

Lecture 29 Sound Effects

Spatial Interference

The equation for a traveling wave

$$D(x, t) = A \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi t}{T}\right) = A \sin(kx - \omega t)$$

tells us that if we are standing a distance $r_1(x)$ from a one dimensional wave the wave displacement at a time t will be

$$D_1(x_1, t) = A \sin(kx_1 - \omega t)$$

if a second identical source is a distance $r_2(x)$ away

$$D_2(x_2, t) = A \sin(kx_2 - \omega t)$$

and the total wave displacement is

$$D_{1+2}(x, t) = A \sin(kx_1 - \omega t) + A \sin(kx_2 - \omega t)$$

$$\text{Using } \sin \theta_1 + \sin \theta_2 = 2 \sin \frac{1}{2}(\theta_1 + \theta_2) \cos \frac{1}{2}(\theta_1 - \theta_2)$$

$$D_{1+2}(x, t) = 2A \sin\left(\frac{k}{2}(x_1 + x_2) - \omega t\right) \cos\left(\frac{k}{2}(x_1 - x_2)\right)$$

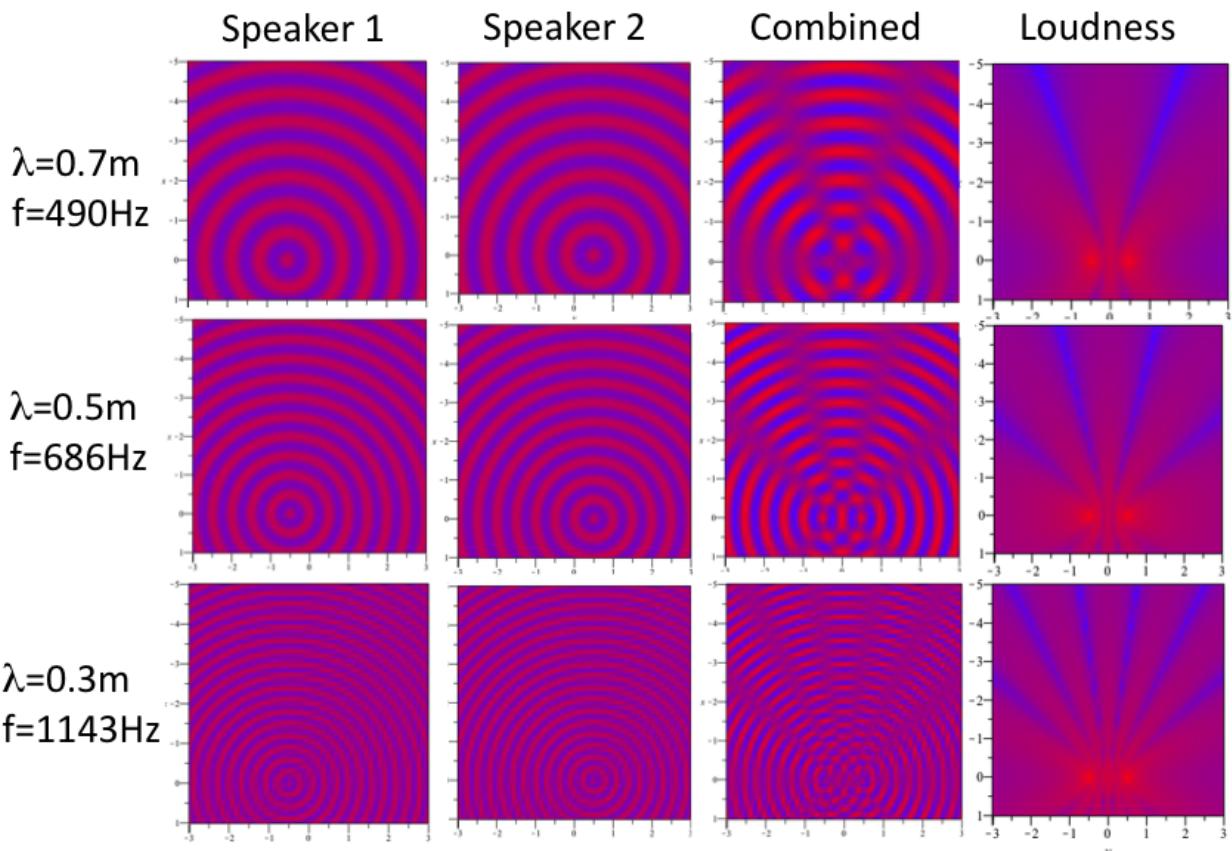
We thus hear a wave that has the same wavelength and frequency as that coming from the source, but the amplitude will depend on the distance between the sources, being maximum ($2A$) when $\frac{k}{2}(x_1 - x_2)$ is 0 or a whole number multiple of π which corresponds to the waves at that point being in phase. The amplitude is minimum (0) when $\frac{k}{2}(x_1 - x_2)$ is a multiple of $\frac{\pi}{2}$ which corresponds to the waves at that point being out of phase.

Interference between two speakers

If the waves propagate from the source in all 3 dimensions then we need to take

into account that as we showed in lecture 28 , $A \propto \frac{1}{r}$.

To determine β or perceived loudness we need to remember that it depends logarithmically on intensity. I have factored these considerations in the following calculations. The patterns are for two speakers separated by 1m.



```
In [31]: import numpy as np
# Compute wavelength
vs = 340 #m/s, sound velocity
f = 490 #Hz=s^-1
l = vs/f #wavelength
print(l,'m')

# Compute the angle for 490 Hz
theta=np.arctan(5/2)
print(theta*180/np.pi)
```

0.6938775510204082 m
68.19859051364818

The Doppler effect

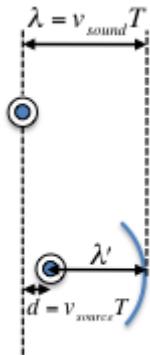
[Doppler Effect \(\[https://en.wikipedia.org/wiki/Doppler_effect\]\(https://en.wikipedia.org/wiki/Doppler_effect\)\).](https://en.wikipedia.org/wiki/Doppler_effect)

[Sheldon Cooper explains \(\[http://www.youtube.com/watch?feature=player_embedded&v=Tn35SB1_NYI\]\(http://www.youtube.com/watch?feature=player_embedded&v=Tn35SB1_NYI\)\).](http://www.youtube.com/watch?feature=player_embedded&v=Tn35SB1_NYI)

Doppler Effect with moving source

When a source of sound moves a stationary observer hears an apparent shift in the frequency. The origin of this effect can be seen nicely in [this animation](#) (<http://upload.wikimedia.org/wikipedia/commons/c/c9/Dopplereffectssourcемovingrightatmach0.7.gif> from Wikipedia).

For a source moving **towards** an observer



$$\lambda' = \lambda - d = \lambda - v_{source}T = \lambda - v_{source}\frac{\lambda}{v_{sound}} = \lambda(1 - \frac{v_{source}}{v_{sound}})$$

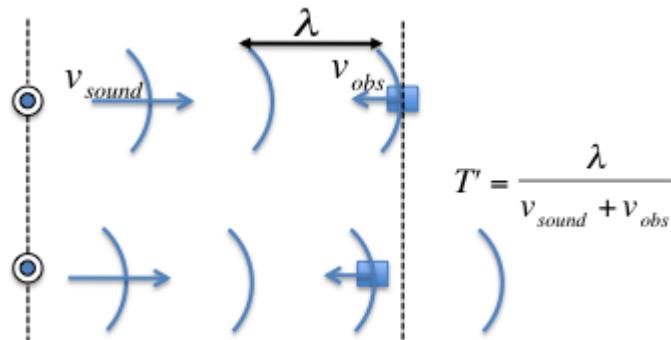
$$f' = \frac{v_{sound}}{\lambda'} = \frac{v_{sound}}{\lambda(1 - \frac{v_{source}}{v_{sound}})} = \frac{f}{(1 - \frac{v_{source}}{v_{sound}})}$$

If the source is moving **away** from the observer

$$\lambda' = \lambda(1 + \frac{v_{source}}{v_{sound}})$$

$$f' = \frac{f}{(1 + \frac{v_{source}}{v_{sound}})}$$

Doppler effect with moving observer



A doppler effect also occurs when an observer moves towards a source, but here the wavelength does not change, instead it is the effective velocity that changes and leads to an apparent change in the frequency of the sound. When the observer moves **towards** the source of the sound

$$T' = \frac{\lambda}{v_{sound} + v_{obs}}$$

$$f' = \frac{v_{sound} + v_{obs}}{\lambda} f = \frac{v_{sound} + v_{obs}}{v_{sound}} f$$

When the observer moves **away** from the source of the sound

$$f' = \frac{v_{sound} - v_{obs}}{\lambda} = \frac{v_{sound} - v_{obs}}{v_{sound}} f$$

Doppler Effect with moving observer and source

For an observer moving towards a source which is also moving towards it

$$f' = \frac{v_{sound} + v_{obs}}{\lambda'} = \frac{v_{sound} + v_{obs}}{\lambda(1 - \frac{v_{source}}{v_{sound}})} = \frac{(v_{sound} + v_{obs})v_{sound}}{\lambda(v_{sound} - v_{source})} = f \frac{(v_{sound} + v_{obs})}{(v_{sound} - v_{source})}$$

A general formula for the doppler effect is

$$f' = f \frac{(v_{sound} \pm v_{obs})}{(v_{sound} \mp v_{source})}$$

Top part of the \pm or \mp sign is for a source or observer moving towards each other, the bottom part is for motion away from each other.

4 cases:

- Source moves towards, observer moves towards

$$f' = f \frac{(v_{sound} + v_{obs})}{(v_{sound} - v_{source})}$$

- Source moves away, observer moves away

$$f' = f \frac{(v_{sound} - v_{obs})}{(v_{sound} + v_{source})}$$

- Source moves towards, observer moves away

$$f' = f \frac{(v_{sound} - v_{obs})}{(v_{sound} - v_{source})}$$

- Source moves away, observer moves towards

$$f' = f \frac{(v_{sound} + v_{obs})}{(v_{sound} + v_{source})}$$

Double Doppler effect when sound bounces off a moving object

If we project sound at a moving object and wait for it to come back its frequency is shifted by two doppler effects. In the first part of the process the source is stationary and the observer is moving, the moving object is hit by sound of frequency f'

$$f' = f \frac{(v_{sound} \pm v_{obs})}{(v_{sound})}$$

Now when the sound is returning to us it is a case of a moving source and stationary observer, we apply the formula for that on to the already shifted frequency f' to get f'' the frequency we would hear

$$f'' = f' \frac{(v_{sound})}{(v_{sound} \mp v_{source})} = f \frac{(v_{sound})}{(v_{sound} \mp v_{source})} \frac{(v_{sound} \pm v_{obs})}{(v_{sound})}$$

Of course in this case $v_{source} = v_{obs} = v_{object}$ so

$$f'' = f \frac{(v_{sound} \pm v_{object})}{(v_{sound} \mp v_{object})}$$

This double doppler effect, used with either sound ([sonar](https://en.wikipedia.org/wiki/Sonar)) or radio waves ([radar](https://en.wikipedia.org/wiki/Radar)) can be used to detect the speed of an object from the frequency of the reflected waves.

Sonic Boom

When a moving object moves faster than the speed of sound [the wavefronts pile up](http://upload.wikimedia.org/wikipedia/commons/d/d1/Dopplereffectssourcemovingrightatmach1.0.gif) creating a shock wave, which is heard as a sonic boom.

A video of a [sonic boom](http://www.youtube.com/watch?v=SsDEfu8s1Lw).

More on sonic booms on [wikipedia](http://en.wikipedia.org/wiki/Sonic_boom).

In []: