Review

Probability

probability of event A : $P(A)$

independent events : $P(A \cap B) = P(A) \cdot P(B)$

expectation : $E(X) = \sum_{\substack{\text{events} \\ X}} X \cdot P(X)$

E.g. rolling a 6-sided die

$$E = 1 \cdot \tfrac{1}{6} + 2 \cdot \tfrac{1}{6} + 3 \cdot \tfrac{1}{6} + 4 \cdot \tfrac{1}{6} + 5 \cdot \tfrac{1}{6} + 6 \cdot \tfrac{1}{6}$$

$$\Rightarrow E = 21/6 = 3.5$$

note: $E(X)$ has "units" of X

Exponential decay

assuming : atoms are independent

timesteps are independent

dt is not too small, not too large

mean-field approximation

so many atoms s.t. actual # that decay is approx average # that decay

$\Rightarrow$ difference eqn : $N(t+dt) = N(t)(1-\delta)$

$\Rightarrow$ ODE: $\frac{dN}{\delta t} = -\tilde{\delta} N(t)$

$\downarrow$ soln

$N(t) = N_0 \exp(-\tilde{\delta} t)$

Chemical kinetics

assuming: well-mixed

"good" dt

reactions only happen when molecules in close proximity

mean-field approximation

independence of molecules

$\Rightarrow$ e.g. $jA + kB \underset{r_2}{\overset{r_1}{\rightleftharpoons}} lC$.

↳ forward reaction: $r_1 [A]^j [B]^k$ ☆

↳ backward reaction: $r_2 [C]^l$ ☆

↳ $\frac{d[A]}{\delta t} = \left(\text{change in \# due to reaction}\right)$ ☆ $+ \ldots$

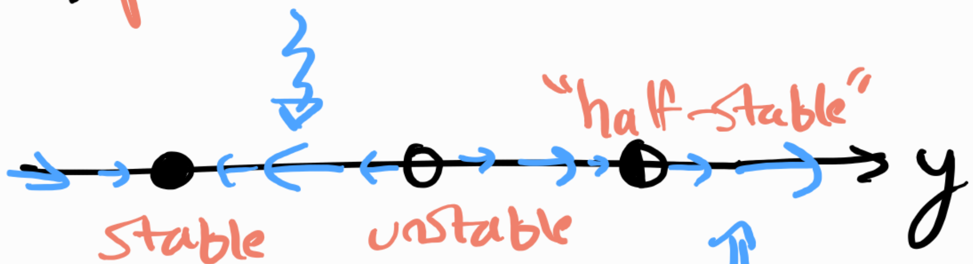
$\frac{d[A]}{\delta t} = -j r_1 [A]^j [B]^k + j r_2 [C]^l$

1D autonomous ODEs

$$\frac{dy}{dt} = f(y) \Rightarrow \text{fixed points}$$

$$y^* \text{ s.t. } f(y^*) = 0$$

$\Rightarrow$ phase line



determined by sign of $\frac{dy}{dt} = f(y)$

$\Rightarrow$ linear stability analysis

how does perturbation $\varepsilon(t) = y(t) - y^*$ behave?

$$\rightsquigarrow \frac{d\varepsilon}{dt} = \frac{dy}{dt} = f(y^* + \varepsilon)$$

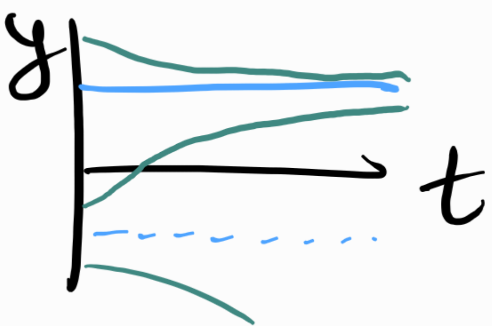
$$\rightsquigarrow \frac{d\varepsilon}{dt} \approx \varepsilon f'(y^*)$$

note: only "true" for locally $\varepsilon \ll 1$

$$\rightsquigarrow \varepsilon(t) = \varepsilon_0 \exp(\underbrace{f'(y^*)}t)$$

sign of $f'(y^*)$ determines decay/growth

$\Rightarrow$ solutions $y(t)$



Important tips

→ models are built on assumptions
  ⇝ what are they?
  ⇝ are they true?
  ⇝ does my data agree with them?
    how do I determine that?

→ Visualize your data

→ think about units
  ⇝ probability has no units!
  ⇝ think about timescale

Quiz

Imagine you are running a "well-mixed" microbe farm. The microbes (M) exhibit reactions

$$M \xrightarrow{\alpha} 0 \quad \text{die}$$

$$2M \xrightarrow{\beta} 5M \quad \text{mate \& proliferate}$$

$$3M \xrightarrow{\gamma} M \quad \text{fight}$$

Let the reaction rates be:

$$\alpha = 2.88, \ \beta = 1.92, \ \gamma = 0.8$$

(a) Write down the kinetic eqns for this system

What will happen if the initial pop of microbes is: (b) 0.5

(c) 1.2

(d) 3.0

$$\frac{dM}{dt} = -\alpha M + 3\beta M^2 - 2\gamma M^3$$

$$= -2.88M + 5.76M^2 - 1.6M^3$$

$$= -1.6x(x-0.6)(x-3) = f(M)$$

fixed pts:

$M=0$, $M=0.6$, $M=3$