Lecture 23 - Fluids in Motion

Laminar vs. Turbulent Flow

A smooth flow of a fluid is called laminar flow. Flow at higher velocities frequently becomes turbulent.

Most of this lecture will focus on laminar flow.

Equation of continuity

im.png If we consider a steady flow of fluid in an enclosed tube the amount of mass which flows past a point in a time interval Δt

$$\frac{\Delta m}{\Delta t}$$

is constant. If we express the mass in terms of volume and density

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

and use the fact that

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

We can say that

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

which for an incompressible fluid gives

$$A_1 v_1 = A_2 v_2$$

Bernoulli's principle

Bernoulli's principle due to Daniel Bernoulli connects the velocity of a flowing fluid to the pressure.

im.png The pressure is a force per unit area, the work done by the pressure at the left end of the tube in moving the the fluid at the end of the tube through Δl does work equal to

$$W_1 = P_1 A_1 \Delta l_1$$

At the other end the force exerted on the fluid is in the opposite direction to the motion and the work done is

$$W_2 = -P_2 A_2 \Delta l_2$$

The mass m of fluid moved changes it height by $y_2 - y_1$ and the work done by gravity on the fluid in is

$$W_g = -mg(y_2 - y_1)$$

The net work done is the sum of these three works

$$W = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

The net work is also equal to the change in kinetic energy

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The mass of fluid is given by $m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$ and substituting this and rearranging gives us Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Torricelli's theorem

im.png

A special case of Bernoulli's equation is Toricelli's theorem, which was actually discovered a century before Bernoulli.

If we apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

to the above situation we can see that $P_1 = P_2$, and v_2 can be approximated as zero.

So then

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

$$v_1 = \sqrt{2g(y_2 - y_1)}$$

The magnitude of the velocity of the water exiting the tap is the same as if it had fallen from the top of the water.

Range

If we want to compute the horizontal range of water pouring from a hole, we need to apply the same equations as in projectile motion:

$$x = v_x\sqrt{\frac{2y_1}{g}} = \sqrt{2g(y_2 - y_1)}\sqrt{\frac{2y_1}{g}}$$

$$x = 2\sqrt{y_1(y_2 - y_1)}$$

Max range when

$$\frac{dx}{dy_1} = 0$$

$$x = 2z^{\frac{1}{2}}$$

$$z = y_1(y_2 - y_1)$$

$$\frac{dx}{dy_1} = \frac{dx}{dz}\frac{dz}{dy_1}$$

$$\frac{dx}{dy_1} = 2 \times \frac{1}{2}\frac{y_2 - 2y_1}{\sqrt{y_1(y_2 - y_1)}} = 0$$

$$y_2 = 2y_1$$

$$y_1 = \frac{1}{2}y_2$$

Bernoulli's theorem and lift

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

At a constant height

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

The pressure in a fast moving column of air will be less than that outside it, so an object in the column will experience a force that will tend to keep it in the column.

Lift on an airfoil

im.png The airfoil produces dynamic lift by compressing the streamlines above the wing and expanding them below.

From the continuity equation

$$A_1 v_1 = A_2 v_2$$

We can see that the velocity on top of the wing is greater than it is below. Note that the time taken for the air to pass over both surfaces is not the same.

Venturi tube

We can also apply the equation

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

to understand a Venturi tube

Viscosity

viscosity. The viscosity is a property of the fluid and can greatly affect the rate of flow.

im.png

Viscosity can be defined from looking at a fluid trapped between two plates in which one plate moves with velocity v with respect to the other. The fluid in contact with each plate must have the same velocity as the plate it is in contact with and the fluid in between is subject to a velocity gradient. The viscosity η is defined in terms of the force require to maintain that velocity gradient (which is required to move the plate).

$$F = \eta A \frac{v}{l}$$

Poiseulle's equation

In a cylindrical tube of radius R and length l the **volume** rate of flow Q is related to the change in pressure from the beginning of the tube P_1 to the end of the tube P_2 by Poiseulle's equation

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta l}$$

Surface tension and capillarity

Surface tension, γ, is due to attractive forces between the molecules in a liquid, and is the force F per unit length l

$$\gamma = \frac{F}{l}$$

At at an interface between a liquid and another medium the liquid forms a contact angle which is determined by the balance of forces, including surface tension. Surface tension in thin tubes leads to capillarity where the movement up or down the walls of the tube is due the balance of surface tension with gravitational and viscous forces.

If you eliminate viscosity, as occurs in a superfluid, you can see some very interesting effects.

And finally

The aerodynamic role of the dimples on a golf ball is an interesting case.

Armed with Bernoulli's equation we can come to an understanding of why the dimples, which convert the laminar flow to a turbulent one actually make the ball go further!

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