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Зад 1 $3y' - \frac{2x}{x^2+2}y = (x+1)e^{-x}(2+x^2)\frac{1}{y^2}$, $y(0) = 1$

$y' = \frac{2x}{3(x^2+2)}y + \frac{(x+1)e^{-x}(2+x^2)}{3} \cdot \frac{1}{y^2} \quad \Big| \cdot y^2$

$x^2+2 \neq 0 \Rightarrow x^2 \neq -2 \Rightarrow \forall x$

$y \neq 0$

$y^2 y' = \frac{2x}{3(x^2+2)}y^3 + \frac{(x+1)e^{-x}(2+x^2)}{3}$

полагаме $z = y^3$ $\quad z' = 3y^2 y'$ $\quad y^2 y' = \frac{z'}{3}$

$\frac{z'}{3} = \frac{2x}{3(x^2+2)}z + \frac{(x+1)e^{-x}(2+x^2)}{3} \quad \Big| \cdot 3$

Сведохме уравнението до линейно уравнение от първи ред
$\Rightarrow$ Можем да използваме формулата

$z(x) = e^{\int a(x)dx}\left(c + \int b(x)e^{-\int a(x)dx}dx\right)$

$a(x) = \frac{2x}{(x^2+2)}$ $\quad b(x) = (x+1)e^{-x}(2+x^2)$

$\cancel{\int \frac{2x}{3x}}$ $\quad \int \frac{2x}{x^2+2}dx = \int \frac{1}{t}dt = \ln(|t|) = \ln(|x^2+2|) = \ln(x^2+2)$

$t = x^2+2$

$e^{\int a(x)dx} = e^{\int \frac{2x}{x^2+2}dx} = e^{\ln(x^2+2)} = x^2+2$

$e^{-\int a(x)dx} = e^{-\int \frac{2x}{x^2+2}dx} = e^{-\ln(x^2+2)} = \frac{1}{x^2+2}$

$$\int b(x)e^{-\int a(x)dx}\,dx = \int (x+1)e^{-x}(2+x^2)\cdot e^{-\int \frac{2x}{x^2+2}dx}\,dx =$$

$$= \int (x+1)e^{-x}(2+x^2)\cdot e^{-\ln(x^2+2)}\,dx = \int (x+1)e^{-x}\cancel{(2+x^2)}\cdot \frac{1}{\cancel{x^2+2}}\,dx =$$

$$= \int (x+1)e^{-x}\,dx = (x+1)(-e^{-x}) - \int -e^{-x}\,dx =$$

$$= (x+1)(-e^{-x}) + \int e^{-x}\,dx = (x+1)(-e^{-x}) - e^{-x} = -xe^{-x} - 2e^{-x}$$

$$z(x) = e^{\int \frac{2x}{x^2+2}dx}\left(c + \int (x+1)e^{-x}(2+x^2)\cdot e^{-\int \frac{2x}{x^2+2}dx}\,dx\right) =$$

$$= (x^2+2)\left(c + (-xe^{-x} - 2e^{-x})\right) = (x^2+2)\left(c - (xe^{-x} + 2e^{-x})\right)$$

$$z(x) = y^3$$

$$\Rightarrow \boxed{y = \sqrt[3]{(x^2+2)\left(c-(xe^{-x}+2e^{-x})\right)}}$$

$$y(x) = \underline{1} \quad x = 0$$

$$\underline{1} = \sqrt[3]{(0+2)(c-(0.1+2.1))} = \sqrt[3]{2(c-2)} = \sqrt[3]{2c-4}$$

$$\underline{1} = 2c-4 \qquad 2c = 5 \qquad c = \frac{5}{2} = 2{,}5$$

$$\boxed{c = \frac{5}{2}}$$