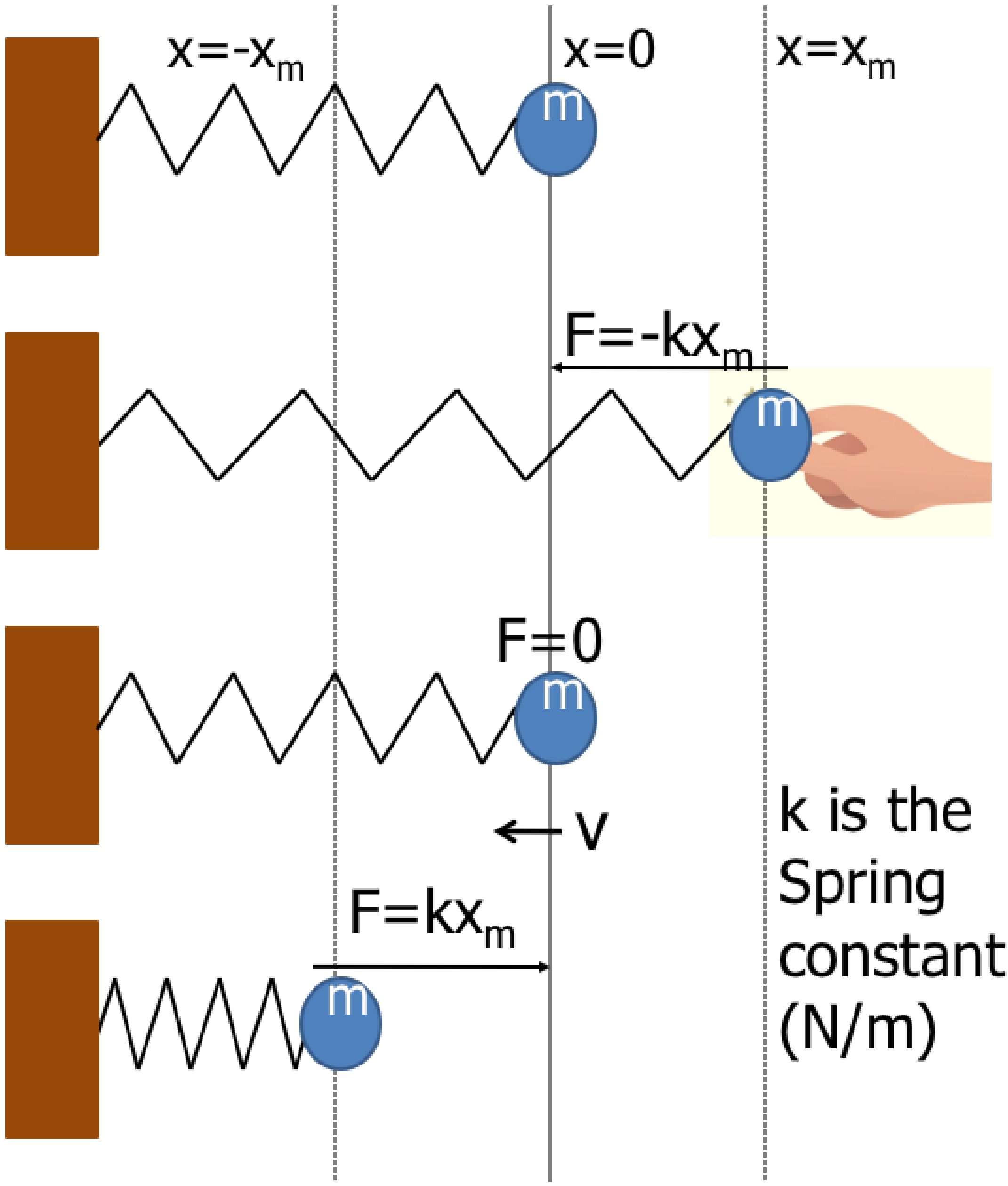
Lecture 24 - Simple Harmonic Motion

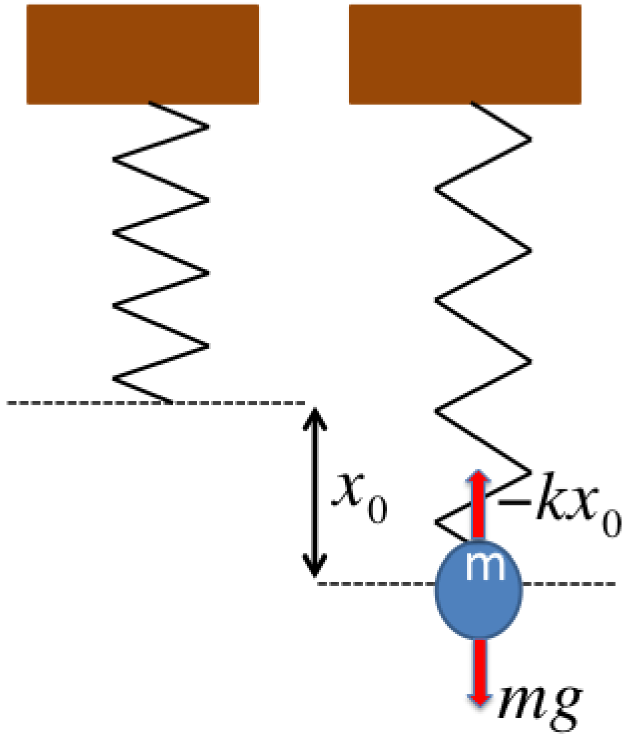
We will now begin to consider oscillatory motion, beginning with the simplest example, [simple harmonic motion (http://en.wikipedia.org/wiki/Simple_harmonic_motion)](http://en.wikipedia.org/wiki/Simple_harmonic_motion).

Restoring force of a spring



A vertical spring

A vertical spring will also execute simple harmonic motion, though it's mean position will be modified by the balance of the gravitational force and the spring force.



SHM as a function of time

These [animations (http://www.animations.physics.unsw.edu.au//jw/SHM.htm)](http://www.animations.physics.unsw.edu.au//jw/SHM.htm) from physclips show the form of displacement, velocity and acceleration of an object in SHM as a function of time.

Equations of motion for SHM

Starting from Newton's 2nd Law

$ma = \Sigma F$

$m\frac{d^2x}{dt^2} = -kx$

Based on our previous observations we might guess that the displacement will be able to be expressed as trigonometric function of time.

$x = A\cos(\omega t + \phi)$

A is the amplitude, $\omega = \frac{2\pi}{T} = 2\pi f$ and ϕ allows us to change the starting point of the motion.

$\frac{dx}{dt} = v = -\omega A \sin(\omega t + \phi)$

$\frac{d^2x}{dt^2} = a = -\omega^2 A\cos(\omega t + \phi)$

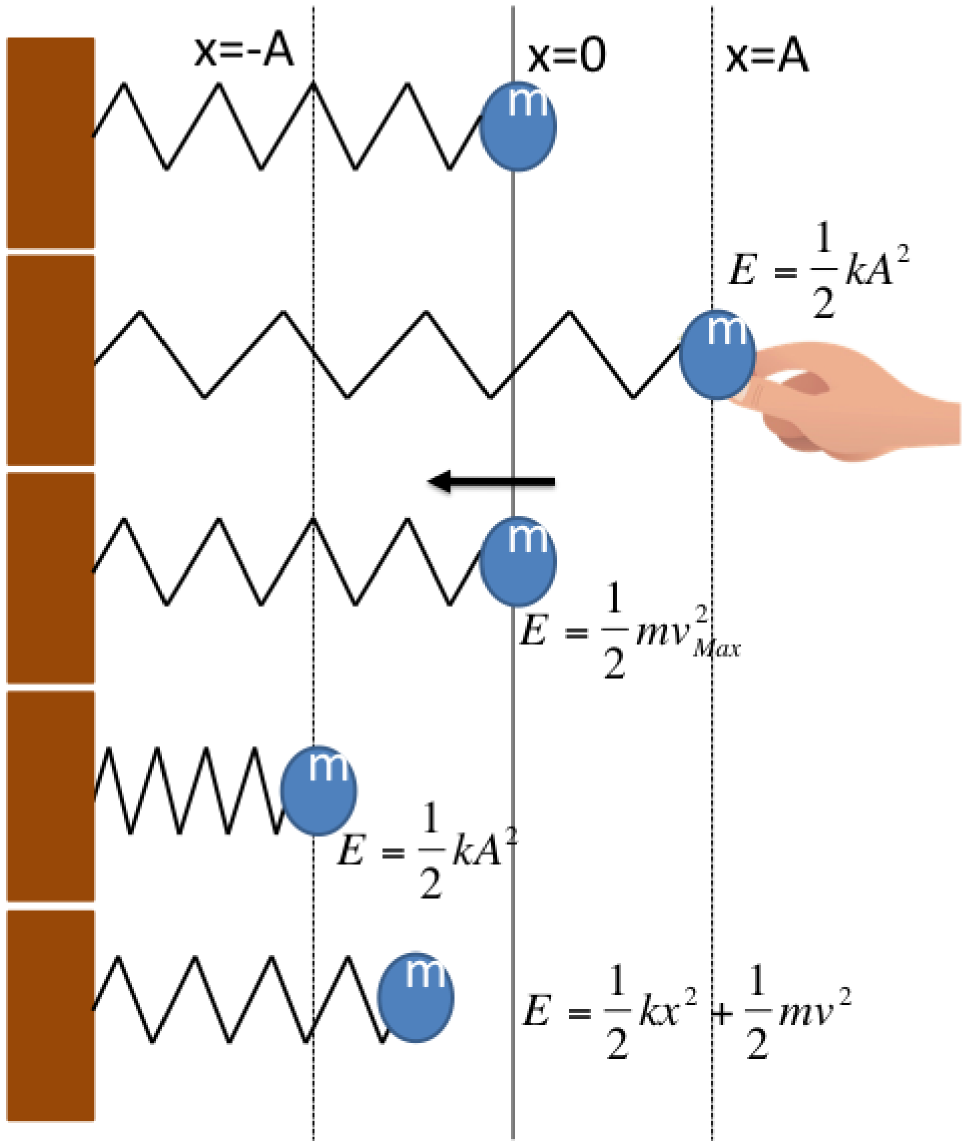
$-m\omega^2 A\cos(\omega t + \phi) = -kA\cos(\omega t + \phi)$

which is true if

$\omega^2 = \frac{k}{m}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Energy in SHM



$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$

Simple harmonic motion with multiple springs

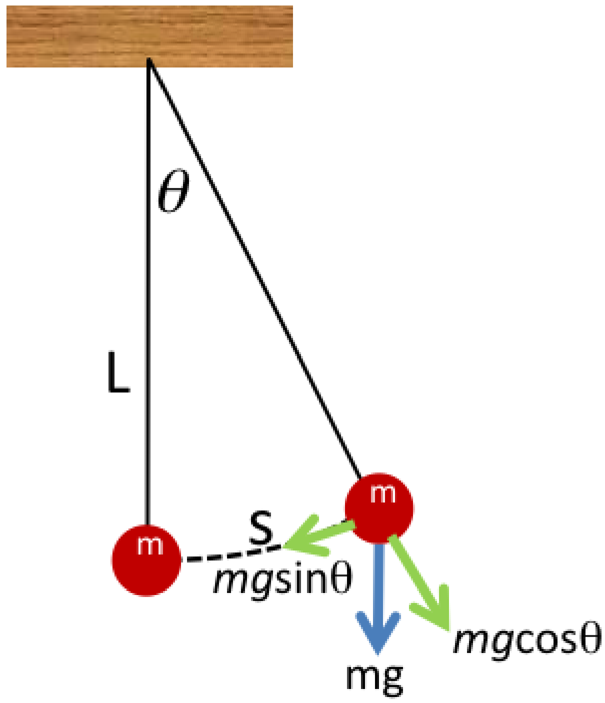
There is a wikipedia page on [multiple springs (https://en.wikipedia.org/wiki/Series_and_parallel_springs)](https://en.wikipedia.org/wiki/Series_and_parallel_springs) which has a detailed explanation of the effective spring constants of springs in series and parallel. The key result is that for springs in parallel

$k_{eq} = k_1 + k_2$

and for springs in series

$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

Simple Pendulum



The restoring force above is $F = -mg\sin\theta$

For small angles $\sin\theta = \theta$ so $F \approx -mg\theta$

and using the relation $s = l\theta$ gives $F \approx -\frac{mg}{L}s$

This is essentially the same as $F = -kx$ with $k = \frac{mg}{L}$

So $\omega = \sqrt{\frac{k}{m}} \rightarrow \omega = \sqrt{\frac{g}{l}}$

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

$T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$

Coming Up

- Physical Pendulum
- Torsional Pendulum
- Damped Harmonic Motion
- Forced Harmonic Motion

In [1]:
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(797+585)*0.5
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Out[1]:
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691.0
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In [ ]: