FORMULA SHEET

LINEAR ALGEBRA

Elementary Row Operations

1. Interchange rows

$$Row\ 1 \leftrightarrow Row\ 3$$

$$\left[\begin{array}{ccc|c} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{array}\right] = \left[\begin{array}{ccc|c} i & j & k & l \\ e & f & g & h \\ a & b & c & d \end{array}\right]$$

2. Multiply rows by a non-zero constant

$$Row\ 1 = 2 \cdot Row\ 1$$

$$\left[\begin{array}{ccc|c} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 2a & 2b & 2c & 2d \\ e & f & g & h \\ i & j & k & l \end{array}\right]$$

3. Add rows together

$$Row\ 1 = Row\ 1 + Row\ 2$$

$$\left[\begin{array}{ccc|c} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} a+e & b+f & c+g & d+h \\ e & f & g & h \\ i & j & k & l \end{array}\right]$$

Matrix Representation of Systems

$$\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases} \leftrightarrow \begin{array}{c} \left[\begin{array}{cc|c} A & B & C \\ D & E & F \end{array}\right] \\ \begin{array}{ccc} x & y & b \end{array} \end{array}$$

Gauss-Jordan Elimination Solutions

One solution

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & b_1 \\ 0 & 1 & \cdots & 0 & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & b_m \end{array}\right] \end{array}$$

Infinite solutions

$$\left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & b_1 \\ 0 & 1 & \cdots & 0 & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{array}\right]$$

No solutions

$$\left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & b_1 \\ 0 & 1 & \cdots & 0 & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & b_m \end{array}\right]$$

Types of Matrices

Symmetric

$$\begin{bmatrix} a & b & c & d \\ b & e & f & g \\ c & f & h & i \\ d & g & i & j \end{bmatrix}$$

Triangular

$$\begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \\ 0 & 0 & 0 & j \end{bmatrix}$$

Diagonal

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & h & 0 \\ 0 & 0 & 0 & j \end{bmatrix}$$

Identity

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

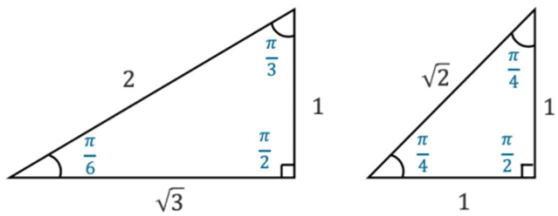
FORMULA SHEET

TRIGONOMETRY

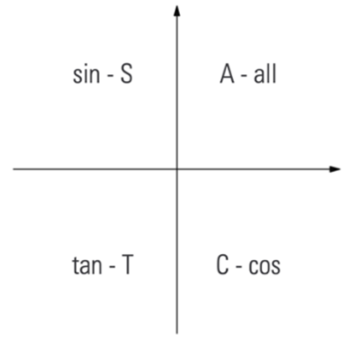
Arc Length Formula

$$a = r\theta$$

Special Triangles



CAST Rule



Compound Angle Formulas

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - cos\ (A) sin\ (B)$$
$$\cos(A+B) = \cos(A)\cos(B) - sin\ (A) sin\ (B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$

$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Double Angle Formulas

$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\cos(2A) = \cos^2(A) - \sin^2(A)$$
$$\cos(2A) = 1 - 2\sin^2(A)$$
$$\cos(2A) = 2\cos^2(A) - 1$$

$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Identities

$$c^2 = a^2 + b^2$$
$$\sin^2(x) + \cos^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$
$$1 + \cot^2(x) = \csc^2(x)$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\sin(-x) = -\sin(x)$$
$$\cos(-x) = \cos(x)$$
$$\tan(-x) = -\tan(x)$$

Co-Function Identities

$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot(x)$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan(x)$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec(x)$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc(x)$$

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)}$$

Sum Identities

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

FORMULA SHEET

TRIGONOMETRY *(continued)*

Trigonometric Functions

$$y = A\sin\big(k\,(x-p\,)\big)+q$$

$$y = A\cos\big(k\,(x-p\,)\big)+q$$

$A = amplitude$

$\frac{2\pi}{k} = period$

$p = phase\ shift$

$q = vertical\ translation$

Hyperbolic Functions

$$\cosh(x) = \frac{e^{x}+e^{-x}}{2}$$

$$\sinh(x) = \frac{e^{x}-e^{-x}}{2}$$

$$\tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

Reciprocal Functions

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^{x}+e^{-x}}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^{x}-e^{-x}}$$

$$\coth(x) = \frac{1}{\tanh(x)} = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

Hyperbolic Identities

$\cosh^2(x)+\sinh^2(x) = \cosh(2x)$

$\cosh^2(x)-\sinh^2(x) = 1$

$\cosh(x)+\sinh(x) = e^{x}$

$\cosh^2(x)-\sinh^2(x) = 1$

$\tanh^2(x)+\operatorname{sech}^2(x) = 1$

$\coth^2(x)-\operatorname{csch}^2(x) = 1$

$\cosh(x) = \cosh(-x)$

$\sinh(x) = -\sinh(-x)$

$\tanh(x) = -\tanh(-x)$

$\coth(x) = -\coth(-x)$

$\operatorname{sech}(x) = \operatorname{sech}(-x)$

$\operatorname{csch}(x) = -\operatorname{csch}(-x)$

FORMULA SHEET

GEOMETRY AND ALGEBRA OF VECTORS

Unit Vector, $\vec{p}$

$$\vec{p} = \frac{\vec{v}}{|\vec{v}|}$$

Vector Magnitude

$$|\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Dot Product

$$\vec{u} \cdot \vec{v} = (u_x, u_y, u_z) \cdot (v_x, v_y, v_z) = u_x v_x + u_y v_y + u_z v_z$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| cos\theta$$

Cross Product

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = (u_y v_z - u_z v_y , u_z v_x - u_x v_z , u_x v_y - u_y v_x)$$

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| sin\theta$$

Applications of Vector Operations

Work

$$W = \vec{F} \cdot \vec{d}$$

Torque

$$\|\tau\| = \|\vec{r} \times \vec{f}\|$$

Area of a Parallelogram

$$A = \|\vec{u} \times \vec{v}\|$$

Volume of a Parallelepiped

$$V = \|(\vec{a} \times \vec{b}) \cdot c\|$$

Equations of Lines

Vector

$$\vec{r} = \vec{r_0} + t\vec{m} , t \in R$$

Scalar

$$Ax + By + C = 0$$

Symmetric

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}, a, b, c \neq 0$$

Parametric

2D

$$x = x_0 + ta$$

$$y = y_0 + tb$$

$$t \in R$$

3D

$$x = x_0 + ta$$

$$y = y_0 + tb$$

$$z = z_0 + tc$$

$$t \in R$$

FORMULA SHEET

FUNCTIONS

Interval Notation

(a, b)

$a < x < b$

$[a, b]$

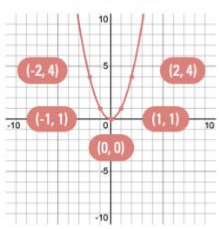
$a \leq x \leq b$

8 Log Properties

1. $\log(ab) = \log(a) + \log(b)$
2. $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
3. $\log(a^x) = x\log(a)$
4. $\log_a(a) = 1$
5. $\log_a(1) = 0$
6. $\log_a b = \frac{\log_c b}{\log_c a}$
7. $\log_a b = \frac{1}{\log_b a}$
8. $a^{\log_a b} = b$

Common Base Graphs

Quadratic Functions

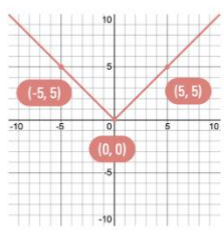


$f(x) = x^2$

$D: (-\infty, \infty)$

$R: [0, \infty)$

Absolute Value Functions

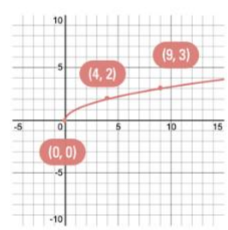


$f(x) = |x|$

$D: (-\infty, \infty)$

$R: [0, \infty)$

Radical Functions

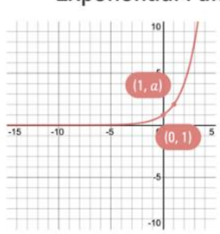


$f(x) = \sqrt{x}$

$D: [0, \infty)$

$R: [0, \infty)$

Exponential Functions

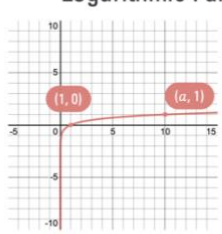


$f(x) = a^x$

$D: (-\infty, \infty)$

$R: (0, \infty)$

Logarithmic Functions



$f(x) = \log_a(x)$

$D: (0, \infty)$

$R: (-\infty, \infty)$

FORMULA SHEET

LIMITS, DERIVATIVES, AND INTEGRALS

Derivative by First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Common Derivatives

Constant Derivatives

$$\left(\frac{d}{dx}\right) c = 0$$

Logarithmic Derivatives

$$\left(\frac{d}{dx}\right) e^x = e^x \qquad \left(\frac{d}{dx}\right) log_a(x) = \frac{1}{xln(a)}$$

$$\left(\frac{d}{dx}\right) ln(x) = \frac{1}{x} \qquad \left(\frac{d}{dx}\right) a^x = a^x\, ln(a)$$

Trigonometric Derivatives

$$\left(\frac{d}{dx}\right) sin(x) = cos(x) \qquad \left(\frac{d}{dx}\right) sec(x) = sec(x)\tan(x) \qquad \left(\frac{d}{dx}\right) sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\left(\frac{d}{dx}\right) cos(x) = -sin(x) \qquad \left(\frac{d}{dx}\right) csc(x) = -csc(x)\,cot(x) \qquad \left(\frac{d}{dx}\right) cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\left(\frac{d}{dx}\right) tan(x) = sec^2(x) \qquad \left(\frac{d}{dx}\right) cot(x) = -csc^2(x) \qquad \left(\frac{d}{dx}\right) tan^{-1}(x) = \frac{1}{1+x^2}$$

Hyperbolic Derivatives

$$\left(\frac{d}{dx}\right) sinh(x) = cosh(x) \qquad \left(\frac{d}{dx}\right) sech(x) = -tanh(x)\,sech(x)$$

$$\left(\frac{d}{dx}\right) cosh(x) = sinh(x) \qquad \left(\frac{d}{dx}\right) csch(x) = -csch(x)\,coth\,(x)$$

$$\left(\frac{d}{dx}\right) tanh(x) = sech^2(x) \qquad \left(\frac{d}{dx}\right) coth(x) = -csch^2(x)$$

Derivative Rules

Multiplication by a Constant

$$\left(\frac{d}{dx}\right)(cf(x)) = c\left(\frac{d}{dx}\right) f(x)$$

Power Rule

$$\left(\frac{d}{dx}\right) x^n = nx^{n-1}$$

Product Rule

$$\left(\frac{d}{dx}\right)(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

Addition/Subtraction Rule

$$\left(\frac{d}{dx}\right)(f(x) \pm g(x)) = \left(\frac{d}{dx}\right) f(x) \pm \left(\frac{d}{dx}\right) g(x)$$

Quotient Rule

$$\left(\frac{d}{dx}\right)\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Chain Rule

$$\left(\frac{d}{dx}\right) f(g(x)) = f'(g(x))\, g'(x)$$

#MacEng EMBER

FORMULA SHEET

LIMITS, DERIVATIVES, AND INTEGRALS *(continued)*

Sigma Notation

$$\sum_{i=n}^{m} a_i = a_n + a_{n+1} + \cdots + a_{m-1} + a_m$$

Properties of Summations

1. $\sum_{i=i_0}^{n} c a_i = c \sum_{i=i_0}^{n} a_i$
2. $\sum_{i=i_0}^{n} (a_i \pm b_i) = \sum_{i=i_0}^{n} a_i \pm \sum_{i=i_0}^{n} b_i$

Special Summations Formulas

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Common Antiderivatives

Constant & First-Degree Polynomial

$\int a\,dx = ax + c$

$\int x\,dx = \frac{x^2}{2} + c$

Logarithmic Antiderivatives

$\int e^x\,dx = e^x + c$

$\int \ln(x)\,dx = x\ln(x) - x + c$

$\int \frac{1}{x}\,dx = ln(x) + c$

$\int a^x\,dx = \frac{a^x}{\ln(a)} + c$

Trigonometric Antiderivatives

$\int \cos(x)\,dx = \sin(x) + c$

$\int \sin(x)\,dx = -\cos(x) + c$

$\int \tan(x)\,dx = -\ln|\cos(x)| + c$

$\int \sec^2(x)\,dx = \tan(x) + c$

$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + c$

$\int \cot(x)\,dx = \ln|\sin(x)| + c$

Hyperbolic Antiderivatives

$\int \sinh(x)\,dx = \cosh(x) + c$

$\int \cosh(x)\,dx = \sinh(x) + c$

$\int \tanh(x)\,dx = \ln|\cosh(x)| + c$

Integral Rules

Multiplication by a Constant

$$\int c\,f(x)\,dx = c\int f(x)\,dx$$

Additive/subtractive rule

$$\int \big(f(x) \pm g(x)\big)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Power rule:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$