

15.77

An earthquake-produced surface wave can be approximated by a sinusoidal transverse wave.

Assuming a frequency of 0.60 Hz (typical of earthquakes, which actually include a mixture of frequencies), what amplitude is needed so that objects begin to leave contact with the ground?

$$D = A \sin(kx - \omega t)$$

we make $x = 0$ (x is not relevant here).

$$a = \frac{d^2 D}{dt^2} = -\omega^2 A \sin(\omega t)$$

Free body diagram: $N - mg = ma$ $N = m(g+a)$. When $N=0$ we have the object not touching the ground, so this happens when $g=-a$

$$a = a_{max} = \omega^2 A \rightarrow A = \frac{g}{\omega^2} = \frac{g}{(2\pi f)^2}$$

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In [5]: import numpy as np
g = 9.8 #m/s^2
f = 0.6 #Hz

A = g/(2*np.pi*f)**2

print(A, ' m')
```

0.6895469442325766 m

16.60

Two trains emit 577 Hz whistles. One train is stationary. The conductor on the stationary train hears a 3.7 Hz beat frequency when the other train approaches.

- What is the speed of the moving train?

This is a doppler problem of a moving source (train approaching) towards an observer at rest (second train). The fact that the observer is at rest. It is then combined with a temporal interference problem (beats are destructive interference in time!).

- Doppler formula for the frequency: $f' = \frac{f}{(1 - \frac{v_{source}}{v_{sound}})}$
- Beats frequency $f_b = |f_1 - f_2|$

So the only point left to remember is to remember that for a given beat frequency there can be two solutions, due to the absolute value. But we know that here f' will be larger than f , so we take $f' = f_b + f$

And having f' now we can solve for v_{source} :

$$v_{source} = v_{sound}(1 - \frac{f}{f'})$$

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In [6]: fb= 3.7 #Hz
f = 577 # Hz
vs = 343 #m/s
fp = f+fb
v = vs*(1-f/fp)

print(v, ' m/s')
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2.1854658171173034 m/s

16.61

A wave on the surface of the ocean with wavelength 44 m is traveling east at a speed of 18 m/s relative to the ocean floor.

- If, on this stretch of ocean surface, a powerboat is moving at 12 m/s (relative to the ocean floor), how often does the boat encounter a wave crest, if the boat is traveling west?
- How often does the boat encounter a wave crest, if the boat is traveling east?

This is another doppler problem. Now we have a moving observer (the power boat) towards/away a stationary source (the water wave is moving, but its source is not). They give us the velocity and the wavelength so as we can compute its frequency and period.

$$T = v/\lambda = 1/f$$

So for the first question we use the formula of the moving observer towards a stationary source. (wave moves east and boat moves west)

$$T' = \frac{\lambda}{v_{wave} + v_{obs}}$$

And for the second we use the formula of the observer moving away

$$T' = \frac{\lambda}{v_{wave} - v_{obs}}$$

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In [7]: v = 18 #m/s
l = 44 #m
vo = 12 #m/s

T = v/l
#towards
Tp= l/(vo+v)

print(Tp, ' s')

# away
Tp= l/(v-vo)

print(Tp, ' s')
```

1.466666666666666 s
7.333333333333333 s

16.65

A factory whistle emits sound of frequency 720 Hz.

- When the wind velocity is 13.8 m/s from the north, what frequency will observers hear who are located, at rest, due north of the whistle? Assume T=20°C.
- What frequency will observers hear who are located, at rest, due south of the whistle?

Again this is a doppler problem. But with a twist. The observer and source are both at rest, but the medium (air) is not. In this case we can do everything on the reference frame of the air. This means that in that reference frame both the observer and emitter are moving! We now look, on the reference frame of the air what is the velocity of the factory and that of the observers.

- First part: As the observer is due north of the factory, its velocity is away from the factory (equal to v_{air}) and the velocity of the factory in this new reference frame is towards the observer (also with v_{air}).
 - Source moves towards, observer moves away

$$f' = f \frac{(v_{sound} - v_{obs})}{(v_{sound} - v_{source})}$$

- Second part: Now the observer moves towards the source and the source moves away from the observer:
 - Source moves away, observer moves towards

$$f' = f \frac{(v_{sound} + v_{obs})}{(v_{sound} + v_{source})}$$

As we see as the velocity of the source and observer is exactly the same, we actually have the same result in all cases, which is that the sound frequency does not change! so really the wind moving will not make any difference to the frequency we hear.

In []:

