Question 1 solution

image.png Two blocks of mass m_1 and m_2 are connected by a rope which runs over a frictionless pulley of negligible mass. One block hangs from the rope, while the other rests on a plane with coefficients of static and kinetic friction μ_s and μ_k inclined at an angle of θ to the horizontal.

- A. (5 points) Add arrows indicating the direction of all of the forces acting on both m_1 and m_2 to the diagram.

- B. (5 points) Assuming that the incline is sufficiently steep that the blocks move when they are released from rest, find an expression for the downwards acceleration, a in terms of m_1, m_2, g, μ_s or μ_k and θ.

  $m_1 a = m_1 g - T$

  $m_2 a = T + m_2 g \sin\theta - \mu_k m_2 g \cos\theta$

  $(m_1 + m_2)a = m_1 g + m_2 g \sin\theta - \mu_k m_2 g \cos\theta$

  $a = \frac{m_1 g + m_2 g \sin\theta - \mu_k g \cos\theta}{m_1 + m_2}$

  For parts C-G consider a case where m_1 =4 kg and m_2=1 kg, μ_s=0.2 and μ_k=0.15.

- C. (5 points) If $\theta = 30^o$ what is the velocity and displacement of m_1, 0.5\,s after the system has been released from rest?

  $a = \frac{4 + 1\sin 30^o - 0.15\cos 30^o}{4+1} \times 9.8 = 8.57\mathrm{ms}^{-2}$

  $v = at = 8.57 \times 0.5 = 4.28\mathrm{ms}^{-1}$ down

  $y = \frac{1}{2}at^2 = 0.5 \times 8.57 \times 0.5^2 = 1.07m$ down

- D. (5 points) What is the magnitude of the tension in the rope during the motion in part (C)?

  $T = m_1 g - m_1 a = 4 \times (9.8 - 8.57) = 4.92\mathrm{N}$

- E. (5 points) How much work is done by gravity on **the two block system** during the motion in part (C)?

  $W_g = m_1 g d + m_2 g d \sin 30^o = 4 \times 9.8 \times 1.07 + 1 \times 9.8 \times 1.07 \times \sin 30^o = 47.187\mathrm{J}$

- F. (5 points) How much work is done by friction during the motion in part (C)?

  $W_{Fr} = \mu_k m_2 g \cos\theta d = 0.15 \times 1 \times 9.8 \times \cos 30^o \times 1.07 = 1.36\mathrm{J}$

- G. (5 points) How much work is done by the normal force on m_2 during the motion in part (C) ?

  $W_N = 0\mathrm{J}$

Problem 2 Solution

2rockets.png

A 200 kg rocket is fired straight up from the ground with an initial velocity of 100 ms^{-1} after which it is subject to a constant gravitational acceleration of 9.8 ms^{-2} down. A 15kg projectile is fired from a cannon 1 km away at the same time as the rocket is launched. 10 seconds later the projectile hits the rocket.

- A. (5 points) At what height above the ground does the collision take place?

  Solve using rocket equation of motion

  $y = v_0 - \frac{1}{2}gt^2 = 100 \times 10 - 0.5 \times 9.8 \times 10^2 = 510\mathrm{m}$

- B. (5 points) At what angle θ above the horizontal must the cannon be fired in order to hit the rocket?

  Equations of motion of projectile

  $x = v_0 \cos\theta t$

  $y = v_0 \sin\theta t - \frac{1}{2}gt^2$

  These give

  $1000 = v_0 \cos\theta \times 10$

  $1000 = v_0 \sin\theta \times 10$

  So

  $\theta = 45^o$

- C. (5 points) What is the initial velocity of the projectile launched from the cannon?

  $v_{0x} = 100\mathrm{ms}^{-1}$

  $v_0 = \frac{v_{0x}}{\cos 45^o} = 141\mathrm{ms}^{-1}$

- D. (5 points) What is the kinetic energy of the rocket when it is hit by the projectile?

  Can use either conservation of mechanical energy

  $KE = \frac{1}{2}mv_0^2 - mgh = 0.5 \times 200 \times 100^2 - 200 \times 9.8 \times 510 = 400\mathrm{J}$

  of find velocity of rocket when it is hit

  $v = 100 - 9.8 \times 10 = 2\mathrm{ms}^{-1}$

  $KE = \frac{1}{2}mv^2 = 400\mathrm{J}$

- E. (5 points) What is the kinetic energy of the projectile when it hits the rocket?

  $KE = \frac{1}{2}mv_0^2 - mgh = 0.5 \times 15 \times 100^2 \times 2 - 15 \times 9.8 \times 510 = 75030\mathrm{J}$

- F. (10 points) If instead of starting with the velocity found in part B the projectile was fired with a velocity of 500 ms^{-1} find the required angle θ above the horizontal for the projectile to the hit the rocket and the height at which the collision occurs in this case.

  Equations of motion for projectile are now:

  $x = 500\cos\theta t$

  $y = 500\sin\theta t - \frac{1}{2}gt^2$

  The angle can be found by considering when the y coordinates of the two objects are the same

  $500\sin\theta t - \frac{1}{2}gt^2 = 100t - \frac{1}{2}gt^2$

  $\sin\theta = \frac{1}{5}$

  $\theta = 11.54^o$

  The time at which the collision takes place can now be found using the equation for the x motion of the projectile.

  $t = \frac{1000}{500\cos\theta} = 2.04\mathrm{s}$

  The height can now be found by substituting back in the y equation of motion

  $y = 500 \times \frac{1}{5} \times 2.04 - 0.5 \times 9.8 \times 2.04^2 = 183.6\mathrm{m}$

Problem 3 Solution

ring.png

A 500 kg car is traveling with a constant speed of 50 ms^{-1} in a circular path around the inside edge of a cylinder. The radius of the curve is 40 m. The coefficient of static friction between the road and the tires is $\mu = 0.2$.

A. (5 points) How long does it take for the car to go around the cylinder once?

$T = \frac{2\pi r}{v} = \frac{2\pi \times 40}{50} = 5.03\mathrm{s}$

B. (5 points) What is the magnitude of the normal force exerted by the wall of the cylinder on the car.

$F_N = \frac{mv^2}{r} = \frac{500 \times 50^2}{40} = 31250\mathrm{N}$

C. (5 points) What is the magnitude and direction of the frictional force on the car. Give the direction as one of the following, up, down, towards the center of the cylinder, radially outwards, in the direction the car is moving, opposite to the direction the car is moving.

$F_{Fr} = mg = 500 \times 9.8 = 4900\mathrm{N}$ up

D. (5 points) How much work is done by the normal force in one complete circuit of the track?

$0\mathrm{J}$

E. (5 points) How much work is done by the frictional force in one complete circuit of the track?

OJ

F. (5 points) The car starts to slow down. At what speed does it start to slip down the wall of the cylinder?

The car starts to slip when the maximum frictional force is less than the weight of the car. The minimum velocity for this not to happen can be found from

$\mu\frac{mv^2}{r} = mg$

$v^2 = \frac{rg}{\mu} = \frac{40 \times 9.8}{0.2}$

$v = 44.3\mathrm{ms}^{-1}$

Additional problem

A smooth block of mass 100g is sliding along the edge of a smooth cone with constant speed. The height of the cone is 20cm, and half of it's apex angle is 30^o.

A. (5 points) Draw a free body diagram which represents all the forces acting on the block.

1block.png

B. (5 points) What is the magnitude of the normal force acting on the block?

$F_N \sin(30^o) = mg$

$F_N = \frac{0.981\mathrm{ms}^{-2}}{0.5} = 1.962\mathrm{N}$

C. What is the magnitude of the horizontal component of the normal force acting on the block?

$F_{Nx} = F_N \cos(30^o) = 1.7N$

D. (5 points) What is the speed of the block?

$\frac{mv^2}{r} = F_N \cos(30^o)$

$r = 0.2\tan 30^o$

$v^2 = 0.2g$

$v = 1.4\mathrm{ms}^{-1}$

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