Latent Growth Curve Modeling With lavaan

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Introduction

Rationales of Longitudinal Research [1]

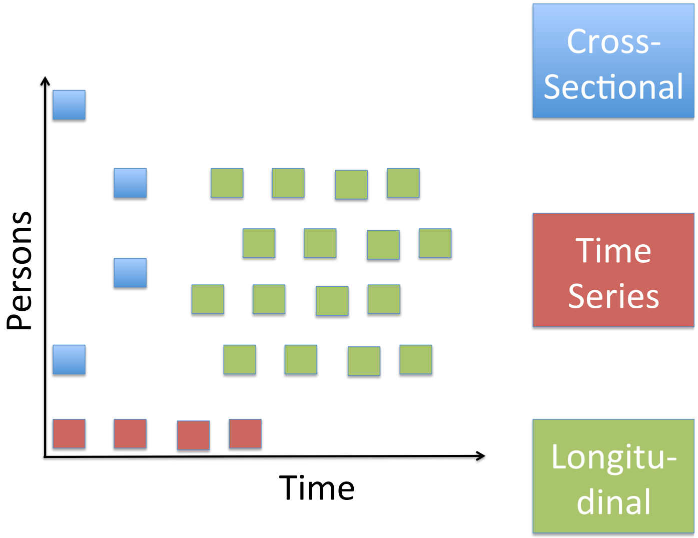
1. **Direct identification of intraindividual change**
2. **Direct identification of interindividual differences (similarity) in intraindividual change**
3. **Analysis of interrelationships in behavioral change**
4. Analysis of causes (determinants) of intraindividual change
5. **Analysis of causes (determinants) of interindividual differences in intraindividual change**

[1] Baltes and Nesselroade (1979). *History and rationale of longitudinal research.*

Goals of Longitudinal Research

1. Within person change?
2. Between person differences in change?
3. Variables change together?
4. Causes of within person change?
5. Causes of between person differences in change?

Types of Data



Core Ingredients for Measuring Change

- Longitudinal data: multiple persons measured at multiple time points
- (Statistical) models:
  - Mathematical description of our assumptions (often derived from a theory) before seeing the data
  - At least: How do people change and how do they differ in their change (rationales 1,2).
- "...all repeated measures analyses should start with the question,"What is your model for change"" [2]
- One popular class of models for longitudinal data are latent growth curve models
- Goal: Introduction of the Latent Growth Curve Modelling and it's implementation in lavaan

[2] McArdle (2009). *Latent Variable Modeling of Differences and Changes with Longitudinal Data.*

Outline

Some Terminology

- Given a variable of interest, for example, fluid intelligence
- We observed it for multiple persons on multiple occasions.
- $y_{i,t}$ is our observation for this variable for person i at time point t.
- In statistical modeling we assume that $y_{i,t}$ is a realization of a corresponding random variable $Y_{i,t}$.
- A longitudinal model describes possible distributions for $Y_{i,t}$ for all time points t and persons i.

Latent Growth Curve Models

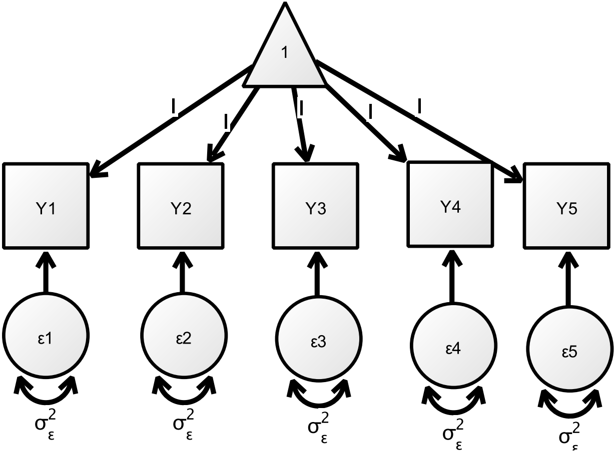
Constant Model

- A very easy (unrealistic) longitudinal model is the constant model.
- For every random variable $Y_{i,t}$ the same distribution is assumed.
-
$$Y_{i,t} = I + \epsilon_{i,t}$$
with $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $I \in \mathbb{R}$
- Alternatively,
$$Y_{i,t} \sim \mathcal{N}(I, \sigma_\epsilon^2)$$
- This encodes the assumption that the variable of interest is the same across all persons and time points.
- All observed differences are caused by measurement error.

SEM: Constant Model

For five measurement occasions $t_j \in \{t_1, \ldots, t_5\}$:

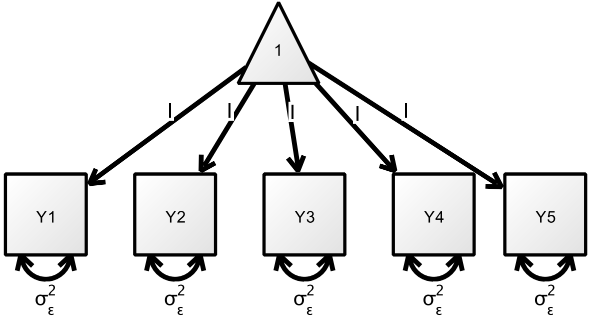
$$Y_{i,j} = 1 \cdot I + \epsilon_{i,j}$$
$$\epsilon_{i,j} \sim \mathcal{N}(0, \sigma_\epsilon^2)$$



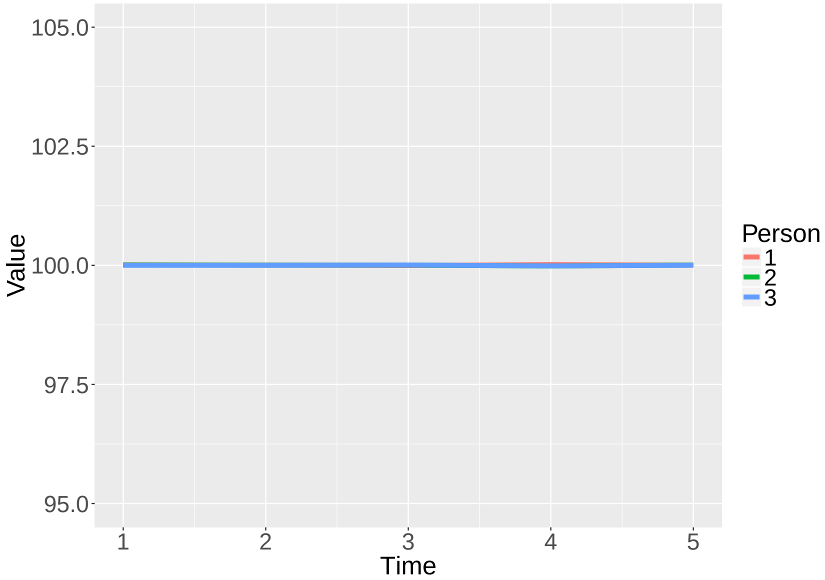
Example: Constant Model Compact Representation

For five measurement occasions $t_j \in \{t_1, \ldots, t_5\}$:

$$Y_{i,j} = 1 \cdot I + \epsilon_{i,j}$$
$$\epsilon_{i,j} \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

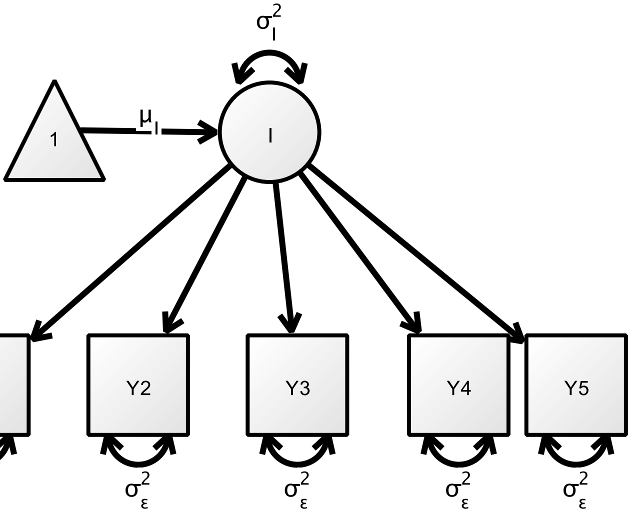


Constant Model Implied Trajectories $I = 100$, $\sigma_\epsilon^2 = 0$



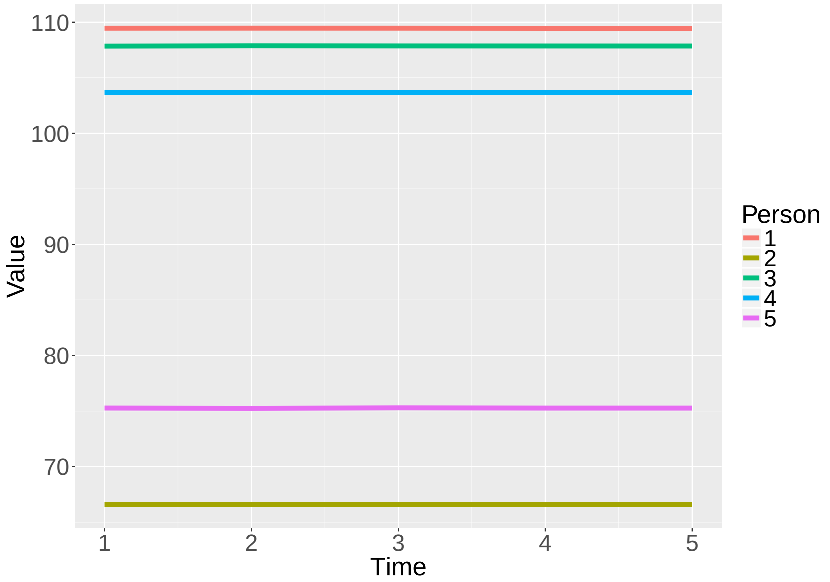
First Extension: Random Intercept

- Each person may have their own intercept I
- Results in random intercept model

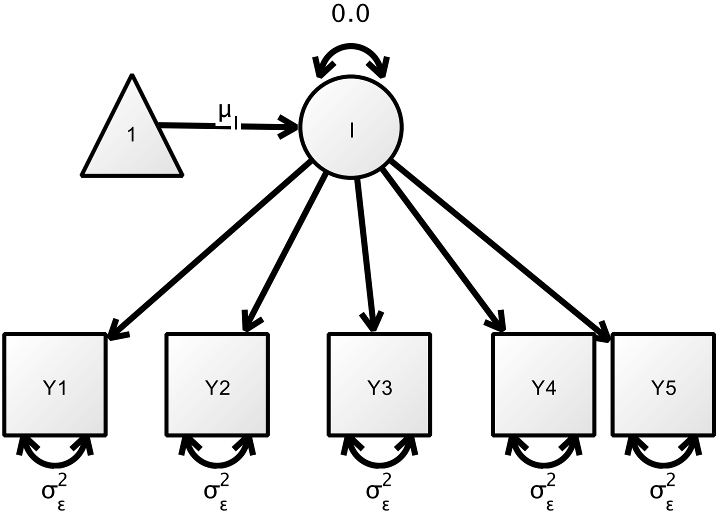


Trajectories: Random Intercept Model

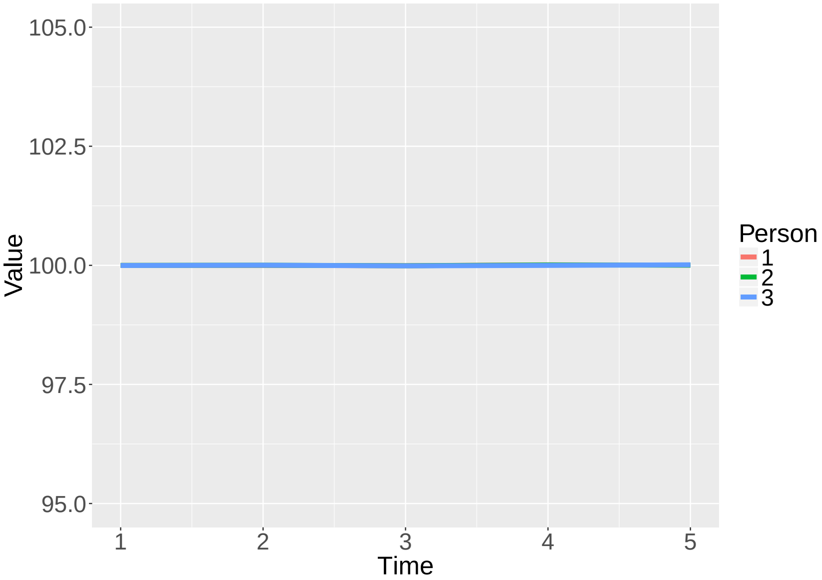
$\mu_I = 100, \sigma_I^2 = 400, \sigma_\epsilon^2 = 0$



Constant as Special Case of Random Intercept

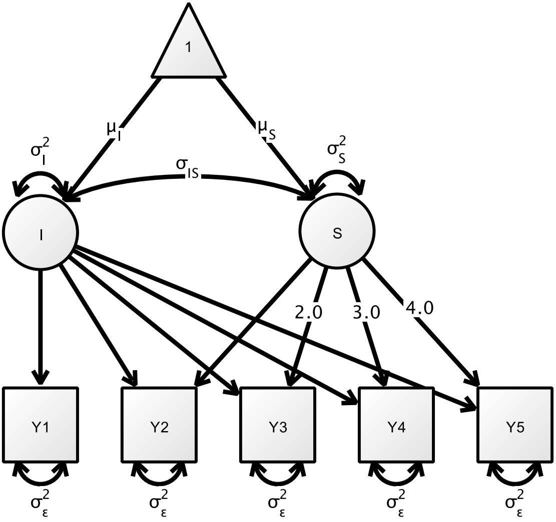


Constant Model Implied Trajectories $I = 100$, $\sigma_\epsilon^2 = 0$



Next Extension: Random Slope

- Allow every person to have a different linear trend
- Results in latent growth curve model



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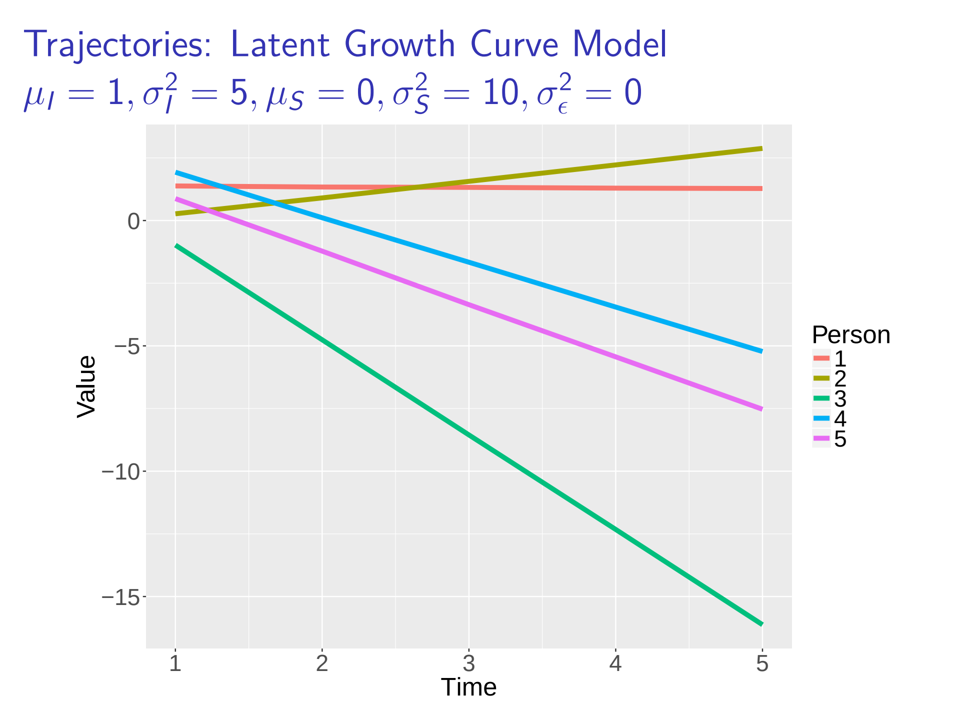

Trajectories: Latent Growth Curve Model
$\mu_I = 1, \sigma_I^2 = 5, \mu_S = 0, \sigma_S^2 = 10, \sigma_\epsilon^2 = 0$
Value
0
−5
−10
−15
Time
1
2
3
4
5
Person
1
2
3
4
5

Which Model To Trust?

- Plot
- Fit Indices

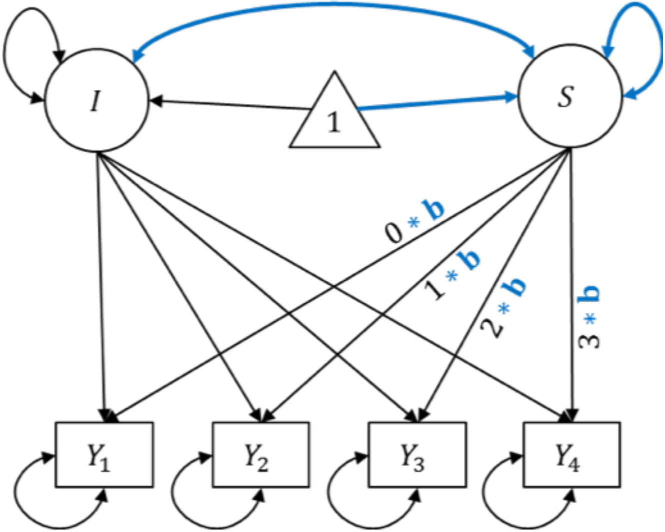
Identification

- For a regular LGCM, we need at least 3 time points
- We estimate 4 ($\mu_I, \sigma_I^2, \mu_S, \sigma_S^2, \sigma_\epsilon^2$) parameters –> Need at least 4 unique covariance matrix entries –> Need at least 3 time points
- For more complex models, we typically need more time points

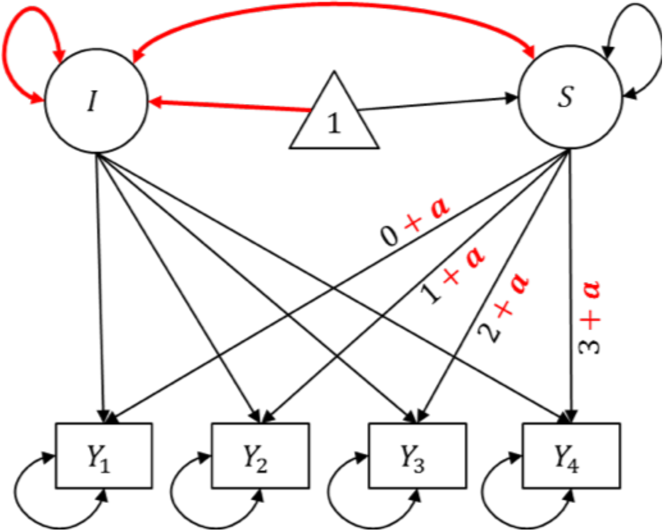
Choosing Time

- We use the time variable t as predictor
- Scaling time i.e. $t^* = a + bt$, will not change the fit but the parameters –> conclusions
- a shifts the origin of time
- b changes the unit of time (seconds vs minutes)

Effect of Changing Units



Effect of Changing Origin



Solution

- Always interpret results conditional on scaling
- μ_I the mean at time point 0
- μ_S average change if 1 time unit (seconds, hours, months) elapses
- σ_I^2 the variance at time point 0
- σ_S^2 the variance in change if 1 time unit elapses
- σ_{IS} the covariance between intercept and slope

Practical Advise

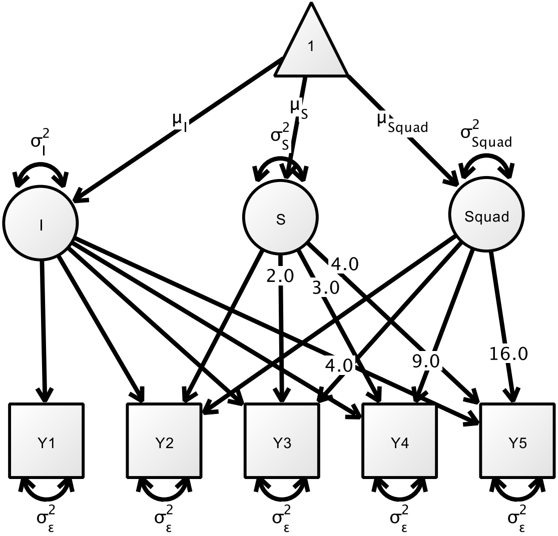
- Unit change is not so drammatic but keep in mind for interpretation.
- Is there a natural origin?

Advanced Latent Growth Curve Models

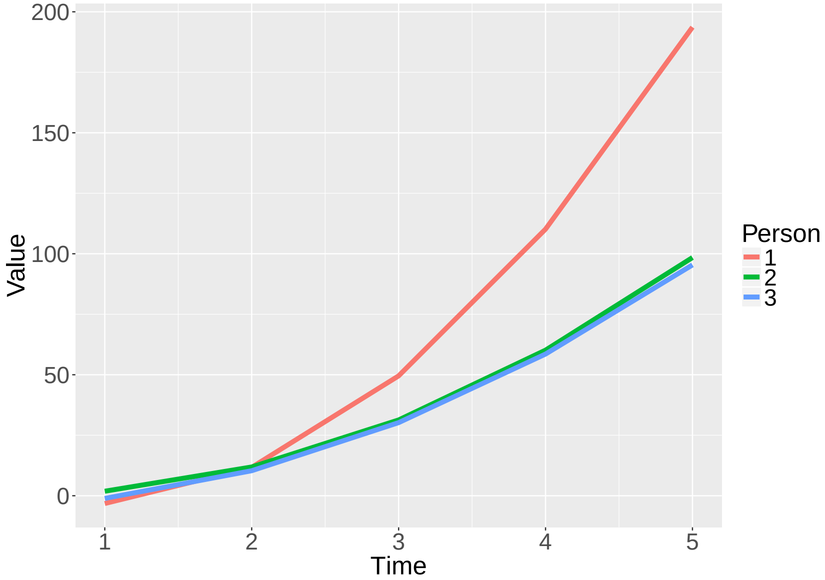
Other Growth Forms

- Real growth is probably not linear
- LGCM allows modeling of nonlinear trajectories
- Polynomials of arbitrary degree are in principal possible

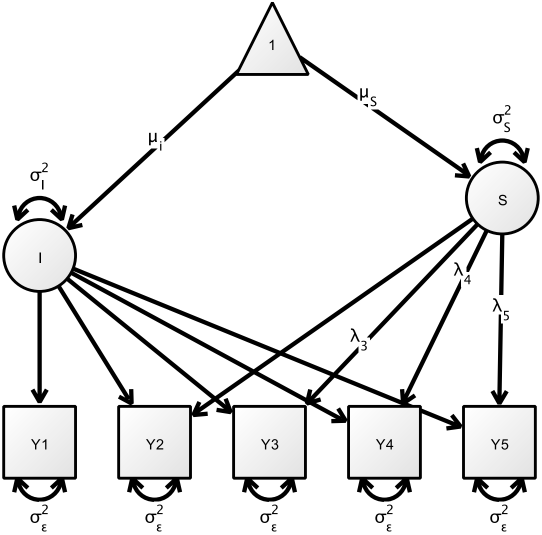
Example: Quadratic + Linear Growth



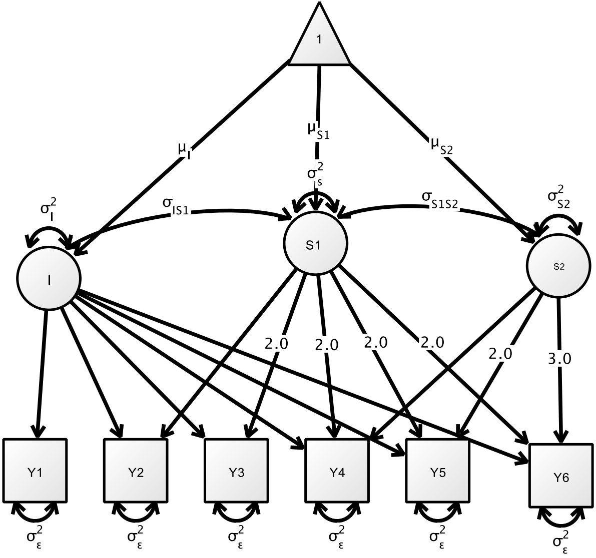
Trajectories: Quadtratic LGCM



Estimating the Form of Change



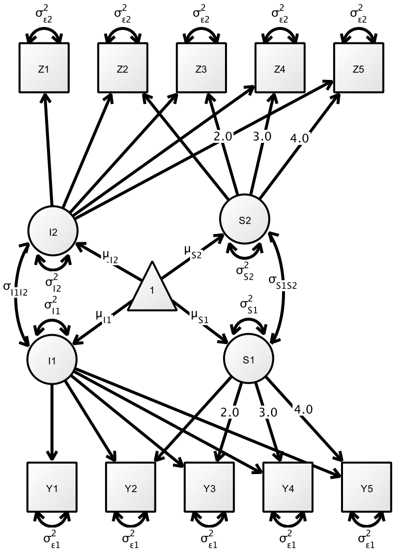
Piecewise Slopes Model



- Demonstrate: Trajectories

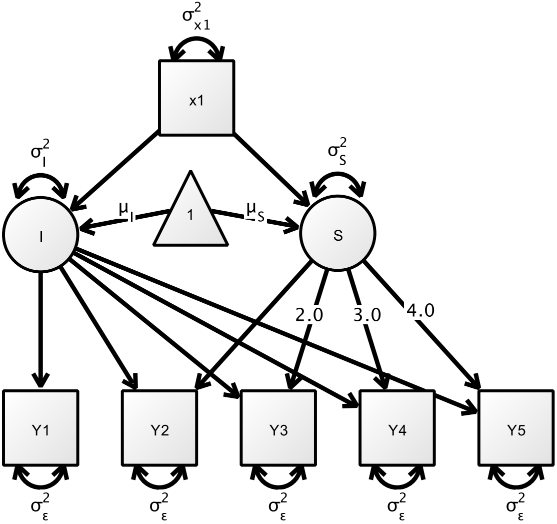
Bivariate Latent Growth Curve Model

- Model both variables using a LGCM
- Allow intercepts and slopes to covary
- Captures interrelationships in behavioral change (3)

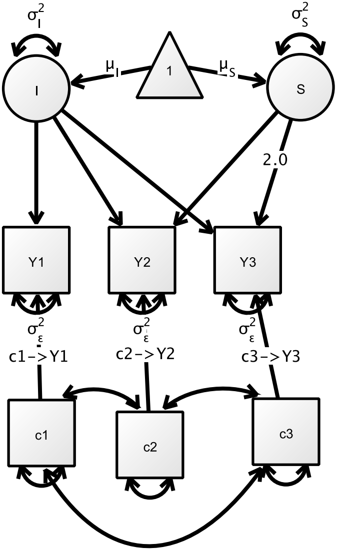


LGCM with time-invariant covariates

- Use time-invariant covariates (for example, parents education) to predict intercept and slope



LGCM with time-variant covariates



Exercise

- Read Example
- Do Exercises
- All at https://github.com/marjoleinF/LVMbasic

Further Readings

Longitudinal Structural Equation Modeling

- Bollen and Curran (2006). *Latent Curve Models: A Structural Equation Perspective.*
- Duncan, Duncan, and Stycker (2006). *Introduction to Latent Variable Growth Curve Modeling: Concepts, Issues, and Applications.*
- Todd D. Little (2006). *Longitudinal Structural Equation Modeling.*
- Jason T. Newsom (2015). *Longitudinal Structural Equation Modeling : A Comprehensive Introduction.*