Q1

a) $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$\frac{2\pi r}{v} = 2 \times 60 \times 60 = 7200\text{s}$

$v = \frac{2\pi r}{7200}$

$\frac{(2\pi)^2 r^2}{7200^2 r} = \frac{GM}{r^2}$

$r^3 = \frac{7200^2 GM}{(2\pi)^2}$

$r^3 = 5.24 \times 10^{20}\text{m}^3$

$r = 8061\text{km}$

$h = r - r_e = 1681\text{km}$

b) $v = \frac{2\pi r}{7200} = \frac{2\pi 8.061\times10^6}{7200} = 7034\text{m/s}$

c) $-\frac{GMm}{8.061\times10^6} + \frac{GMm}{6.380\times10^6} = 1.3 \times 10^{10}\text{J}$

Q2

a) $a = 0.5\text{ms}^{-2}$

$Fr - Tr = \frac{1}{2}m_{pulley}r^2\alpha = \frac{1}{2}m_{pulley}r^2\frac{a}{r}$

$F - T = \frac{1}{2}m_{pulley}a$

$m_{gift}a = T - m_{gift}g \rightarrow T = m_{gift}(a+g)$

$F = m_{gift}(a+g) + \frac{1}{2}m_{pulley}a$

$F = 4(10.3) + \frac{1}{2} \times 2 \times 0.5 = 41.7\text{N}$

b) $\alpha = \frac{a}{r} = \frac{0.5}{0.2} = 25\text{s}^{-2}$

c) $F_N = m_{elf}g - F = 20 \times 9.8 - 41.7 = 154.3\text{N}$

d) $F = m_{gift}g = 4 \times 9.8 = 39.2\text{N}$

e) $\omega = \frac{v}{r} = \frac{0.5}{0.2} = 2.5\text{s}^{-1}$

Q3

a) $\theta = \tan^{-1}\frac{30}{60} = 26.57^o$

b)

Vertical forces, up is positive $T\sin\theta + Fh_y = m_{sign}g + m_{rod}g$

Horizontal forces, right is positive $-T\cos\theta + Fh_x = 0$

Torques $0.6T\sin\theta = 0.8m_{sign}g + 0.4m_{rod}g$

$T = \frac{0.8\times 3\times 9.8+0.4\times 1\times 9.8}{0.6\times \sin 26.57^o} = 102.24\text{N}$

c) $Fh_y = 4.981 - 102.24\sin 26.57^o = -6.53\text{N}$

So force points down.

d) $Fh_x = 102.24\cos 26.57^o = 91.4\text{N}$

Force points to the right

Q4

Weight of displaced fluid (which include the air from the half of the ball that sits above the water).

$(\frac{1}{2}1000 + \frac{1}{2}1.2)\frac{4}{3}\pi 0.1^3$

Weight of ball including dense air

$1800\frac{4}{3}\pi(0.1^3 - 0.09^3) + \rho_{air}\frac{4}{3}\pi 0.09^3$

At equilibrium these two are equal, so

$\rho_{air} = \frac{(\frac{1}{2}1000+\frac{1}{2}1.2)\frac{4}{3}\pi 0.1^3 - 1800\frac{4}{3}\pi(0.1^3-0.09^3)}{\frac{4}{3}\pi 0.09^3}$

$\rho_{air} = 17.55\text{kg m}^{-3}$

b) $m_{air} = 17.55\times\frac{4}{3}\pi 0.09^3 = .054\text{kg}$

c) $n = \frac{54}{29} = 1.85\text{moles}$

d) $PV = nRT$

$P = \frac{1.85\times 8.314\times 293}{\frac{4\pi}{3}0.09^3} = 1471\text{kPa} = 14.6\text{atm}$

Q5

a) $kx_0 = mg$

$x_0 = \frac{0.5\times 9.8}{10} = 0.49\text{m}$

b)

$\Delta PE = \int_{x_0}^{0} F_{Net}\, dx = \int_{x_0}^{0} (kx - mg)\, dx = -\frac{1}{2}kx_0^2 + mgx_0 = -\frac{1}{2} \times 10 \times 0.49^2 + 0.5 \times 9.8 \times 0.49 = 1.2\text{J}$

If we took the spring in the other direction then

$\Delta PE = \int_{x_0}^{2x_0} F_{Net}\, dx = \int_{x_0}^{2x_0} (kx - mg)\, dx = \frac{3}{2}kx_0^2 - mgx_0 = \frac{3}{2} \times 10 \times 0.49^2 - 0.5 \times 9.8 \times 0.49 = 1.2\text{J}$

So you can see that there is the same amount of energy stored in either direction, which is important for simple harmonic motion to occur.

c) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10}{0.5}} = 0.71\text{Hz}$

d) $v_{max} = \omega A = 2\pi \times 0.71 \times 0.49 = 2.19\text{m/s}$ occurs at displacement $x = 0\text{m}$ from x_0

e) $a_{max} = \omega^2 A = (2\pi \times 0.71)^2 \times 0.49 = 9.8\text{m/s}^2$ occurs at displacements $x = -0.49\text{m}, x = 0.49\text{m}$ from x_0

Q6

a) Diagram should have pressure nodes wherever there are displacement antinodes and vice versa.

b) $v = f\lambda$

$\lambda = 0.4\text{m}$

$f = \frac{330}{0.4} = 825\text{Hz}$

c) To get 3 nodes (one at each ear on oone more in the middle) the wavelength must be halved, hence frequency is doubled, $f = 1650\text{Hz}$

d) I will accept another half wavelength, 0.2m in front of his ears/nose. However the exact answer is at a distance of $x = \frac{\lambda}{4}\sqrt{11}$ which is obtained by trigonometry solving for the long side of the triangle being $l_2 = \frac{6\lambda}{4}$ and the top side being $l_1 = \frac{5\lambda}{4}$

Q7

a) For an adiabatic process

$P_i V_i^\gamma = P_f V_f^\gamma$ where in this case $\gamma = \frac{7}{5}$

$P_f = P_i(\frac{V_i}{V_f})^\gamma = 101.3\text{kPa} \times 5^{7/5} = 964.2\text{kPa}$

b) As the gas is ideal

$P_i V_i = nRT_i$

$P_f V_f = nRT_f$

$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i}$

$T_f = \frac{964.2}{101.3} \times \frac{1}{5} \times 275 = 523.5\text{K}$

c) For an isothermal process

$W = nRT \ln \frac{V_f}{V_i}$

$PV = nRT$

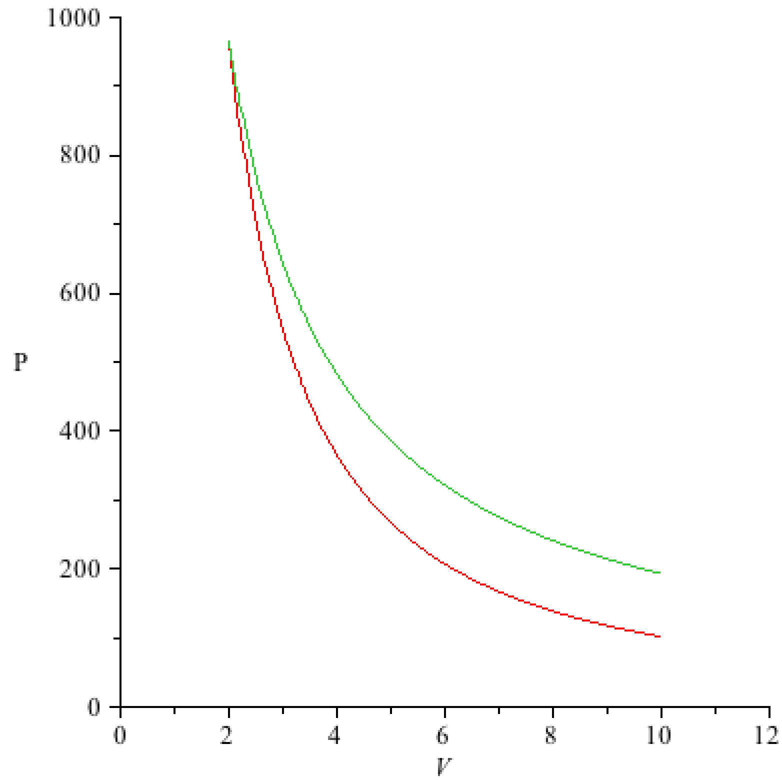
$W = 964.2 \times 10^3 \times 2 \ln(5) = 3103\text{kJ}$

d) For an isothermal process

$P_i V_i = P_f V_f$

$P_f = \frac{V_f}{V_i} P_i = \frac{964.2}{5} = 192.8\text{kPa}$

e)



Red is first process, green is second

Q 8

a) $COP = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = 274/24 = 11.42$

$Q_L = 11.42W = 11.42\text{kJ}$

b) 1st Law gives $Q_H = Q_L + W = 12.42\text{kJ}$

c) $e = 1 - \frac{T_c}{T_H} = 1 - \frac{274}{298} = 8\%$

d) PV=nRT, so $n = 101300 * 64/(8.314 * 274) = 2846$

e) One should not that opposite to the efficiency of a heat engine, the coefficient of performance increases if the efficiency decreases. I know this was not covered in class so I will give credit to this part regardless of. $c = \frac{T_H}{T_H - T_C} = 12.4$ So now with a new c of 13.6 we have that $T_H = 295.7K$

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