

Task 1

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```
[3]: import numpy as np
import matplotlib.pyplot as plt
from numpy.linalg import inv
from numpy.fft import fft, ifft
```

```
[4]: N = 10
k = np.arange(N)
mu = np.arange(N)

K = np.outer(k, mu)
W = np.exp(+1j * 2*np.pi/N * K)
X_test = np.array([6, 2, 4, 3, 4, 5, 0, 0, 0, 0])
x_test = 1/N * np.matmul(W, X_test)

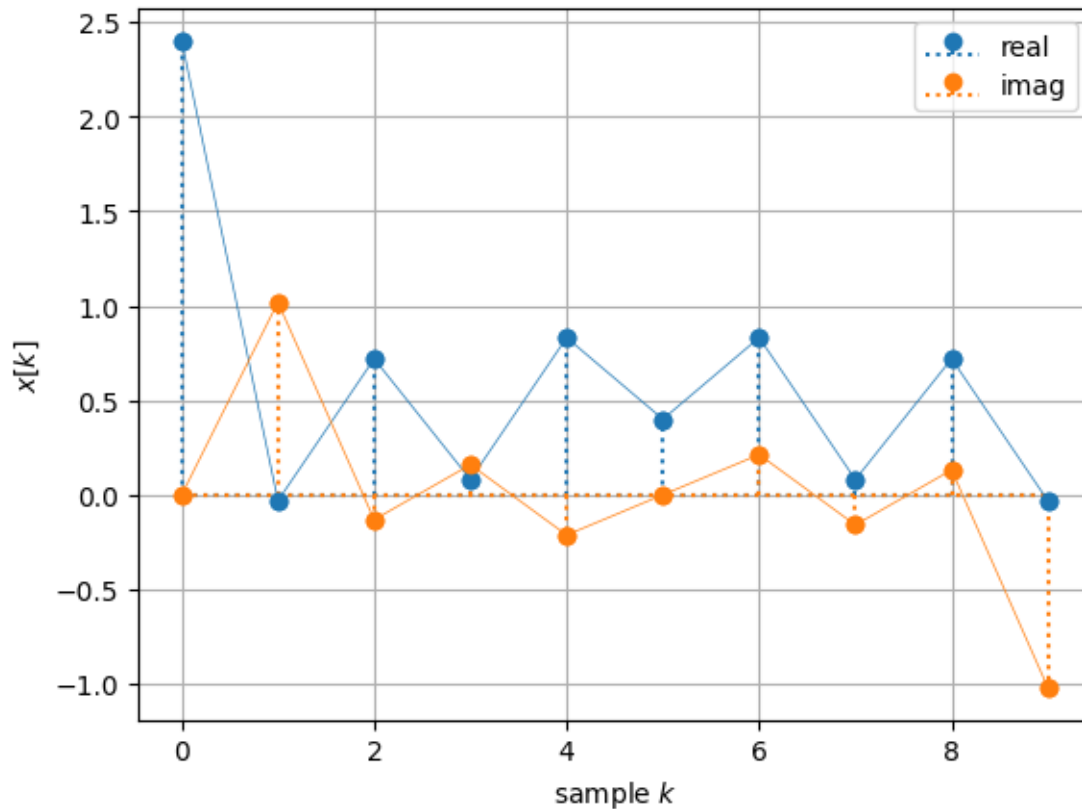
plt.stem(k, np.real(x_test), label='real',
         markerfmt='C0o', basefmt='C0:', linefmt='C0:')
plt.stem(k, np.imag(x_test), label='imag',
         markerfmt='C1o', basefmt='C1:', linefmt='C1:')

plt.plot(k, np.real(x_test), 'C0o-', lw=0.5)
plt.plot(k, np.imag(x_test), 'C1o-', lw=0.5)
plt.xlabel(r'sample $k$')
plt.ylabel(r'$x[k]$')
plt.legend()
plt.grid(True)

print(np.allclose(ifft(X_test), x_test))
print('DC is 1 as expected: ', np.mean(x_test))
```

True

DC is 1 as expected: (0.6+8.881784197001253e-17j)



```
[14]: x_test2 = X_test[0] * W[:, 0] + X_test[1] * W[:, 1] + X_test[2] * W[:, 2]
      ↪ +X_test[3] * W[:, 3] + X_test[4] * W[:, 4] + X_test[5] * W[:, 5]
```

```
[15]: x_test2 *= 1/N
      print(np.allclose(x_test, x_test2))
```

True

```
[12]: K
```

```
[12]: array([[ 0,  0,  0,  0,  0,  0,  0,  0,  0,  0],
            [ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9],
            [ 0,  2,  4,  6,  8, 10, 12, 14, 16, 18],
            [ 0,  3,  6,  9, 12, 15, 18, 21, 24, 27],
            [ 0,  4,  8, 12, 16, 20, 24, 28, 32, 36],
            [ 0,  5, 10, 15, 20, 25, 30, 35, 40, 45],
            [ 0,  6, 12, 18, 24, 30, 36, 42, 48, 54],
            [ 0,  7, 14, 21, 28, 35, 42, 49, 56, 63],
            [ 0,  8, 16, 24, 32, 40, 48, 56, 64, 72],
            [ 0,  9, 18, 27, 36, 45, 54, 63, 72, 81]])
```

[29]: W

```
[29]: array([[ 1.          +0.00000000e+00j,  1.          +0.00000000e+00j,
              1.          +0.00000000e+00j,  1.          +0.00000000e+00j,
              1.          +0.00000000e+00j,  1.          +0.00000000e+00j,
              1.          +0.00000000e+00j,  1.          +0.00000000e+00j],
            [ 1.          +0.00000000e+00j,  0.80901699+5.87785252e-01j,
              0.30901699+9.51056516e-01j, -0.30901699+9.51056516e-01j,
              -0.80901699+5.87785252e-01j, -1.          +1.22464680e-16j,
              -0.80901699-5.87785252e-01j, -0.30901699-9.51056516e-01j,
              0.30901699-9.51056516e-01j,  0.80901699-5.87785252e-01j],
            [ 1.          +0.00000000e+00j,  0.30901699+9.51056516e-01j,
              -0.80901699+5.87785252e-01j, -0.80901699-5.87785252e-01j,
              0.30901699-9.51056516e-01j,  1.          -2.44929360e-16j,
              0.30901699+9.51056516e-01j, -0.80901699+5.87785252e-01j,
              -0.80901699-5.87785252e-01j,  0.30901699-9.51056516e-01j],
            [ 1.          +0.00000000e+00j, -0.30901699+9.51056516e-01j,
              -0.80901699-5.87785252e-01j,  0.80901699-5.87785252e-01j,
              0.30901699+9.51056516e-01j, -1.          +3.67394040e-16j,
              0.30901699-9.51056516e-01j,  0.80901699+5.87785252e-01j,
              -0.80901699+5.87785252e-01j, -0.30901699-9.51056516e-01j],
            [ 1.          +0.00000000e+00j, -0.80901699+5.87785252e-01j,
              0.30901699-9.51056516e-01j,  0.30901699+9.51056516e-01j,
              -0.80901699-5.87785252e-01j,  1.          -4.89858720e-16j,
              -0.80901699+5.87785252e-01j,  0.30901699-9.51056516e-01j,
              0.30901699+9.51056516e-01j, -0.80901699-5.87785252e-01j],
            [ 1.          +0.00000000e+00j, -1.          +1.22464680e-16j,
              1.          -2.44929360e-16j, -1.          +3.67394040e-16j,
              1.          -4.89858720e-16j, -1.          +6.12323400e-16j,
              1.          -7.34788079e-16j, -1.          +8.57252759e-16j,
              1.          -9.79717439e-16j, -1.          +1.10218212e-15j],
            [ 1.          +0.00000000e+00j, -0.80901699-5.87785252e-01j,
              0.30901699+9.51056516e-01j,  0.30901699-9.51056516e-01j,
              -0.80901699+5.87785252e-01j,  1.          -7.34788079e-16j,
              -0.80901699-5.87785252e-01j,  0.30901699+9.51056516e-01j,
              0.30901699-9.51056516e-01j, -0.80901699+5.87785252e-01j],
            [ 1.          +0.00000000e+00j, -0.30901699-9.51056516e-01j,
              -0.80901699+5.87785252e-01j,  0.80901699+5.87785252e-01j,
              0.30901699-9.51056516e-01j, -1.          +8.57252759e-16j,
              0.30901699+9.51056516e-01j,  0.80901699-5.87785252e-01j,
              -0.80901699-5.87785252e-01j, -0.30901699+9.51056516e-01j],
            [ 1.          +0.00000000e+00j,  0.30901699-9.51056516e-01j,
              -0.80901699-5.87785252e-01j, -0.80901699+5.87785252e-01j,
              0.30901699+9.51056516e-01j,  1.          -9.79717439e-16j,
              0.30901699-9.51056516e-01j, -0.80901699-5.87785252e-01j,
              -0.80901699+5.87785252e-01j,  0.30901699+9.51056516e-01j],
```

```
[ 1.          +0.00000000e+00j,  0.80901699-5.87785252e-01j,
  0.30901699-9.51056516e-01j, -0.30901699-9.51056516e-01j,
 -0.80901699-5.87785252e-01j, -1.          +1.10218212e-15j,
 -0.80901699+5.87785252e-01j, -0.30901699+9.51056516e-01j,
  0.30901699+9.51056516e-01j,  0.80901699+5.87785252e-01j]])
```

```
[44]: import pandas as pd
      df = pd.DataFrame(K)
```

```
[42]: latex_code = df.to_latex(header = False, index=False)
      print(latex_code)
```

```
\begin{tabular}{rrrrrrrrrr}
\toprule
\midrule
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \\\
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & \\\
0 & 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 & \\\
0 & 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 & \\\
0 & 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 & \\\
0 & 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 & 54 & \\\
0 & 7 & 14 & 21 & 28 & 35 & 42 & 49 & 56 & 63 & \\\
0 & 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 & 72 & \\\
0 & 9 & 18 & 27 & 36 & 45 & 54 & 63 & 72 & 81 & \\\
\bottomrule
\end{tabular}
```

```
[33]: df_w = pd.DataFrame(W)
```

```
[43]: latex_code = df_w.to_latex(header = False, index=False)
      print(latex_code)
```

```
\begin{tabular}{rrrrrrrrrr}
\toprule
\midrule
1.000000+0.000000j & 1.000000+0.000000j & 1.000000+0.000000j & & & & & & & & \\
1.000000+0.000000j & 1.000000+0.000000j & 1.000000+0.000000j & & & & & & & & \\
1.000000+0.000000j & 1.000000+0.000000j & 1.000000+0.000000j & & & & & & & & \\
1.000000+0.000000j & \\\
1.000000+0.000000j & 0.809017+0.587785j & 0.309017+0.951057j & & & & & & & & \\
-0.309017+0.951057j & -0.809017+0.587785j & -1.000000+0.000000j & & & & & & & & \\
-0.809017-0.587785j & -0.309017-0.951057j & 0.309017-0.951057j & & & & & & & & \\
0.809017-0.587785j & \\\
1.000000+0.000000j & 0.309017+0.951057j & -0.809017+0.587785j & & & & & & & & \\
-0.809017-0.587785j & 0.309017-0.951057j & 1.000000-0.000000j & & & & & & & & \\
0.309017+0.951057j & -0.809017+0.587785j & -0.809017-0.587785j & & & & & & & & \\
\end{tabular}
```

```

0.309017-0.951057j \\
1.000000+0.000000j & -0.309017+0.951057j & -0.809017-0.587785j &
0.809017-0.587785j & 0.309017+0.951057j & -1.000000+0.000000j &
0.309017-0.951057j & 0.809017+0.587785j & -0.809017+0.587785j &
-0.309017-0.951057j \\
1.000000+0.000000j & -0.809017+0.587785j & 0.309017-0.951057j &
0.309017+0.951057j & -0.809017-0.587785j & 1.000000-0.000000j &
-0.809017+0.587785j & 0.309017-0.951057j & 0.309017+0.951057j &
-0.809017-0.587785j \\
1.000000+0.000000j & -1.000000+0.000000j & 1.000000-0.000000j &
-1.000000+0.000000j & 1.000000-0.000000j & -1.000000+0.000000j &
1.000000-0.000000j & -1.000000+0.000000j & 1.000000-0.000000j &
-1.000000+0.000000j \\
1.000000+0.000000j & -0.809017-0.587785j & 0.309017+0.951057j &
0.309017-0.951057j & -0.809017+0.587785j & 1.000000-0.000000j &
-0.809017-0.587785j & 0.309017+0.951057j & 0.309017-0.951057j &
-0.809017+0.587785j \\
1.000000+0.000000j & -0.309017-0.951057j & -0.809017+0.587785j &
0.809017+0.587785j & 0.309017-0.951057j & -1.000000+0.000000j &
0.309017+0.951057j & 0.809017-0.587785j & -0.809017-0.587785j &
-0.309017+0.951057j \\
1.000000+0.000000j & 0.309017-0.951057j & -0.809017-0.587785j &
-0.809017+0.587785j & 0.309017+0.951057j & 1.000000-0.000000j &
0.309017-0.951057j & -0.809017-0.587785j & -0.809017+0.587785j &
0.309017+0.951057j \\
1.000000+0.000000j & 0.809017-0.587785j & 0.309017-0.951057j &
-0.309017-0.951057j & -0.809017-0.587785j & -1.000000+0.000000j &
-0.809017+0.587785j & -0.309017+0.951057j & 0.309017+0.951057j &
0.809017+0.587785j \\
\\bottomrule
\\end{tabular}

```

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