Final Exam

- Dec 13 (monday) 11:15-13:45 (2.5 hours)
- Review session Th Dec 9 6 pm?
- 8 problems
  - 2 MT1
  - 2 MT2
  - 1 Simple Harmonic motion
  - 1 Fluids
  - 1 Ideal Gas
  - 1 Thermodynamics (Heat engines)
- 3 pages of formulas
- Calculator! (no excuses)
- Grading:
  - Attendance/lecture quizzes 5%
  - HW 15 %
  - Recitations 5 %
  - MT1 20%
  - MT2 20%
  - whatever is larger (Mt1+Mt2)*0.2 or 0.4*MT1 or 0.4*MT2
  - Final 35%
  - There was no project hence 35% for final. However, if you present a report of an experiment/video experiment where a concept discussed in class is presented and explained I will give an extra 5% and make the final 30%. This needs to be sent before the final exam date.

Q1

A satellite in a circular geostationary orbit above the Earth's surface (meaning it has the same angular velocity as the Earth) explodes in to three pieces of equal mass. One piece remains in a circular orbit at the same height as the satellite's orbit after the collision, but the velocity of this piece is in the opposite direction to the velocity of the satellite before the explosion. After the explosion the second piece falls from rest in a straight line directly to the Earth. The third piece has an initial velocity in the same direction as the original direction of the satellite, but now has a different magnitude of velocity. The diagram is obviously not remotely to scale!

- (a) (10 points) At what height above the Earth does the satellite orbit?

$G\frac{mM_E}{r^2} = \frac{mv^2}{r}$

$v_{circular} = \sqrt{\frac{GM_E}{r}}$

$M_E = 5.98 \times 10^{24}\text{kg}$

$R_E = 6380\text{km}$

$G = 6.67 \times 10^{-11} \mathrm{Nm^2kg^{-2}}$

$\frac{2\pi r}{v}$ = 1 day = 86,400 s

Using the equation for the velocity $v = \sqrt{\frac{GM_E}{r}}$

$2\pi \frac{r^{3/2}}{\sqrt{GM_E}}$ = 86,400 s

$r = (86,400 \frac{\sqrt{GM_E}}{2\pi})^{2/3} = 4.23 \times 10^7 \text{ m} = 42,300\,\mathrm{km}$

(35,900 km above surface)

- (b) (5 points) What is the magnitude of the velocity of the satellite along it's original circular path?

$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{6.67\times10^{-11}\times5.98\times10^{24}}{4.23\times10^{7}}} = 3071\mathrm{m\,s^{-1}}$

- (c) (5 points) For the piece that falls to the ground, what is the magnitude of the velocity when it strikes the ground? You may neglect any effects due to air resistance.

$\frac{1}{2}mv^2 = \frac{-GMm}{4.23\times10^{7}} - \frac{-GMm}{6.38\times10^{6}}$

$v^2 = 1.06 \times 10^8$

$v = 1.03 \times 10^4 \mathrm{m\,s^{-1}}$

- (d) (5 points) What happens to the third piece? Does it fall to the ground, continue in an orbit around the Earth (not necessarily a circular one), or escape the gravitational pull of the Earth? Justify your answer.

From conservation of momentum the velocity of the third piece must be 4 times the original velocity. The escape velocity is $\sqrt{2}$ times the circular velocity, so the third piece escapes the gravitational pull of the earth.

Q2

An inclined plane has a coefficient of static friction $\mu_s = 0.4$ and kinetic friction $\mu_k = 0.3$.

- (a) (5 points) What is the maximum inclination angle θ for which a stationary block on the plane will remain at rest?

The condition for when static friction and the gravitational force down the plane exactly cancel is given by

$mg \sin\theta = \mu_s mg \cos\theta$

$\tan\theta = \mu_s = 0.4$

$\theta = 21.8^o$

- (b) (10 points) If a ball ($I = \frac{2}{5}Mr^2$) is placed on the incline when it has the inclination angle you found in part (a) and rolls through a distance of 1.5m, what is the translational velocity of the ball when it reaches the bottom of the incline?

For rolling

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2\frac{v^2}{r^2} = \frac{7}{10}mv^2$$

$$\frac{7}{10}mv^2 = mg1.5\sin\theta$$

$$v = \sqrt{\frac{10}{7} \times 1.5 \times 9.8 \times \sin 21.8^o}$$

$$v = 2.79\mathrm{m\,s^{-1}}$$

- (c) (10 points) What percentage of the kinetic energy of the ball is rotational while it is rolling down the incline?

$$\frac{\frac{2}{5}}{\frac{2}{5}+1} = 0.29 = 29\%$$

Q3

A plane is flying horizontally with a constant speed of 100m/s at a height h above the ground, and drops a 50kg bomb with the intention of hitting a car that has just begun driving up a 10° incline which starts a distance l in front of the plane. The speed of the car is a constant 30m/s. For the following questions use the coordinate axes defined in the figure, where the origin is taken to be the initial position of the car.

- (a) (5 points) What is the initial velocity of the bomb relative to the car? Write your answer in unit vector notation.

$$v_x = 100 - 30\cos(10^o) = 70.46\mathrm{ms^{-1}}$$

$$v_y = -30\sin(10^o) = -5.21\mathrm{ms^{-1}}$$

$$\vec{v} = 70.46\mathrm{ms^{-1}}\hat{i} - 5.21\mathrm{ms^{-1}}\hat{j}$$

- (b) (5 points) Write equations for both components (x and y) of the car's displacement as a function of time, taking t=0s to be the time the bomb is released.

$$x = 30\cos(10^o)t = 29.54t\ \mathrm{m}$$

$$y = 30\sin(10^o)t = 5.21t\ \mathrm{m}$$

- (c) (5 points) Write equations for both components (x and y) of the bomb's displacement as a function of time, taking t=0s to be the time the bomb is released.

$$x = 100t - l\ \mathrm{m}$$

$$y = h - \frac{1}{2}gt^2\ \mathrm{m}$$

- (d) (5 points) If the bomb hits the car at time t=10s what was the height of the plane above the ground h when it dropped the bomb?

$y = 52.1\text{m}$

$52.1 = h - \frac{1}{2}g10^2$

$h = 52.1 + 50 \times 9.81 = 542.61\text{m}$

- (e) (5 points) What is the horizontal displacement of the plane relative to the car when the bomb hits the car at t=10s.

0m

(f) (5 points) How much kinetic energy does the bomb have when it hits the car?

$\frac{1}{2}mv_0^2 + 24525 = \frac{1}{2} \times 50 \times 100^2 + 240590 = 250000 + 240590 = 490590\text{J}$

Q4

A 90cm tall cylindrical steel oil drum weighs 15kg and has a external volume of 0.2m^3. When full the drum contains 190L of crude oil. The density of crude oil is 900kg/m^3 and of water is 1000kg/m^3.

- (a) (5 points) What is the external diameter of the oil drum?

$\pi\frac{d^2}{4}h = 0.2$

$d = 0.53\text{m}$

- (b) (10 points) If the oil drum has fallen overboard and is floating upright in the sea, what length of the oil drum x sticks up above the water? As a simplifying approximation you may assume that the all the steel is on the sides of the drum, and the top and bottom do not contribute to the mass of the drum.

The internal volume of the drum is $190\text{L} = 0.19\text{m}^3$, which means that the steel has a volume of 0.01m^3 and thus a density of 1500kg/m^3. The average density of the barrel is then

$\frac{1500\times0.01+900\times0.19}{0.01+0.19} = 930\text{kg/m}^3$

The equilibrium is found when the bouyant force equals the gravitational force.

$\rho_{water}(h - x)\pi r^2 g = 930h\pi r^2 g$

$0.9 - x = 0.9\frac{930}{1000}$

$x = 0.063\text{m} = 6.3\text{cm}$

- (c) (5 points) What is the pressure at the bottom of the drum? You may assume standard atmospheric pressure for the air above the drum.

$P = P_{atm} + \rho gh = 1.013 \times 10^5 + 1000 \times 9.8 \times 0.837 = 1.095 \times 10^5\text{Pa}$

Q5

The system shown in the figure is initially in static equilibrium. The inclined plane with an inclination angle of θ is frictionless. The mass m_2 which is hanging from the rope is heavier than the mass m_1 which lies on the inclined plane. The pulley in this problem has mass m_3 and radius r. The mass m_1 is attached to a spring with spring constant k, the other end of this spring is fixed. For all of the following questions try to simplify your answer as much as possible.

- (a) (5 points) Find an expression for x_0, the distance that the spring is extended from it's natural length in terms of some or all of the variables, m_1, m_2, m_3, r, θ and k.

$x_0 = (m_2 - m_1 \sin\theta)\frac{g}{k}$

- (b) (5 points) The mass m_2 is now pulled down by a distance A. Write an expression for the work done on the system during this process in terms of some or all of the variables, m_1, m_2, m_3, θ, r, k and A.

$W_{net} = \frac{1}{2}kA^2$

- (c) (5 points) The system is now released. Write an expression for the frequency of the subsequent simple harmonic motion in terms of some or all of the variables, m_1, m_2, m_3, θ, k, r and A.

Consider the sum of the forces on each object

$m_2 a = m_2 g - T_2$

$\frac{1}{2}m_3 a = T_2 - T_1$

$m_1 a = T_1 - m_1 g \sin\theta - kx$

$(m_1 + m_2 + \frac{1}{2}m_3)a = m_2 g - m_1 g \sin\theta - kx$

The solution to this equation of motion is an oscillation centered around x_0 with $\omega^2 = \frac{k}{m_1 + m_2 + \frac{1}{2}m_3}$

and the frequency is

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_1 + m_2 + \frac{1}{2}m_3}}$

- (d) (5 points) Find an expression for the magnitude of the maximum velocity of the system in terms of some or all of the variables, m_1, m_2, m_3, θ, k, r and A. At which value or values of the displacement of m_2 from it's original equilibrium position does this occur?

$v_{max} = \omega A = \sqrt{\frac{k}{m_1 + m_2 + \frac{1}{2}m_3}}A$

This occurs when the displacement from $x_0 = 0$

- (e) (5 points) Find an expression for the magnitude of the maximum acceleration of the system in terms of some or all of the variables, m_1, m_2, m_3, θ, k, r and A. At which value or values of the displacement of m_2 from it's original equilibrium position does this occur?

$$a_{max} = \omega^2 A = \frac{k}{m_1 + m_2 + \frac{1}{2} m_3} A$$

This occurs when the displacement from $x_0 = \pm A$

Q6

The speed of sound in air is $v = 331 + 0.6T\,\mathrm{ms}^{-1}$ where T is the temperature in ˚C. Two organ pipes of length 0.6m which are open at both ends are used to produce sounds of slightly different frequencies by heating one of the tubes above the room temperature of 20˚C.

- (a) (5 points) What is the lowest frequency sound that can be produced in the room temperature pipe?

  $v_{sound} = 343\mathrm{m\,s}^{-1}$

$$f = \frac{v}{2l} = \frac{343}{2\times 0.6} = 285.8\mathrm{Hz}$$

- (b) (5 points) Draw on the figure the form of the standing wave amplitude for both displacement and pressure that produces this sound.

Remember, on the open ends the displacement is zero and the pressure change is max/min.

- (c) (5 points) To produce beats with a beat frequency of 1Hz with the sound produced in (a) using the first harmonic of the heated pipe what should the temperature of the heated pipe be?

$$\Delta f = 1\mathrm{Hz}$$

$$f_T = 286.8\mathrm{Hz}$$

$$v_T = 286.8 \times 2 \times 0.6 = 344.2\mathrm{m\,s}^{-1}$$

$$T = \frac{344.2 - 331}{0.6} = 22^{\mathrm{o}}\mathrm{C}$$

- (d) (5 points) At what speed should a person running toward the tubes be moving so that the sound from the room temperature tube has an apparent frequency equal to the actual frequency of sound produced by the heated tube.

$$f' = \frac{v_{sound} + v_{obs}}{v_{sound}} f$$

$$286.8 = \frac{343 + v_{obs}}{343} 285.8$$

$$v_{obs} = 1.2\mathrm{m\,s}^{-1}$$

- (e) (5 points) What is the frequency of the second harmonic that the room temperature pipe produces?

$$f = 2 \times 285.8 = 571.6\mathrm{Hz}$$

Q7

3 moles of a ideal monatomic gas are heated from atmospheric pressure 20˚C to 120˚C at constant volume. The gas is then heated further from 120˚C to 220˚C at constant pressure.

- (a) (5 points) How much does the internal energy of the gas change in each process?

For both processes

$$\Delta E_{int} = \frac{3}{2}nR\Delta T = \frac{3}{2} \times 3 \times 8.314 \times 100 = 3741.3\text{J}$$

- (b) (5 points) How much heat is added to the gas in each process?

For an ideal monatomic gas expanded at constant volume

$$Q = \frac{3}{2}nR\Delta T = \frac{3}{2} \times 3 \times 8.314 \times 100 = 3741.3\text{J}$$

At constant pressure

$$Q = \frac{5}{2}nR\Delta T = \frac{5}{2} \times 3 \times 8.314 \times 100 = 6235.5\text{J}$$

- (c) (5 points) How much work is done by the gas in each process?

At constant volume $W = 0J$

At constant pressure $W = nR\Delta T = 2494.2\text{J}$

- (d) (5 points) What is the entropy change of the gas in each process?

For the constant volume process

$$\Delta S = \int \frac{dQ}{T} = \frac{3}{2}nR\int_{T_1}^{T_2} \frac{dT}{T} = \frac{3}{2}nR\ln\frac{T_2}{T_1} = \frac{3}{2} \times 3 \times 8.314 \ln\frac{393}{293} = 10.985\text{J/K}$$

For the constant pressure process

$$\Delta S = \int \frac{dQ}{T} = \frac{5}{2}nR\int_{T_1}^{T_2} \frac{dT}{T} = \frac{5}{2}nR\ln\frac{T_2}{T_1} = \frac{5}{2} \times 3 \times 8.314 \ln\frac{493}{393} = 14.136\text{J/K}$$

- (e) (5 points) What is the final pressure and volume of the gas?

$$PV = nRT$$

$$P_{initial} = 1.013 \times 10^5\text{Pa}$$

$$V_{initial} = \frac{3\times8.314\times293}{1.013\times10^5} = 0.072\text{m}^3$$

$$P_{final} = \frac{3\times8.314\times393}{V_{initIal}} = 1.386 \times 10^5\text{Pa}$$

$$V_{final} = \frac{3\times8.314\times493}{P_{final}} = 0.090\text{m}^3$$

Q8

The diagram shows the P-V diagram for a 40% efficient ideal Carnot engine. Assume the gas used in this Carnot engine is an ideal diatomic gas.

- (a) (5 points) For every Joule of work obtained from the engine, how much heat needs to be added to engine?

$e = \frac{W}{Q_H}$

$Q_H = \frac{1}{0.4} = 2.5\mathrm{J}$

- (b) (5 points) For every Joule of work obtained from the engine how much heat is lost to the environment?

$Q_H = W + Q_L$

$Q_L = 1.5\mathrm{J}$

- (c) (5 points) At points B and D the gas in the system has the same volume, but different temperatures. If the gas at point D is at twice atmospheric pressure, what is the pressure of the gas at point B?

$P_B V_B = nRT_H$

$P_D V_D = nRT_L$

$\frac{P_B V_B}{P_D V_D} = \frac{T_H}{T_L}$

$P_B = 2P_{atm}\frac{T_H}{T_L}$

As $e = 1 - \frac{T_L}{T_H} = 0.4$

$\frac{T_L}{T_H} = 0.6$

$P_B = 3.33P_{atm}$

- (d) (5 points) If the volume of the gas at point B is 1L what is the volume of the gas at point C?

The expansion from B to C is adiabatic so $P_B V_B^\gamma = P_C V_C^\gamma$

For a diatomic gas $\gamma = \frac{7}{5}$

We also know that $P_B V_B = nRT_H$ and $P_C V_C = nRT_L$

so

$nRT_H V_B^{\gamma-1} = nRT_L V_C^{\gamma-1}$

$V_C^{\gamma-1} = \frac{T_H}{T_L} V_B^{\gamma-1}$

$V_C^{2/5} = \frac{10}{6}$

$V_C = 3.6\mathrm{L}$

- (e) (5 points) How much does the net entropy of the engine and the environment change for every Joule of work done by this Carnot engine?

$$\Delta S = 0 \frac{\mathrm{J}}{\mathrm{K}}$$

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