Answers to exercises multigroup LVMs

```
library("lavaan")
```

Exercise 4.1: Measurement invariance between the WAIS and WAIS-IV

We read in the data:

```
WAIS.cor <- lav_matrix_lower2full(c(
  1.00,
  0.31, 1.00,
  0.36, 0.40, 1.00,
  0.51, 0.46, 0.45, 1.00,
  0.29, 0.40, 0.33, 0.43, 1.00,
  0.39, 0.29, 0.27, 0.36, 0.33, 1.00,
  0.32, 0.27, 0.29, 0.33, 0.24, 0.28, 1.00,
  0.22, 0.32, 0.15, 0.22, 0.27, 0.12, 0.26, 1.00
))
WAIS.means <- c(7.83, 5.50, 5.67, 21.50, 7.67, 8.00, 6.50, 34.83)
WAIS.sds <- c(2.69, 1.50, 2.36, 6.06, 1.85, 2.18, 5.97, 9.94)
WAIS.cov <- cor2cov(WAIS.cor, sds=WAIS.sds)

WAISIV.cor <- lav_matrix_lower2full(c(
  1.00,
  0.46, 1.00,
  0.58, 0.55, 1.00,
  0.63, 0.43, 0.73, 1.00,
  0.27, 0.51, 0.37, 0.33, 1.00,
  0.45, 0.38, 0.37, 0.43, 0.13, 1.00,
  0.33, 0.52, 0.49, 0.41, 0.29, 0.43, 1.00,
  0.15, 0.27, 0.16, 0.09, 0.12, 0.25, 0.23, 1.00
))
WAISIV.means <- c(15.17, 15.00, 11.83, 21.67, 12.17, 17.83, 18.67, 45.83)
WAISIV.sds <- c(4.93, 4.10, 5.20, 6.54, 2.72, 5.35, 9.36, 10.44)
WAISIV.cov <- cor2cov(WAISIV.cor, sds=WAISIV.sds)

WAIS.names <- c("Compr", "Arith", "Simil", "Vocab", "DigSpan", "PictCompl",
                "BlockDes", "Cod")

names(WAIS.means) <- names(WAIS.sds) <- colnames(WAIS.cov) <-
  rownames(WAIS.cov) <- names(WAISIV.means) <- names(WAISIV.sds) <-
  rownames(WAISIV.cov) <- colnames(WAISIV.cov) <- WAIS.names

WAIS.cov.list <- list(WAIS.cov, WAISIV.cov)
WAIS.mean.list <- list(WAIS.means, WAISIV.means)
WAIS.n.list <- list(WAIS.n = 200, WAISIV.n = 200)
```

```
WAIS.mod <- '
  ## verbal comprehension
  VC =~ Simil + Vocab + Compr

  ## Perceptual reasoning
  PR =~ PictCompl + BlockDes

  ## Working memory
  WM =~ Arith + DigSpan + Cod
'
```

a) Considering that both WAIS versions were administered to the same respondents, one would expect their latent verbal comprehension, Perceptual Reasoning and Working Memory scores to be the same between tests. Thus, the latent means and variances can be expected to be equal. Therefore, the standardized LV approach seems the most appropriate for these data.

Note that when equality restrictions on parameters across groups are applied in a model identified using the standardized-LV approach in lavaan, the latent mean and variance will be fixed to 1 and 0, respectively, in the first group. In the second group, the loading and intercept of the first item will be fixed to the respective values in the first group, to identify the model. This allows for differences in the latent mean and variance to be present. For the current example, this may not be completely appropriate, but the current design (testing measurement invariance between two different tests in the same individuals) is quite rare.

b) We fit the configural invariance model to the datasets:

```
fit.indices <- c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr", "aic", "bic")
WAIS.conf.fit <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                     sample.mean = WAIS.mean.list, std.lv = TRUE,
                     sample.nobs = WAIS.n.list, meanstructure = TRUE)
pars <- parameterestimates(WAIS.conf.fit, standardized = TRUE)
pars[order(pars$op, decreasing = TRUE), c(1:3, 5:7, 9, 13)]
```

```
##          lhs op       rhs group    est     se pvalue std.all
## 1         VC =~     Simil     1  1.393  0.170  0.000   0.592
## 2         VC =~     Vocab     1  4.659  0.415  0.000   0.771
## 3         VC =~     Compr     1  1.725  0.190  0.000   0.643
## 4         PR =~ PictCompl     1  1.198  0.191  0.000   0.551
## 5         PR =~  BlockDes     1  3.026  0.509  0.000   0.508
## 6         WM =~     Arith     1  1.018  0.113  0.000   0.681
## 7         WM =~   DigSpan     1  1.147  0.140  0.000   0.621
## 8         WM =~       Cod     1  4.123  0.779  0.000   0.416
## 34        VC =~     Simil     2  4.460  0.313  0.000   0.860
## 35        VC =~     Vocab     2  5.499  0.397  0.000   0.843
## 36        VC =~     Compr     2  3.516  0.318  0.000   0.715
## 37        PR =~ PictCompl     2  3.194  0.398  0.000   0.599
## 38        PR =~  BlockDes     2  6.707  0.713  0.000   0.718
## 39        WM =~     Arith     2  3.761  0.308  0.000   0.920
## 40        WM =~   DigSpan     2  1.495  0.198  0.000   0.551
## 41        WM =~       Cod     2  3.085  0.778  0.000   0.296
## 23     Simil ~1               1  5.670  0.166  0.000   2.409
## 24     Vocab ~1               1 21.500  0.427  0.000   3.557
## 25     Compr ~1               1  7.830  0.190  0.000   2.918
## 26 PictCompl ~1               1  8.000  0.154  0.000   3.679
## 27  BlockDes ~1               1  6.500  0.421  0.000   1.092
## 28     Arith ~1               1  5.500  0.106  0.000   3.676
## 29   DigSpan ~1               1  7.670  0.130  0.000   4.156
```

```
## 30       Cod ~1             1 34.830  0.701  0.000   3.513
## 31        VC ~1             1  0.000  0.000     NA   0.000
## 32        PR ~1             1  0.000  0.000     NA   0.000
## 33        WM ~1             1  0.000  0.000     NA   0.000
## 56     Simil ~1             2 11.830  0.367  0.000   2.281
## 57     Vocab ~1             2 21.670  0.461  0.000   3.322
## 58     Compr ~1             2 15.170  0.348  0.000   3.085
## 59 PictCompl ~1             2 17.830  0.377  0.000   3.341
## 60  BlockDes ~1             2 18.670  0.660  0.000   2.000
## 61     Arith ~1             2 15.000  0.289  0.000   3.668
## 62   DigSpan ~1             2 12.170  0.192  0.000   4.485
## 63       Cod ~1             2 45.830  0.736  0.000   4.401
## 64        VC ~1             2  0.000  0.000     NA   0.000
## 65        PR ~1             2  0.000  0.000     NA   0.000
## 66        WM ~1             2  0.000  0.000     NA   0.000
## 9      Simil ~~     Simil   1  3.602  0.419  0.000   0.650
## 10     Vocab ~~     Vocab   1 14.834  2.454  0.000   0.406
## 11     Compr ~~     Compr   1  4.226  0.518  0.000   0.587
## 12 PictCompl ~~ PictCompl   1  3.293  0.451  0.000   0.696
## 13  BlockDes ~~  BlockDes   1 26.304  3.282  0.000   0.742
## 14     Arith ~~     Arith   1  1.202  0.179  0.000   0.537
## 15   DigSpan ~~   DigSpan   1  2.090  0.272  0.000   0.614
## 16       Cod ~~       Cod   1 81.310  8.768  0.000   0.827
## 17        VC ~~        VC   1  1.000  0.000     NA   1.000
## 18        PR ~~        PR   1  1.000  0.000     NA   1.000
## 19        WM ~~        WM   1  1.000  0.000     NA   1.000
## 20        VC ~~        PR   1  0.907  0.106  0.000   0.907
## 21        VC ~~        WM   1  0.837  0.067  0.000   0.837
## 22        PR ~~        WM   1  0.820  0.118  0.000   0.820
## 42     Simil ~~     Simil   2  7.014  1.237  0.000   0.261
## 43     Vocab ~~     Vocab   2 12.323  1.991  0.000   0.290
## 44     Compr ~~     Compr   2 11.823  1.394  0.000   0.489
## 45 PictCompl ~~ PictCompl   2 18.276  2.247  0.000   0.642
## 46  BlockDes ~~  BlockDes   2 42.184  7.143  0.000   0.484
## 47     Arith ~~     Arith   2  2.580  1.642  0.116   0.154
## 48   DigSpan ~~   DigSpan   2  5.127  0.578  0.000   0.696
## 49       Cod ~~       Cod   2 98.930 10.059  0.000   0.912
## 50        VC ~~        VC   2  1.000  0.000     NA   1.000
## 51        PR ~~        PR   2  1.000  0.000     NA   1.000
## 52        WM ~~        WM   2  1.000  0.000     NA   1.000
## 53        VC ~~        PR   2  0.758  0.066  0.000   0.758
## 54        VC ~~        WM   2  0.647  0.062  0.000   0.647
## 55        PR ~~        WM   2  0.751  0.076  0.000   0.751
```

```
fitMeasures(WAIS.conf.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##    62.187    34.000     0.002     0.968     0.064     0.037 17617.734 17833.273
```

Model fit is good according to SRMR and CFI. RMSEA indicates adequate fit. All standardized loadings are substantial in both versions, in both versions Coding is the weakest indicator. The strongest indicator is Vocabulary in the WAIS and Arithmetics in the WAIS-IV. The three factors correlate strongly in both groups, but more strongly in the WAIS than in the WAIS-IV group.

We inspect standardized residuals:

```r
residuals(WAIS.conf.fit, type = "cor")
```

```
## $`Group 1`
## $`Group 1`$type
## [1] "cor.bollen"
##
## $`Group 1`$cov
##           Simil  Vocab  Compr  PctCmp BlckDs Arith  DigSpn Cod
## Simil      0.000
## Vocab     -0.006  0.000
## Compr     -0.020  0.015  0.000
## PictCompl -0.025 -0.025  0.069  0.000
## BlockDes   0.017 -0.025  0.024  0.000  0.000
## Arith      0.063  0.021 -0.056 -0.017 -0.014  0.000
## DigSpan    0.022  0.029 -0.044  0.049 -0.019 -0.023  0.000
## Cod       -0.056 -0.048 -0.004 -0.068  0.087  0.037  0.012  0.000
##
## $`Group 1`$mean
##     Simil     Vocab     Compr PictCompl  BlockDes     Arith   DigSpan       Cod
##         0         0         0         0         0         0         0         0
##
##
## $`Group 2`
## $`Group 2`$type
## [1] "cor.bollen"
##
## $`Group 2`$cov
##           Simil  Vocab  Compr  PctCmp BlckDs Arith  DigSpn Cod
## Simil      0.000
## Vocab      0.005  0.000
## Compr     -0.035  0.027  0.000
## PictCompl -0.020  0.047  0.125  0.000
## BlockDes   0.021 -0.049 -0.060  0.000  0.000
## Arith      0.038 -0.072  0.035 -0.033  0.024  0.000
## DigSpan    0.064  0.030  0.015 -0.118 -0.007  0.003  0.000
## Cod       -0.005 -0.072  0.013  0.117  0.070 -0.002 -0.043  0.000
##
## $`Group 2`$mean
##     Simil     Vocab     Compr PictCompl  BlockDes     Arith   DigSpan       Cod
##         0         0         0         0         0         0         0         0
```

WAIS-IV has two residual correlations with absolute values larger than .1: the correlations of Picture Completion with Digit Span and Coding.

All in all, configural invariance appears tenable.

b) Assess whether loadings, intercepts and residual variances are equal between the two WAIS versions.

When we use the standardized LV identification method, and apply an equality restriction on the loadings between groups, **lavaan** will keep the latent variance of the first group fixed at one, but will freely estimate the latent variance in the second group.

```r
WAIS.metr.fit <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                     sample.mean = WAIS.mean.list,
                     sample.nobs = WAIS.n.list, std.lv = TRUE,
                     meanstructure = TRUE,
```

```r
                      group.equal = "loadings")
pars <- parameterestimates(WAIS.metr.fit, standardized = TRUE)
pars[pars$op == "=~" | pars$rhs %in% c("VC", "PR", "WM"), c(1:3, 5, 7, 14)]
```

```
##    lhs op       rhs group   est std.all
## 1   VC =~     Simil     1 1.831   0.715
## 2   VC =~     Vocab     1 2.742   0.502
## 3   VC =~     Compr     1 1.651   0.618
## 4   PR =~ PictCompl     1 1.262   0.574
## 5   PR =~  BlockDes     1 2.813   0.478
## 6   WM =~     Arith     1 1.295   0.842
## 7   WM =~   DigSpan     1 0.685   0.390
## 8   WM =~       Cod     1 1.617   0.168
## 17  VC ~~        VC     1 1.000   1.000
## 18  PR ~~        PR     1 1.000   1.000
## 19  WM ~~        WM     1 1.000   1.000
## 20  VC ~~        PR     1 0.930   0.930
## 21  VC ~~        WM     1 0.750   0.750
## 22  PR ~~        WM     1 0.683   0.683
## 34  VC =~     Simil     2 1.831   0.806
## 35  VC =~     Vocab     2 2.742   0.878
## 36  VC =~     Compr     2 1.651   0.724
## 37  PR =~ PictCompl     2 1.262   0.582
## 38  PR =~  BlockDes     2 2.813   0.728
## 39  WM =~     Arith     2 1.295   0.834
## 40  WM =~   DigSpan     2 0.685   0.623
## 41  WM =~       Cod     2 1.617   0.387
## 50  VC ~~        VC     2 4.688   1.000
## 51  PR ~~        PR     2 5.934   1.000
## 52  WM ~~        WM     2 6.567   1.000
## 53  VC ~~        PR     2 3.940   0.747
## 54  VC ~~        WM     2 3.713   0.669
## 55  PR ~~        WM     2 4.926   0.789
```

Note that now the latent variance are freely estimated for the WAIS-IV, and are much larger than for the WAIS (where they are fixed to 1).

This may indicate that the measurement precision of the WAIS-IV is much higher than that of the WAIS. It cannot be due to latent differences between the two 'groups', because the two tests were administered to the same individuals.

```r
fitMeasures(WAIS.metr.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##   132.004    39.000     0.000     0.896     0.109     0.093 17677.551 17873.133
```

```r
lavTestLRT(WAIS.conf.fit, WAIS.metr.fit)
```

```
## Chi-Squared Difference Test
##
##               Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## WAIS.conf.fit 34 17618 17833  62.187
## WAIS.metr.fit 39 17678 17873 132.004     69.817       5  1.119e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All fit indices indicate a lack of metric invariance.

```r
lavTestScore(WAIS.metr.fit)$uni
```

```
##
## univariate score tests:
##
##    lhs op   rhs     X2 df p.value
## 1 .p1. == .p34. 22.090  1   0.000
## 2 .p2. == .p35. 27.669  1   0.000
## 3 .p3. == .p36.  0.052  1   0.819
## 4 .p4. == .p37.  0.680  1   0.410
## 5 .p5. == .p38.  0.680  1   0.410
## 6 .p6. == .p39. 27.254  1   0.000
## 7 .p7. == .p40. 12.023  1   0.001
## 8 .p8. == .p41. 11.753  1   0.001
```

Five out of eight loadings seem non-invariant. Which parameters are the most invariance-offending?

```r
pars <- parameterestimates(WAIS.metr.fit)
pars[pars$label %in% c(".p1.", ".p2.", ".p6.", ".p7.", ".p8."), c(1:3, 5:6)]
```

```
##    lhs op     rhs group label
## 1   VC =~   Simil     1  .p1.
## 2   VC =~   Vocab     1  .p2.
## 6   WM =~   Arith     1  .p6.
## 7   WM =~ DigSpan     1  .p7.
## 8   WM =~     Cod     1  .p8.
## 34  VC =~   Simil     2  .p1.
## 35  VC =~   Vocab     2  .p2.
## 39  WM =~   Arith     2  .p6.
## 40  WM =~ DigSpan     2  .p7.
## 41  WM =~     Cod     2  .p8.
```

Vocabulary and Arithmetics seem the worst-offending loadings; let's lift those equality restrictions:

```r
WAIS.metr.fit2 <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                      sample.mean = WAIS.mean.list,
                      sample.nobs = WAIS.n.list, std.lv = TRUE,
                      auto.fix.first = TRUE,
                      meanstructure = TRUE,
                      group.equal = "loadings",
                      group.partial = c("VC =~ Vocab", "WM =~ Arith"))
fitMeasures(WAIS.metr.fit2, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##    71.647    37.000     0.001     0.961     0.068     0.050 17621.194 17824.759
```

```r
lavTestLRT(WAIS.conf.fit, WAIS.metr.fit2)
```

```
## Chi-Squared Difference Test
##
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WAIS.conf.fit  34 17618 17833 62.187
## WAIS.metr.fit2 37 17621 17825 71.647     9.4601       3    0.02376 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
lavTestScore(WAIS.metr.fit2)$uni
```

```
##
## univariate score tests:
##
##    lhs op   rhs    X2 df p.value
## 1 .p1. == .p34. 6.010  1   0.014
## 2 .p3. == .p36. 6.010  1   0.014
## 3 .p4. == .p37. 0.605  1   0.437
## 4 .p5. == .p38. 0.605  1   0.437
## 5 .p7. == .p40. 2.423  1   0.120
## 6 .p8. == .p41. 2.423  1   0.120
```

Model fit improved, but not enough.

It may seem odd that we have three identical values for the six remaining modification indices. We lifted two equality restrictions, one for the VC and one for the WM factor. For each latent factor, there are now two equality restrictions left. Releasing any of the two will yield an identical improvement of model fit.

Next, we lift the equality restriction on the loading of Similarity:

```r
WAIS.metr.fit3 <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                      sample.mean = WAIS.mean.list,
                      sample.nobs = WAIS.n.list, std.lv = TRUE,
                      meanstructure = TRUE,
                      group.equal = "loadings",
                      group.partial = c("VC =~ Vocab", "WM =~ Arith",
                                        "VC =~ Simil"))
fitMeasures(WAIS.metr.fit3, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##    65.316    36.000     0.002     0.967     0.064     0.042 17616.863 17824.419
```

```r
lavTestLRT(WAIS.conf.fit, WAIS.metr.fit3)
```

```
## Chi-Squared Difference Test
##
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WAIS.conf.fit  34 17618 17833 62.187
## WAIS.metr.fit3 36 17617 17824 65.316     3.1289       2     0.2092
```

Fit has improved. Next, we restrict item intercepts to equality:

```r
WAIS.scal.fit <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                     sample.mean = WAIS.mean.list, std.lv = TRUE,
                     sample.nobs = WAIS.n.list, meanstructure = TRUE,
                     group.equal = c("intercepts", "loadings"),
                     group.partial = c("VC =~ Vocab", "WM =~ Arith",
                                       "VC =~ Simil"))
fitMeasures(WAIS.scal.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##   222.241    41.000     0.000     0.797     0.149     0.117 17763.788 17951.387
```

```r
lavTestLRT(WAIS.metr.fit3, WAIS.scal.fit)
```

```
## Chi-Squared Difference Test
##
##                Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
```

```
## WAIS.metr.fit3 36 17617 17824  65.316
## WAIS.scal.fit  41 17764 17951 222.241     156.93       5  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
lavTestScore(WAIS.scal.fit)$uni
```

```
##
## univariate score tests:
##
##      lhs op   rhs     X2 df p.value
## 1   .p3. == .p36.  0.000  1   1.000
## 2   .p4. == .p37.  8.224  1   0.004
## 3   .p5. == .p38.  8.224  1   0.004
## 4   .p7. == .p40.  2.153  1   0.142
## 5   .p8. == .p41.  2.153  1   0.142
## 6  .p23. == .p56.  2.052  1   0.152
## 7  .p24. == .p57. 93.438  1   0.000
## 8  .p25. == .p58. 32.113  1   0.000
## 9  .p26. == .p59. 16.203  1   0.000
## 10 .p27. == .p60. 16.203  1   0.000
## 11 .p28. == .p61.  3.357  1   0.067
## 12 .p29. == .p62.  2.683  1   0.101
## 13 .p30. == .p63.  0.037  1   0.847
```

```r
pars <- parameterestimates(WAIS.scal.fit)
pars[pars$label %in% c(".p24.", ".p25.", ".p26.", ".p27."), c(1:3, 5:6)]
```

```
##          lhs op rhs group label
## 24     Vocab ~1         1 .p24.
## 25     Compr ~1         1 .p25.
## 26 PictCompl ~1         1 .p26.
## 27  BlockDes ~1         1 .p27.
## 57     Vocab ~1         2 .p24.
## 58     Compr ~1         2 .p25.
## 59 PictCompl ~1         2 .p26.
## 60  BlockDes ~1         2 .p27.
```

Vocabulary seems to have the worst variance-offending item intercept, we release those restirctions:

```r
WAIS.scal.fit2 <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                      sample.mean = WAIS.mean.list, std.lv = TRUE,
                      sample.nobs = WAIS.n.list, meanstructure = TRUE,
                      group.equal = c("loadings", "intercepts"),
                      group.partial = c("VC =~ Vocab", "WM =~ Arith",
                                        "VC =~ Simil", "Vocab ~1"))
fitMeasures(WAIS.scal.fit2, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##    99.437    40.000     0.000     0.933     0.086     0.070 17642.984 17834.574
```

```r
lavTestLRT(WAIS.scal.fit2, WAIS.metr.fit3)
```

```
## Chi-Squared Difference Test
##
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WAIS.metr.fit3 36 17617 17824 65.316
## WAIS.scal.fit2 40 17643 17835 99.436     34.121       4  7.039e-07 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We should lift additional restrictions:

```r
lavTestScore(WAIS.scal.fit2)$uni
```

```
##
## univariate score tests:
##
##      lhs op   rhs     X2 df p.value
## 1    .p3. == .p36.  0.000  1   1.000
## 2    .p4. == .p37.  7.398  1   0.007
## 3    .p5. == .p38.  7.398  1   0.007
## 4    .p7. == .p40.  2.221  1   0.136
## 5    .p8. == .p41.  2.221  1   0.136
## 6   .p23. == .p56. 12.844  1   0.000
## 7   .p25. == .p58. 12.844  1   0.000
## 8   .p26. == .p59. 15.451  1   0.000
## 9   .p27. == .p60. 15.451  1   0.000
## 10 .p28. == .p61.  2.279  1   0.131
## 11 .p29. == .p62.  1.904  1   0.168
## 12 .p30. == .p63.  0.010  1   0.919
```

```r
WAIS.scal.fit3 <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                      sample.mean = WAIS.mean.list, std.lv = TRUE,
                      sample.nobs = WAIS.n.list, meanstructure = TRUE,
                      group.equal =c("loadings", "intercepts"),
                      group.partial =  c("VC =~ Vocab", "WM =~ Arith",
                                         "VC =~ Simil", "Vocab ~1",
                                         "Compr ~ 1"))
fitMeasures(WAIS.scal.fit3, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##    85.705    39.000     0.000     0.948     0.077     0.064 17631.252 17826.834
```

```r
lavTestLRT(WAIS.scal.fit3, WAIS.metr.fit3)
```

```
## Chi-Squared Difference Test
##
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WAIS.metr.fit3 36 17617 17824 65.316
## WAIS.scal.fit3 39 17631 17827 85.705     20.389       3   0.000141 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
lavTestScore(WAIS.scal.fit3)$uni
```

```
##
## univariate score tests:
##
##      lhs op   rhs     X2 df p.value
## 1    .p3. == .p36.  0.000  1   1.000
## 2    .p4. == .p37.  7.359  1   0.007
## 3    .p5. == .p38.  7.359  1   0.007
## 4    .p7. == .p40.  2.228  1   0.136
## 5    .p8. == .p41.  2.228  1   0.136
## 6   .p23. == .p56.  0.000  1   1.000
```

```
## 7  .p26. == .p59. 16.705  1   0.000
## 8  .p27. == .p60. 16.705  1   0.000
## 9  .p28. == .p61.  2.243  1   0.134
## 10 .p29. == .p62.  1.875  1   0.171
## 11 .p30. == .p63.  0.011  1   0.915
```

```
WAIS.scal.fit4 <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                      sample.mean = WAIS.mean.list, std.lv = TRUE,
                      sample.nobs = WAIS.n.list, meanstructure = TRUE,
                      group.equal = c("loadings", "intercepts"),
                      group.partial =  c("VC =~ Vocab", "WM =~ Arith",
                                         "VC =~ Simil", "Vocab ~1",
                                         "Compr ~ 1", "PictCompl ~ 1"))
fitMeasures(WAIS.scal.fit4, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic       bic
##    67.775    38.000     0.002     0.967     0.063     0.046 17615.322 17814.895
```

```
lavTestLRT(WAIS.scal.fit4, WAIS.metr.fit3)
```

```
## Chi-Squared Difference Test
##
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WAIS.metr.fit3 36 17617 17824 65.316
## WAIS.scal.fit4 38 17615 17815 67.775     2.4588       2     0.2925
```

Fit is acceptable. We continue with testing equality of measurement error variances:

```
WAIS.uni.fit <- cfa(WAIS.mod, sample.cov = WAIS.cov.list,
                    sample.mean = WAIS.mean.list, std.lv = TRUE,
                    sample.nobs = WAIS.n.list, meanstructure = TRUE,
                    group.equal = c("loadings", "intercepts", "residuals"),
                    group.partial = c("VC =~ Vocab", "WM =~ Arith",
                                       "VC =~ Simil", "Vocab ~1",
                                       "Compr ~ 1", "PictCompl ~ 1"))
```

```
## Warning in lav_model_estimate(lavmodel = lavmodel, lavpartable = lavpartable, :
## lavaan WARNING: the optimizer warns that a solution has NOT been found!
```

We get a warning about negative LV variances, which indicates that restricting all measurement error variances to equality is not a good idea.

Considering the large number of non-invariant parameters, measurement invariance does not seem tenable. I would rather go back and inspect the parameter estimates of the configural invariant model:

```
pars <- parameterestimates(WAIS.conf.fit, standardized = TRUE)
col_names <- c("lhs", "op", "rhs", "group", "est", "se", "pvalue", "std.all")
pars[pars$op == "=~", col_names]
```

```
##    lhs op       rhs group   est    se pvalue std.all
## 1   VC =~     Simil     1 1.393 0.170      0   0.592
## 2   VC =~     Vocab     1 4.659 0.415      0   0.771
## 3   VC =~     Compr     1 1.725 0.190      0   0.643
## 4   PR =~ PictCompl     1 1.198 0.191      0   0.551
## 5   PR =~  BlockDes     1 3.026 0.509      0   0.508
## 6   WM =~     Arith     1 1.018 0.113      0   0.681
## 7   WM =~   DigSpan     1 1.147 0.140      0   0.621
## 8   WM =~       Cod     1 4.123 0.779      0   0.416
```

```
## 34   VC =~      Simil      2 4.460 0.313      0    0.860
## 35   VC =~      Vocab      2 5.499 0.397      0    0.843
## 36   VC =~      Compr      2 3.516 0.318      0    0.715
## 37   PR =~ PictCompl      2 3.194 0.398      0    0.599
## 38   PR =~  BlockDes      2 6.707 0.713      0    0.718
## 39   WM =~      Arith      2 3.761 0.308      0    0.920
## 40   WM =~    DigSpan     2 1.495 0.198      0    0.551
## 41   WM =~        Cod     2 3.085 0.778      0    0.296
```

Standardized loadings are higher for the VC and PR indicators in the WAIS-IV, compared to the WAIS. For the WM indicators, we see that in the WAIS, the indicators have similar standardized loadings, while in the WAIS-IV, Arithmetics has a much higher (standardized) loading than the other two subtests.

```
pars[pars$op == "~~" & !pars$lhs %in% c("VC", "WM", "PR"), col_names]
```

```
##           lhs op       rhs group    est     se pvalue std.all
## 9       Simil ~~     Simil     1  3.602  0.419  0.000   0.650
## 10      Vocab ~~     Vocab     1 14.834  2.454  0.000   0.406
## 11      Compr ~~     Compr     1  4.226  0.518  0.000   0.587
## 12 PictCompl ~~ PictCompl     1  3.293  0.451  0.000   0.696
## 13  BlockDes ~~  BlockDes     1 26.304  3.282  0.000   0.742
## 14      Arith ~~     Arith     1  1.202  0.179  0.000   0.537
## 15    DigSpan ~~   DigSpan     1  2.090  0.272  0.000   0.614
## 16        Cod ~~       Cod     1 81.310  8.768  0.000   0.827
## 42      Simil ~~     Simil     2  7.014  1.237  0.000   0.261
## 43      Vocab ~~     Vocab     2 12.323  1.991  0.000   0.290
## 44      Compr ~~     Compr     2 11.823  1.394  0.000   0.489
## 45 PictCompl ~~ PictCompl     2 18.276  2.247  0.000   0.642
## 46  BlockDes ~~  BlockDes     2 42.184  7.143  0.000   0.484
## 47      Arith ~~     Arith     2  2.580  1.642  0.116   0.154
## 48    DigSpan ~~   DigSpan     2  5.127  0.578  0.000   0.696
## 49        Cod ~~       Cod     2 98.930 10.059  0.000   0.912
```

We observe the reverse pattern for the residual variances: Standardized residual variances are higher for the VC and PR indicators in the WAIS, compared to the WAIS-IV. For the WM indicators, we see that in the WAIS, the indicators have more similar standardized residual variances, while in the WAIS-IV, Arithmetics has a much lower (standardized) loading than the other two subtests.

```
pars[pars$op == "~1" & !pars$lhs %in% c("VC", "WM", "PR"), col_names]
```

```
##           lhs op rhs group   est    se pvalue std.all
## 23      Simil ~1         1  5.67 0.166      0   2.409
## 24      Vocab ~1         1 21.50 0.427      0   3.557
## 25      Compr ~1         1  7.83 0.190      0   2.918
## 26 PictCompl ~1          1  8.00 0.154      0   3.679
## 27   BlockDes ~1         1  6.50 0.421      0   1.092
## 28      Arith ~1         1  5.50 0.106      0   3.676
## 29    DigSpan ~1         1  7.67 0.130      0   4.156
## 30        Cod ~1         1 34.83 0.701      0   3.513
## 56      Simil ~1         2 11.83 0.367      0   2.281
## 57      Vocab ~1         2 21.67 0.461      0   3.322
## 58      Compr ~1         2 15.17 0.348      0   3.085
## 59 PictCompl ~1          2 17.83 0.377      0   3.341
## 60   BlockDes ~1         2 18.67 0.660      0   2.000
## 61      Arith ~1         2 15.00 0.289      0   3.668
## 62    DigSpan ~1         2 12.17 0.192      0   4.485
```

```
## 63       Cod ~1          2 45.83 0.736      0   4.401
```

The intercepts seem lower for the WAIS than for the WAIS-IV, indicating that the the subtests of the WAIS-IV may be easier. Perhaps the tests have different numbers of items and different scoring rules. But the intercept differences should be interpreted relative to the indicator item.

c)

```r
pars[pars$op == "~~" & pars$lhs %in% c("VC", "WM", "PR"), col_names]
```

```
##    lhs op rhs group   est    se pvalue std.all
## 17  VC ~~  VC     1 1.000 0.000     NA   1.000
## 18  PR ~~  PR     1 1.000 0.000     NA   1.000
## 19  WM ~~  WM     1 1.000 0.000     NA   1.000
## 20  VC ~~  PR     1 0.907 0.106      0   0.907
## 21  VC ~~  WM     1 0.837 0.067      0   0.837
## 22  PR ~~  WM     1 0.820 0.118      0   0.820
## 50  VC ~~  VC     2 1.000 0.000     NA   1.000
## 51  PR ~~  PR     2 1.000 0.000     NA   1.000
## 52  WM ~~  WM     2 1.000 0.000     NA   1.000
## 53  VC ~~  PR     2 0.758 0.066      0   0.758
## 54  VC ~~  WM     2 0.647 0.062      0   0.647
## 55  PR ~~  WM     2 0.751 0.076      0   0.751
```

We see stronger correlations between the latent factors in the WAIS, than in the WAIS-IV.

```r
pars[pars$op == "~1" & pars$lhs %in% c("VC", "WM", "PR"), col_names]
```

```
##    lhs op rhs group est se pvalue std.all
## 31  VC ~1           1   0  0     NA       0
## 32  PR ~1           1   0  0     NA       0
## 33  WM ~1           1   0  0     NA       0
## 64  VC ~1           2   0  0     NA       0
## 65  PR ~1           2   0  0     NA       0
## 66  WM ~1           2   0  0     NA       0
```

All in all, there does not seem to be measurement invariance between the WISC and the WISC-IV. Thus, observed subtest scores are not comparable between the two test versions. However, the results do indicate that the WAIS-IV is an improvement over the WAIS: It has better measurement precision, and the latent factors seem more separate traits than they were in the WAIS.