

Lecture 24 - Simple Harmonic Motion

We will now begin to consider oscillatory motion, beginning with the simplest example, [simple harmonic motion](#).

Restoring force of a spring



A vertical spring

A vertical spring will also execute simple harmonic motion, though its mean position will be modified by the balance of the gravitational force and the spring force.



SHM as a function of time

These [animations](#) from physclips show the form of displacement, velocity and acceleration of an object in SHM as a function of time.

Equations of motion for SHM

Starting from Newton's 2nd Law

$$ma = \Sigma F$$

$$m \frac{d^2x}{dt^2} = -kx$$

Based on our previous observations we might guess that the displacement will be able to be expressed as trigonometric function of time.

$$x = A \cos(\omega t + \phi)$$

A is the amplitude, $\omega = \frac{2\pi}{T} = 2\pi f$ and ϕ allows us to change the starting point of the motion.

$$\frac{dx}{dt} = v = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = a = -\omega^2 A \cos(\omega t + \phi)$$

$$-m\omega^2 A \cos(\omega t + \phi) = -kA \cos(\omega t + \phi)$$

which is true if

$$\omega^2 = \frac{k}{m}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Energy in SHM



$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)}$$

Simple harmonic motion with multiple springs

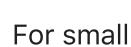
There is a wikipedia page on [multiple springs](#) which has a detailed explanation of the effective spring constants of springs in series and parallel. The key result is that for springs in parallel

$$k_{eq} = k_1 + k_2$$

and for springs in series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Simple Pendulum



The restoring force above is $F = -mg \sin \theta$

For small angles $\sin \theta = \theta$ so $F \approx -mg\theta$

and using the relation $s = l\theta$ gives $F \approx -\frac{mg}{L}s$

This is essentially the same as $F = -kx$ with $k = \frac{mg}{L}$

$$\text{So } \omega = \sqrt{\frac{k}{m}} \rightarrow \omega = \sqrt{\frac{g}{l}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

$$T = \frac{1}{f} = 2\pi \sqrt{\frac{l}{g}}$$

Coming Up

- Physical Pendulum
- Torsional Pendulum
- Damped Harmonic Motion
- Forced Harmonic Motion

In [1]:

(797+585)*0.5

Out[1]: 691.0

In []: