

Lecture 22 - Static Fluids

Phases of Matter

- Solids - Will maintain a fixed shape and will not flow.
- Liquids - Does not maintain a fixed shape, will flow, but is not readily compressible.>
- Gases - Does not maintain a fixed shape, will flow, and will expand or contract depending on the container it is in
- Plasma - Ionized atoms in which electrons are stripped from the nucleus.

Typically the progression amongst these [phases](#) is associated with increasing temperature or internal energy.

Here we are concerned with fluids, in other words matter which flows, ie. either gasses or liquids.

Density and specific gravity

The [density](#) of a substance is defined as

$$\rho = \frac{m}{V}$$

The [specific gravity](#) of a substance is its density relative to the density of water at 4.0°C which is 1.00g/cm³ = 1 × 10³kg/m³

So we can say that the specific gravity of a substance is equal to its density in g/cm³.

Pressure

[Pressure](#) is defined as the force per unit area in a direction perpendicular to the surface of an object

$$P = \frac{F}{A}$$

There are **lots** of units for pressure.

The SI unit is N/m² or pascal (Pa).

You will also be familiar with psi (lb/in²)

$$1\text{psi} = 6.895 \times 10^3 \text{ Pa}$$

Some other common units are:

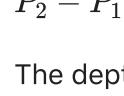
$$1 \text{ atm} = 101.3 \text{ kPa}$$

$$1 \text{ Torr} = 1 \text{ mmHg} = 133 \text{ Pa}$$

$$1 \text{ bar} = 1.0 \times 10^5 \text{ Pa}$$

Click [here](#) for a conversion table.

Pressure as a function of depth in a static fluid



At a given depth in a static fluid the pressure must be the same in every direction, otherwise the fluid would not be static. The force due to the fluid on any solid surface it touches must be perpendicular to the surface, otherwise there would be some flow of the fluid over the surface.

To quantify the force due to pressure on a surface of area A at a depth h we should consider its origin, which is gravitational force acting on liquid above the point at which we want to know the pressure.

So the pressure due to the **water only** will be

$$P = \frac{F}{A} = \frac{\rho Ahg}{A} = \rho gh$$

We need to also take into account the external pressure, so we should come up with a more general formula which will let us take that into account.

We can consider a slab of water of thickness dy at height y above the bottom. The pressure acting on the bottom is P , the pressure on the top at height $y + dy$ differs by dP (making it $P + dP$).

Gravity acts on the slab and the force due to gravity on the slab is $dF_G = \rho g A dy$

The equilibrium condition tells us that all the forces must balance so

$$PA - (P + dP)A - \rho g A dy = 0$$

giving us a differential form of the relationship between pressure and depth

$$\frac{dP}{dy} = -\rho g$$

to an open container of fluid of uniform density

$$\int_{P_1}^{P_2} dP = - \int_{y_1}^{y_2} \rho g dy$$

$$P_2 - P_1 = -\rho g(y_2 - y_1)$$

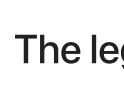
The depth $h = y_2 - y_1$ and the pressure at the top P_2 is atmospheric pressure P_0

We can therefore say that the pressure at depth h , $P = P_1$ is

$$P = P_0 + \rho gh$$

We can see that we need to take into account the pressure of the atmosphere above the container!

Container with hose



If we now consider the same situation but with a hole in the bottom, to which a hose is attached the pressure change is again

$$\int_{P_1}^{P_2} dP = - \int_{y_1}^{y_2} \rho g dy$$

$$P_2 - P_1 = -\rho g(y_2 - y_1)$$

so the pressure difference is again ρgh , but we should remember that the atmospheric pressure is the same at both ends, so if you want to find the effective pressure on the water at the end of the hose you should not add on the atmospheric pressure.

Non uniform density - atmospheric pressure at elevation

In the previous example we consider atmospheric pressure to be the same at both ends. This is very good approximation for small height differences. Over large distances we need some way to account for changes in density. An approximation made mostly for mathematical convenience is to treat the density of the air as proportional to pressure.

$$\frac{\rho}{\rho_0} = \frac{P}{P_0}$$

Here P_0 is the atmospheric pressure at the Earth's surface, 101.3 kPa and ρ_0 is the density, 1.29 kg/m³

$$\frac{dP}{dy} = -\rho g = -P \frac{\rho_0}{P_0} g$$

$$\frac{dP}{P} = -\frac{\rho_0}{P_0} g dy$$

$$\int_{P_0}^P \frac{dP}{P} = -\frac{\rho_0}{P_0} g \int_0^y dy$$

$$\ln P - \ln P_0 = -\frac{\rho_0}{P_0} gy$$

$$\ln \frac{P}{P_0} = -\frac{\rho_0}{P_0} gy$$

$$P = P_0 e^{-(\rho_0 g / P_0) y}$$

Gauge Pressure

If you measure the pressure in your tires you are actually measuring the pressure difference between the atmospheric pressure. The absolute pressure is therefore the sum of the gauge pressure P_G , which is what you measure, and the atmospheric pressure P_0

$$P = P_0 + P_G$$

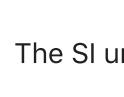
There are many different methods for [measuring pressure](#).

Pascal's Principle

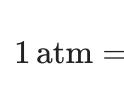
Pascal's principle, due to [Blaise Pascal](#) is an important principle that is the basis of the [hydraulic press](#)

Pascal's Principle: If an external pressure is applied to a confined fluid the pressure at every point within the fluid increases by that amount.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Buoyancy



Earlier we looked at the forces on an element within a fluid. Now we will look at an object that has displaced some of the fluid. In this case there is a force we call the buoyant force which reflects the difference between the force on the top and bottom of the object due to the fluid. In a fluid of uniform density ρ_f

$$F_B = F_2 - F_1 = \rho_f g A(h_2 - h_1) = \rho_f g A \Delta h = \rho_f V g = m_f g$$

We can note from this equation that the buoyant force does not depend on the depth of the object or on the density of the object. The buoyant force is determined by the weight of the fluid which is displaced by the object.

This is a statement of [Archimedes' principle](#) discovered by [Archimedes](#) in the 3rd century BC.

The legendary experiment of Archimedes

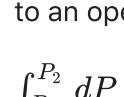
Legend has it that Archimedes arrived at his principle while working on a task for King Hiero II who wished to know if a crown he had asked to be made from pure gold actually was...without damaging it!

The most likely way that he achieves this was by the application of his principle to a [balance](#) with the crown on one side and a pure gold brick on the other.

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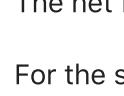
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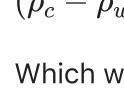
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The net force of an object is determined by the sum of the buoyant force and the gravitational force. If an object is more dense than the water then it will sink to a point where either the fluid density is the same as the object density, or to the bottom where it can be supported by normal force.

$$The net force on an object will be \vec{F}_B + \vec{F}_G = \rho_f V_0 g - \rho_0 V_0 g.$$

For the scale to balance in air

$$\rho_c V_c g l_1 = \rho_b V_b g l_2$$

Which would mean that

$$V_c g l_1 = V_b g l_2$$

which can only be true if $\rho_c = \rho_b$. If the scale tips the densities are not equal and the fraud is exposed!

Sinking and Floating

For objects less dense than water Archimedes' principle also applies.

An object floats when the gravitational force balances the buoyant force

$$F_B = mg$$

$$\rho_f V_{displ} g = \rho_0 V_0 g$$

$$\frac{V_{displ}}{V_0} = \frac{\rho_0}{\rho_f}$$

The fraction of the volume which is displaced, or equivalently, the fraction of the object which is submerged, is determined by ratio of the object's density to that of the fluid.

