Mariya Eggensperger
CST 370, Spring 2017
Dr. Feiling Jia
Design/Analysis
of Algorithms

CST 370 Homework (Complexity Analysis)

1. (20 points) Consider the following algorithm.

ALGORITHM
Mystery(n)
//Input: A nonnegative integer n S← 0
for *i* ← 1 to *n* do
S←S + i*i return S

a. What does this algorithm compute?

The algorithm computes : $S(n) = \sum_{i=1}^{n} i^2$

b. What is its basic operation?

The basic operation is multiplication.

c. How many times is the basic operation executed?

The operation is executed : $C(n) = \sum_{i=1}^{n} 1 = n$

d. What is the worst case running time of this algorithm in asymptotic notation?

The worst case running time of this algorithm in asymptotic notation is $C(n) = n \in \Theta(n)$

2. (20 points) Consider the following algorithm.

ALGORITHM
Secret(A[0...n-1])
//Input: An array A[O..n - 1] of *n* real numbers
minval←A[O]; *maxval*←A[O]
for i←1to *n*- 1 do
if (A[i] < *minval)*
minval ←A[i]
if (*A[i]* > *maxval)*
maxval ←A[i]
return *maxval* - *minval*

a. What does this algorithm compute?

The algorithm computes the difference between the array's largest and smallest values.

b. What is its basic operation?

The basic operation is the comparison of elements.

c. How many times is the basic operation executed?

$$C(n) = \sum_{i=1}^{n-1} 2 = 2(n-1)$$

d. What is the worst case running time of this algorithm in asymptotic notation?

The worst case running time of this algorithm is: $\Theta(n)$

3. (10 points) Compute the following sums.

a. 1 + 3 + 5 + 7.... 999

Sum is :

$$\frac{(1+999)500}{2} \approx 250{,}000$$

b. 2 + 4 + 8 + 16 + . . . + 1024

$$2 \quad \frac{2^{\wedge}10-1}{2-1} = 2046$$

4. (20 points) Climbing stairs Problem: Find the number of different ways to climb an n-stair stair- case if each step is either one or two stairs.

It would seem this is a Fibonacci sequence problem. Therefore, assuming that S(n) is the number of ways to climb an n stair staircase.

$S(n) = S(n-1) + S(n-2)$ for $n \geq 3$ and $S(1) = 1$, $S(2) = 2$

It’s concluded:

$S(n) = F(n+1)$ for $n \geq 1$

5. (30 points) Solve the following recurrence relations and prove by induction.

a. x(n) = x(n-1) + 5 for n > 1, x(1) = 0

$$\begin{aligned}
x(n) &= x(n-1)+5 \\
&= [x(n-2)+5]+5 = x(n-2)+5*2 \\
&= [x(n-3)+5]+5*2 = x(n-3)+5*3 \\
&= \text{. . . etc} \\
&= [x(n-i)+5*i]
\end{aligned}$$

solution $\mathbf{= x(1) + 5 * (n-1) = 5(n-1)}$

b. x(n) = 3x(n-1)   for n > 1, x(1) = 4

x(n) = 3x(n-1)
= 3[3x(n-2)] = 3^2x(n-2)
= 3^2[3x(n-3)] = 3^3x(n-3)
= . . . etc
= [3^ix(n-i)]
Solution: 3^n-1x(1) = 4*3^n-1

c. x(n) = x(n-1) + n   for n > 0, x(0) = 0

x(n) = x(n-1) + n
= [x(n − 2) + (n − 1)] + n = x(n − 2) + (n − 1) + n
= [x(n − 3) + (n − 2)] + (n − 1) + n = x(n − 3) + (n − 2) + (n − 1) + n
= . . . etc
= x(n − i)+(n − i + 1) + (n − i + 2) + ... +
Solution = x(0) + 1 + 2 + ... + n = $\frac{n(n + 1)}{2}$