Lecture 30 - Temperature and ideal gas laws

Atomic Theory of Matter

To introduce the concept of temperature we need to start from the atomic theory, which considers matter to be composed of atoms.

A useful unit when discussing the mass of atoms is the atomic mass unit [u]

$1\mathrm{u} = 1.6605 \times 10^{-27}\mathrm{kg}$

Atoms and molecules are in continual motion, the amount of motion is proportional to the temperature, which we can think of as a measure of energy.

Brownian motion, named after Robert Brown who observed it as random movements of pollen or dust suspended in water was explained by Albert Einstein in 1905 as a product of the thermal motion of molecules or atoms.

We can consider temperature to be a measure of the energy contained in the motion of the microscopic constituents of a material.

Temperature Scales

To be able to measure temperature we need a scale. The temperature scale in common usage in the United States, Fahrenheit, $^{\circ}\mathrm{F}$ is based on what may now seem to be fairly arbitrary reference points.

Most of the world uses the Celsius scale, $^{\circ}\mathrm{C}$, for everyday measurements. This is based on dividing the difference between the freezing point and boiling point of water in to 100 degrees and fixing $0^{\circ}C$ as the freezing point.

For thermodynamics an absolute temperature scale, in which 0 is the complete absence of thermal energy, is appropriate. This scale is the Kelvin Scale. In this scale water freezes at $273.15\mathrm{K}$. Note that we do not use a degree symbol for temperatures in Kelvin.

To convert between Farenheit and Celsius you can use the fact that water freezes at $32^{\circ}\mathrm{F}$ and boils at $212^{\circ}F$ to deduce that:

$T(^{\circ}\mathrm{C}) = \frac{5}{9}(T(^{\circ}\mathrm{F}) - 32)$

or

$T(^{\circ}\mathrm{F}) = \frac{9}{5}T(^{\circ}\mathrm{C}) + 32$

Thermal Equilibrium

If two objects with different temperatures are brought in to contact with one another thermal energy will flow from one to another until the temperatures are the same, and we then say that the objects are in thermal equilibrium.

The zeroth law of thermodynamics states that:

"If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other."

This law may seem obvious but this is only because we already have some expectation from our everyday experience that temperature is a general quantity that we can measure with a thermometer. This law allows us to declare this formally, and turns out to be important, in conjunction with the First and Second Laws (we'll talk about these later!), for establishing the true definition of thermal equilibrium.

Thermal Expansion

Most, but not all, materials expand when heated. The change in length of material due to linear thermal expansion is

$\Delta l = \alpha l_0 \Delta T$

α is the coefficient of linear expansion of the material, measured in $(^{\circ}\mathrm{C})^{-1}$

The length of the object after it's temperature has been changed by ΔT is

$l = l_0(1 + \alpha\Delta T)$

A material expands in all directions, and if we are interested in the volume changes of a rectangular object, that is isotropic, meaning it expands in the same way in all directions, then

$\Delta V = \beta V_0 \Delta T$

$V_0 = l_0 w_0 h_0 \rightarrow V = l_0(1+\alpha\Delta T)w_0(1+\alpha\Delta T)h_0(1+\alpha\Delta T)$

$\Delta V = V - V_0 = V_0(1+\alpha\Delta T)^3 - V_0 = V_0[3(\alpha\Delta T) + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3]$

If $\alpha\Delta T >> 1$ then $\beta \approx 3\alpha$

Coefficients of thermal expansion can be found here or your textbook.

Negative Thermal Expansion

Some materials in certain temperature ranges do not expand with temperature. An example is water, which actually decreases in volume as it's temperature is increased from $0^{\circ}\mathrm{C}$ to $4^{\circ}\mathrm{C}$. This has an important effect, it explains why the surface of lakes freeze, which helps prevent the flow of heat out of the lake. If the thermal expansion was normal between $0^{\circ}\mathrm{C}$ and $4^{\circ}\mathrm{C}$ the circulation effect would continue below $4^{\circ}\mathrm{C}$ and the whole lake would freeze, starting from the bottom.

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Char Laws

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Charle Law states that at consta pressu the volum of a gas is propo to the tempe

$V \propto T$

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Gay Luss law

Gay Lussa Law states that for a fixed volum the press is propo to the tempe

$P \propto T$

Ideal Gas Law

The combina of the previous 3 laws implies that

$PV \propto T$

Our previous laws were for systems of constant mass, but we can see that the amount of mass should effect the volume (at a given pressure or the pressure (at a given volume).

$PV \propto mT$

Measurir the amount of mass in moles will allow us to write the ideal gas law in terms of a universal constant A mole of gas is a given number of molecule Avagadro number, $N_A = 6.02 \times 10^{23}$. If we have a certain mass m of a gas which has a certain molecula mass (measure in atomic mass units, u, which are also the number of grams per mole.), the the number of moles n is given by

$n = \frac{m\,[}{\text{molecu}\,[\text{g}/\text{mol}}$

and

$PV = nRT$ where $R = 8.314$

This equation is the ideal gas law

Ideal Gas Law for a numb of mole

The ideal gas law can also be written in terms of the numbe of molecu N

$PV = nRT = \frac{N}{N_A}R = NkT$

where $k = \frac{R}{N_A} \; 8.314$

$= \frac{}{6.0 \times 1} = 1.38 \times 10^{-2}$

is the Boltzm Consta

Using the Ideal Gas Law to deter Abso Zero

If $PV = nR$ the absolu zero temper occurs when $P = 0$. In practic most gases will liquefy before this point, but we can measu the pressu of fixed volume of gas at a couple of referen points and extrapo down to zero pressu to get an estima for absolu zero.

What makes a gas ideal?

There are a number of conditio which must be satisfied for a gas to be consider ideal

- The mus be a larg num of mol and they shou mov in ranc dire with a rang of diffe spe
- The spac betv mol shou be muc grea thar the size of the mol
- Mol are assu to inte only thro colli
- The colli are assu to be elas

Ideal gas law from a molecu perspe

We can work out the pressure exerted by an ideal gas on it's container by starting from the change in momentu of a molecule when it strikes the container wall

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The average force due to one molecule is then

$F_{molecule} = \frac{\Delta(mv)}{\Delta t} = \frac{2mv_x}{2l/v_x} = \frac{mv_x^2}{l}$

The net force on the wall will be the sum of the forces from all N molecule

$F_{net} = \frac{m}{l}\Sigma_{i=}$

$\frac{\Sigma_{i=1..N}v_{xi}^2}{N} = \bar{v_x^2} \rightarrow F_{net} = \frac{m}{l}N\bar{v}$

$v^2 = v_x^2 + v_y^2 + v_z^2 \rightarrow \bar{v^2} = \bar{v_x^2} + \bar{v_y^2} + \bar{v_z^2} = 3\bar{v_x^2}$

$F_{net} = \frac{m}{l}N\frac{\bar{v}}{3}$

$P = \frac{F}{A} = \frac{1}{3}\frac{Nm\bar{v}}{Al} = \frac{1}{3}\frac{Nm\bar{v}}{V}$

$PV = \frac{2}{3}N(\frac{1}{2}m\bar{v^2}) = NkT$

$\bar{KE} = \frac{1}{2}m\bar{v^2} = \frac{3}{2}kT$

Maxwe Boltzm Distrib

If we look at a simulatio of particles moving accordin to the kinetic theory we can get some idea of the distributi of the speeds of the particles

The Maxwell-Boltzmar Distributi gives the probabili $f(v)$ that a particula particle in an ideal gas has a given speed v.

$f(v) = 4\pi(\frac{m}{2\pi kT})^{\frac{3}{2}}v^2e^{-\frac{1}{2}}$

It was originally derived by James Clerk Maxwell based on symmetr argumen later Ludwig Boltzmar derived it on a more general basis.

Temper depend

The distributio of speeds is highly temperatu dependen as can be seen in simulation and is predicted by the Maxwell distributio

The plot below is for He atoms at various temperatu

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Mass depend

The speed distributio is also a function of the mass of the molecules the plot below is for the first 4 noble gasses at room temperatu

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In [ ]: