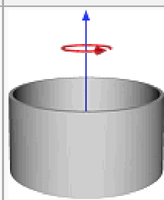
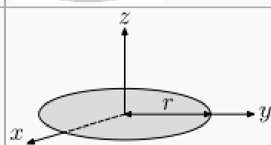
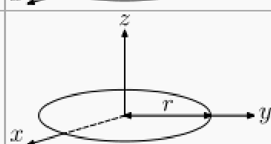
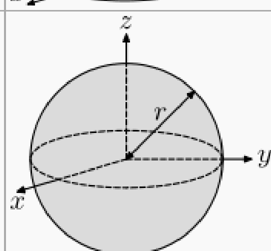
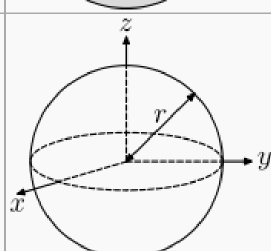
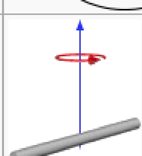
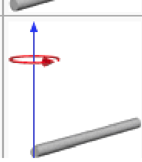
PHY141 Midterm 2 Fall 2023, Nov - a.k.a. Post Halloween Hangover

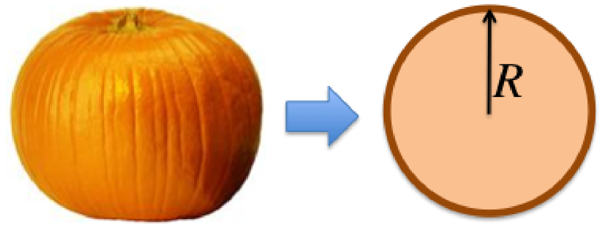
Name:

You have 80 minutes to complete these exam. You may consult two handwritten sheet of notes during the exam. All working and answers should be completed on the exam sheets. You may use the front and back sides of the exam sheets for your answers. Partial credit will be given for answers which, while finally incorrect, contain sensible, legible, and physically correct working. Below is some information which might help you in answering these questions.

Description	Figure	Moment(s) of inertia	Comment
Thin cylindrical shell with open ends, of radius *r* and mass *m*		$I = mr^2$ [1]	This expression assumes the shell thickness is negligible. It is a special case of the next object for $r_1 = r_2$. Also, a point mass (m) at the end of a rod of length r has this same moment of inertia and the value r is called the radius of gyration.
Thin, solid disk of radius *r* and mass *m*		$I_z = \frac{mr^2}{2}$ $I_x = I_y = \frac{mr^2}{4}$	This is a special case of the previous object for h=0.
Thin circular hoop of radius *r* and mass *m*		$I_z = mr^2$ $I_x = I_y = \frac{mr^2}{2}$	This is a special case of a torus for b=0. (See below.), as well as of a thick-walled cylindrical tube with open ends, with $r_1 = r_2$ and h=0.
Ball (solid) of radius *r* and mass *m*		$I = \frac{2mr^2}{5}$ [1]	A sphere can be taken to be made up of a stack of infinitesimal thin, solid discs, where the radius differs from 0 to r.
Sphere (hollow) of radius *r* and mass *m*		$I = \frac{2mr^2}{3}$ [1]	Similar to the solid sphere, only this time considering a stack of infinitesimal thin, circular hoops.
Rod of length *L* and mass *m*		$I_{\mathrm{center}} = \frac{mL^2}{12}$ [1]	This expression assumes that the rod is an infinitely thin (but rigid) wire. This is a special case of the previous object for *w = L* and *h = 0*.
Rod of length *L* and mass *m* (Axis of rotation at the end of the rod)		$I_{\mathrm{end}} = \frac{mL^2}{3}$ [1]	This expression assumes that the rod is an infinitely thin (but rigid) wire. This is also a special case of the thin rectangular plate with axis of rotation at the end of the plate: *h = L* and *w = 0*.

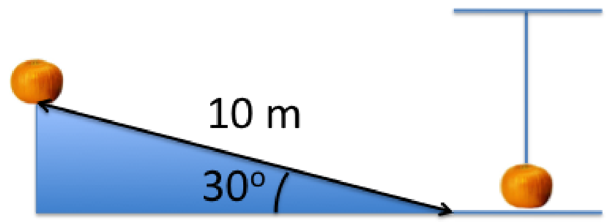
Moment of inertia of candies around sofa $I_{candies} = \sum_{candy} m_{candy} r^2_{candy}$

Question 1. (40 points)



A legend says that after Halloween, once kids are not paying attention to them, all pumpkins play and do physics demos together. Lets help them with the theory!

(a) (5 points) We can approximate a non-hollowed-out pumpkin of mass m as a uniform sphere of radius R which accounts for half of the mass of the pumpkin and a thin but dense spherical shell, also of radius R, which accounts for the other half of the mass of the pumpkin. Write an expression for the moment of inertia of the pumpkin in terms of m and R.



(b)(5 points) The pumpkin rolls down an 30^o incline of length 10m without slipping. Draw a free body diagram for the pumpkin in this situation.

(b) (5 points) In the situation described in (b), what is the magnitude of the pumpkin's velocity at the end of the slope?

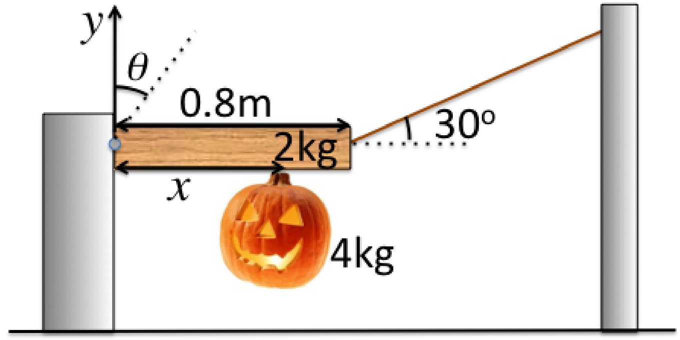
(c) (5 points) What is the direction of the angular momentum vector of the pumpkin while it is rolling down the slope? (ie. up, down, left, right, in to the page, out of the page, down the slope, up the slope, etc..)

(d) (10 points) At the bottom of the slope the pumpkin continues to roll on the flat with the same speed until it hits a second pumpkin which is suspended from a rope. The collision between the two pumpkins is elastic. After the collision the first pumpkin rolls halfway back up the slope before it comes to rest and starts rolling back down. What is the ratio of the mass of the second pumpkin to the mass of the first pumpkin?

(e) (5 points) What is the velocity of the second pumpkin immediately after the collision?

(f) (5 points) What maximum height above the ground will the second pumpkin reach?

Question 2. (30 points)



After Halloween, a Jack-O'-Lantern of mass 4kg is left be suspended as shown in the diagram using a hinged uniform beam of mass 2kg and length 0.8m and a massless string. The beam is level and the string makes an angle of 30^o with the horizontal.

(a) (5 points) Draw a free body diagram with all the forces acting on the system in this situation.

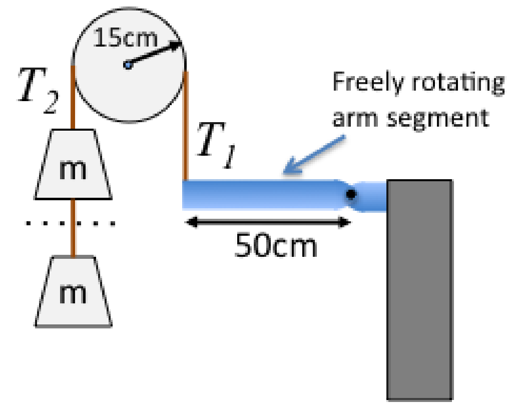
(b) (10 points) The maximum tension the string can support without breaking is 50N, and the string is at the edge of breaking. What is the distance from the hinge, x, that the Jack-O'-Lantern is hang on.

(b) (10 points) In the situation described in (a), (ie. when the tension in the string is 50N), what is the magnitude of the net force on the hinge? What is the direction of the force (give your answer in terms of the angle θ from the y axis shown in the diagram)?

(d) (10 points) If I would like to hang the Jack-O'-Lantern at the far end of the beam from the hinge using the same string as was used in parts (a) and (b), what is the minimum angle the string should make with the horizontal instead of the 30^o angle it makes in parts (a) and (b).

Question 3. (35 points)

A hinged arm is held on one end by a fixed post, and on the other end by a rope which is run over a pulley of mass 2kg and radius 15cm and then tied to a weight of mass m. A second identical weight of mass m is tied to the bottom of the first weight. The system is in equilibrium. The length of the piece of the arm that is free to rotate is 50cm long and weighs 8kg. It can be considered to be a uniform rod. The tensions in the ropes T_1 and T_2 are labelled on the diagram.



(a) (5 points) What is the mass of each weight, m?

(b) (5 points) What is the vertical component of the force exerted on the freely rotating arm segment by the hinge? If appropriate, specify a direction.

(c) (5 points) What is the horizontal component of the force exerted on the freely rotating arm segment by the hinge? If appropriate, specify a direction.

(d) (5 points) What is the moment of inertia for rotation of the freely rotating arm segment around the hinge?

(e) (10 points) The rope is now cut at the position of the dotted line, so that the bottom mass now falls to the ground. When the rope is cut, what is the *initial* acceleration of the weight that is still attached to the arm? Give magnitude and direction.

(f) (5 points) What are the values of the tensions T_1 and T_2 just after the rope has been cut.