Linear momentum in more than one dimension

Inelastic collisions

A bullet hitting an object is a good example of an **inelastic** collision.

Ballistic Pendulum

We can however use conservation of momentum for the collision and conservation of energy for the **subsequent motion** in a ballistic pendulum to find the velocity of a bullet fired in to a block from the height that the bullet and block rises to.

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$mv = (M+m)v'$

$\frac{1}{2}mv^2 \neq \frac{1}{2}(M+m)v'^2$

but $\frac{1}{2}(M+m)v'^2 = (M+m)gh \rightarrow \frac{1}{2}v'^2 = gh$

so $v = \frac{M+m}{m}\sqrt{2gh}$

Multidimensional collisions

The equation of conservation of momentum is a vector equation.

$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v'}_A + m_B\vec{v'}_B$

Therefore if we have a collision involving more than one dimension we need to consider conservation of each component of momentum, for example in 2 dimensions

$m_Av_{Ax} + m_Bv_{Bx} = m_Av'_{Ax} + m_Bv'_{Bx}$

$m_Av_{Ay} + m_Bv_{By} = m_Av'_{Ay} + m_Bv'_{By}$

Perfectly Elastic Collisions in 2 dimensions

If a collision is perfectly elastic we can add a 3rd equation

$m_Av_{Ax} + m_Bv_{Bx} = m_Av'_{Ax} + m_Bv'_{Bx}$

$m_Av_{Ay} + m_Bv_{By} = m_Av'_{Ay} + m_Bv'_{By}$

$\frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2 = \frac{1}{2}m_Av`^2_A + \frac{1}{2}m_Bv`^2_B$

Billiards Question

When we hit a billiard ball straight on all the momentum of the incident ball should be transferred to the 2nd ball.

elastic1.gif

However we know already from experience that if we hit a billiard ball at an angle this does not happen. Can we use our conservation of momentum principle to predict the trajectories of billiard balls after a collision? If one ball hits another head on, it should come to rest. **But if it comes in at a small angle where should it go?** elastic1.gif

Bil (o pu

Co of mo v_A $+$

0 $+$

e

v'_B v'_B

Billia (con

Squar the conse of mome equati

$v'^2_{Ay} = v'^2_{By}$

$v_A^2 - 2v_A + v'^2_{Ax} = v'^2_{Bx}$

Add these togeth

$v_A^2 + v'^2_A - 2v_A = v'^2_B$

,

Use kinetic energ equati to elimin v'_B

$v_A^2 + v'^2_A - 2v_A = v_A^2 - v'^2_A \rightarrow v'^2_A = v_A$

$v'^2_A = v_A$ $\theta \rightarrow v'_A = v_A$ θ

and conse of energ equati gives us $v'_B = v_A$ θ

Perfe Inela Colli in 2D

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In a perfec inelast collisic the two object stick togeth after the collisic so conse of mome now is writter

$m_A\vec{v}_A + m_B = (m_A + m_B)\vec{v}'$

Compa betwee the two collisio

All real collisions lie somewhe between the extremes we have looked at, but let's consider now the two extreme cases for a collision of 2 objects of equal mass colliding at right angles with equal velocities

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Momentu must be conserve for both the x and y directions

$v_A = v'_{Ax} + v'_{Bx}$

$v_B = v'_{Ay} + v'_{By}$

Squaring and adding our momentu equations lead to

$2v^2 = v'^2_A + v'^2_B + 2v'_{Ax}v'_B + 2v'_{Ay}v'_B$

In the case of an elastic collision

$\frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv = \frac{1}{2}m_Av + \frac{1}{2}m_Bv$

$2v^2 = v'^2_A + v'^2_B$

which means that

$2v'_{Ax}v'_{Bx} + 2v'_{Ay}v'_B = 0$

And by inspectio we can see that the result is a 90^o change of direction for each object with no change in the speed of the objects.

Perfe Inela Colli (con

For an inelast collisic

$2v^2 = 2v'^2 + 2v'^2_x + 2v'^2_y = 4v'^2$

$v' = \frac{v}{\sqrt{2}}$

and inspec shows that the x and y compo of the velocit are equal.

Cen of Mas

We have looke at two quite differ 2 dimer collis Howe there is one way of lookir at them whicl make them look more simila If we consi the motic of the cente of mass of the objec instea of the motic of the indivi mass we can see that from this persp the motic is ident

image

The Useful of Cente of Mass

We can actually consider the motion of any extende mass to be compos of different kinds of motion with respect to the center of mass.

So far we have dealt with translati motion, and we can now make explicit our implicit assumpt till now that we consider the translati motion of an object's center of mass. We will now also see in the coming lectures that we can add to this translati motion other motions of the mass around the center of mass, for example rotationa or vibratior motion.

In general the displace vector for the center of mass of a system of particles m_i can be written as

$\vec{r}_{CM} = \frac{\Sigma_i m_i\vec{r}_i}{\Sigma_i m_i} = \frac{\Sigma_i m_i\vec{r}_i}{M}$

Different with respect to time gives

$M\frac{d\vec{r}_{CM}}{dt} = \Sigma_i m_i$ or $M\vec{v_{CM}} = \Sigma_i m_i$

and doing it once more

$M\vec{a_{CM}} = \Sigma_i m_i$

$\Sigma_i m_i\vec{a}_i = \Sigma_i F_i = \Sigma\vec{F}_{ex}$

Cer of Ma (co

so we have obta a new form of New Secc Law that work for a syste of parti

$Ma = \Sigma$

the total mom of a syste can also be writt in thes term

$\vec{P} = M$

Center of Mass for Contin Object

$\vec{r}_{CM} = \frac{\Sigma_i m_i\vec{r}_i}{M}$

can be expresse in integral form

$\vec{r}_{CM} = \frac{1}{M}\int\vec{r}\,dm$

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We can consider a uniform rod to be a one dimensio object.

If we want to find the COM we could place the origin of our coordina system at the center of the rod and then

$\vec{r}_{CM} = \frac{1}{M}\int\vec{r}\,dm = \frac{1}{M}\int_{-l/2}^{l/2}\lambda x\,dx = \frac{1}{M}\frac{\lambda}{2}((\frac{l}{2})^2 - (-\frac{l}{2})^2) = 0$

COM of contin object (cont.)

On the other hand if we were to place the origin at the left end of the rod then we could show that

$\vec{r}_{CM} = \frac{1}{M}\int\vec{r}\,dm = \frac{1}{M}\int_0^l\lambda x\,dx = \frac{1}{M}\frac{\lambda}{2}l$

and as $M = \lambda l$

$\vec{r}_{CM} = \frac{l}{2}\hat{i}$

Snooke Video

snookervi

Clearly there is more to this game than we discussed above! To understan better the way we can manipulat the path of a billiard ball we will need to study rotational motion..

In [ ]: