Lecture 21 - Physics of Materials

Corbel arch

An object can be in stable equilibrium if a vertical line from the center of gravity lies within it's center of support.

We will demonstrate this concept by building a corbel arch.

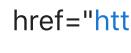![im.png](im.png)

True Arch

The weakness of a corbel arch as compared to a true arch which should contain a catenary curve, is that not all the tensile stresses are converted in to compressive stresses. What does this mean and why it is important? We need to know a little about the properties of materials.

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Stress

When an object is exposed to a external force, it can deform. The relative size of this deformation, is called a **strain** $\varepsilon = \frac{\Delta l}{l_0}$. Often, when dealing with material properties it can be more useful to consider the force as a force per unit area, or **stress** $\sigma = \frac{F}{A}$

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href="http://www.doitpoms.ac.uk/tlplib/dislocations/dislocation_motion.php" class="urlextern" title="http://www.doitpoms.ac.uk/tlplib/dislocations/dislocation_motion.php">These videos of bubble rafts from DoITPoMs at Cambridge are useful for visualizing the way materials respond to the various kinds of stress.

Elastic Deformation

Many materials exhibit elastic behavior, where they will recover from a strain to their original shape.

This can be divided in to two regimes, the linear elastic regime, below the proportional limit, where objects obey Hooke's Law

$$F = k\Delta l$$

and their strain is linearly proportional to the stress. As the stress is increased this is followed by a non-linear elastic regime where the stress and strain are not linearly related.

The upper limit of this non-linear elastic regime is the elastic limit. Up to this limit the object will revert to it's original shape when the stress is removed, beyond it the object will deform in a non-reversible fashion, until it eventually breaks.

The material specific plot of stress vs strain is called a stress strain curve.

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Elastic Moduli

There are a set of elastic moduli that link the deformation of an object to particular types of stress.

Young's modulus, E, is the constant that relates stress along a direction to strain in that direction in the elastic regime.

$$\sigma = E\varepsilon$$

$$\frac{F}{A} = E\frac{\Delta l}{l_0}$$

This formula can be applied to either tensile stress or compressive stress.

For shear stresses a different elastic modulus, the shear modulus, G applies. G is usually smaller than E.

$$\sigma = G\varepsilon$$

$$\frac{F}{A} = G\frac{\Delta l}{l_0}$$

Note: for this equation Δl is perpendicular to l_0!

An object subject to uniform pressure change will have a volume change

$$\frac{\Delta V}{V_0} = -\frac{1}{B}\Delta P$$

B is the bulk modulus.

Strength of materials

Different materials, depending on their micro-structure have different strengths with relation to the different strains. By strength we refer to the maximum stress that can be withstood without the material breaking.

Ultimate tensile strength for ductile materials is usually determined by necking, brittle materials, like concrete typically have very low tensile strength, but have high compressive strength. Structures under compressive stress can collapse well before the material itself fails due to buckling.

Shear failure can be greatly assisted by imperfections that allow dislocation glide.

Truss bridge

A truss bridge uses connected elements to spread the load out, some elements are under compression, others are under tension. In the diagram below we ignore the mass of the elements themseleves.

Frequently different materials are used for compression and tension elements. Long truss bridges get very heavy, a better design for these applications is a suspension bridge.

Suspension bridge

The simplest kind of suspension bridge is one that hangs under it's own weight in the shape of the catenary, the equation for which is $y = a\cosh(\frac{x}{a})$. An inverted catenary is the ideal shape for an arch which supports only it's own weight.

A more normal suspension bridge, and also more typical arches, support weight besides their own and the ideal shape in these situations is closer to that of a parabola, in the case where the mass of the cables can be neglected the shape is entirely parabolic.

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