers'} \\ \text{distortions} \\ (-19\%)}} - \underbrace{\sum_{i:s^i>0} \pi^i y_1^i g_1^i}_{\substack{\text{Savers'} \\ \text{consumption} \\ (-55\%)}} - \underbrace{\sum_{i:s^i>0} \pi^i y_1^i \rho^i}_{\substack{\text{Savers'} \\ \text{income effects} \\ (29\%)}}\Big], \tag{38}$$

where $\bar{y}_1 = \sum_{i:s^i<0} \pi^i y_1^i$ *and* $\tau\Delta_r = \tau\left(1 - \frac{1+r}{1+r_b}\right)$.

Proof. See Appendix A.6. ■

The first part of Proposition 4 shows that the welfare effects of delayed taxation can be decomposed into four parts. We use our calibrated model in Section 3 to provide numerical values of these parts, expressed as a share of the total marginal welfare gain on the left-hand side.[10] The first part is the welfare effect of providing a subsidized loan of $\tau\bar{y}_1$, which is the amount of taxes accrued by the average borrower. The second term is the (positive) welfare effect of a compensated decrease in the effective period-1 tax rate of $\tau\Delta_r$ among borrowers. In other words, a central source of additional welfare gains is that delayed taxation reduces tax distortions. The third and fourth term is an "error terms" due to the fact that the loan is uniform but delayed taxation conditions on period-1 income. The third term is effectively the covariance between income and the marginal welfare effect of another dollar of consumption in period 1. Among constrained borrowers, this covariance will be negative. That is, one benefit of a uniform loan relative to delayed taxation is that effective loan amounts are better targeted. The fourth term accounts for the fact that the income effects of a loan are allocated differently. This is due to the fact that the loan implied by delayed taxation is proportional to period-1 income while the uniform loan is not. This term can thus be written as the covariance between period-1 income and the income effect parameter. This term is positive if income effects are decreasing in income levels, that is, if the magnitude of the marginal propensity to earn out of unearned income is

[10] Specifically, we consider the case when the private borrowing rate, r_b equals 10%. We use the expressions for the various derivatives derived in Appendix D and the allocations in the benchmark economy.

decreasing in the level of earnings. In other words, the advantage of uniform loans that it better targets those with lower marginal utilities of consumption may be offset by larger income effects. In line with our intuition, in our numerical exercise, the three last terms of (37) sum up to a positive quantity.

The second part of Proposition 4 shows that the welfare effect of marginally delaying taxation can be decomposed into five terms. The first term is the welfare effects of an age-dependent reduction in the period-1 tax rate. The second term is the welfare gains from avoiding the accompanying negative mechanical effects of a lower period-1 tax rate. That is, with delayed taxation, we lower the effective period-1 tax rate by changing only the timing of the payment. Since the tax will still be paid later, we avoid the mechanical effect on tax revenues. The third term addresses the fact that an age-dependent tax cut would induce a positive substitution effect for all workers, while delayed taxation only increases the labor supply of borrowers. Hence, the third term subtracts the positive substitution effect of an age-dependent tax cut among net savers. The fourth term subtracts the welfare gains that age-dependent taxation would produce by increasing period-1 consumption of *net savers*. We posit that this term has a modest effect since net savers are able to smooth their consumption across periods, and hence, their marginal utility of consumption in period-1 will not be particularly high. The final term subtracts the welfare *losses* that would arise due to the income effects on labor supply of net savers.

Note that if everyone borrows, and the income tax is optimally age-dependent, it follows from Proposition 4 that:

$$\frac{1}{\lambda}\frac{dW}{d(1-\delta)}\Big|_{\delta=1} = \tau\Delta_r\left(\frac{1}{\lambda}\frac{dW}{d(1-\tau_1)} + \sum_i \pi^i y_1^i\right) = \tau\Delta_r \sum_i \pi^i y_1^i,$$

where the last equality follows because $\frac{1}{\lambda}\frac{dW}{d(1-\tau_1)} = 0$ under an optimal age-dependent tax. Thus, we can establish Corollary 1.

Corollary 1 *When all agents borrow ($s^i < 0$), starting from no delayed taxation ($\delta = 1$) but optimal age-dependent tax rates (τ_1, τ_2), the money-metric welfare effects of marginally delaying taxation, $d(1-\delta) > 0$ equals*

$$\frac{1}{\lambda}\frac{dW}{d(1-\delta)}\Big|_{\delta=1} = \tau_1\Delta_r \sum_i \pi^i y_1^i, \tag{39}$$

where λ is the Lagrangian of the optimal age-dependent tax rate problem.

The intuition for (39) is that if you delay taxation slightly, you can afford a slightly higher tax rate in period 1 while leaving the effective wedge in period 1 unaffected. This increases the total tax revenue. Note that since marginal taxes are optimally age-dependent in the pre-reform situation, there is no welfare gain from changing the wedge in period 1 by introducing a small amount of delayed taxation.

Before moving on to the quantitative analysis, it is important to clarify a point from our theoretical framework. We set the interest rate on delayed taxes, r_{dtax}, equal to the net saving rate, r. This assumption implies two things: first, for workers whose marginal borrowing rate is equal to r, delayed taxation does not affect their behavior (since for them delayed taxation leaves the present value of the tax rate unchanged); second, the government incurs no additional costs by delaying tax collection, since r is also assumed to be the government's interest rate. This means that delayed taxation does not affect the behavior of net savers in partial equilibrium, nor does it alter outcomes in a frictionless financial environment where $r_b = r$.

In Appendix B, we explore a variation of our model that allows r_{dtax} to deviate from the government's borrowing rate, $r_{gov} = r$, and consider scenarios without financial frictions where $r_b = r$. If the government has the flexibility to set any $r_{dtax} \in \mathbb{R}$, it could emulate any age-based tax system characterized by $(\tau_1, \tau_2) \in \mathbb{R}^2_+$ by selecting $\tau = \tau_2$, assigning $r_{dtax} = -1$, and adjusting $\delta = \tau_1/\tau_2$.[11] Nevertheless, this approach overlooks the heterogeneity of individual borrowing rates, $R'(s^i)$, and fails to exploit the strategic potential of delayed taxation —its ability to tailor tax rates to an individual's borrowing status. In section 3.5 of our quantitative analysis, we explore the implications of treating r_{dtax} as a policy instrument.

3 A quantitative investigation of delayed taxation

3.1 Calibration

We assume that the utility function has the form

$$u(c) - v(\ell) = \frac{c^{1-\sigma} - 1}{1 - \sigma} - \xi \frac{\ell^{1+\frac{1}{k}}}{1 + \frac{1}{k}}, \tag{40}$$

which implies that σ is the inverse of the elasticity of intertemporal substitution and k is the constant consumption elasticity of labor supply, while ξ is a scaling parameter reflecting the intensity of the disutility of labor. In the simulations below, we choose $\sigma = 5$ and $k = 0.5$. We also set $\xi = 1$. Our baseline analyses consider constant social welfare weights.

We calibrate our model to Norwegian workers between the ages of 20 and 30 in 1990 who are employed and not in school. There are 100 agents. For each decile in the 1990 wage distribution, there are 10 agents corresponding to their decile in the 2011 wage distribution. We set π_i, $i = 1, \ldots, 100$ equal to the population fraction of each of these types. Our calibration assumes perfect foresight to obtain heterogeneity in expected inflation-adjusted wage trajectories.[12] We set the exogenous revenue requirements, M_t, equal to 15% of GDP_t (sum of labor income in period t) measured in the benchmark economy when $r_b = 3\%$. The revenue requirements thus do not vary by r_b or policy choices.

Appendix Table A.1 provides summary statistics for the wage trajectories. We see that the median cumulative real wage growth is 0.88. Annualizing this over a 21-year period yields annual real wage growth of 5.16%. This masks considerable heterogeneity. The fifth percentile of cumulative real wage growth is -28% and the ninety-fifth percentile is 464%.

[11] If $\tau_2 = 0$, δ becomes undefined, but this is inconsequential since δ loses relevance if $\tau = \tau_2 = 0$.

[12] See, e.g., Herbst and Hendren (2023) who argue that young workers (students) possess substantial private knowledge about their future earnings, academic persistence, employment, and likelihood of loan repayment beyond what is captured by observable characteristics.

In our calibration, there are no exogenous non-labor income or endowments. Accordingly, all variation in savings incentives (and thus the degree of financial friction) comes from differences in earnings trajectories. For example, those who start in the bottom decile and end in the top decile will want to borrow the most.

We set the base interest rate (faced by the government) to 3%. Since we are modeling periods that are 21 years apart, the cumulative interest rates enter the budget constraints. That is, the present value in period 1 of \$1 in period 2 is 1.03^{-21}. Accordingly, we set $\beta = 1.03^{-21}$ so that a net saver facing an interest rate of 3% would choose the same amount of consumption in the two periods.

Policies. For age-dependent taxation (AD), the government may choose different tax rates, i.e. we allow $\tau_1 \neq \tau_2$, but we require $\tau_t \geq 0$. Allowing $\tau_1 \neq \tau_2$ is not an option under either the benchmark policy or delayed taxation. We also require $G_1 = G_2 = G \geq 0$, except when we introduce an age-dependent tax *and* transfer system (AD T&T).

Under delayed taxation, we restrict the fraction of period 1 taxes payable in period 1 to be within $[0, 1]$. $1 - \delta \leq 1$ implies that agents cannot borrow from the government in excess of the amount of taxes they accrue, which is typically binding when financial frictions are severe. Imposing $1 - \delta \geq 0$ is not a binding constraint. In the presence of financial frictions, the optimizing government will not force workers to save an amount proportional to their accrued taxes. In other words, "social security" contributions do not arise in our model.[13]

Individual-level delayed taxes. We further impose that agents dissave rather than delay taxes if they weakly prefer to. This occurs when $R'(s^i) \leq 1 + r_{dtax}$.

3.2 Main numerical results with unrestricted government borrowing

We first consider delayed taxation and related reforms when the government is not subject to any borrowing limits. We do not believe this is the most realistic implementation of delayed taxation, but it is nevertheless useful to understand the welfare gains and allocations under unrestricted reforms like this. Later, in Section 3.4, we explore what welfare gains can be achieved when government borrowing is limited.

Figure 1 summarizes the key findings for the case when the annualized private borrowing rate is 10%. We see that delayed taxation increases money-metric welfare by almost 9%. Age-dependent taxation also offers significant but more moderate welfare gains of about 4.2%. The third column shows us that implementing delayed taxation on top of age-dependent taxation more than doubles the welfare gains, indicating that age-dependent taxation is not a substitute for delayed taxation. The fourth column shows the welfare gains from a fully age-dependent tax *and* transfer scheme, that is, allowing both τ_t and G_t to vary by age. The final column shows that the highest welfare gains arises when the government is able to completely remove financial frictions. This is done by offering unlimited unconditional loans at the government interest rate

[13] One way to achieve an optimal $1 - \delta < 0$ is to make the government "paternalistic". In Choukhmane and Palmer (2023), for example, the government uses a different time discount factor (i.e., a higher *beta*) than consumers when calculating welfare.

of 3%.

Figure 1: Welfare Effects of Different Policy Reforms With Unrestricted Government Borrowing

This figure plots the money-metric welfare effects of implementing different reforms when the private borrowing rate is 10% and government borrowing is unconstrained. The welfare gain measure is the exogenous revenue shock (scaled by GDP, the sum of present-value labor earnings) the government needs to be equally well of in the benchmark (no policies) economy. We consider delayed taxation (DT), age-dependent taxation (AD), DT and AD simultaneosly, fully age-dependent taxes *and transfers* (letting G_t vary with t), and a subsidized lending scheme in which the government lends to households at the government interest rate (which equals the interest rate on net savings).

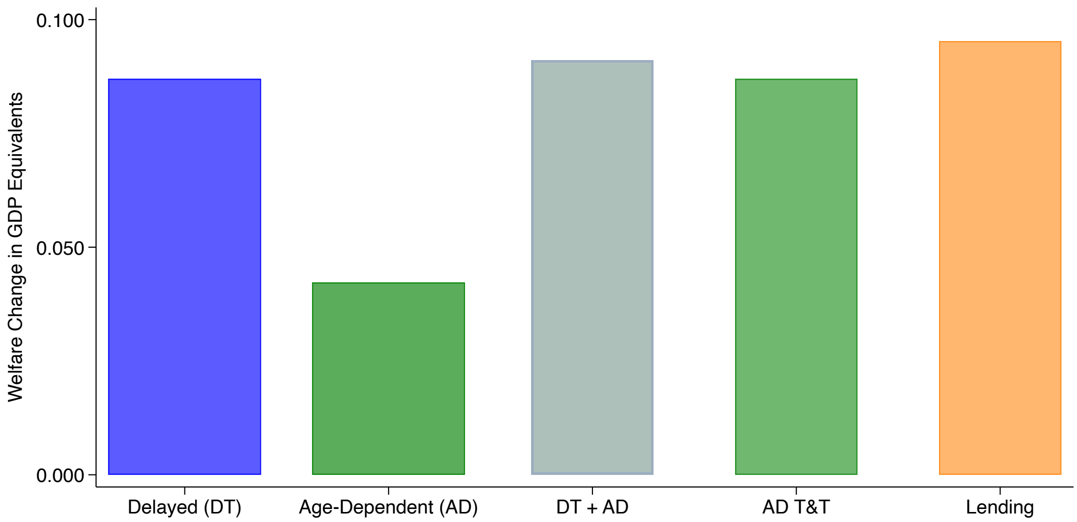


Table 1 shows the optimal tax policies and allocations for the case where the private borrowing rate, r_b, is 10%. We see that the government chooses to maximally delay taxes with $1-\delta = 100\%$. We further see that delayed taxation involves a lower marginal tax rate of 57% as opposed to 61% in the benchmark economy. This is partially because delayed taxation improves intertemporal consumption smoothing and thus lowers the optimal unconditional transfer, G.

We see that allowing age-dependent taxation (different τ_1 and τ_2) leads to a a substantial intertemporal tax wedge of $\tau_2 - \tau_1 = 0.34$. When AD is implemented together with DT, the $\tau_2 - \tau_1$ wedge is reduced to only 0.13. This indicates that age-dependency is less desirable in the presence of delayed taxation. Relatedly, we see that implementing AD on top of DT only modestly increases money-metric welfare from 8.70 to 9.11, but implementing DT on top of AD more than doubles welfare.

Table 1: Optimal Taxation with Financial Frictions: The Case with No Government Borrowing Constraints

This table provides summary statistics for the calibrated economy when $r_b = 10\%$. The present value function calculates present values according to the government's discount rate. The (money-metric) welfare gap is the exogenous shock to revenue that the government must experience in the benchmark economy to be equally well off as with the given policy. This number is measured as a fraction of the baseline economy's GDP. Govt loan limit is the maximal loan given under the subsidized lending reform. Govt borrowing is the amount the government borrows to satisfy its first-period budget constraint. Liab to govt $> G_2$ is the fraction of workers that owe the government (in delayed taxes or subsidized loans, with interest) more than the period-2 lump-sum transfer.

	Tax schedule and allocations with $r_b = 10\%$					
	Benchmark	Delayed Taxation	Age-Dependent	DT + AD	AD (T&T)	Lending
τ_1	0.60	0.57	0.25	0.49	0.48	0.51
τ_2	0.60	0.57	0.69	0.62	0.66	0.51
G_1	0.51	0.49	0.37	0.48	0.75	0.43
G_2	0.51	0.49	0.37	0.48	0.00	0.43
$1-\delta$	0.00	1.00	0.00	1.00	0.00	0.00
r_{dtax}	0.03	0.03	0.03	0.03	0.03	0.00
Govt loan limit	0.00	0.00	0.00	0.00	0.00	0.55
Govt Borrowing	0.09	0.36	0.24	0.33	0.48	0.39
Δ Welfare, % GDP (Benchmark, $r_b = 10\%$)	0.00	8.70	4.22	9.11	8.70	9.53
means						
$l_1 w_1$	0.89	0.73	0.93	0.78	0.79	0.72
$l_2 w_2$	1.63	1.96	1.61	1.88	1.83	2.05
PV($l_1 w_1$,$l_2 w_2$)	1.77	1.78	1.79	1.79	1.77	1.82
PV($l_1 w_1 \tau_1$,$l_2 w_2 \tau_1$)	1.06	1.02	0.84	1.01	1.03	0.94
l_1	0.88	0.73	0.94	0.77	0.79	0.72
l_2	0.78	0.95	0.77	0.90	0.86	0.98
$(1-\delta^i) l_1 w_1 \tau_1$	0.00	0.18	0.00	0.12	0.00	0.00
s	-0.01	0.00	0.08	0.01	0.19	-0.22
Liab to govt $> G_2$	0.00	0.23	0.00	0.14	0.00	0.43

Another interesting statistic is the fraction of workers for whom liabilities to the government (delayed taxes and subsidized loans, with interest) exceed second-period lump-sum transfers, G_2. For delayed taxation, this fraction is about 23%. For the government lending policy, the fraction is 43%. Hence, if the government may use G_2 as collateral, the upper bound for defaults would be much lower for delayed taxation.

3.3 Marginal Value of Public Funds

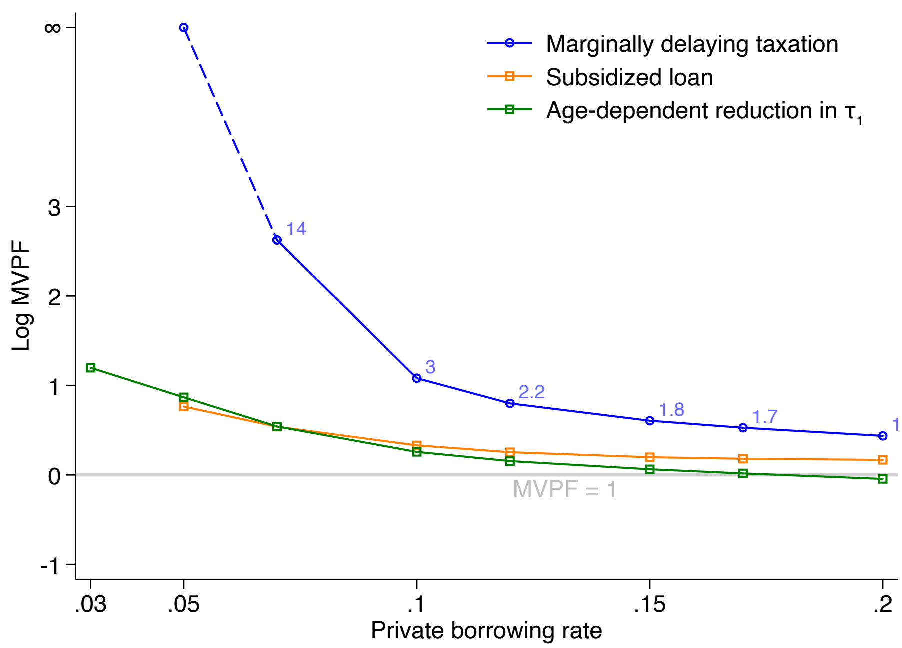
This subsection quantitatively evaluates delayed taxation using the marginal value of public funds methodology (Hendren and Sprung-Keyser 2022, Finkelstein and Hendren 2020). We calculate the MVPF of marginally changing some policy parameter, ω, as

$$MVPF = \frac{\sum_i \pi^i \frac{dV^i}{d\omega}/\lambda}{\frac{dR}{d\omega}}, \tag{41}$$

where V_i is an individual's life-time utility, λ is the Langrange multiplier on the government budget constraint in the benchmark economy, and R is present-value government tax revenues. The MVPF is thus the ratio of money-metric utility gains from changing ω to the marginal net cost faced by the government. If the numerator is positive but net cost is negative (e.g., if the tax reform increases revenues), the ratio is set to infinity (Hendren and Sprung-Keyser, 2022).

Figure 2: Marginal Value of Public Funds

This figure plots the MVPF of three marginal reforms: (i) marginal delayed taxation, (ii) age-dependent reduction in the period-1 tax rate, offering a uniform loan at the government interest rate subject to a marginal uniform loan limit. See section 3.3 for the methodology.



We value the MVPF using the analytical expressions derived in Section 2. These expressions are written in terms of elasticities and not allocations. Hence, we derive analytical expressions for the elasticities that depend only on allocations (i.e., consumption and labor supply of an agent) in Appendix D. We can thus plug in the numerical allocations for the simulated benchmark economy (or any economy for that matter) to obtain the exact MVPFs for different tax reforms.

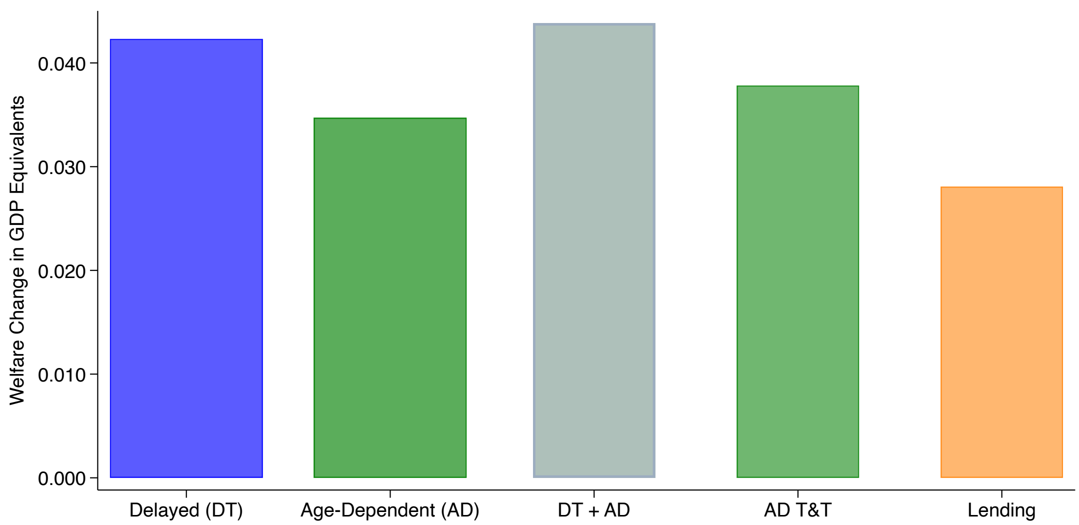
We provide the calculated MVPFs under different assumptions on the baseline private borrowing rate in Figure 2. This shows that the MVPF of delayed taxation exceeds that of age-dependent taxation and providing subsidized loans. When r_b is low (e.g., 5%), the MVPF of delayed taxation is *infinite*. This occurs because when the private borrowing rate is low, the income effects of reducing the present-value period-1 tax rate for constrained borrowers is small. Hence, the substitution effects dominate, and marginally delaying taxation increases government tax revenues. Marginal delayed taxation thus pays for itself and is a Pareto-improving policy (Hendren and Sprung-Keyser, 2022) when the private borrowing rate is modest.

3.4 Reforms when the government is borrowing constrained

An important source of welfare gains from both age-dependent and delayed taxation is improved consumption smoothing, which is facilitated by government borrowing. In our model, the government can borrow an unlimited amount at r_{gov}, and thus is not concerned with the amount it must borrow to finance the various reforms. In practice, this may not be the case.

Figure 3: Welfare Effects of Different Policy Reforms When Government Borrowing is Restricted

This figure plots the money-metric welfare effects of implementing different reforms when the private borrowing rate is 10% and government cannot increase net borrowing by more than 10% of period-1 GDP in the Benchmark economy. The welfare gain measure is the exogenous revenue shock (scaled by GDP, the sum of present-value labor earnings) the government needs to be equally well of in the benchmark (no policies) economy. We consider delayed taxation (DT), age-dependent taxation (AD), DT and AD simultaneously, fully age-dependent taxes *and transfers* (letting G_t vary with t), and a subsidized lending scheme in which the government lends to households at the government interest rate (which equals the interest rate on net savings).



We proceed by directly addressing the question of optimal tax policy in the presence of government borrowing constraints. The net amount that a government borrows is denoted B. If the government saves, then B is negative. When implementing any tax reform, the maximal increase in borrowing is proportional to period-1 GDP in the benchmark economy. That is,

$$B - B^*(r_b) \leq b \times GDP_1^*(r_b), \tag{42}$$

where $B^*(r_b)$ is the amount of government borrowing in the benchmark economy with a (private) borrowing rate of r_b. $GDP_1^*(r_b)$ is the amount of period-1 labor earnings. The government borrows at $r_{gov} = 3\%$. We consider moderate values of the parameter b from 0 to 0.5, where a value of 0.5 implies that the government can increase borrowing (decrease net saving) by at most 50% of period-1 GDP. Tying the borrowing constraints to the benchmark GDP as opposed to the new GDP ensures that all policies are subject to the same nominal borrowing limit.

Table 2: Optimal Taxation with Financial Frictions when Government Faces Borrowing Constraints

This table provides summary statistics for the calibrated economy when $r_b = 10\%$ and the government borrowing limit, b, is 50%. The present value function calculates present values according to the government's discount rate. For a given policy reform, the money-metric welfare measure is the exogenous change in government tax revenue that would be required for the government of the benchmark economy to be as well off as it would be with the reform. It is expressed as a percentage of the benchmark economy's GDP (PV labor earnings).

	Benchmark	Delayed Taxation	Age-Dependent	DT + AD	AD (T&T)	Lending
τ_1	0.60	0.52	0.39	0.46	0.46	0.55
τ_2	0.60	0.52	0.66	0.57	0.58	0.55
G_1	0.51	0.43	0.43	0.43	0.47	0.48
G_2	0.51	0.43	0.43	0.43	0.33	0.48
$1-\delta$	0.00	0.26	0.00	0.18	0.00	0.00
r_{dtax}		0.03		0.03		
Govt loan limit						0.08
Govt Borrowing	0.09	0.18	0.18	0.18	0.18	0.18
Δ Welfare, % GDP (Benchmark, $r_b = 10\%$)	0.00	4.23	3.47	4.38	3.78	2.81
means						
$l_1 w_1$	0.89	0.87	0.91	0.88	0.89	0.85
$l_2 w_2$	1.63	1.74	1.61	1.70	1.68	1.82
PV($l_1 w_1$,$l_2 w_2$)	1.77	1.80	1.77	1.79	1.79	1.83
PV($l_1 w_1 \tau_1$,$l_2 w_2 \tau_1$)	1.06	0.94	0.93	0.93	0.92	1.01
l_1	0.88	0.87	0.91	0.88	0.89	0.84
l_2	0.78	0.86	0.77	0.83	0.82	0.88
$(1-\delta^i) l_1 w_1 \tau_1$	0.00	0.09	0.00	0.05	0.00	0.00
s	-0.01	-0.00	0.04	0.01	0.02	-0.06
Liab to govt $> G_2$	0.00	0.00	0.00	0.00	0.00	0.00

We present our main results for the case where $b = 0.1$ and $r_b = 10\%$ in Table 2. We see that the government now optimally chooses to allow only about 26% of taxes to be delayed. Relative to the unconstrained scenario, the government now faces more pressing tradeoffs in implementing delayed taxation: once the borrowing limit is reached, any increase in $1-\delta$ must be financed either by higher tax rates in period 1 (more distortions) or by lower transfers, G (less redistribution). Similar trade-offs apply to age-dependent taxation. The only way to finance a reduction in τ_1 is to reduce G.

We find that delayed taxation offers the highest welfare gains, equal to about 4.23% of the GDP of the benchmark economy. Interestingly, this welfare effect is also larger than for *AD T&T*, which is a more flexible age-dependent tax system that also allows the government to choose age-dependent lump-sum transfers in addition to age-dependent (marginal) tax rates. This is all the more surprising given that delayed taxation, in which the government chooses only three parameters (δ, τ, G), outperforms a scenario in which the government chooses four parameters

$(\tau_1, \tau_2, G_1, G_2)$.

Figure 4: Welfare Effects of Delayed and Age-Dependent Taxation With Government Borrowing Constraints

This figure plots the money-metric welfare effects (measured in terms of the GDP of the benchmark economy) of implementing either delayed taxation or age-dependent taxation. We do this for different values of b, which is defined as the maximum relative increase in borrowing relative to the benchmark economy. For example, if $b = 0$, the government can introduce delayed taxation but cannot itself borrow more than it did before implementing delayed taxation.

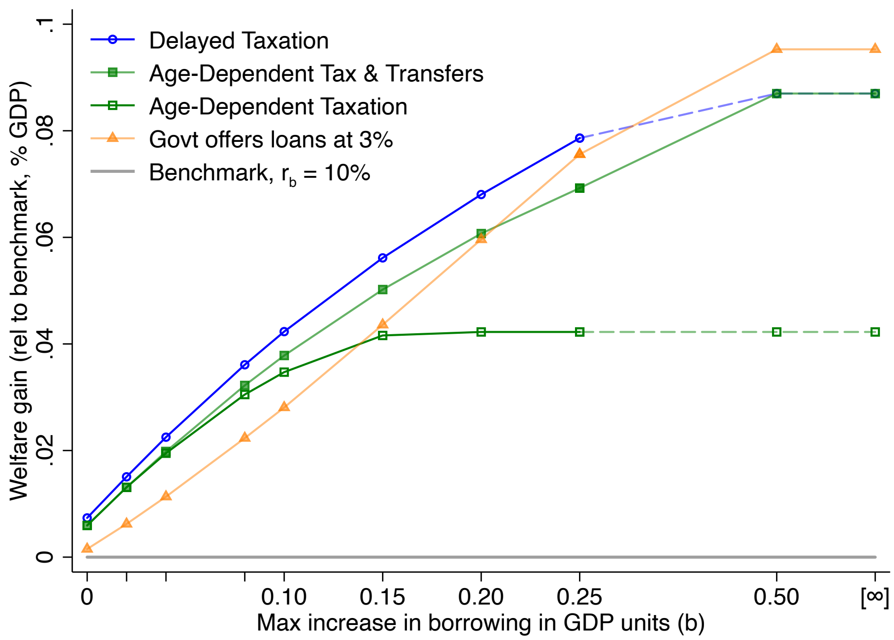
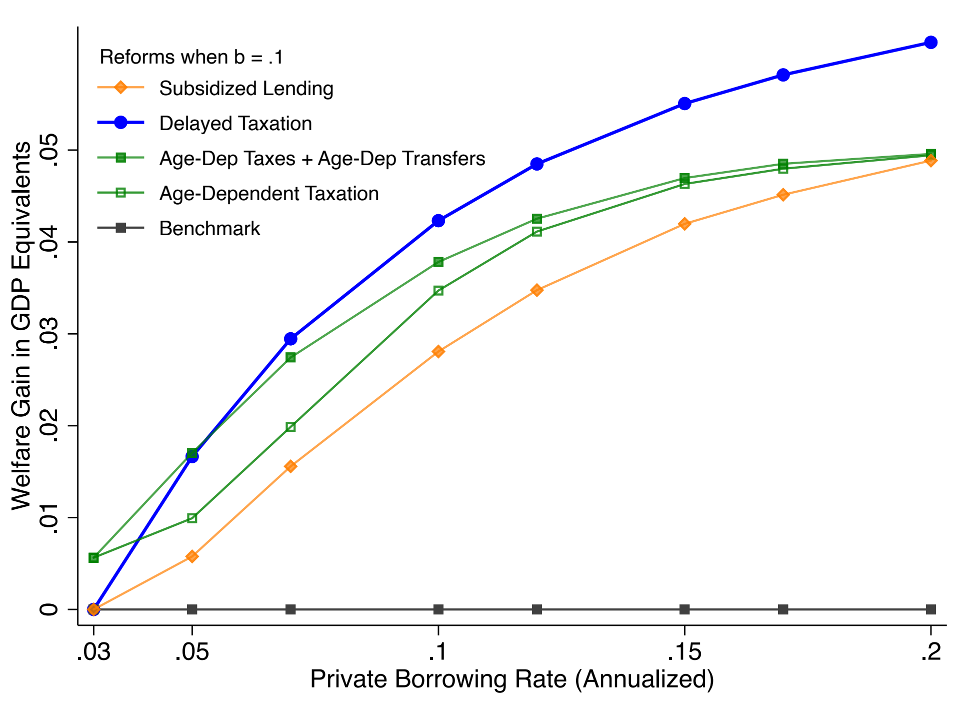


Figure 5 shows how the welfare gains under a government borrowing constraint of $b = 0.10$ varies with the severity of financial frictions. Naturally, there are no gains from either delayed taxation or lending when the interest rates on savings and borrowing are equal. However, as the borrowing rate increases, delayed taxation produces noticeable gains, exceeding those of both age-dependent taxation and a fully age-dependent tax and transfer scheme.

Figure 5: Welfare Effects from Implementing Delayed and Age-Dependent Taxation under Different Assumptions on Marginal Borrowing Rates

This figure plots the monetary-metric welfare gains from implementing different policies when the government can only increase borrowing by 50% relative to the benchmark economy. These welfare gains are provided for different interest rates on borrowing, with $r_b = 3\%$ equaling the marginal interest rate that the government and net savers face.



We also note that the amount of delayed taxes, $(1 - \delta^i)\ell_1 w_1 \tau_1$, never exceeds lump-sum transfers. Hence, the no-default assumption is not restrictive. The government could simply subtract the delayed taxes (with interest) from the lump sum transfer in period 2.

We illustrate how welfare varies with the degree of government borrowing constraints, b, in Figure 4 for the case where $r_b = 10\%$. We see that delayed taxation offers larger welfare gains than all the other reform options whenever $b < \infty$. Interestingly, all *tax-reform* policies provide non-negligible welfare gains of about 0.5% of GDP even when the government cannot increase borrowing at all. In order to engage in delayed taxation or lending when $b = 0$, the government must either raise nominal tax rates or decrease transfers. This causes the lending reform to have virtually zero welfare gains, but there are still significant gains from delayed taxation. This emphasizes that delayed taxation, through reducing the distortionary effects of income taxes, is more attractive than a pure lending policy.

3.5 Unmodeled aspects of the various policies

Our analyses highlight the potential benefits of delayed taxation and show that, in most cases, it generates more welfare than age-dependent taxation. Our modeling does not take into account the possibility that borrowers may default. One concern might be that agents who acquire a significant amount of delayed taxation might choose to evade tax payment by moving abroad and refusing to pay tax liabilities. This problem also applies to age-dependent taxation: those who benefit from young tax rates when they are young may choose to move to a tax jurisdiction with age-independent tax rates when they are old. In our framework, the possibility of default could *increase* the welfare gains from delayed taxation. This happens because the government, if it could, would like to charge a lower interest rate on delayed taxes than the government interest rate. We show this when we consider "DT+", which is a more flexible form of delayed taxation in which the government is free to choose the interest rate on delayed taxes. When given this freedom, the government chooses a lower interest rate, and when the government is subject to tight borrowing constraints, it optimally chooses to set $r_{dtax} = -1$ (see Appendix Figure A.4), which is equivalent to ex ante loan forgiveness. Nevertheless, one avenue for reducing defaults is to ensure that no (or very few workers) delay more taxes than the lump-sum transfer they receive in the second period. In this case, the government could use these transfers as collateral to ensure no defaults.

3.6 HSV tax function

Most modern tax systems feature a progressive marginal tax schedule that cause average tax rates to be lower for lower-income younger workers. This leaves the question of whether delayed taxation still has material welfare gains on top of a progressive marginal tax schedule. We investigate this by extending our model such that the benchmark economy features a nonlinear marginal tax schedule as in Heathcote et al. (2020):

$$T_t(y_t) = y_t - \lambda_t y_t^{1-\tau_t} + G_t. \tag{43}$$

We provide the relevant budget constraints and first-order conditions in Appendix F. Note that unlike Heathcote et al. (2020), we include a lump sum transfer, $G_t \geq 0$ in the tax and transfer schedule to stay consistent with our previous analyses. In the benchmark case, and when implementing delayed taxation, the t subscripts on the tax schedule are redundant as λ, τ, and G cannot vary with t. Age-dependent taxation allows λ and τ to vary with t. Fully age-dependent taxes and transfers (AD T&T) allows G to vary with t.

The numerical results using the HSV tax schedule as the benchmark policy reveal *improved* welfare gains from delayed taxation relative to using a linear benchmark tax schedule. Considering the case when the private borrowing rate is 10%, we see that delayed taxation provides higher welfare gains than age-dependent taxation and subsidized lending when government faces borrowing constraints. We provide detailed allocations when the borrowing rate is 10% and the government may at most increase borrowing by 50% in Appendix Table A.3. This shows that under the HSV benchmark, delayed taxation improves welfare by 3.05% relative to 2.58% over the linear tax benchmark. These findings indicate that a more flexible (i.e., nonlinear) tax system is a complement rather than a substitute to delayed taxation. The fact that the delayed taxation welfare gains do not vanish under the HSV tax schedule is perhaps not surprising given that the original benchmark was already progressive due to the lump-sum transfers, G_t.

Relatedly, Ferriere, Grübener, Navarro, and Vardishvili (2023) show that adding transfers on top of a log-linear tax system materially increases welfare by decoupling the progressivity of average and marginal tax rates. We find this to be true also in our setting, as Appendix Table A.3 shows that the optimal tax and transfer schedule features significant lump-sum transfers.

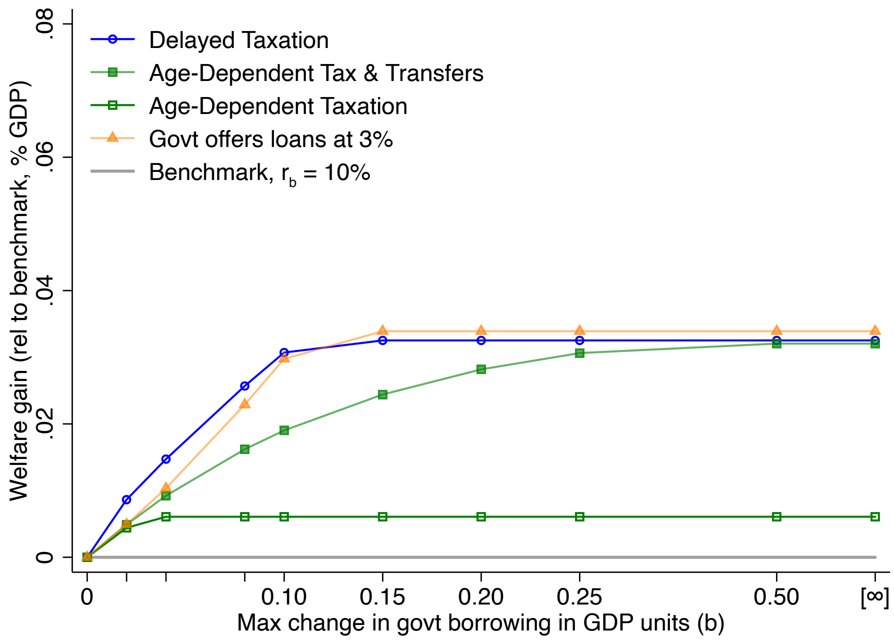
3.7 U.S. calibration

A distinct feature of our calibration is that a large majority of workers see real wage increases. Accordingly, a majority wishes to borrow and thus the extent of financial frictions is severe. This creates an important source of welfare gains for all of the policies we consider (delayed and age-dependent taxation as well as subsidized lending). However, it is useful to verify that the results from our existing horse-race generalizes to other settings. We therefore use data from the Panel Study of Income Dynamics (PSID) obtained from the replication package in Weinzierl (2011). To match our empirical setting, we consider workers aged 20–30 in 1980 for whom Weinzierl (2011) has calculated effective wages in both 1980 and 2001 (i.e., the same 21-year gap). We use then use these wage trajectories to provide numerical solutions.

Figure 6 shows how the welfare gains from the different policies compare and how they vary with the severity of government borrowing restrictions. The first finding is that welfare gains are smaller than in our main exercise, but still substantial. The money-metric welfare gains from delayed taxation approach 3.3% of benchmark GDP as the government borrowing restrictions are relaxed. One observation is that subsidized lending creates the highest welfare gains for more modest borrowing constraints. This is driven by lower wage growth in the PSID data, causing less workers to want to borrow, which means that the government needs to borrow less to satisfy workers' borrowing demand.

Figure 6: Welfare Effects using U.S. PSID Data

This figure plots the money-metric welfare effects (measured in terms of the GDP of the benchmark economy) of implementing either delayed taxation or age-dependent taxation for the 1980 cohort of young workers in the PSID. We do this for different values of b, which is defined as the maximum relative increase in borrowing relative to the benchmark economy.



4 Does delayed taxation already exist?

We define delayed taxation as a scheme in which tax payment, but not tax accrual, is (substantially) delayed. As we show theoretically, the benefits of delayed taxation arise when this wedge is large enough to substantially reduce marginal borrowing rates between the time of tax accrual and tax payment. While the use of this time wedge as a policy tool is novel to our paper, there are several tax schemes in place today that have similar ingredients. Below, we discuss some of these tax schemes and contrast them with our notion of delayed taxation.

Non-withheld taxes. In the absence of employer withholding, most employees would face a modest tax delay. If taxes are due in April of the following tax year, then taxes on earnings in January are delayed by about 15 months. While most developed countries require employers to withhold taxes, not all countries require the entire tax to be withheld. In Sweden, for example, the progressive portion of the income tax is sometimes not withheld, meaning that some high-income Swedish workers face a modest lag in taxation on marginal earnings.

Installment plans. While many countries allow for the use of installment plans to delay the payment of tax liabilities, these tax deferral systems are typically not designed in a way that resembles our concept of delayed taxation. In the U.S., for example, employers are required to withhold federal income taxes, which means that taxes are paid immediately. The IRS does offer (up to) 10-year installment plans for taxpayers in adverse financial circumstances, but only for the balance due, i.e., the taxes owed less what has already been withheld. Thus, unless a taxpayer expects tax liabilities to substantially exceed withholding amounts, the option to enter into an installment plan will not dampen behavioral responses to labor income taxes.

Social Security. Mandating workers to save a portion of their income for retirement effectively amounts to a negative tax deferral. If we assume that Social Security contributions are about 8% of pre-tax income and taxes are about 24%, then workers are effectively paying 133% of their taxes today and getting 33% back in retirement.

The idea that social security contributions can reduce welfare in the presence of financial frictions is not new. Hubbard et al. (1986) argue that when there are liquidity constraints, social security contributions lead to reduced welfare and excessive saving. In discussing the paper, Larry Summers notes that "reversing the direction of transfers" under social security would seem a natural way to reduce welfare losses from financial frictions. However, the key point that negative Social Security taxes (i.e., delayed taxation) can also reduce welfare losses from the distortionary effects of income taxation is missing.[14]

Capital gains. In most jurisdictions, capital gains are not taxed until realized. This effectively allows taxpayers to delay their taxes indefinitely, especially if there is a step-up in basis at death, as in the U.S. However, the underlying mechanism through which delayed capital gains taxation affects behavior is very different from delayed income taxation. Since unrealized capital gains are, in principle, difficult to consume, capital gains deferral does not facilitate intertemporal consumption smoothing in the same way as delayed income taxation.

Student stipends. A case of delayed taxation is when student financial aid is based on income levels. In the U.S., for example, the generosity of financial aid generally depends on parental income. Thus, if parents work and earn more, the financial aid package may consist to a greater extent of student loans. If we consider the family as a single economic unit, this is essentially delayed taxation: higher earned income results in the accrual of a (de facto) tax to be paid in the future. In Norway, the mix of financial support provided by the government does not depend on parental income, but rather, mechanically, on the student's own earnings while in school. We discuss this in the next section.

Retirement savings accounts. Tax-deductible contributions to retirement accounts where distributions are taxed as income (such as traditional IRAs in the U.S) also lower effective tax rates.[15] However, these savings incentives are often capped, which means that many workers (who contribute the maximal amount) see no effect on their effective marginal tax rate. In addition, they may be irrelevant for the labor supply decisions of constrained workers whose marginal saving

[14] It is also useful to note that the presence of an income limit ("maximum taxable earnings") for Social Security taxes limits the labor supply distortions for high-income earners. Social security contributions do not affect the marginal effective tax rates of these earners, but only serve to exacerbate financial frictions that may *increase* labor supply through an income effect.

[15] In the U.S., contributions to traditional IRAs are income-tax deductible and distributions are later taxable as income. Under the assumption that marginal savings go into a traditional IRA account, this savings scheme lowers the effective marginal tax rate in two ways. First, the nominal rate changes to the rate that workers expect to face during retirement when the savings are distributed. Since retirement incomes are generally lower than working-age incomes and the tax system is progressive, this channel lowers the nominal tax rate. Second, the effective tax rate is reduced because savings in IRA accounts are not subject to capital taxation and thus grow at a higher (pre-tax) rate of return.

likely go towards reduced borrowing as opposed to IRA contributions.

5 De facto Delayed Taxation in Norway

5.1 Empirical setting

Norwegian students receive monthly transfers from the government to pay for housing and other consumption while pursuing higher education. Importantly, these transfers are a mix of stipends and loans. If students earn above a certain threshold, each additional NOK of earnings causes a reduction in the stipend amount which is offset by an equal increase in the student's loan balance.

Our empirical study focuses on the years 2004 to 2011. During these years, most Norwegian students faced an earnings threshold ranging from NOK 104,500 ($17,000) in 2004 to NOK 140,823 in 2011. Monthly transfers ranged from NOK 8000 ($1,300) in 2004 to NOK 9785 in 2011. These transfers are initially given as a loan, but 40% can be forgiven (converted to a stipend) as long as students pass their courses and stay below the earnings thresholds mentioned above. Students are notified of the amount of the transfer at the beginning of the academic year. These notification letters include a breakdown of the transfers, noting the amount (40% of the total) that will be given as a conversion loan, and stating that the conversion of the loan to a stipend is contingent upon income being below an income threshold. The following year, students are notified how much of their loan has been converted, based on grades reported by the educational institutions and income reported to the tax authorities. Loans must generally be repaid within 20 years of graduation. No interest is charged while the student is receiving aid. Thereafter, interest rates are slightly above the risk-free rate and loan payments can be delayed at the (former) student's discretion for up to a total of 3 years.[16]

This study is facilitated by administrative data hosted by Statistics Norway. The key data are derived from tax returns, including data on income, assets and debts of individuals. The sample consists of students who received the standard student support for full-time studies for at least one full financial year during 2004-2011. We restrict the sample to students who received a strictly positive grant after conversion. This excludes students who are ineligible for debt conversion because, for example, they live at home with their parents. This ensures that nearly all students in our sample are subject to income-contingent debt conversion.

Summary statistics are given in Table 3. The average student is 23 years old. This is reasonable given that high school graduation occurs at age 18 and that we condition on students being enrolled in higher education for both semesters within a given year. The summary statistics reveal a significant spread in the amount of liquid assets available to students. While students at the 25th percentile have only NOK 8,000 ($1,300) in liquid assets, students at the 75th percentile have almost *ten* times more. A similar spread can be observed in the liquid assets of the students' parents. We also see that the average student earns around NOK 100,000 ($17,000), which is

[16]These generous terms differ from those offered in the U.S., where Gopalan, Hamilton, Sabat, and Sovich (2021) document debt responses to minimum wage increases that are consistent with either student debt aversion or very high perceived interest rates.

a direct consequence of our sample restrictions caused by focusing on students around the debt conversion threshold. Four years later, the average student has a much higher income of around NOK 360,000 ($60,000).

Table 3: Summary Statistics

This table presents summary statistics. The main sample period is 2004-2011. The financial variables are denominated in NOK. The USD/NOK exchange rate was around 6 in 2010. The main sample is restricted to students who had earned income within 50,000 of the debt conversion threshold. Liquid assets consist of deposits, investment funds and ownership of public shares. Labor earnings are censored to be below NOK 1,000,000 in 2010 NOK. The Bottom Tax Threshold is only considered for the years 2005-2011.

	N	Mean	p25	p50	p75
Liquid Assets$_{t-1}$	230,906	57,522	7,989	29,296	77,099
Liquid Assets$_{t-1}$ (Parents)	214,419	429,326	59,805	176,471	460,545
Age	231,036	23.4	22	23	25
Labor Earnings$_t$	231,036	101,394	81,156	98,536	118,966
Labor Earnings$_{t+4}$	229,027	357,506	226,244	372,615	464,829
Debt-Conversion Threshold$_t$	231,036	120,162	108,680	116,983	128,360
Bottom Tax Threshold$_t$	198,815	36,706	29,600	39,900	39,900

Salience. In order to meaningfully compare the implied elasticity of the debt conversion threshold with that of the regular tax thresholds, the conversion threshold must be similarly salient. As one of the authors is a former participant in this program, we certainly believe this to be the case. Beyond anecdotal evidence, however, it is useful to consider the magnitude of the effective tax increase. A 50 percentage point reduction in the "net of debt" wage is unlikely to go unnoticed. Moreover, students are informed of the existence of such a cap in a loan agreement letter that they must sign, and they also receive letters informing them of any conversion that has occurred. Even if students do not expect to receive a reduction in their debt through conversion, they will want to read these letters to confirm that their institution has accurately recorded and reported their academic progress. Non-passing grades in courses also reduce debt conversion. Students are also informed of their annual student debt balances when they receive their annual prefilled tax returns, which also include information about their income tax liabilities.

5.2 Bunching methodology

The purpose of the bunching method is to estimate earnings elasticity,

$$e = \frac{\Delta y^*/y^*}{\Delta\tau/(1-\tau)}, \tag{44}$$

where Δy^* is the reduction in earnings of the marginal buncher who is at an interior optimum at the debt-conversion threshold (i.e., the kink). The bunching mass is denoted B. By construction (see Saez 2010 and Kleven 2016 for graphical intuition), B equals $\int_{y^*}^{y^*+\Delta y^*} h_0(y)dy$, where $h_0(y)$ is the counterfactual (absent a kink) probability density function of earnings. We apply the standard approximation

$$B = \int_{y^*}^{y^*+\Delta y^*} h_0(y)dy \approx h_0(y^*)\Delta y^*. \tag{45}$$

Dividing through by y^*, we may write the (approximated) relative change in earnings of the marginal buncher as

$$\frac{\Delta y^*}{y^*} = \frac{B}{h_0(y^*)y^*} = \frac{b}{y^*}. \tag{46}$$

This is equation represents one of the central insights in the bunching literature, namely that the marginal buncher's earnings reduction caused by the kink is proportional to the excess mass at the kink.

We empirically estimate b, the relative excess mass at the threshold, using the methodology in Chetty et al. (2011), which we call the bunching estimate. The empirical analog of y^* is the (average) debt conversion threshold, expressed in the same units (thousands) as the empirical earnings bins.[17] We write our estimated compensated labor earnings elasticity as

$$\hat{e} = \frac{\hat{b}/y^*}{\widehat{\Delta\tau}/(1-\tau)}, \tag{47}$$

where $\widehat{\Delta\tau}$ is the estimated change in the effective nominal tax rate occurring at the debt-conversion threshold, and τ is the at-threshold after-tax keep rate of $1-\tilde{\tau} = 0.75$.

In a standard model without adjustment frictions, the estimator $\hat{e}$ is considered to estimate the Frisch elasticity (Saez 2010 and Kleven 2016). When preferences are additively separable as in our calibration (equation 40), this implies that $\hat{e}$ identifies the structural Frisch elasticity, k. However, this is not true in the presence of financial frictions and delayed taxation.

In our two-period model, the FOC for period 1 labor supply from equation (8) can be written as:

$$u'(c_1^i) \cdot w_1^i(1 - \tau_1(1-\delta)\Delta_r^i) = v'(\ell_1^i). \tag{48}$$

Differentiating this expression with respect to τ_1, keeping $u'(c_1)$ constant and value it at the threshold where $1-\delta = 0$, we obtain

$$\varepsilon_{\ell_1, 1-\tau_1}^{i,F} = \left(1 - (1-\delta)\Delta_r^i\right) \frac{v'(\ell_1^i)}{\ell_1^i v''(\ell_1^i)} = \left(1 - (1-\delta)\Delta_r^i\right) k, \tag{49}$$

where $\Delta_r^i = 1 - \frac{1+r}{R'(s^i)}$ is the interest rate wedge and k is the "structural" Frisch elasticity. In our empirical setting, the marginal tax is fully delayed, i.e., $1-\delta = 1$, and hence,

$$\varepsilon_{\ell_1, 1-\tau_1}^{i,F} = \left(1 - \Delta_r^i\right) k. \tag{50}$$

[17] Alternatively, we could multiply $\hat{B}$ and thus $\hat{b}$ by the width of the earnings bins (NOK 1,000), and let y^* equal the threshold in NOK.

Our estimator estimates a scaled-down structural Frisch elasticity, where the scaling depends on local average marginal interest rates. Furthermore, we allow our estimator to be biased downward by a factor of ζ due to, for example, labor supply adjustment frictions. We denote these factors as

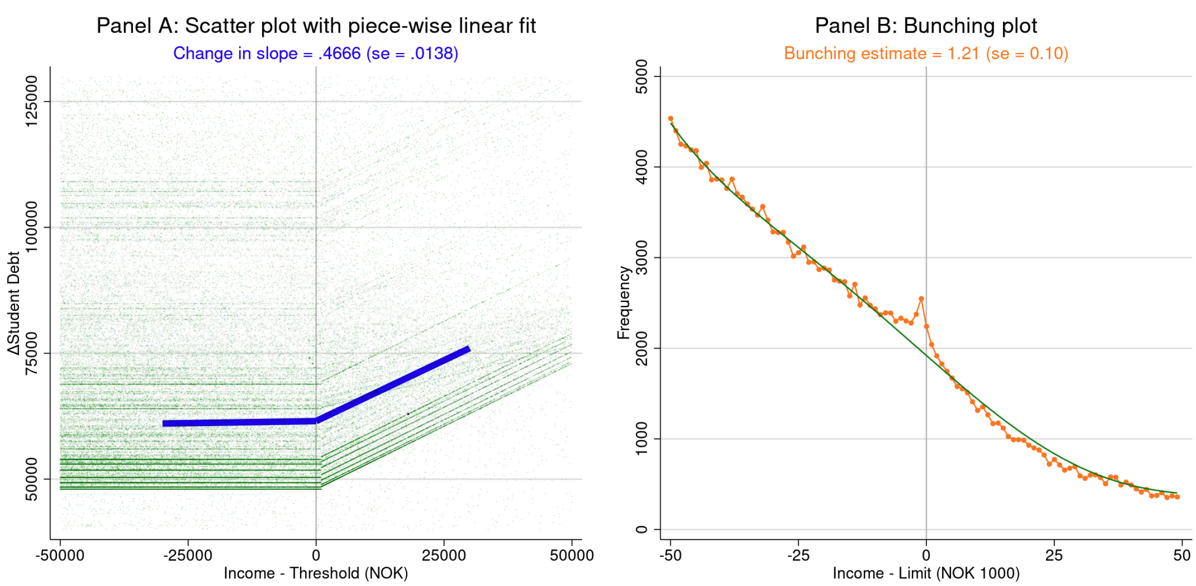
$$E[\hat{e}] = E[\zeta \varepsilon^{i,F}_{\ell_1,1-\tau_1}] = \underbrace{E\left[1-\Delta^i_r\right]}_{\text{Delayed taxation effect}} \cdot \underbrace{\zeta}_{\text{bias}} \cdot \underbrace{k}_{\text{structural Frisch}}. \tag{51}$$

5.3 Bunching at the debt-conversion threshold

Figure 7 summarizes the empirical analysis. Panel A verifies that earnings above the threshold lead to an increase in debt in the next period. Most students are on the expected kinked trajectory, where each additional NOK of earnings increases debt by NOK 0.50. The blue fitted line illustrates how we obtain our first-stage measure of the effect of excess earnings on debt accumulation. We find that the slope increases by 0.47. This is close to the nominal increase of 0.50 because there are very few non-compliers.[18] In terms of the previous notation, this means that $\widehat{\Delta\tau} = 0.47$.

FIGURE 7: VERIFYING THE EFFECT OF EXCESS EARNINGS ON FUTURE DEBT AND EXAMINING BUNCHING RESPONSES

Panel (A) shows a scatterplot in green of the relationship between debt accumulation and student earnings around the debt conversion threshold. The fitted blue line illustrates the estimate of the effect of earnings above the threshold and accumulated debt. Panel (B) provides a graphical illustration of the bunching estimate. The orange fitted line shows the actual distribution of students around the conversion threshold. The fitted green line shows the estimated counterfactual distribution. The bunching estimate provides the relative excess mass (actual versus counterfactual) of students near the threshold. This is done using the Stata .ado file provided by Chetty, Friedman, Olsen, and Pistaferri (2011). This program calculates the excess bunching between NOK -10,000 and NOK 6,000. Standard errors are calculated from bootstrapping (1,000 replications). All plots show statistics from the pooled sample years 2004-2011.



In Panel (B), the yellow dotted line shows the distribution of students around the earnings

[18] Some non-compliers exist, for example, because they may have moved in with their parents during the fall semester, which would exclude them from receiving a conversion for fall semester loans. Such moves must be reported to the Educational Loan Fund, but not to the tax authorities from which we receive address data.

threshold. The green line is the counterfactual distribution, which is a 5th order polynomial fitted to the non-bunching region. By comparing the actual and counterfactual distributions, we obtain a measure of the excess mass of individuals near the threshold. This provides a bunching estimate, b, of 1.21, which means that there are 121% more individuals around the threshold than the counterfactual distribution implies. Dividing 1.21 by the average threshold amount (120.162 in NOK 1,000s), per equation 47, we obtain a remarkably low elasticity of labor earnings to the net-of-tax (or net-of-debt-increase) wage of 0.0162.[19] The standard error is 0.0013.[20]

This analysis shows that students are remarkably unresponsive to de facto delayed taxation. We show that the results are virtually identical when considering students employed in likely highly flexible hospitality and sales positions in Figure A.2. We also find qualitatively similar results when we consider bunching around the conversion cap. Here, additional earnings no longer increase student debt because students are no longer eligible for *any* conversion from loans to stipends. We report these results in Figure A.1. We find that the bunching estimate is negative, in line with theory, but statistically close to zero (t-stat=-1.64).

5.4 Determinants of non-bunching

We now examine potential determinants of this (non-)bunching behavior. Our main approach is to plot students' characteristics against their position relative to the conversion threshold.[21] This is a visual exercise in which we try to draw conclusions from visual breaks in the relationship between a given characteristic and students' earnings that occur around the conversion threshold.

In Figure 8, Panel (A), we see that the amount of ex-ante liquid wealth drops sharply just above the threshold. This suggests that non-bunchers have less liquid assets, which is consistent with these students being financially constrained. Panel (B) of Figure 8 shows how future labor earnings vary with the student's position relative to the threshold. This shows no sharp increase or decrease in realized future earnings above the threshold, suggesting that non-bunchers do not differ significantly in terms of medium-term earnings prospects.

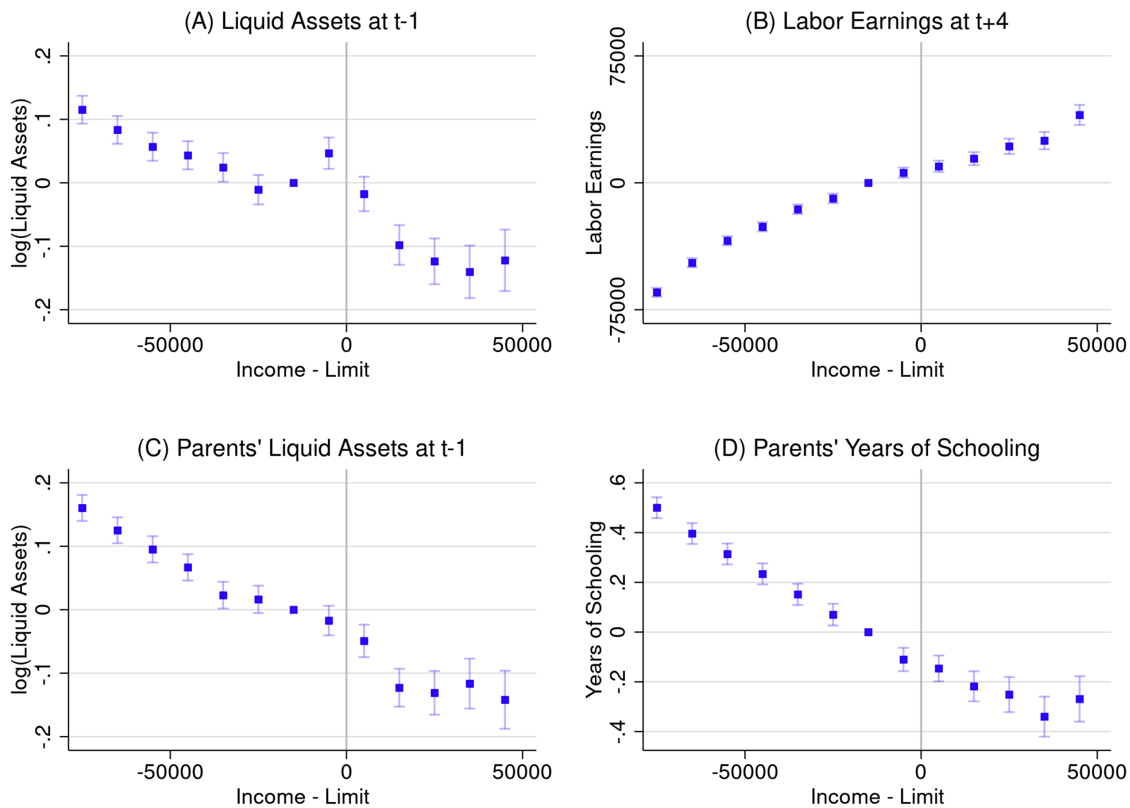
[19] These calculations do not adjust for the fact that any debt accumulated while in school is interest-free. Adjusting for a 3-year 3% interest discount would increase the elasticity by about 9%.

[20] We ignore the (very small) estimation error in $\widehat{\Delta\tau}$.

[21] Another application of this type of analysis can be found in the concurrent work of Bastani and Waldenström (2021), who examine how ability covaries with taxpayers' position relative to a regular tax threshold to infer the ability gradient in tax responsiveness.

Figure 8: Characteristics of Students Below and Above the Income-contingent Debt-conversion Threshold

The graphs below show the financial characteristics of students near the threshold. Panel A looks at students' liquid assets. These consist of deposits, stocks, bonds, and mutual fund holdings. Panel B shows future log labor earnings measured 4 years later. Panel C shows the amount of liquid assets held by the student's parents. Panel D shows the educational attainment of the parents, measured as the maximum number of years of schooling among the set of parents. Standard errors, used to provide 95% confidence intervals, are clustered at the student level.



Taken together, these results highlight financial frictions as a key channel driving the insensitivity to the conversion threshold. Those who earn above the threshold have similar future earnings prospects, but hold significantly less liquid assets. Holding fewer assets may both causally affect the extent to which agents are constrained and be a proxy for financial frictions, as it indicates a preference for smoothing consumption toward the present.

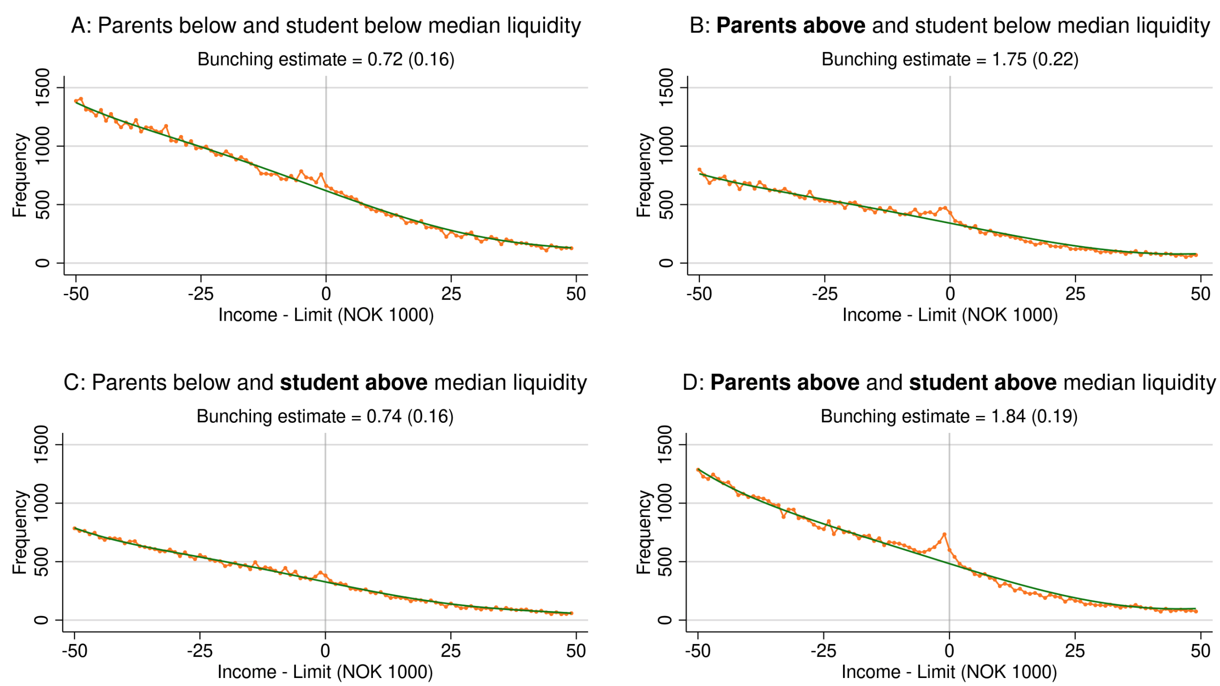
To investigate this liquidity channel further, in panel (C) of figure 8 we also show how *parents'* liquidity correlates with the student's earnings location. This documents a notable negative relationship between parents' financial resources and the child's in-school labor earnings. This suggests that parents play an important role in determining the amount of time students are able to devote to their studies. More relevant to the present study is the finding that parental wealth declines just above the earnings threshold. This suggests that non-bunchers have access to fewer financial resources, which is consistent with financial frictions playing a key role in the observed non-responsiveness to the conversion threshold. However, wealth may be a proxy for human capital, which influences tax responsiveness (Bastani and Waldenström, 2021). Therefore, we plot parental education on the y-axis in panel (D). This shows that there is no break in the relationship between educational attainment, measured by the maximum number of years of schooling of the parents, and the position of the child relative to the conversion threshold. This argues against the hypothesis that fewer resources, in a human capital rather than a financial sense, can explain the irresponsiveness to the threshold. If anything, extrapolating from the relationship below the threshold, non-bunchers may have more highly educated parents. To the extent that this is correlated with students' lifetime wealth, it may explain some of the students' desire to front-load consumption by taking on higher student loans.

5.5 Bunching heterogeneity

We proceed with a complementary, more standard approach to examine the heterogeneity in the earnings sensitivity to the threshold in Figure 9. This approach splits the sample into subsets based on student and parental characteristics to compute heterogeneous bunching elasticities. We see that the largest contribution to the total excess mass in the previous figure 7 comes from students who themselves and their parents have above-average liquid assets. Figure 9 also suggests that the main driver of bunching responses is parents' rather than students' own liquid assets. Moving from the left to the right panel, which improves parental liquidity, more than doubles the bunching estimates.[22]

Figure 9: Heterogeneity in Bunching by Amount of Liquid Assets

These plots calculate the bunching elasticity for different subsamples. Students are divided into four subsamples based on whether their and their parents' $\text{LiquidAssets}_{t-1}$ are below or above the median. These medians are computed separately for each year in the sample.



What can this heterogeneity tell us about how the severity of financial constraints varies

[22]In this case, it doesn't matter whether we compare the excess mass in terms of students or earnings, since the bin widths and thresholds are the same.

with liquidity? In Figure 9, we see that the elasticity increases from 0.72 to 1.84 as we move from below to above the median for both student and parent resources. Using our expression (51) for the expectation of the estimator $\hat{e}$, we can write

$$\frac{1.84}{0.72} = \frac{\mathbb{E}[\hat{e}_{\text{above}}]}{\mathbb{E}[\hat{e}_{\text{below}}]} = \frac{E_{\text{below}}\left[(1-\tau_1)(1-\Delta_r^i)\right] \quad \cdot \quad \zeta \quad \cdot k}{E_{\text{below}}\left[(1-\tau_1)(1-\Delta_r^i)\right] \quad \cdot \quad \zeta \quad \cdot k}. \tag{52}$$

From this expression, we get that

$$\frac{1.84}{0.72} = \frac{E_{\text{below}}[1-\Delta_r^i]}{E_{\text{below}}[1-\Delta_r^i]} = \frac{E_{\text{below}}[R'(s^i)]}{E_{\text{below}}[R'(s^i)]}, \tag{53}$$

Given an average maturity for these loans of about 10 years, we find that the gross annual interest rate $(1+r_b)$ is $(1.84/0.72)^{\frac{1}{10}} = 1.0984$ times greater for the below-median liquidity group, roughly a 10 percentage point difference. This is a substantial difference in marginal borrowing rates. While our theoretical framework allows us to calculate implied differences in marginal interest rates, we cannot derive them directly from the data. First, while we can calculate average interest rates on debt, we do not observe marginal interest rates. For example, in practice, students may have a choice between not borrowing and accumulating credit card debt at interest rates close to 20%. If the marginal rate at which they would borrow is 10%, these students would borrow 0, and thus we would not observe any (realized) interest rates for them.

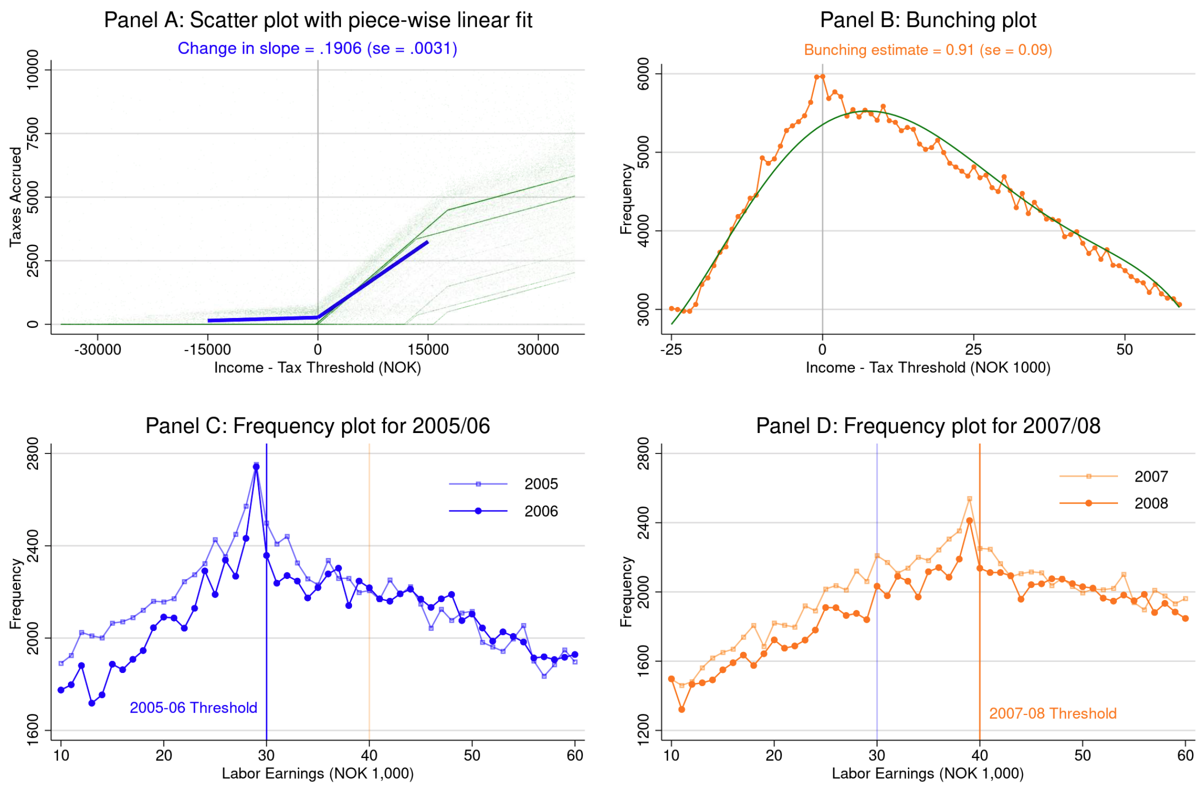
5.6 Analysis of bunching at a regular tax threshold

In this section, we repeat the introductory analyses performed in Figure 7 using a *tax* threshold instead of the debt conversion threshold. The purpose of this exercise is to obtain a reference estimate of the implied labor earnings elasticity at a tax threshold at which marginally accrued taxes are not delayed. We focus on the first tax threshold in the progressive income tax system. This threshold was NOK 30,000 in 2005-06 and NOK 40,000 in 2007-2011.[23] At this threshold, the marginal income tax increases from 0 to about 25 percent for most taxpayers.

[23] We omit 2004. In that year the threshold was only NOK 23,000, which significantly reduces the size of the left tail we can use to estimate a counterfactual distribution.

Figure 10: Bunching at a Regular Tax Threshold

The first and second plots show the relationship between labor income ("pensionable income") and taxes accrued in that year (payable in the same or next year) in the form of a scatterplot and binscatterplot, respectively. The third plot shows the distribution of students around the income tax threshold. The fourth plot calculates the bunching elasticity in terms of the implied excess fraction of students in the NOK 1,000 bin directly to the left of the threshold using the Stata .ado file provided by Chetty, Friedman, Olsen, and Pistaferri (2011). This program calculates the excess bunching between NOK -10,000 and NOK 6,000. Standard errors are calculated from bootstrapping (N=1,000). All plots are statistics from the pooled sample years.



In Figure 10 we examine this complementary empirical setting. Panel (A) provides a scatterplot that verifies the presence of an increase in the marginal income tax rate by plotting total taxes accrued in the year against income. It also provides the fitted line, from which we infer an average increase in the marginal tax rate of 19 percentage points at the threshold. The coefficient is lower than the nominal increase of 25 percentage points because some individuals may be eligible for higher standard deductions.

Panel (B) illustrates how the bunching estimate of $b = 0.91$ is calculated. While this bunching estimate is smaller than that found at the debt conversion threshold, this one-to-one comparison is uninformative for two reasons. First, we must divide 0.91 by the threshold (36.706 in NOK 1,000) to obtain a relative reduction in earnings for the marginal buncher of 2.48%. This is already larger than the reduction we found at the debt conversion threshold of 1.00% (1.21 divided by 120.162). Second, we need to take into account the fact that this is in response to a smaller increase in the marginal (nominal) tax rate. Dividing 2.48% by the relative reduction in the after-tax rate of 19.6%/100%, we get a more substantial elasticity of 0.13.

In Panel (B), we see that the bunching mass occurs at the mode of the distribution. If the location of the mode is not driven by students' responses to the tax threshold, then the co-location of the mode and the threshold could lead to an upward bias in the bunching estimate. To address this concern, we show in panels (C) and (D) that the location of the mode is driven by the location of the tax threshold. From 2005 to 2006 and from 2007 to 2008, there was no change in the mode of the distribution. However, when the tax threshold increased from 2006 to 2007, the mode followed exactly. This reassures us that there is indeed substantial responsiveness to the tax threshold that is not driven by a coincidental co-location of the mode and the threshold.

The elasticity of 0.13 is eight times greater than the elasticity of 0.0162 found in the analysis of responsiveness to the debt conversion threshold. For these differences to be consistent with the same structural Frisch elasticity, k, we need

$$\frac{0.13}{0.0162} = \frac{E_{\text{delayed}}\left[(1-(1-0)\Delta_r^i)\right]\ \zeta\ k}{E_{\text{regular}}\left[(1-(1-1)\Delta_r^i)\right]\ \zeta\ k} = (1 - E_{\text{delayed}}[\Delta_r^i]). \tag{54}$$

which implies an average annual interest rate over 10 years of $\left(\frac{0.13}{0.162}\right)^{\frac{1}{10}} - 1 = 23\%$. This figure is comparable to average credit card rates, which are slightly above 20%.[24] This suggests that some students are willing to borrow from the Education Loan Fund at an interest rate higher than that offered by financial institutions. This may be partly due to credit rationing, but probably mainly due to the fact that the loan fund does not require payments while students are still in school, and generally has a long maturity, with the additional option of delaying payments for up to three years.

We can use this implied elasticity to get an idea of how much bunching would be caused by the debt conversion threshold in the absence of financial frictions. In other words, how much bunching would there be in Figure 7 if students responded to the debt conversion threshold as if it were a regular income tax threshold? To find out, we reverse the calculation used to derive labor supply elasticities from the bunching estimates. This yields a counterfactual bunching estimate of 23.43.[25] This is considerably larger than the empirical bunching estimate of 1.21.

[24]Source: Statistics Norway's Statistics on Interest Rates in Banks and Credit Institutions, source table 12844, 2019Q4: 21.6%

[25]=0.13*(120162/1000)*(75/50)

Figure 11: Contrasting Characteristics of Bunchers at the Delayed Tax and Regular Tax Thresholds

This figure shows how financial characteristics vary across the delayed tax threshold (blue squares) and the regular tax threshold (orange triangles). Panel A looks at the propensity to take out unsecured loans, defined as having interest expenses of more than NOK 1,000. For this sample, we exclude students who, according to the tax authorities, own a house, car or boat. Panel B considers log liquid assets, where liquid assets consist mainly of deposits, but also stocks and bonds. For each threshold, using observations for 2007-2011, we regress the y-variable on earnings-bin fixed effects. We control for year fixed effects and third-order polynomials in the student's age and (max) years of parental education. The bin width is NOK 2000 and is based on the distance between labor earnings and the applicable threshold.

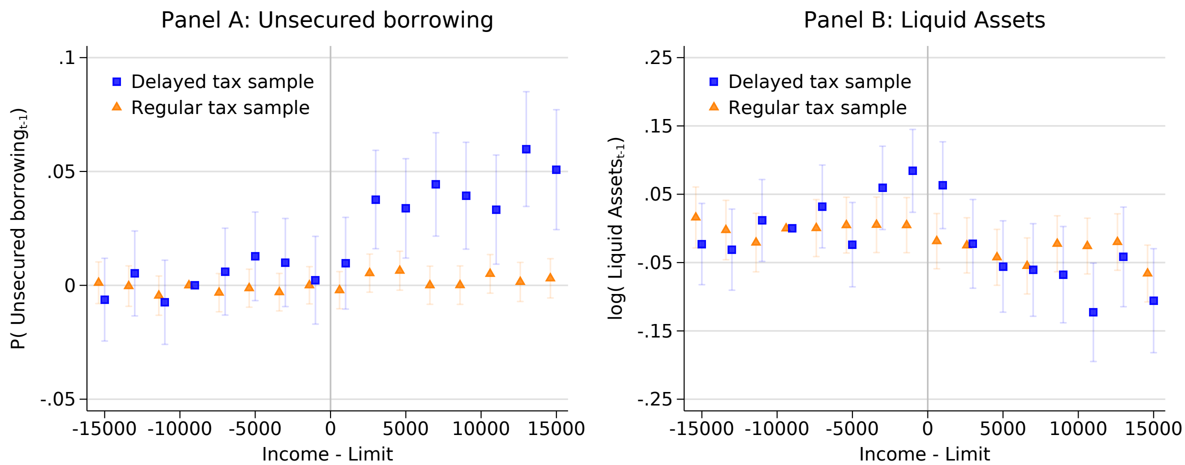


Figure 8 shows that those who bunch at the *delayed tax* threshold have more liquid wealth. This is intuitive because more constrained students have higher marginal borrowing rates on average and are thus less sensitive to a tax that is payable in the future. However, it is useful to show that proxies for financial constraints matter more for delayed tax bunching than for regular tax bunching to rule out the possibility that financial frictions are simply correlated with labor supply adjustment frictions. Accordingly, we empirically investigate whether regular-tax bunchers also appear to be more or less constrained, and we contrast this with the characteristics of bunchers at the delayed-tax threshold in Figure 11.

Panel A shows that the delayed tax non-bunchers are significantly more likely to take on unsecured debt, such as credit cards or consumer loans. In the regular tax sample, however, there does not appear to be a systematic relationship between whether someone bunches and their unsecured borrowing. Panel B looks at liquid assets. In the delayed tax sample, we see that bunchers have more liquidity and that those earning above the threshold have significantly less liquid assets. In the regular tax sample, we do not see strong deviations for bunchers, but moving to the right, we see that non-bunchers appear to have less liquid assets.

5.7 Accounting for differences in observables when comparing elasticities.

In this section, we pool the samples used to examine bunching at the debt conversion (delayed tax) and regular tax thresholds. We develop a regression-based approach that allows us to compare the underlying elasticities while holding observables fixed.[26] This addresses the fact that higher earning students in the debt-conversion sample may have different characteristics than those in the lower earning regular tax sample. We want to address the fact that differences in observable characteristics, such as occupation, may partially explain differences in bunching behavior due to, for example, differences in labor supply adjustment frictions.

We first define the individual-level elasticity as

$$e_i = \left[\underbrace{\frac{1[y_i \in BR_s] - \hat{P}^{cf}(y_i \in BR_s)}{\hat{P}^{cf}(y_i \in BR_s)/N_s^{\text{BR bins}}}}_{b_i} \cdot 1 \Big/ y_s^* \right] \Big/ (\widehat{\Delta \tau_s}/(1-\tau_s)), \tag{55}$$

where $\hat{P}^{cf}$ denotes the estimated (counterfactual) probabilities of being in the bunching region in the absence of any tax or debt conversion kinks. This is estimated using the frequencies in the earnings bins around the bunching region as in Saez (2010). $N_s^{\text{BR bins}}$ is the number of bins in the bunching region. b_i as defined by the curly braces is the individual-level excess mass, the sample-specific mean of which provides an estimate of the relative excess mass at the given threshold (see Appendix C.2). By multiplying further by the ratio of bin widths to the mean threshold value and dividing by the relative change in the after-tax wage, we obtain e_i. The s sample mean, $\hat{E}$, of e_i provides an estimate of the implied earnings elasticity. For the delayed tax sample, this mean is about 0.0155, which is very close to our baseline estimate of 0.0162.[27]

We then estimate regression equations of the following form.

$$e_i = \alpha + \beta \mathbb{1}[\text{regular tax sample}]_i + \gamma' X_i + \varepsilon_i, \tag{56}$$

where y_i is the individual's labor earnings and X_i is a vector of individual-level observables, such as the worker's 4-digit occupation code, if available. We report the results of varying the contents of X_i in Table 4. To find the estimated relative increase in e in the regular versus delayed tax samples, we divide $\hat{\beta}$ by the delayed-tax sample mean of e_i.

The main finding is that the relative difference in labor supply elasticities is about 7.20 (CI = [6.04, 8.36]) once we control for sex, age, parental education, and 4-digit occupation fixed effects. When we include narrower 2-digit industry interacted with 4-digit occupation code fixed effects, the relative difference is 6.10 (CI = [4.90, 7.30]). This is slightly less than the relative difference obtained by simply contrasting the implied elasticities from the bunching analyses, but the qualitative implications are the same: to rationalize a 6.1 times higher elasticity, we need an average marginal interest rate of 19.82%=$6.1^{1/10}$-1.

[26]See Ring and Thoresen (2021) for a related method that uses regressions of a bunching indicator on observables to infer bunching heterogeneity.

[27]The new estimate for the regular tax sample is about 0.2, which is larger than our baseline estimate for the regular tax threshold of 0.13. However, the graphical evidence in Figure 10 shows that this is likely to be a very conservative estimate. Differences arise because in the regression-based approach we take the simpler approach of estimating P^{cf}s using the observed number of observations in the two income bins just below and the two income bins just above the bunching region, BR_s (as in Saez 2010), rather than estimating a higher order polynomial (as in Chetty et al. 2011).

Table 4: Regression-based Approach to Account for Differences on Observables in Delayed and Regular Tax Samples

This table presents the results of the regression-based approach to comparing labor supply elasticities in the delayed and regular tax samples. The estimated relative elasticity difference is calculated as the coefficient on 1[regular tax sample] divided by $\hat{E}[e_i \mid s = delayed]$. We only keep observations for which we observe an employer-employee relationship, and thus can assign NACE and occupation codes based on the student's highest paid job within the year. Standard errors are shown in parentheses.

	(1)	(2)
Estimated Relative Difference in Elasticity		
$\widehat{\frac{e_{regular} - e_{delayed}}{e_{delayed}}}$	7.20	6.10
	(.59)	(.61)
Underlying Regression Coefficients		
1[regular tax sample]	0.0969***	0.0787***
	(0.0093)	(0.0094)
Male	0.0360***	0.0414***
	(0.0100)	(0.0102)
Age	-0.0434***	-0.0410***
	(0.0022)	(0.0022)
College, parents	0.0501**	0.0442**
	(0.0204)	(0.0205)
Years of schooling, parents	0.0056	0.0070**
	(0.0035)	(0.0036)
N	393443	390177
R2	0.01	0.02
$\widehat{E}[e_i \mid s = regular]$	0.2031	0.2032
$\widehat{E}[e_i \mid s = delayed]$	0.0156	0.0154
FEs	4-Digit Occ	4-Digit Occ × NACE2

We note that the methodology developed here can be used in other settings where one wishes to compare responses to different kinks. It is important to note that the standard definition of relative excess mass, b, produces a measure that is not invariant with respect to bin width. The implied elasticity, however, is invariant because y^* is expressed in bin-width units.

6 Discussion

When workers are unable to borrow against future earnings, the timing of cash flows (and especially tax payments) becomes an important policy tool. This is recognized in the existing literature on age-dependent taxation, but not fully exploited. We propose a simple policy tool, delayed taxation, that both addresses the welfare losses from credit market imperfections and exploits them by reducing the distortionary effects of income taxation. Delayed taxation allows agents to delay the payment, but not the accrual, of income taxes. This decoupling is a novel feature of our study. From a horizontal equity perspective, it is attractive in that substantial welfare gains can be achieved without conditioning tax rates on taxpayer characteristics such as age.

Our numerical analyses show that the welfare gains can be substantial. While we emphasize the benefits of delayed taxation over age-dependent taxation, this is simply because age-dependent taxation provides a natural benchmark. Rather, our quantitative analyses emphasize the (absolute) benefits of age-dependent taxation and its relative benefits compared to reducing financial frictions through a government credit program. Overall, the results highlight delayed taxation as a promising new tool in optimal taxation and a fertile ground for further theoretical and empirical research. Towards the latter, we make some progress by studying a *de-facto* delayed tax scheme affecting young workers in Norway. The empirical results confirm one of the key mechanisms in our model, namely that delayed taxation reduces the distortionary effects of income taxation when workers are financially constrained.

References

Akerlof, G. A. (1978): "The economics of tagging as applied to the optimal income tax, welfare programs, and manpower planning," *The American Economic Review*, 68, 8–19.

Andreoni, J. (1992): "IRS as loan shark tax compliance with borrowing constraints," *Journal of Public Economics*, 49, 35–46.

Atkinson, A. B. and J. E. Stiglitz (1980): *Lectures on Public Economics*, McGraw-Hill Book Company.

Bachas, N., O. S. Kim, and C. Yannelis (2021): "Loan guarantees and credit supply," *Journal of Financial Economics*, 139, 872–894.

Bäckman, C., P. van Santen, et al. (2020): *The Amortization Elasticity of Mortgage Demand*, Aarhus BSS, Aarhus University, Department of Economics and Business Economics.

Bastani, S., S. Blomquist, and L. Micheletto (2013): "The Welfare Gains of Age-Related Optimal Income Taxation," *International Economic Review*, 54, 1219–1249.

Bastani, S. and H. Selin (2014): "Bunching and non-bunching at kink points of the Swedish tax schedule," *Journal of Public Economics*, 109, 36–49.

Bastani, S. and D. Waldenström (2021): "The ability gradient in tax responsiveness," *Journal of Public Economics Plus*, 2, 100007.

Bernstein, A. (2021): "Negative home equity and household labor supply," *Journal of Finance, forthcoming.*

Best, M. C., J. Cloyne, E. Ilzetzki, and H. Kleven (2018): "Estimating the elasticity of intertemporal substitution using mortgage notches," Tech. rep., National Bureau of Economic Research.

Blomquist, S. and L. Micheletto (2008): "Age-Related Optimal Income Taxation," *The Scandinavian Journal of Economics*, 110, 45–71.

Brown, J. and D. A. Matsa (2020): "Locked in by leverage: Job search during the housing crisis," *Journal of Financial Economics*, 136, 623–648.

Bruze, G., A. K. Hilsløv, and J. Maibom (2024): "The Long-Run Effects of Individual Debt Relief," Tech. rep., CESifo.

Chetty, R., J. N. Friedman, T. Olsen, and L. Pistaferri (2011): "Adjustment costs, firm responses, and micro vs. macro labor supply elasticities: Evidence from Danish tax records," *The Quarterly Journal of Economics*, 126, 749–804.

Choukhmane, T. and C. Palmer (2023): "How Do Consumers Finance Increased Retirement Savings?" *Unpublished Working Paper.*

Dávila, E. and B. M. Hébert (2019): "Optimal Corporate Taxation Under Financial Frictions," *NBER Working Paper.*

de Silva, T. (2023): "Insurance versus Moral Hazard in Income-Contingent Student Loan Repayment," *Available at SSRN 4614108.*

DeFusco, A. A., S. Johnson, and J. Mondragon (2020): "Regulating household leverage," *The Review of Economic Studies*, 87, 914–958.

DeFusco, A. A. and A. Paciorek (2017): "The interest rate elasticity of mortgage demand: Evidence from bunching at the conforming loan limit," *American Economic Journal: Economic Policy*, 9, 210–40.

Donaldson, J. R., G. Piacentino, and A. Thakor (2019): "Household debt overhang and unemployment," *The Journal of Finance*, 74, 1473–1502.

Doornik, B. F. N. V., A. R. Gomes, D. Schoenherr, and J. Skrastins (2021): "Financial Access and Labor Market Outcomes: Evidence from Credit Lotteries," *Available at SSRN 3800020.*

Eaton, J. and H. S. Rosen (1980): "Optimal redistributive taxation and uncertainty," *The Quarterly Journal of Economics*, 95, 357–364.

Fack, G. and C. Landais (2016): "The effect of tax enforcement on tax elasticities: Evidence from charitable contributions in France," *Journal of Public Economics*, 133, 23–40.

Ferriere, A., P. Grübener, G. Navarro, and O. Vardishvili (2023): "On the optimal design of transfers and income tax progressivity," *Journal of Political Economy Macroeconomics*, 1, 276–333.

Finkelstein, A. and N. Hendren (2020): "Welfare analysis meets causal inference," *Journal of Economic Perspectives*, 34, 146–167.

Gervais, M. (2012): "On the optimality of age-dependent taxes and the progressive US tax system," *Journal of Economic Dynamics and Control*, 36, 682–691.

Golosov, M. and A. Tsyvinski (2015): "Policy implications of dynamic public finance," *Annual Review of Economics*, 7, 147–171.

Gopalan, R., B. H. Hamilton, J. Sabat, and D. Sovich (2021): "Aversion to student debt? Evidence from low-wage workers," *Evidence from low-wage workers (May 11, 2021).*

Heathcote, J. (2005): "Fiscal policy with heterogeneous agents and incomplete markets," *The Review of Economic Studies*, 72, 161–188.

Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal tax progressivity: An analytical framework," *The Quarterly Journal of Economics*, 132, 1693–1754.

——— (2020): "Optimal progressivity with age-dependent taxation," *Journal of Public Economics*, 189, 104074.

Hendren, N. and B. Sprung-Keyser (2022): "The case for using the MVPF in empirical welfare analysis," Tech. rep., National Bureau of Economic Research.

Herbst, D. and N. Hendren (2023): "Opportunity Unraveled: Private Information and the Missing Markets for Financing Human Capital," *NBER Working Paper 29214. Updated May 2023.*

Hubbard, R. G., K. L. Judd, R. E. Hall, and L. Summers (1986): "Liquidity constraints, fiscal policy, and consumption," *Brookings Papers on Economic Activity*, 1986, 1–59.

Kleven, H. J. (2016): "Bunching," *Annual Review of Economics*, 8, 435–464.

Kostøl, A. and A. Myhre (2020): "Labor supply responses to learning the tax and benefit schedule," *IZA Discussion Paper.*

Le Barbanchon, T. (2020): "Taxes Today, Benefits Tomorrow," Tech. rep., Working Paper.

Lockwood, B. B. (2020): "Optimal income taxation with present bias," *American Economic Journal: Economic Policy*, 12, 298–327.

Lozachmeur, J.-M. (2006): "Optimal age-specific income taxation," *Journal of Public Economic Theory*, 8, 697–711.

Ndiaye, A. (2020): "Flexible retirement and optimal taxation," *Working Paper.*

Ring, M. and T. O. Thoresen (2021): "Wealth Taxation and Charitable Giving," *Working Paper.*

Saez, E. (2010): "Do taxpayers bunch at kink points?" *American Economic Journal: Economic Policy*, 2, 180–212.

Seim, D. (2017): "Behavioral responses to wealth taxes: Evidence from Sweden," *American Economic Journal: Economic Policy*, 9, 395–421.

Sheshinski, E. (1972): "The Optimal Linear Income-tax," *The Review of Economic Studies*, 39, 297–302.

SMITH, P. (1991): "Lessons from the British Poll Tax Disaster," *National Tax Journal*, 44, 421–436.

SØGAARD, J. E. (2019): "Labor supply and optimization frictions: Evidence from the danish student labor market," *Journal of Public Economics*, 173, 125–138.

STANTCHEVA, S. (2020): "Dynamic Taxation," *Annual Review of Economics*, 12, 801–831.

VARIAN, H. R. (1980): "Redistributive taxation as social insurance," *Journal of public Economics*, 14, 49–68.

WEINZIERL, M. (2011): "The surprising power of age-dependent taxes," *The Review of Economic Studies*, 78, 1490–1518.

YU, P. C. (2021): "Optimal retirement policies with present-biased agents," *Journal of the European Economic Association*, 19, 2085–2130.

ZATOR, M. (2019): "Working More to Pay the Mortgage: Household Debt, Consumption Commitments, and Labor Supply," *Working paper*.

A Proofs

A.1 Proof of Proposition 1

Assuming $\tau_1 = \tau_2 = \tau$, differentiating the Lagrangian of the government's optimization problem with respect to τ and setting it equal to zero, yields

$$-\sum_i \pi^i \Big(\delta u_1'(\cdot) y_1^i + \Big([1-\delta](1+r)y_1^i + y_2^i\Big) u_2'(\cdot)\beta\Big) + \lambda \sum_i \pi^i \left(y_1^i + \tau \frac{dy_1^i}{d\tau} + \frac{1}{1+r}\left[y_2^i + \tau \frac{dy_2^i}{d\tau}\right]\right) = 0. \tag{A1}$$

Substituting in for g_t^i and rearranging yields

$$-\sum_i \pi^i \Big(\delta g_1^i y_1^i + \Big([1-\delta]y_1^i + \frac{1}{1+r} y_2^i\Big) g_2^i\Big) + \sum_i \pi^i \left(y_1^i + \tau \frac{dy_1^i}{d\tau} + \frac{1}{1+r}\left[y_2^i + \tau \frac{dy_2^i}{d\tau}\right]\right) = 0. \tag{A2}$$

Setting $\varepsilon_{t,\tau}^i = \frac{1-\tau}{y_t^i}\frac{dy_t^i}{d(1-\tau)} = -\frac{1-\tau}{y_t^i}\frac{dy_t^i}{d\tau}$, we may rewrite as

$$\sum_i \pi^i \Big(\delta g_1^i y_1^i + \Big([1-\delta]y_1^i + \frac{1}{1+r} y_2^i\Big) g_2^i\Big) = \sum_i \pi^i \left(y_1^i + \frac{1}{1+r} y_2^i - \frac{\tau}{1-\tau} y_1^i \varepsilon_{1,1-\tau}^i - \frac{1}{1+r}\frac{\tau}{1-\tau} y_2^i \varepsilon_{2,1-\tau}^i \right). \tag{A3}$$

Equation (23) follows by re-arrangement and using the operator $\mathbb{E}_t[x] = \sum_i \pi^i y_t^i x,\ t = 1, 2$. Condition (25) follows by noting that the the FOC for G can be written as:

$$\sum_i \pi^i \Big(u'(c_1) + \beta u'(c_2)\Big) = \lambda \sum_i \pi^i \left(1 + \frac{1}{1+r} - \tau \frac{dy_1^i}{dG} - \tau \frac{1}{1+r}\frac{dy_2^i}{dG}\right). \tag{A4}$$

A.2 Proof of Proposition 2

Forming the Lagrangian expression of the government optimization problem defined above and letting λ denote the multiplier attached to the government's budget constraint, the first-order condition with respect to τ_1 is

$$-\sum_i \pi^i y_1^i \Big(\delta u_1'(\cdot) + [1-\delta]u_2'(\cdot)\beta(1+r)\Big) + \lambda \sum_i \pi^i \left[y_1^i + \tau_1 \frac{dy_1^i}{d\tau_1} + \frac{\tau_2}{1+r}\frac{dy_2^i}{d\tau_1}\right] = 0, \tag{A5}$$

where the envelope theorem is invoked on the utility terms, V_i. Let $\varepsilon_1^i = \frac{1-\tau_1}{y_1^i}\frac{dy_1^i}{d(1-\tau_1)}$ and $\varepsilon_{2,1}^i = \frac{1-\tau_1}{y_2^i}\frac{dy_2^i}{d(1-\tau_1)}$. We can then write:

$$-\sum_i \pi^i y_1^i \Big(\delta u_1'(\cdot) + [1-\delta]u_2'(\cdot)\beta(1+r)\Big) + \lambda \sum_i \pi^i y_1^i \Big[1 - \frac{\tau_1}{1-\tau_1}\varepsilon_1^i - \frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^i\Big] = 0. \tag{A6}$$

Reorganizing and using $\mathbb{E}_1[x] = \sum_i \pi^i y_1^i x$ and the definition of g_t^i, $t = 1, 2$, we can rewrite as

$$-\mathbb{E}_1\left[\delta g_1^i + [1-\delta]g_2^i\right] + \mathbb{E}_1\left[1 - \frac{\tau_1}{1-\tau_1}\varepsilon_{1,1-\tau_1}^i - \frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^i\right] = 0. \tag{A7}$$

Alternatively, we may write it as

$$\frac{\tau_1}{1-\tau_1} = \frac{\mathbb{E}_1\left[1 - \delta g_1^i + [1-\delta]g_2^i\right]}{\mathbb{E}_1\left[\varepsilon_1^i\right]} - \frac{\tau_2}{1-\tau_1}\frac{\mathbb{E}_1\left[\frac{y_2^i}{y_1^i}\varepsilon_{2,1}^i\right]}{\mathbb{E}_1\left[\varepsilon_1^i\right]} = 0. \tag{A8}$$

The formula for τ_2 is directly derived from the first order condition:

$$-\sum_i \pi^i y_2^i \beta u_2'(\cdot) + \lambda \sum_i \pi^i \left[\tau_1 \frac{dy_1}{d\tau_2} + \frac{1}{1+r}\left(y_2^i + \tau_2 \frac{dy_2^i}{d\tau_2}\right)\right] = 0, \tag{A9}$$

which we may rewrite as

$$-\frac{1}{1+r}\mathbb{E}_1\left[\frac{y_2^i}{y_1^i}g_2^i\right] + \mathbb{E}_1\left[-\frac{\tau_1}{1-\tau_2}\varepsilon_{1,1-\tau_2}^i + \frac{1}{1+r}\left(\frac{y_2^i}{y_1^i} - \frac{\tau_2}{1-\tau_2}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_2}^i\right)\right] = 0. \tag{A10}$$

The conditions for G_1 and G_2 follow from the first-order conditions for G_1 and G_2 which are:

$$\sum_i \pi^i u_1'(\cdot) = \lambda \sum_i \pi^i \left[1 - \tau_1 \frac{dy_1^i}{dG_1} - \frac{1}{1+r}\tau_2 \frac{dy_2^i}{dG_1}\right], \tag{A11}$$

$$\beta \sum_i \pi^i u_2'(\cdot) = \lambda \sum_i \pi^i \left[\frac{1}{1+r} - \tau_1 \frac{dy_1^i}{dG_2} - \frac{1}{1+r}\tau_2 \frac{dy_2^i}{dG_2}\right]. \tag{A12}$$

These expressions may be further simplified under the assumption that $R'(s^i)$ is well defined, i.e., that $s^i \neq 0$. In that case, changing G_2 by dG_2 is equivalent to changing G_1 by the agent's present value of dG_2. Hence, $R'(s^i)\frac{dy_t^i}{dG_2} = \frac{dy_t^i}{dG_1}$. Hence, the LHS of the last equation becomes

$$\beta \sum_i \pi^i u_2'(\cdot) = \lambda \sum_i \pi^i \left[\frac{1}{1+r} - \left(\tau_1 \frac{dy_1^i}{dG_1} + \frac{1}{1+r} \tau_2 \frac{dy_2^i}{dG_1} \right) \frac{1}{R'(s^i)} \right]. \tag{A13}$$

Now define $\rho^i = \frac{d}{dG_1}\left(\tau_1 y_1^i + \frac{\tau_2 y_2^i}{1+r}\right)$ as an income effect parameter that provides the change in present-value tax revenues from increasing period-1 unearned income.

A.3 Proof of Lemma 1

We assume that $R'(s^i)$ is well-defined and constant for marginal changes in economic incentives. When using the piecewise-linear parametric formulation of $R(s)$ where the marginal interest rate is higher for $s < 0$, we require that $s^i \neq 0$. Under these assumptions, substituting the Euler equation into the FOC for ℓ_1 and re-organizing the life-time budget constraint reveals that δ and τ only enter in a multiplicative manner, which implies that their effect on labor supply is closely related. More formally, we start with the FOC for ℓ_1. Define $\bar{\delta}^i = \delta + (1-\delta)\frac{1+r}{R'(s^i)}$. Using equation (D11), we can write:

$$\frac{d\ell_1^i}{d(\bar{\delta}^i \tau_1)} = \frac{u'(c_1^i)w_1^i \left[1 + \left(1 - \frac{[w_2^i(1-\tau_2)]^2 u''(c_2^i)}{v''(\ell_2^i)}\right) \frac{u''(c_1^i)}{u''(c_2^i)} \frac{1}{\beta R'(s^i)^2}\right] - \ell_1^i w_1^i u_1''(c_1^i) w_1^i \left(1 - \tilde{\tau}_1^i\right)}{v''(\ell_1^i)\left[1 + \left(1 - \frac{[w_2^i(1-\tau_2)]^2 u''(c_2^i)}{v''(\ell_2^i)}\right) \frac{u''(c_1^i)}{u''(c_2^i)} \frac{1}{\beta R'(s^i)^2}\right] - u_1''(c_1^i) w_1^{i2} \left(1 - \tilde{\tau}_1^i\right)^2}. \tag{A14}$$

From this, we see that marginal changes in δ and τ_1 only affect l_1 through the term $\tilde{\tau}_1^i = \tau_1[1 - (1-\delta)\Delta_r^i]$. By allowing *either* δ or τ_1 to vary, we obtain expressions for $\frac{d\ell_1}{d(1-\tau_1)}$ and $\frac{d\ell_1}{d(1-\delta)}$ that directly lead to (30).

The proof of (31) has the following steps. We substitute the Euler equation (6) into the period-1 intratemporal FOC (8) to obtain an expression that relates ℓ_1 and ℓ_2. We differentiate this to obtain an expression that relates $d\ell_1$ and $d\ell_2$. This allows us to substitute out $d\ell_1$ in (A14) and replace it with an expression for $d\ell_2$. Then the above logic applies, since marginal changes in δ and τ_1 only affect l_2 through the term $\tilde{\tau}_1^i$.

We may also note that the optimal solution $(c_1, c_2, \ell_1, \ell_2)$ to the individual's problem is given by the solution to the following set of equations:

$$u_1'(c_1)w_1^i\left(1 - \tau_1\bar{\delta}^i\right) = v'(\ell_1) \tag{A15}$$

$$\frac{u_1'(c_1)}{\beta(1+r_b)} w_2^i[1-\tau_2] = v'(\ell_2) \tag{A16}$$

$$c_1 + \frac{c_2}{1+r_b} = w_1\ell_1(1-\tau_1\bar{\delta}^i) + \frac{w_2\ell_2(1-\tau_2) + G_2}{1+r_b}. \tag{A17}$$

The first condition is just (8), obtained by inserting the intertemporal FOC (6) into (4), the second condition is obtained by inserting (6) into (5), and the third constraint is just the life-time budget constraint.[28] Thus, the optimal individual allocation (and any comparative statics exercise) only depends on τ_1 and δ through the term $\tau_1\bar{\delta}^i$. Note that $\frac{d(\tau_1\bar{\delta}^i)}{d\tau_1} = \bar{\delta}^i$ and $\frac{d(\tau_1\bar{\delta}^i)}{d\delta} = \tau_1\left(1 - \frac{1+r}{R'(s)}\right)$. Thus, we have that

$$\frac{d\ell_1}{d\tau_1} = \frac{d\ell_1}{d(\tau_1\bar{\delta}^i)}\frac{d(\tau_1\bar{\delta}^i)}{d\tau_1} = \bar{\delta}^i\frac{d\ell_1}{d(\tau_1\bar{\delta}^i)} \tag{A18}$$

$$\frac{d\ell_1}{d\delta} = \frac{d\ell_1}{d(\tau_1\bar{\delta}^i)}\frac{d(\tau_1\bar{\delta}^i)}{d\delta} = \tau_1\left(1 - \frac{1+r}{R'(s)}\right)\frac{d\ell_1}{d(\tau_1\bar{\delta}^i)} \tag{A19}$$

Substituting from (A18) into (A19), we get:

$$\frac{d\ell_1}{d\delta} = \tau_1\left(1 - \frac{1+r}{R'(s)}\right)\frac{1}{\bar{\delta}^i}\frac{d\ell_1}{d\tau_1}.$$

The proof for ℓ_2 is analogous.

A.4 Proof of Proposition 3

We first differentiate the Lagrangian of the government's optimization problem with respect to $1-\delta$ and invoke the envelope theorem on the V_i terms.

$$\sum_i \pi^i\left(u'(c_1)\tau_1 y_1 - \beta u'(c_2)(1+r)\tau_1 y_1\right) + \lambda\sum_i \pi^i\left(\tau_1\frac{dy_1^i}{d(1-\delta)} + \tau_2\frac{1}{1+r}\frac{dy_2^i}{d(1-\delta)}\right) = 0. \tag{A20}$$

We then assume $s^i \neq 0$ and use Lemma 1 to modify the terms in the parenthesis in the second summation term.

$$\left(\tau_1^2\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\frac{dy_1^i}{d(1-\tau_1)} + \tau_2\tau_1\frac{1}{1+r}\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\frac{dy_2^i}{d(1-\tau_1)}\right). \tag{A21}$$

We use the Slutsky equation to rewrite $\frac{dy_1^i}{d(1-\tau_1)} = \left(\frac{dy_1^i}{d(1-\tau_1)}\right)^c + y_1\frac{dy_1}{dG_1}$ and the cross-price Slutsky equation to rewrite $\frac{dy_2^i}{d(1-\tau_1)} = \left(\frac{dy_2^i}{d(1-\tau_1)}\right)^c + y_1\frac{dy_2}{dG_1}$. The expression above becomes

$$\left(\tau_1^2\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\left\{\left(\frac{dy_1^i}{d(1-\tau_1)}\right)^c + y_1\frac{dy_1}{dG_1}\right\} + \tau_2\tau_1\frac{1}{1+r}\left[1 - \frac{1+r}{R'(s^i)}\right]\frac{1}{\bar{\delta}^i}\left\{\left(\frac{dy_2^i}{d(1-\tau_1)}\right)^c + y_1\frac{dy_2}{dG_1}\right\}\right). \tag{A22}$$

[28] The latter is derived by noticing that

$$\begin{aligned} c_1 + \frac{c_2}{1+r_b} &= y_1(1-\delta\tau_1) + G_1 + \frac{y_2(1-\tau_2)+G_2}{1+r_b} - \frac{1+r}{1+r_b}(1-\delta)\tau_1 y_1 \\ &= y_1 - \delta\tau_1 y_1 - \frac{1+r}{1+r_b}(1-\delta)\tau_1 y_1 + \frac{y_2(1-\tau_2)+G_2}{1+r_b} \\ &= y_1 - \bar{\delta}^i\tau_1 y_1 + \frac{y_2(1-\tau_2)+G_2}{1+r_b} \\ &= y_1(1-\bar{\delta}^i\tau_1) + \frac{y_2(1-\tau_2)+G_2}{1+r_b} \end{aligned}$$

Further rearranging and using elasticity notation yields

$$y_1^i \frac{\tau_1}{1-\tau_1}\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\ \left(\left\{\tau_1\varepsilon_{1,1-\tau_1}^{i,c}+(1-\tau_1)\tau_1\frac{dy_1}{dG_1}\right\}+\frac{1}{1+r}\left\{\tau_2\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^{i,c}+(1-\tau_1)\tau_2\frac{dy_2}{dG_1}\right\}\right). \tag{A23}$$

Further using the definitions $\rho^i = \frac{d}{dG_1}\left(\tau_1 y_1^i + \frac{\tau_2 y_2^i}{1+r}\right)$, we may now rewrite the government's FOC with respect to $1-\delta$ as

$$\tau_1 \sum_i \pi^i y_1^i \left(\ \overline{\lambda}\ \Delta_r^i \cdot u'(c_1^i)\ \ +\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\left(\frac{\tau_1}{1-\tau_1}\varepsilon_{1,1-\tau_1}^{i,c}+\frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^{i,c}+\rho^i\right)\right)=0. \tag{A24}$$

Using the definitions of g_1^i and g_2^i and re-arranging yields:

$$\tau_1\mathbb{E}_1\left(g_1^i-g_2^i\right)=-\mathbb{E}_1\left[\tau_1\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\left(\frac{\tau_1}{1-\tau_1}\varepsilon_{1,1-\tau_1}^{i,c}+\frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^{i,c}+\rho^i\right)\right]. \tag{A25}$$

Using Lemma 1 yields (32).

A.5 Proof of Lemma 2

Part i) Using the derivations from the proof of Proposition 3 (see Appendix A.4), we have that:

$$\frac{1}{\lambda}\frac{dW}{d(1-\delta)}=\tau_1\mathbb{E}_1\left(g_1^i-g_2^i\right)+\mathbb{E}_1\left[\tau_1\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\left(\frac{\tau_1}{1-\tau_1}\varepsilon_{1,1-\tau_1}^{i,c}+\frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^{i,c}+\rho^i\right)\right]. \tag{A26}$$

Now divide by τ_1 and use the fact that that $g_2^i = \frac{\beta(1+r)u'(c_2^i)}{\lambda} = \frac{u'(c_1^i)}{\lambda}\frac{1+r}{R'(s^i)} = g_1^i\frac{1+r}{R'(s^i)}$:

$$\frac{dW}{\lambda d(1-\delta)}\frac{1}{\tau_1}=\mathbb{E}_1\left(g_1^i\left[1-\frac{1+r}{R'(s^i)}\right]\right)+\mathbb{E}_1\left[\left[\frac{\Delta_r^i}{1-(1-\delta)\Delta_r^i}\right]\left(\frac{\tau_1}{1-\tau_1}\varepsilon_{1,1-\tau_1}^{i,c}+\frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^{i,c}+\rho^i\right)\right]. \tag{A27}$$

Setting $\delta = 1$ yields:

$$\frac{dW}{\lambda d(1-\delta)}\frac{1}{\tau_1}=\mathbb{E}_1\left(g_1^i\left[1-\frac{1+r}{R'(s^i)}\right]\right)+\mathbb{E}_1\left[\left(1-\frac{1+r}{R'(s^i)}\right)\left(\frac{\tau_1}{1-\tau_1}\varepsilon_{1,1-\tau_1}^{i,c}+\frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon_{2,1-\tau_1}^{i,c}+\rho^i\right)\right]. \tag{A28}$$

Given the piece-wise linear return technology, we have that $R'(s^i) = 1+r$ when $s^i > 0$ and $R'(s) = 1 + r_b$ when $s^i < 0$. Remember $s^i \neq 0$ by assumption. Letting $\Delta_r = 1 - \frac{1+r}{1+r_b} > 0$ we get:

$$\frac{dW}{\lambda d(1-\delta)}\frac{1}{\tau_1\Delta_r} = \sum_{i:s^i<0}\pi^i y_1^i g_1^i + \sum_{i:s^i<0}\pi^i y_1^i \rho^i + \sum_{i:s^i<0}\pi^i y_1^i\left(\frac{\tau_1}{1-\tau_1}\varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon^{i,c}_{2,1-\tau_1}\right), \tag{A29}$$

$$= \sum_{i:s^i<0}\pi^i y_1^i (g_1^i + \rho^i) + \sum_{i:s^i<0}\pi^i y_1^i\left(\frac{\tau_1}{1-\tau_1}\varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon^{i,c}_{2,1-\tau_1}\right). \tag{A30}$$

Part ii) We calculate the welfare effect as if everyone accepts the loan. Because, if $s^i < 0$, then agents would strictly prefer to accept, if $s^i > 0$, then agents are indifferent.

$$\frac{dW}{dx} = \sum_i \pi^i\left(u'(c_1^i) - (1+r)\beta u'(c_2^i)\right) + \lambda\sum_i\pi^i\left(\frac{dy_1^i}{dx} + \frac{1}{1+r}\frac{dy_2^i}{dx}\right). \tag{A31}$$

From the agent's perspective, as long as $s^i \neq 0$, a loan of dx is equivalent to $dG_1 = \left(1 - \frac{1+r}{R'(s^i)}\right)dx$. This follows from using the period-2 budget constraint to replace s^i in the period-1 budget constraint. Therefore, we can rewrite the above expression as

$$\frac{dW}{dx} = \sum_i \pi^i\left(u'(c_1^i) - (1+r)\beta u'(c_2^i)\right) + \lambda\sum_i\pi^i\left(1 - \frac{1+r}{R'(s^i)}\right)\left(\tau_1\frac{dy_1^i}{dG_1} + \frac{1}{1+r}\tau_2\frac{dy_2^i}{dG_1}\right). \tag{A32}$$

Further rearranging and using the definition of $\rho^i = \tau_1\frac{dy_1^i}{dG_1} + \frac{1}{1+r}\tau_2\frac{dy_2^i}{dG_1}$,

$$\frac{1}{\lambda}\frac{dW}{dx} = \sum_i\pi^i\left(g_1^i - g_2^i\right) + \sum_i\pi^i\left(1 - \frac{1+r}{R'(s^i)}\right)\rho^i. \tag{A33}$$

The last step follows by realizing that $g_2^i = \frac{\beta(1+r)u'(c_2^i)}{\lambda} = \frac{u'(c_1^i)}{\lambda}\frac{1+r}{R'(s^i)} = g_1^i\frac{1+r}{R'(s^i)}$ and that $R'(s) = 1+r$ when $s > 0$.

Part iii) Equation (A7) in the proof of Proposition 2 provides the money-metric welfare effect of a marginal increase in $1-\tau_1$:

$$\frac{dW}{d(1-\tau_1)\lambda} = \mathbb{E}_1\left[\delta g_1^i + [1-\delta]g_2^i\right] - \mathbb{E}_1\left[1 - \frac{\tau_1}{1-\tau_1}\varepsilon^i_{1,1-\tau_1} - \frac{1}{1+r}\frac{\tau_2}{1-\tau_1}\frac{y_2^i}{y_1^i}\varepsilon^i_{2,1-\tau_1}\right]. \tag{A34}$$

This holds at any baseline tax system, including an age-independent one where $\tau_1 = \tau_2$. When $\delta = 1$, the expression above simplifies to:

$$\frac{dW}{d(1-\tau_1)\lambda} = \sum_i \pi^i y_1^i g_1^i + \sum_i \pi^i y_1^i \left(\frac{\tau_1}{1-\tau_1} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau_2}{1-\tau_1} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right) + \sum_i \pi^i y_1^i \rho^i - \sum_i \pi^i y_1^i, \tag{A35}$$

$$= \sum_i \pi^i y_1^i (g_1^i + \rho^i) + \sum_i \pi^i y_1^i \left(\frac{\tau_1}{1-\tau_1} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau_2}{1-\tau_1} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right) - \sum_i \pi^i y_1^i. \tag{A36}$$

A.6 Proof of Proposition 4

Multiplying (35) in Lemma 2 by $\bar{y}_1 = \sum_{i:s^i<0} \pi^i y_1^i$ yields:

$$\frac{\bar{y}_1}{\Delta_r} \frac{dW}{\lambda dx} = \bar{y}_1 \sum_{i:s^i<0} \pi^i (g_1^i + \rho^i). \tag{A37}$$

Taking the difference between (34) in Lemma 2 and (A37) yields:

$$\frac{dW}{\lambda d(1-\delta)} \frac{1}{\tau_1 \Delta_r} - \frac{\bar{y}_1}{\Delta_r} \frac{dW}{\lambda dx} = \sum_{i:s^i<0} \pi^i (y_1^i - \bar{y}_1)(g_1^i + \rho^i) + \sum_{i:s^i<0} \pi^i y_1^i \left(\frac{\tau}{1-\tau} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau}{1-\tau} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right),$$

which can be written

$$\frac{dW}{\lambda d(1-\delta)} = \frac{\tau_1 \bar{y}_1}{\lambda} \frac{dW}{dx} + \tau_1 \Delta_r \left[\sum_{i:s^i<0} \pi^i (y_1^i - \bar{y}_1)(g_1^i + \rho^i) + \sum_{i:s^i<0} \pi^i y_1^i \left(\frac{\tau}{1-\tau} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau}{1-\tau} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right) \right].$$

This establishes (37). To establish (38), we take the difference between (34) and (36) in Lemma 2 to obtain:

$$\frac{dW}{\lambda d(1-\delta)} \frac{1}{\tau_1 \Delta_r} - \frac{dW}{d(1-\tau_1)\lambda} = \sum_{i:s^i<0} \pi^i y_1^i (g_1^i + \rho^i) + \sum_{i:s^i<0} \pi^i y_1^i \left(\frac{\tau}{1-\tau} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau}{1-\tau} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right) - \left(\sum_i \pi^i y_1^i (g_1^i + \rho^i) + \sum_i \pi^i y_1^i \left(\frac{\tau}{1-\tau} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau}{1-\tau} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right) - \sum_i \pi^i y_1^i \right).$$

Re-arranging we get:

$$\frac{dW}{\lambda d(1-\delta)} = \tau_1 \Delta_r \left(\frac{dW}{d(1-\tau_1)\lambda} - \sum_{i:s^i>0} \pi^i y_1^i (g_1^i + \rho^i) - \sum_{i:s^i>0} \pi^i y_1^i \left(\frac{\tau}{1-\tau} \varepsilon^{i,c}_{1,1-\tau_1} + \frac{1}{1+r} \frac{\tau}{1-\tau} \frac{y_2^i}{y_1^i} \varepsilon^{i,c}_{2,1-\tau_1} \right) + \sum_i \pi^i y_1^i \right).$$

B Extension: Letting the interest rate on delayed taxes be a policy tool

If the interest rate on delayed taxes, r_{dtax}, is constrained to equal the interest rate on net saving, r, delayed taxation has no effect on the behavior of workers whose marginal borrowing rate is r. Delayed taxation does not change the present value of the tax rate. In our framework, this implies that delayed taxation does not affect the behavior of net savers in partial equilibrium, nor does it have any effect in the absence of financial frictions (when $r_b = r$).

As an extension of our framework, we now allow the interest rate on delayed taxes to be a policy tool. That is, we let r_{dtax} differ from $r_{gov} = r$. We also allow for the possibility that there are no financial frictions, i.e. $r_b = r$. A trivial result if the government can choose any $r_{dtax} \in \mathbb{R}$ is that it can replicate any age-dependent marginal tax system characterized by $(\tau_1, \tau_2) \in \mathbb{R}^2_+$ by choosing $\tau = \tau_2$, setting $r_{dtax} = -1$, and choosing $\delta = \tau_1/\tau_2$. [29] However, such a policy does not exploit the fact that there is heterogeneity in marginal borrowing rates, $R'(s^i)$.

Our next proposition examines whether a flexible delayed taxation policy can replicate (and improve upon) an age-dependent marginal tax schedule under lower bound restrictions on the interest rate.

Proposition 5 (When Delayed Taxation Pareto dominates Age-Dependent Taxation)

Assume that policymakers can choose an interest rate on delayed taxes, $\underline{r_{dtax}} < r_{dtax} \leq r$. *Then any optimal age-dependent marginal tax scheme characterized by* $G_1 = G_2 = G$ *and* $1 > \frac{\tau_1}{\tau_2} \geq \frac{1+\underline{r_{dtax}}}{1+r}$ *may be weakly Pareto dominated by a (not necessarily optimal) delayed tax policy with* $1 - \delta < 1$*, which leaves the following slack in the government budget constraint.*

$$\frac{1}{1+r} \sum_{i:s^i<0} \left((1 + r_{dtax})(\ell_1^{i,*} - \ell_1^i) w_1^i \tau_1 + (\ell_2^{i,*} - \ell_2^i) w_2^i \tau_2 \right), \tag{B1}$$

where ℓ_t^i *is the labor supply under the AD scheme and* $\ell_t^{i,*}$ *is the labor supply under the DT scheme.* $\ell_t^{i,*} = \ell_t^i \ \forall i : s^i > 0$ *and* $\ell_t^{i,*}$*, for* i*, such that* $s^i < 0$ *differs from* ℓ_t^i *only because of a lower effective tax rate in period 1 among borrowers,*

$$\tilde{\tau}_1^* = \tau_2 \left[1 - (1 - \delta) \left(1 - \frac{1 + r_{dtax}}{1 + r} \right) \right] \leq \tau_1 \quad \textit{for } i \textit{ s.t. } s^i < 0. \tag{B2}$$

Proof. Suppose there exists an optimal age-dependent tax scheme characterized by G, τ_1, and τ_2, where $\tau_1 < \tau_2$. We consider an alternative delayed tax scheme on top of the benchmark linear tax policy. We want to show that we can make savers just as well off and borrowers strictly better off while leaving slack in the delayed-tax policy's government budget constraint.

Consider a delayed tax scheme with $1 - \delta > 0$, where the age-independent (nominal) tax rate $\tau^* = \tau_2$. Given this $\delta \neq 1$, we set $r_{dtax} \leq r$ such that the effective marginal period-1 tax rate for

[29] If $\tau_2 = 0$, δ is not well defined, but it does not matter since δ becomes irrelevant when $\tau = \tau_2 = 0$.

workers with $R'(s^i) = r$ equals τ_1 from the AD scheme.

$$\tau_1^{i,*} = \tau_2 \left[1 - (1-\delta)\left(1 - \frac{1+r_{dtax}}{1+r}\right)\right] = \tau_1 \quad \text{for all } i \text{ s.t. } R'(s^i) = r \tag{B3}$$

We also set $G^* = G$. This ensures that choices of workers is the same under AD and the new DT policies $\forall i : s^i > 0$. Hence V^i are the same under the AD and DT schemes $\forall i : R'(s^i) = r$. Importantly, (B3) also ensures that the PV tax revenues obtained under AD and DT from all i such that $R'(s^i) = r$ are the same.

We need to ensure that $r_{dtax} > \underline{r_{dtax}}$. From (B3), we that this is equivalent to

$$\frac{\tau_1}{\tau_2} \geq \frac{1+\underline{r_{dtax}}}{1+r}. \tag{B4}$$

Hence if $\underline{r_{dtax}} > -1$, we need $\tau > 0$.

If $R'(s^i) = 1+r$ for all i, then the proof is complete because we have exactly replicated the AD policy. Hence, now we assume that there exists at least one i for which $R'(s^i) = 1 + r_b < 1 + r_{gov}$. We proceed to ensure that V^i increases for those with $s^i < 0$ and that their contribution to tax revenues does not decrease. We want to show that borrowers can be made better off while not violating the government budget constraint. Under the DT policy, net borrowers face an effective tax rate of

$$\tilde{\tau}_1^{i,*} = \tau_2 \left[1 - (1-\delta)\left(1 - \frac{1+r_{dtax}}{1+r}\right)\right] \quad \text{for all } i \text{ s.t. } R'(s^i) = 1 + r_b. \tag{B5}$$

Since $G^* = G$ and $\tilde{\tau}_1^{i,*} < \tau_1$, they are strictly better off under DT than AD, i.e., $V^{i,*} > V^i$ for all i such that $s^i < 0$. We next explore feasibility. The change to PV tax revenues is

$$\sum_{s^i<0} \left(\ell_1^{i,*} w_1^i \tau_2 \left[1 - (1-\delta)\left(1 - \frac{1+r_{dtax}}{1+r}\right)\right] + \frac{\ell_2^{i,*} w_2^i \tau_2}{1+r}\right) - \sum_{s^i<0} \left(\ell_1^i w_1^i \tau_1 + \frac{\ell_2^i w_2^i \tau_2}{1+r}\right). \tag{B6}$$

By virtue of how r_{dtax} is set (equation B3), this revenue change may be rewritten as

$$\sum_{i:s^i<0} \ell_1^{i,*} w_1^i \tau_1 + \frac{1}{1+r} \sum_{i:s^i<0} \ell_2^{i,*} w_2^i \tau_2 - \left(\sum_{i:s^i<0} \ell_1^i w_1^i \tau_1 + \frac{1}{1+r} \sum_{i:s^i<0} \ell_2^i w_2^i \tau_2\right). \tag{B7}$$

or

$$\sum_{i:s^i<0} \left((\ell_1^{i,*} - \ell_1^i) w_1^i \tau_1 + \frac{1}{1+r} (\ell_2^{i,*} - \ell_2^i) w_2^i \tau_2\right), \tag{B8}$$

which is equivalent to (B1). Since the government perceives the tax rates in the same way as financially unconstrained individuals, the only thing that matters is the change in labor supply in periods 1 and 2 by the constrained agents who experience the tax rate as $\tilde{\tau}_1^* < \tau_1$ (see equation B5). The slack in the budget constraint will materialize as long as the direct substitution effect on labor supply in period 1 is not offset by income effects (individuals become wealthier over their lifetimes, which may reduce labor supply in both periods) and intertemporal substitution effects (the tax rate cut in period 1 may be accompanied by a labor supply increase in period 1 and a labor supply reduction in period 2).

■

Note that (i) when there are no financial frictions (i.e., $r_b = r = r_{gov}$), the delayed tax policy in Proposition 5 exactly replicates the allocations under the age-dependent policy and satisfies the budget constraint with equality.

We also note that (ii), for $r_b > r$, this specific Pareto-dominant delayed tax policy does not exist if the behavioral response to a decrease in the effective tax rate is sufficiently negative to cause tax revenue to decrease. However, we do not consider the assumption of a non-negative revenue effect (B1) to be particularly strong. This is because the relevant revenue effect only includes behavioral responses to a tax cut —and not the mechanical negative effects typically caused by a tax cut. For example, if the age-dependent optimal tax rates of the existing tax system coincide with the revenue-maximizing rates, then the marginal behavioral responses would be strictly positive and equal in magnitude to the negative mechanical effects of a marginal tax rate reduction.

C Bunching Analysis Appendix

C.1 Additional figures

Figure A.1: Little Evidence of "Negative-Bunching" at Debt-Conversion-Cap Threshold

Panel (A) provides a scatter plot, in green, of the relationship between debt accumulation and student earnings around the debt-conversion-cap threshold. This is the threshold above which additional earnings do not increase future student debt because there is no more stipends to convert to debt. Panel (B) provides a graphical illustration of how the bunching estimate. See Figure 7 for further info on the methodology.

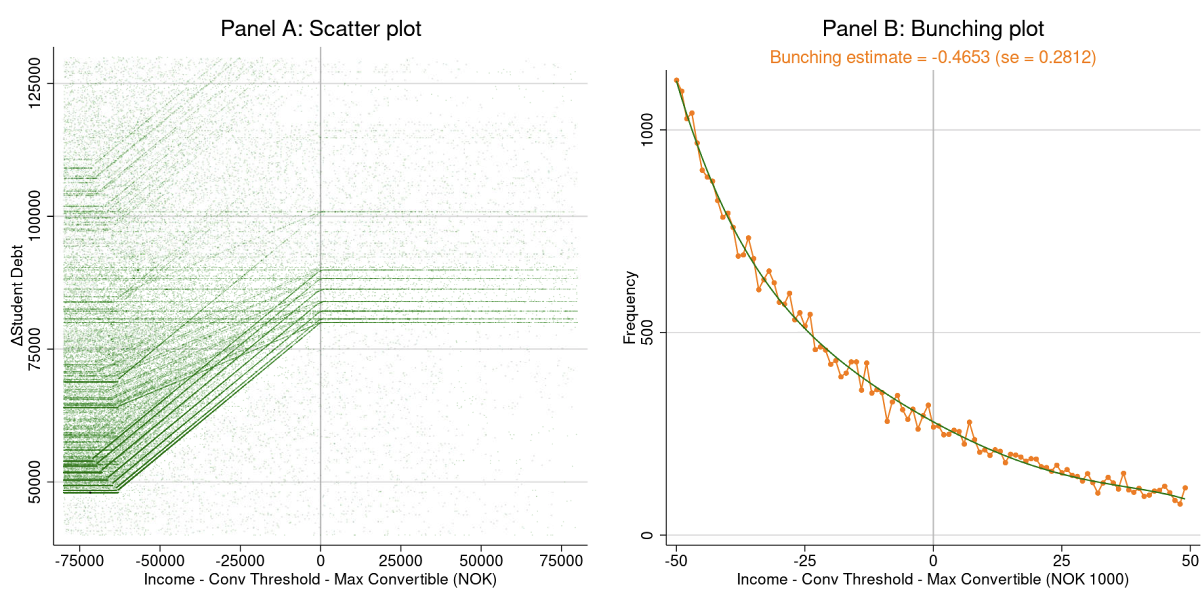


Figure A.2: Bunching at Debt-Conversion Threshold for Workers with Sales and Hospitality Occupations

We repeat the exercise in Panel (B) of Figure 7 on a subset of workers with hospitality (4-digit "STYRK-98"occupation code = 5123, waiters and bartenders) and store sales/clerk jobs (4-digit occupation code = 5221.

C.2 Individual-level bunching elasticities

We expand here on how individual level elasticities are constructed (see main text section 5.7).

In the standard bunching framework, b is the the relative excess mass in the bin right below the kink, where all bunchers in the (wider) bunching region are assumed to be in the one bin below the kink. We discuss how to map this more visual notion of excess mass into relative excess probabilities.

First, the estimated excess mass can be written

$$\hat{B} = \Big[P[y_i \in BR] \cdot N^{sample} - P^{cf}[y_i \in BR] \cdot N^{sample}\Big] \times \text{Bin width.} \tag{C1}$$

Then to obtain $\hat{b}$, we need to divide by the baseline mass. That is, the counterfactual mass of individuals at the single bin right below the kink.

$$\hat{b} = \frac{\Big[P[y_i \in BR] \cdot N^{sample} - P^{cf}[y_i \in BR] \cdot N^{sample}\Big] \times \text{Bin width}}{\frac{P^{cf}[y_i \in BR]}{N^{\text{BR Bins}}} \times N^{sample} \times \text{Bin width}}. \tag{C2}$$

Note that, in the numerator, we divide $P^{cf}[y_i \in BR]$ by the number of bins in the bunching region to get an estimate of the probability that someone is in the single bin right below the kink. We simplify to get

$$\hat{b} = \frac{\Big[P[y_i \in BR] - P^{cf}[y_i \in BR]\Big]}{\frac{P^{cf}[y_i \in BR]}{N^{\text{BR Bins}}}}. \tag{C3}$$

From this we can define an individual level b_i as

$$b_i = \frac{\mathbb{1}[y_i \in BR] - P^{cf}[y_i \in BR]}{\frac{P^{cf}[y_i \in BR]}{N^{\text{BR Bins}}}}, \tag{C4}$$

which satisfies the property that

$$\widehat{\mathbb{E}}[b_i] = \hat{b}, \tag{C5}$$

namely that the sample average of b_i equals $\hat{b}$.

Now, in order to obtain an individual-level elasticity, we divide b_i by y^* (as in equation 47) and then further by the relative change in the after-tax wage.

D Dynamic Uncompensated and Compensated Elasticities

In dynamic economies, Frisch elasticities impose restrictions that are helpful in obtaining simple elasticity expressions in cases where accounting for the full range of substitution effects across periods would be intractable. In this section, we derive unrestricted elasticities that allow for intertemporal substitution in the context of our two-period framework.

D.1 Derivative of period-1 labor supply w.r.t. $\bar{\delta}^i\tau_1$

We first differentiate c_1 using the first-period budget constraint of agent i, allowing τ_1 and δ^i to vary.

$$dc_1 = d\ell_1 w_1(1 - \delta\tau_1) - \ell_1 w_1 d(\delta\tau_1) - ds. \tag{D1}$$

Since $s^i \neq 0$, we can use the period-2 budget constraint to obtain an expression for s^i and differentiate it to obtain

$$ds = \frac{1}{R'(s)}\left[dc_2 + \ell_1 w_1 d\left\{(1-\delta)(1+r)\tau_1\right\} + (1-\delta)(1+r)w_1\tau_1 d\ell_1 - (1-\tau_2)w_2 d\ell_2\right]. \tag{D2}$$

Substituting (D2) into (D1), and using the expression for $\bar{\delta}^i$, yields

$$dc_1 = d\ell_1 w_1(1 - \bar{\delta}^i\tau_1) - \ell_1 w_1 d(\bar{\delta}^i\tau_1) - \frac{1}{R'(s^i)}dc_2 + \frac{1}{R'(s^i)}(1-\tau_2)w_2 d\ell_2. \tag{D3}$$

Now we can differentiate the second period intratemporal FOC (5) to get $d\ell_2 = \frac{u''(c_2)}{v''(l_2)} w_2^i (1-\tau_2) dc_2$ and substitute this into (D3) and collect multiplicative terms on dc_2 to get

$$dc_1 = d\ell_1 w_1(1 - \bar{\delta}^i\tau_1) - \ell_1 w_1 d(\bar{\delta}^i\tau_1) - \frac{1}{R'(s^i)}\left(1 - [(1-\tau_2)w_2]^2 \frac{u''(c_2)}{v''(l_2)}\right) dc_2. \tag{D4}$$

Similarly, we use the differentiated intertemporal FOC, $u''(c_1)dc_1 = \beta u''(c_2)R'(s)dc_2$, to replace dc_2 with an expression that includes dc_1.

$$dc_1 \left[1 + \left(1 - [(1-\tau_2)w_2]^2 \frac{u''(c_2)}{v''(l_2)}\right) \frac{u''(c_1)}{\beta u''(c_2)} \frac{1}{R'(s)^2}\right] = d\ell_1 w_1(1-\bar{\delta}^i\tau_1) - \ell_1 w_1 d(\bar{\delta}^i\tau_1). \quad \text{(D5)}$$

Now, we wish to substitute out dc_1 with a term that only contains dl_1. We obtain this by differentiating the FOC for l_1 (equation 8, which relies on the Euler equation 6):

$$u_1''(c_1^i)w_1^i\left(1-\tau_1\bar{\delta}^i\tau_1\right)dc_1 - u'(c_1^i)w_1^i d\left(\tau_1\bar{\delta}^i\right) = v''(\ell_1^i)d\ell_1^i. \quad \text{(D6)}$$

We then substitute (D6) into (D5). We denote the term in the brackets in (D5) as ι^i.

$$\left[\frac{v''(\ell_1^i)d\ell_1^i}{u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)} + \frac{u'(c_1^i)w_1^i d\left(\tau\bar{\delta}^i\tau_1\right)}{u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)}\right]\iota^i = d\ell_1 w_1(1-\bar{\delta}^i\tau_1) - \ell_1 w_1 d(\bar{\delta}^i\tau_1). \quad \text{(D7)}$$

Then we re-arrange to obtain

$$\frac{d\ell_1^i}{d(\bar{\delta}^i\tau_1)} = \frac{-u'(c_1^i)w_1^i\iota^i - \ell_1^i w_1^i u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)}{v''(\ell_1^i)\iota^i - w_1^i(1-\bar{\delta}^i\tau_1)u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)}. \quad \text{(D8)}$$

Then we re-arrange to obtain

$$\frac{d\ell_1^i}{d(\bar{\delta}^i\tau_1)} = \frac{-u'(c_1^i)w_1^i\iota^i - \ell_1^i w_1^i u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)}{v''(\ell_1^i)\iota^i - w_1^i(1-\bar{\delta}^i\tau_1)u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)}. \quad \text{(D9)}$$

This may also be written as

$$\frac{d\ell_1^i}{d(\bar{\delta}^i\tau_1)} = \frac{-u'(c_1^i)w_1^i\iota^i - \ell_1^i w_1^i u_1''(c_1^i)\tilde{w}_1^i}{v''(\ell_1^i)\iota^i - \tilde{w}_1^i u_1''(c_1^i)\tilde{w}_1^i}. \quad \text{(D10)}$$

Writing out the ι^i and $\bar{w}_1^i$ terms, we get

$$\frac{d\ell_1^i}{d(\bar{\delta}^i\tau_1)} = \frac{-u'(c_1^i)w_1^i\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right] - \ell_1^i w_1^i u_1''(c_1^i)w_1^i\left(1-\bar{\delta}^i\tau_1\right)}{v''(\ell_1^i)\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right] - u_1''(c_1^i)w_1^{i2}\left(1-\bar{\delta}^i\tau_1\right)^2}, \quad \text{(D11)}$$

which depends explicitly on an individual's marginal interest rate, $R'(s^i)$.

D.2 Period 1 income effects

We want to have an expression for $\frac{d\ell_1^i}{d(\bar{\delta}^i\tau_1)}$. All derivations assume $s^i \neq 0$.

(a) We use the period-2 budget constraint to substitute in for s in the period-1 budget constraint to get an expression for c_1 and differentiate.

$$dc_1 = dG_1 + \tilde{w}_1 d\ell_1 + \frac{1}{R'(s)}((1-\tau_2)w_2 d\ell_2 - dc_2) \tag{D12}$$

(b) Now find an expression for $d\ell_2$. We use the implied intratemporal FOC for labor (12), and differentiate it to get

$$d\ell_2 = \frac{w_2(1-\tau_2)v''(\ell_1)}{\beta R'(s)\tilde{w}_1 v''(\ell_2)} d\ell_1. \tag{D13}$$

(c) Now find an expression for dc_2. We differentiate the Euler equation (6).

$$dc_2 = \frac{u''(c_1)}{\beta R'(s)u''(c_2)} dc_1 \tag{D14}$$

(d) Now find an expression for dc_1. We differentiate the intratemporal FOC for ℓ_1, (8), which relies on the intertemporal FOC, (6).

$$dc_1 = \frac{v''(\ell_1)}{u''(c_1)\tilde{w}_1} d\ell_1. \tag{D15}$$

(e) Now substitute the expressions found in steps (b) and (c) into (a).

$$dc_1 = dG_1 + \tilde{w}_1 d\ell_1 + \frac{1}{R'(s)}\left((1-\tau_2)w_2 \frac{w_2(1-\tau_2)v''(\ell_1)}{\beta R'(s)\tilde{w}_1 v''(\ell_2)} d\ell_1 - \frac{u''(c_1)}{\beta R'(s)u''(c_2)} dc_1\right) \tag{D16}$$

(f) Now substitute in the expression for dc_1 found in step (d).

$$\frac{v''(\ell_1)}{u''(c_1)\tilde{w}_1} d\ell_1 = dG_1 + \tilde{w}_1 d\ell_1 + \frac{1}{R'(s)}\left((1-\tau_2)w_2 \frac{w_2(1-\tau_2)v''(\ell_1)}{\beta R'(s)\tilde{w}_1 v''(\ell_2)} d\ell_1 - \frac{u''(c_1)}{\beta R'(s)u''(c_2)} \frac{v''(\ell_1)}{u''(c_1)\tilde{w}_1} d\ell_1\right) \tag{D17}$$

(g) Reorganize by collecting terms on $d\ell_1$.

$$\left(\frac{v''(\ell_1)}{u''(c_1)\tilde{w}_1} - \tilde{w}_1 - \frac{1}{R'(s)}\left[\frac{[w_2(1-\tau_2)]^2 v''(\ell_1)}{\beta R'(s)\tilde{w}_1 v''(\ell_2)} - \frac{1}{\beta R'(s)u''(c_2)} \frac{v''(\ell_1)}{\tilde{w}_1}\right]\right) d\ell_1 = dG_1 \tag{D18}$$

(g) Reorganize by collecting terms on $d\ell_1$.

$$\left(\frac{v''(\ell_1)}{u''(c_1)\tilde{w}_1} - \frac{1}{R'(s)}\left[\frac{[w_2(1-\tau_2)]^2 v''(\ell_1)}{\beta R'(s)\tilde{w}_1 v''(\ell_2)} - \frac{1}{\beta R'(s)u''(c_2)} \frac{v''(\ell_1)}{\tilde{w}_1}\right] - \tilde{w}_1\right) d\ell_1 = dG_1 \tag{D19}$$

$$\left(v''(l_1)\left[1 - u''(c_1)\frac{1}{\beta R'(s)^2}\left[\frac{[w_2(1-\tau_2)]^2}{v''(\ell_2)} - \frac{1}{u''(c_2)}\right]\right] - \tilde{w}_1 u''(c_1)\tilde{w}_1\right) d\ell_1 = u''(c_1)\tilde{w}_1 dG_1 \tag{D20}$$

$$\left(v''(l_1)\left[1+\left(1-\frac{[w_2(1-\tau_2)]^2u''(c_2)}{v''(\ell_2)}\right)\frac{u''(c_1)}{u''(c_2)}\frac{1}{\beta R'(s)^2}\right]-\tilde{w}_1u''(c_1)\tilde{w}_1\right)d\ell_1=u''(c_1)\tilde{w}_1dG_1 \tag{D21}$$

(i) Finally, we may write the income effect term as

$$\frac{d\ell_1^i}{dG_1}=\frac{u''(c_1^i)\tilde{w}_1^i}{v''(\ell_1^i)\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right]-\tilde{w}_1^iu''(c_1^i)\tilde{w}_1^i}, \tag{D22}$$

or using the definition of ι^i,

$$\frac{d\ell_1^i}{dG_1}=\frac{u''(c_1^i)\tilde{w}_1}{v''(\ell_1^i)\iota^i-\tilde{w}_1u''(c_1^i)\tilde{w}_1^i}, \tag{D23}$$

D.3 Slutsky application to obtain period-1 compensated labor supply elasticity

We may use the results in the previous two subsection to get an expression for the compensated period-1 labor supply elasticity, $\varepsilon^i_{1,1-\tau_1}$ as follows.

By Slutsky,

$$\frac{d\ell_1^i}{d\tilde{w}_1^i}=\left(\frac{d\ell_1^i}{d\tilde{w}_1^i)}\right)^c+\frac{d\ell_1^i}{dG_1}\ell_1^i. \tag{D24}$$

If we keep $\bar{\delta}^i$ fixed, $d\tilde{w}_1^i=-\bar{\delta}^iw_1d\tau_1=\bar{\delta}^iw_1d(1-\tau_1)$. Hence, the relevant Slutsky equation becomes

$$\frac{d\ell_1^i}{d(1-\tau_1)}=\left(\frac{d\ell_1^i}{d(1-\tau_1)}\right)^c+\frac{d\ell_1^i}{dG_1}\ell_1^i\bar{\delta}w_1^i. \tag{D25}$$

Substituting in for the LHS using equation (D11) and the second-term on the RHS using (D23), we obtain

$$-\bar{\delta}^i\frac{-u'(c_1^i)w_1^i\iota^i-\ell_1^iw_1^iu_1''(c_1^i)\tilde{w}_1^i}{v''(\ell_1^i)\iota^i-\tilde{w}_1^iu_1''(c_1^i)\tilde{w}_1^i}=\left(\frac{d\ell_1^i}{d(1-\tau_1)}\right)^c+\frac{u''(c_1^i)\tilde{w}_1}{v''(\ell_1^i)\iota^i-\tilde{w}_1u''(c_1^i)\tilde{w}_1^i}\ell_1^iw_1^i\bar{\delta}^i. \tag{D26}$$

Rearrange and cancel out to get

$$\left(\frac{d\ell_1^i}{d(1-\tau_1)}\right)^c=\bar{\delta}^i\frac{u'(c_1^i)w_1^i\iota^i}{v''(\ell_1^i)\iota^i-\tilde{w}_1^iu_1''(c_1^i)\tilde{w}_1^i}. \tag{D27}$$

In terms of an elasticity, we can write it as

$$\varepsilon^c_{1,1-\tau_1}=\bar{\delta}^i\frac{1-\tau_1}{\ell_1^i}\frac{u'(c_1^i)w_1^i\iota^i}{v''(\ell_1^i)\iota^i-\tilde{w}_1^iu_1''(c_1^i)\tilde{w}_1^i}. \tag{D28}$$

We may rewrite this using the intratemporal FOC (8), which says that $u'(c_1^i)\tilde{w}_1^i=v'(\ell_1)$ and thus

$u'(c_1^i)w_1^i = \frac{1}{1-\delta^i\tau 1}v'(\ell_1)$ to get

$$\varepsilon^c_{1,1-\tau_1} = \frac{\bar{\delta}^i}{1-\bar{\delta}^i\tau_1}\frac{1-\tau_1}{\ell_1^i}\frac{v'(\ell_1)\iota^i}{v''(\ell_1^i)\iota^i - \tilde{w}_1^i u_1''(c_1^i)\tilde{w}_1^i}. \tag{D29}$$

Writing out the ι^i terms, we get

$$\varepsilon^c_{1,1-\tau_1} = \frac{\bar{\delta}^i}{1-\bar{\delta}^i\tau_1}\frac{1-\tau_1}{\ell_1^i}\frac{v'(\ell_1)\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right]}{v''(\ell_1^i)\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right] - \tilde{w}_1^i u_1''(c_1^i)\tilde{w}_1^i}. \tag{D30}$$

Writing out the $\tilde{w}_1^i$ and $\bar{\delta}^i$ terms, we get

$$\varepsilon^c_{1,1-\tau_1} = \frac{\frac{\delta+(1-\delta)\frac{1+r}{R'(s^i)}}{1-\left[\delta+(1-\delta)\frac{1+r}{R'(s^i)}\right]\tau_1}\frac{1-\tau_1}{\ell_1^i}v'(\ell_1)\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right]}{v''(\ell_1^i)\left[1+\left(1-\frac{[w_2^i(1-\tau_2)]^2u''(c_2^i)}{v''(\ell_2^i)}\right)\frac{u''(c_1^i)}{u''(c_2^i)}\frac{1}{\beta R'(s^i)^2}\right] - \left[w_1^i\right]^2\left(1-\tau_1\left[\delta+(1-\delta)\frac{1+r}{R'(s^i)}\right]\right)^2 u_1''(c_1^i)}. \tag{D31}$$

E Data for calibration

We first use microdata from the 1990 and 2011 censuses to compute micro-level data on (proxies for) effective hourly wages.

Wages in 1990

- To construct a measure of effective wages in 1990, we first construct a measure of hours worked. The 1990 census ("folke- og boligtellingen") contains categories for usual weekly hours worked, [1,10], [11,20], [20,30], [30,35], or full time (37 hours). We assign numeric values of 5, 15, 25, 32.5, and 37 to these categories. This variable is defined as *typical hours worked*.
- We calculate a measure of *minimum hours worked* as 37.5 hours × 47 weeks × number of reported months of full time work / 12 months.
- The survey also includes information on the number of months worked full-time and part-time. We sum the number of months to get a measure of *total months worked*.
- Our final measure of weekly hours worked is then max(*typical hours worked* × 47 weeks × *total months worked* / 12, *minimum hours worked*).
- We replace hours worked as missing if either the number of months worked is less than 2 or the total hours worked is less than 47 × 7.5 (i.e., we require an average of 7.5 hours worked per week).
- We calculate the 1990 wage as annual labor income divided by the number of hours worked. To avoid low wage outliers, we replace as missing whenever the wage is below 25% of the median wage. This drops about 3.5% of the observations. To avoid high wage outliers, we

winsorize at the 99.9th percentile.

Wages in 2011

- Using the 2011 Census, we calculate hours as the number of contractual weekly hours × 47 weeks.
- We then calculate wages as annual labor earnings / hours worked. We take the same approach to dealing with outliers as for 1990 wages.
- Finally, we deflate the 2011 wages by 1.5576, which is the ratio of the 2011 to 1990 Consumer Price Index from Statistics Norway.

We then keep only observations for which the age in 1990 was between 20 and 30. For individuals under the age of 25, we further require that their age exceeds 7 (first year of education) plus the number of years of education reported in the 2011 census by one year. We also require that we observe effective wages for the individual in both 1990 and 2011.

Wage trajectories for calibration

Using the microdata above, we calculate the median wage within each 1990 decile and each 2011 decile. This gives us 100 different (w_1, w_2) combinations: $i = 1, ..., 100$. For each wage combination, we assign π_i using the empirical probabilities in the microdata above.

Table A.1: Wage Trajectories for Calibration

This table provides summary statistics for the wage trajectories used in our calibration

	Panel A: Exogenous wage heterogeneity				
	p5	p25	p50	p75	p95
w_1	0.53	0.80	0.98	1.20	1.75
w_2	0.76	1.56	1.83	2.55	5.13
w_2/w_1	0.72	1.29	1.88	2.92	5.64

F HSV Benchmark

The HSV tax schedule implies that

$$y_t - T_t(y_t) = \lambda_t y_t^{1-\tau_t}, \tag{F1}$$

$$y_1 - \delta T_1(y_1) = y_1 - \delta[y_1 - \lambda_1 y_1^{1-\tau_1}] = y_1(1-\delta) + \delta\lambda_1 y_1^{1-\tau_1}, \tag{F2}$$

$$\frac{dT_t(y_t)}{dy_t} = 1 - \lambda_t(1-\tau_t)y_t^{-\tau_t}. \tag{F3}$$

$$\tag{F4}$$

The HSV tax function serves to replace the proportional tax rates in the linear tax model studied above. To maintain consistency with the earlier analysis, we also assume the presence of lump-sum transfers $G_t \geq 0$, $t = 1, 2$.[30] Consumption in the two periods are given by:

$$c_1^i = w_1^i \ell_1 - \delta T_1(w_1^i \ell_1) + G_1 - s$$
$$c_2^i = w_2^i \ell_2^i - T_2(w_2^i \ell_2^i) - (1+r)(1-\delta) T_1(w_1^i \ell_1) + G_2$$

or

$$c_1^i = w_1^i \ell_1^i (1-\delta) + \delta \lambda_1 (w_1^i \ell_1^i)^{1-\tau_1} + G_1 - s$$
$$c_2^i = \lambda_2 (w_2^i \ell_2^i)^{1-\tau_2} - (1+r)(1-\delta)[w_1^i \ell_1^i - \lambda_1 (w_1^i \ell_1^i)^{1-\tau_1}] + G_2 + R(s)$$

Let $\widetilde{\tau}_t = \frac{dT_t(y_t)}{dy_t} = 1 - \lambda_t (1-\tau_t)(w_t^i \ell_t^i)^{-\tau_t}$, $t = 1, 2$. The first-order conditions are:

$$(\ell_1) : u_1'(c_1^i) w_1^i \, (1 - \delta \widetilde{\tau}_1) - \beta (1+r)(1-\delta) u_2'(c_2^i) w_1^i \widetilde{\tau}_1 = v'(\ell_1^i), \tag{F5}$$

$$(\ell_2) : u_2'(c_2^i) \Big(1 - \widetilde{\tau}_2\Big) = v'(\ell_2), \tag{F6}$$

$$(s) : u_1'(c_1^i) = \beta u_2'(c_2^i) R'(s^i). \tag{F7}$$

Break out $u'(c_1^i) w_1^i$ in the FOC for ℓ_1 and exploit the FOC for s:

$$u_1'(c_1^i) w_1^i \Big[(1 - \delta \widetilde{\tau}_1) - \frac{1+r}{R'(s)} (1-\delta) \widetilde{\tau}_1 \Big] = v'(\ell_1^i), \tag{F8}$$

which can be written as

$$u_1'(c_1^i) w_1^i \left(1 - \widetilde{\tau}_1 \Big[\delta + (1-\delta) \cdot \frac{1+r}{R'(s)} \Big] \right) = v'(\ell_1^i). \tag{F9}$$

[30] Interestingly, these transfers materially improve upon the HSV tax optimum.

Table A.2: Optimal Taxation with Financial Frictions With a Nonlinear Taxation Benchmark

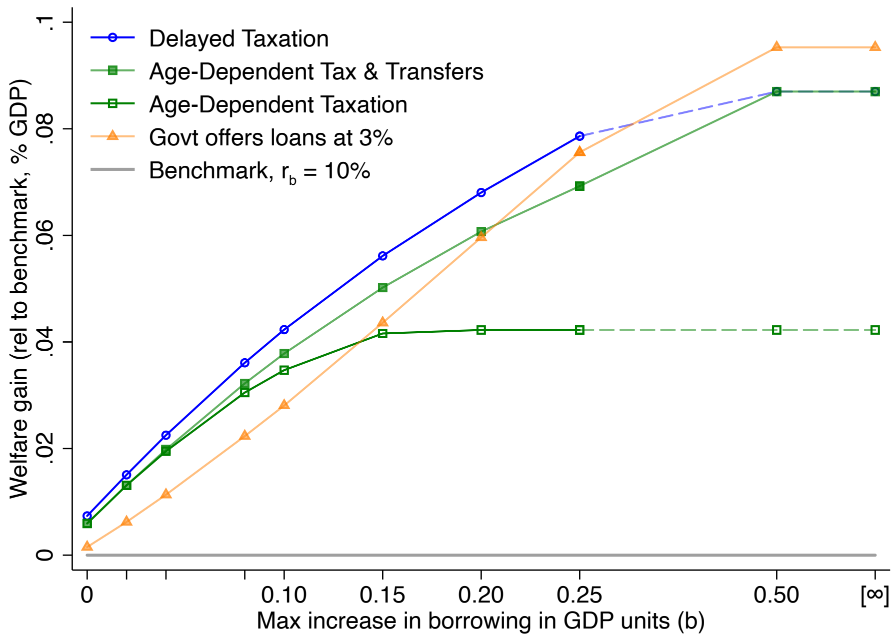
This table provides summary statistics for the calibrated economy when $r_b = 10\%$ and $b = 0.10$. The present value function calculates present values according to the government's discount rate. The (money-metric) welfare measure is the exogenous shock to revenue that the government must experience in the benchmark case to be equally well off as with the policy in a given column. This number is measured as a fraction of the benchmark economy's GDP (present-value labor earnings). The ATRs are calculated per period, and the $t = 1$ ATRs include total taxes accrued in that period (whether delayed or not).

	Tax schedule and allocations with $r_b = 10\%$ and $b = 0.10$				
	Benchmark	Delayed Taxation	Age-Dependent	AD T&T	Lending
λ_1^{HSV}	0.31	0.48	0.51	0.41	0.94
λ_2^{HSV}	0.31	0.48	0.24	0.27	0.94
τ_1^{HSV}	-0.20	-0.00	-0.08	-0.39	0.47
τ_2^{HSV}	-0.20	-0.00	-0.39	-0.30	0.47
G_1	0.59	0.43	0.51	0.61	0.00
G_2	0.59	0.43	0.51	0.48	0.00
$1 - \delta$	0.00	0.28	0.00	0.00	0.00
r_{dtax}	0.03	0.03	0.03	0.03	0.03
Subs Loan Limit	0.00	0.00	0.00	0.00	0.10
Govt Borrowing	0.12	0.21	0.21	0.21	0.21
Δ Welfare, % GDP (Benchmark, $r_b = 10\%$)	0.00	4.33	3.99	4.05	3.29
means					
$l_1 w_1$	0.86	0.86	0.88	0.88	0.86
$l_2 w_2$	1.70	1.75	1.70	1.70	1.76
PV($l_1 w_1$,$l_2 w_2$)	1.78	1.80	1.79	1.80	1.81
PV($l_1 w_1 \tau_1$,$l_2 w_2 \tau_1$)	1.18	0.94	1.06	1.14	0.27
l_1	0.85	0.86	0.88	0.87	0.87
l_2	0.79	0.86	0.80	0.81	0.87
$(1 - \delta^i) l_1 w_1 \tau_1$	0.00	0.09	0.00	0.00	0.00
s	-0.02	-0.00	0.02	0.03	-0.07

Figure A.3: Welfare Effects of Delayed and Age-Dependent Taxation With Government Borrowing Constraints

Panel A repeats Figure 4. Panel B replicates Figure 4 with a nonlinear tax as in main-text section 3.6. This figure plots the money-metric welfare effects (measured in terms of the GDP of the benchmark economy) of implementing either delayed taxation or age-dependent taxation. We do this for different values of b, which is defined as the maximum relative increase in borrowing relative to the benchmark economy. For example, if $b = 0$, the government can introduce delayed taxation but cannot itself borrow more than it did before implementing delayed taxation.

Panel A: Linear Tax Benchmark



Panel A: Nonlinear (HSV) Tax Benchmark

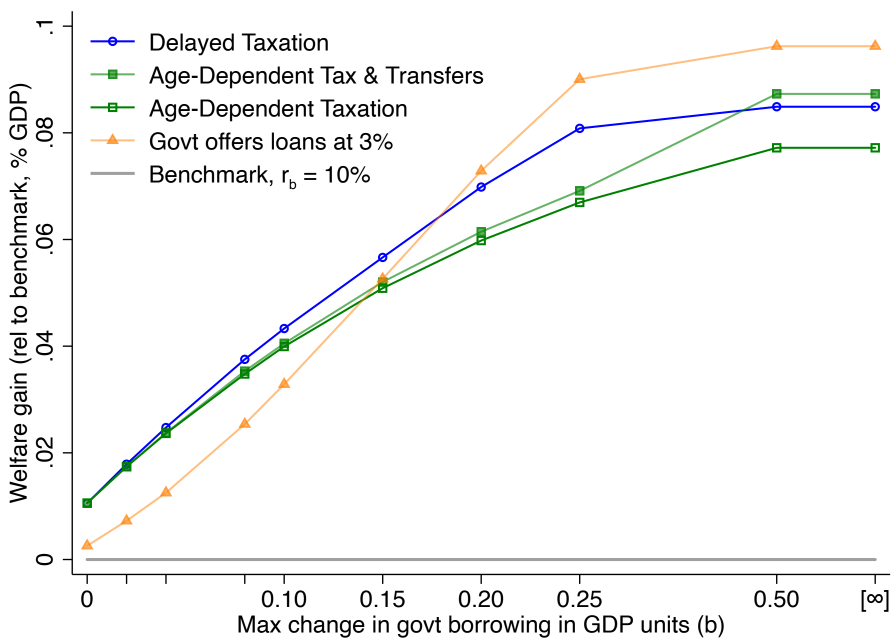


Figure A.4: Optimal interest rate on delayed taxes

This figure shows the optimally chosen $r_{dtax} \geq -1$ in the flexible delayed tax version. We fix $r_b = 10\%$ and vary the government borrowing constraint parameter, b. The y-axis gives the cumulative interest rate, $(1 + r_{dtax})^{21} - 1$, where a value of -1 implies immediate debt forgiveness.