Exercises multigroup LVMs

```
library("lavaan")
```

```
## Warning: package 'lavaan' was built under R version 3.4.4
```

```
## This is lavaan 0.6-1
```

```
## lavaan is BETA software! Please report any bugs.
```

Exercise 4.1: Measurement invariance between the WISC and WISC-IV

We read in the data:

```
WISC.names <- c("Compr", "Arith", "Simil", "Vocab", "DigSpan", "PictCompl", "BlockDes", "Cod")

WISC.cor <- lav_matrix_lower2full(c(
  1.00,
  0.31, 1.00,
  0.36, 0.40, 1.00,
  0.51, 0.46, 0.45, 1.00,
  0.29, 0.40, 0.33, 0.43, 1.00,
  0.39, 0.29, 0.27, 0.36, 0.33, 1.00,
  0.32, 0.27, 0.29, 0.33, 0.24, 0.28, 1.00,
  0.22, 0.32, 0.15, 0.22, 0.27, 0.12, 0.26, 1.00
))
WISC.means <- c(7.83, 5.50, 5.67, 21.50, 7.67, 8.00, 6.50, 34.83)
WISC.sds <- c(2.69, 1.50, 2.36, 6.06, 1.85, 2.18, 5.97, 9.94)
WISC.cov <- cor2cov(WISC.cor, sds=WISC.sds)

WISCIV.cor <- lav_matrix_lower2full(c(
  1.00,
  0.46, 1.00,
  0.58, 0.55, 1.00,
  0.63, 0.43, 0.73, 1.00,
  0.27, 0.51, 0.37, 0.33, 1.00,
  0.45, 0.38, 0.37, 0.43, 0.13, 1.00,
  0.33, 0.52, 0.49, 0.41, 0.29, 0.43, 1.00,
  0.15, 0.27, 0.16, 0.09, 0.12, 0.25, 0.23, 1.00
))
WISCIV.means <- c(15.17, 15.00, 11.83, 21.67, 12.17, 17.83, 18.67, 45.83)
WISCIV.sds <- c(4.93, 4.10, 5.20, 6.54, 2.72, 5.35, 9.36, 10.44)
WISCIV.cov <- cor2cov(WISCIV.cor, sds=WISCIV.sds)

names(WISC.means) <- names(WISCIV.sds) <- names(WISC.sds) <-
  names(WISCIV.sds) <- rownames(WISC.cov) <- colnames(WISC.cov) <-
  rownames(WISCIV.cov) <- colnames(WISCIV.cov) <- WISC.names

WISC.cov.list <- list(WISC.cov, WISCIV.cov)
WISC.mean.list <- list(WISC.means, WISCIV.means)
WISC.n.list <- list(WISC.n = 200, WISCIV.n = 200)
```

a) Fit a two-domensional model, with Verbal Comprehension (Similarities, Vocabulary and Comprehension), Working Memory (Artihmetic, Digit Span and Coding) and Perceptual Reasoning (Picture Completion and Block Design) to both covariance matrices. Perform a multigroup analyses, using edition as the grouping variable.

We fit the configural invariance model to the datasets:

```
WISC.mod <- '
  ## verbal comprehension
  VC =~ Simil + Vocab + Compr

  ## Perceptual reasoning
  PR =~ PictCompl + BlockDes

  ## Working memory
  WM =~ Arith + DigSpan + Cod
'
fit.indices <- c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr", "aic")
WISC.conf.fit <- cfa(WISC.mod, sample.cov = WISC.cov.list,
                     sample.mean = WISC.mean.list,
                     sample.nobs = WISC.n.list, meanstructure = TRUE)
summary(WISC.conf.fit, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after 180 iterations
##
##   Number of observations per group
##   Group 1                                          200
##   Group 2                                          200
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      62.187
##   Degrees of freedom                                34
##   P-value (Chi-square)                           0.002
##
## Chi-square for each group:
##
##   Group 1                                       18.998
##   Group 2                                       43.189
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
##
## Group 1 [Group 1]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   VC =~
##     Simil             1.000                               1.393    0.592
##     Vocab             3.346    0.457    7.328    0.000    4.659    0.771
##     Compr             1.238    0.185    6.693    0.000    1.725    0.643
##   PR =~
##     PictCompl         1.000                               1.198    0.551
##     BlockDes          2.526    0.498    5.077    0.000    3.026    0.508
```

```
##   WM =~
##     Arith             1.000                               1.018    0.681
##     DigSpan           1.126    0.171    6.568    0.000    1.147    0.621
##     Cod               4.049    0.847    4.780    0.000    4.123    0.416
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   VC ~~
##     PR                1.513    0.310    4.876    0.000    0.907    0.907
##     WM                1.187    0.224    5.287    0.000    0.837    0.837
##   PR ~~
##     WM                1.000    0.211    4.751    0.000    0.820    0.820
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Simil             5.670    0.166   34.062    0.000    5.670    2.409
##    .Vocab            21.500    0.427   50.300    0.000   21.500    3.557
##    .Compr             7.830    0.190   41.268    0.000    7.830    2.918
##    .PictCompl         8.000    0.154   52.028    0.000    8.000    3.679
##    .BlockDes          6.500    0.421   15.436    0.000    6.500    1.092
##    .Arith             5.500    0.106   51.985    0.000    5.500    3.676
##    .DigSpan           7.670    0.130   58.780    0.000    7.670    4.156
##    .Cod              34.830    0.701   49.679    0.000   34.830    3.513
##     VC                0.000                               0.000    0.000
##     PR                0.000                               0.000    0.000
##     WM                0.000                               0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Simil             3.602    0.419    8.603    0.000    3.602    0.650
##    .Vocab            14.834    2.454    6.044    0.000   14.834    0.406
##    .Compr             4.226    0.518    8.151    0.000    4.226    0.587
##    .PictCompl         3.293    0.451    7.308    0.000    3.293    0.696
##    .BlockDes         26.304    3.282    8.015    0.000   26.304    0.742
##    .Arith             1.202    0.179    6.722    0.000    1.202    0.537
##    .DigSpan           2.090    0.272    7.691    0.000    2.090    0.614
##    .Cod              81.310    8.768    9.273    0.000   81.310    0.827
##     VC                1.939    0.472    4.108    0.000    1.000    1.000
##     PR                1.436    0.458    3.134    0.002    1.000    1.000
##     WM                1.037    0.231    4.496    0.000    1.000    1.000
##
##
## Group 2 [Group 2]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   VC =~
##     Simil             1.000                               4.460    0.860
##     Vocab             1.233    0.094   13.137    0.000    5.499    0.843
##     Compr             0.788    0.072   10.915    0.000    3.516    0.715
##   PR =~
##     PictCompl         1.000                               3.194    0.599
##     BlockDes          2.100    0.309    6.794    0.000    6.707    0.718
##   WM =~
```

```
##     Arith             1.000                               3.761    0.920
##     DigSpan           0.397    0.062    6.447    0.000    1.495    0.551
##     Cod               0.820    0.216    3.790    0.000    3.085    0.296
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   VC ~~
##     PR               10.806    1.934    5.586    0.000    0.758    0.758
##     WM               10.853    1.637    6.630    0.000    0.647    0.647
##   PR ~~
##     WM                9.022    1.594    5.661    0.000    0.751    0.751
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Simil            11.830    0.367   32.254    0.000   11.830    2.281
##    .Vocab            21.670    0.461   46.977    0.000   21.670    3.322
##    .Compr            15.170    0.348   43.626    0.000   15.170    3.085
##    .PictCompl        17.830    0.377   47.250    0.000   17.830    3.341
##    .BlockDes         18.670    0.660   28.280    0.000   18.670    2.000
##    .Arith            15.000    0.289   51.869    0.000   15.000    3.668
##    .DigSpan          12.170    0.192   63.434    0.000   12.170    4.485
##    .Cod              45.830    0.736   62.238    0.000   45.830    4.401
##     VC                0.000                               0.000    0.000
##     PR                0.000                               0.000    0.000
##     WM                0.000                               0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Simil             7.014    1.237    5.670    0.000    7.014    0.261
##    .Vocab            12.323    1.991    6.190    0.000   12.323    0.290
##    .Compr            11.823    1.394    8.481    0.000   11.823    0.489
##    .PictCompl        18.276    2.247    8.133    0.000   18.276    0.642
##    .BlockDes         42.184    7.143    5.906    0.000   42.184    0.484
##    .Arith             2.580    1.642    1.571    0.116    2.580    0.154
##    .DigSpan           5.127    0.578    8.869    0.000    5.127    0.696
##    .Cod              98.930   10.059    9.835    0.000   98.930    0.912
##     VC               19.891    2.790    7.129    0.000    1.000    1.000
##     PR               10.204    2.546    4.008    0.000    1.000    1.000
##     WM               14.146    2.316    6.109    0.000    1.000    1.000
```

```
fitMeasures(WISC.conf.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic
##    62.187    34.000     0.002     0.968     0.064     0.037 17617.734
```

Note that the tests were made by the sampe individuals, so assuming that the latent means are similar between the two groups (e.g., identifying the scale by setting the latent means to 0) seems like a reasonable assumption.

b) Assess whether configural invariance between the WISC and WISC-IV is tenable.

The model fits well according to the CFI and SRMR, but not according to the χ^2 and RMSEA, though the latter does indicate acceptable fit. The model misfit seems stronger for the WISC-IV (higher χ^2 for group 2). All standardized loadings are substantial in both groups, although the loading for Coding is relatively low in the WISC-IV group, but it is still substantial and significant. The three factors correlate substantially in both groups, but more strongly in the WISC than in the WISC-IV group.

As this is just an exercise, we do not look further for potential sources of misfit, but conclude the configural invariance model fits the data. We continue our analysis by restricting loadings to be equal across the four groups:

c) Assess whether loadings, intercepts and residual variances are equal between the two WISC versions.

```
WISC.metr.fit <- cfa(WISC.mod, sample.cov = WISC.cov.list,
                     sample.mean = WISC.mean.list,
                     sample.nobs = WISC.n.list,
                     meanstructure = TRUE, group.equal = "loadings")
fitMeasures(WISC.metr.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic
##   132.004    39.000     0.000     0.896     0.109     0.093 17677.551
```

```
lavTestLRT(WISC.conf.fit, WISC.metr.fit)
```

```
## Chi Square Difference Test
##
##               Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## WISC.conf.fit 34 17618 17833  62.187
## WISC.metr.fit 39 17678 17873 132.004     69.817       5  1.119e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We did not obtain full metric invariance, according to $\Delta\chi^2$ and ΔCFI. BIC, AIC and RMSEA also indicate loadings are not equal between WISC and WISC-IV.

```
lavTestScore(WISC.metr.fit)
```

```
## $test
##
## total score test:
##
##    test     X2 df p.value
## 1 score 65.047  5       0
##
## $uni
##
## univariate score tests:
##
##    lhs op   rhs     X2 df p.value
## 1 .p2. == .p35. 27.669  1   0.000
## 2 .p3. == .p36.  0.052  1   0.819
## 3 .p5. == .p38.  0.680  1   0.410
## 4 .p7. == .p40. 12.023  1   0.001
## 5 .p8. == .p41. 11.753  1   0.001
```

We see that three out of five restrictions are not tenable (i.e., yield a significant test statistic). Which parameters are those?

```
pars <- parameterestimates(WISC.metr.fit)
pars[pars$label %in% c(".p2.", ".p7.", ".p8."),]
```

```
##    lhs op     rhs block group label   est    se      z pvalue ci.lower
## 2   VC =~   Vocab     1     1  .p2. 1.498 0.107 13.969      0    1.288
## 7   WM =~ DigSpan     1     1  .p7. 0.529 0.063  8.351      0    0.405
## 8   WM =~     Cod     1     1  .p8. 1.248 0.239  5.229      0    0.780
## 35  VC =~   Vocab     2     2  .p2. 1.498 0.107 13.969      0    1.288
```

```
## 40   WM =~ DigSpan     2     2  .p7. 0.529 0.063  8.351      0    0.405
## 41   WM =~     Cod     2     2  .p8. 1.248 0.239  5.229      0    0.780
##    ci.upper
## 2     1.708
## 7     0.653
## 8     1.716
## 35    1.708
## 40    0.653
## 41    1.716
```

Vocabulary, Digit Span and Coding do not seem to have equal loadings in both subtests. We will lift those equality restricitons:

```r
WISC.metr.fit2 <- cfa(WISC.mod, sample.cov = WISC.cov.list,
                     sample.mean = WISC.mean.list,
                     sample.nobs = WISC.n.list,
                     meanstructure = TRUE, group.equal = "loadings",
                     group.partial = c("VC =~ Vocab", "WM =~ DigSpan", "WM =~ Cod"))
fitMeasures(WISC.metr.fit2, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic
##    69.187    36.000     0.001     0.963     0.068     0.047 17620.734
```

```r
lavTestLRT(WISC.conf.fit, WISC.metr.fit2)
```

```
## Chi Square Difference Test
##
##                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WISC.conf.fit  34 17618 17833 62.187
## WISC.metr.fit2 36 17621 17828 69.187     7.0002       2    0.03019 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Even after lifting three out of five restictions, the difference in model fit is significance, so equality of loadings does not seem tenable at all. Let's check whether the intercepts are equal:

```r
WISC.scal.fit <- cfa(WISC.mod, sample.cov = WISC.cov.list,
                     sample.mean = WISC.mean.list,
                     sample.nobs = WISC.n.list, meanstructure = TRUE,
                     group.equal = "intercepts")
fitMeasures(WISC.scal.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic
##   211.365    39.000     0.000     0.807     0.149     0.109 17756.913
```

```r
lavTestLRT(WISC.conf.fit, WISC.scal.fit)
```

```
## Chi Square Difference Test
##
##               Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## WISC.conf.fit 34 17618 17833  62.187
## WISC.scal.fit 39 17757 17953 211.365     149.18       5  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
lavTestScore(WISC.scal.fit)
```

```
## $test
##
```

```
## total score test:
##
##    test      X2 df p.value
## 1 score 117.749  8       0
##
## $uni
##
## univariate score tests:
##
##     lhs op   rhs     X2 df p.value
## 1 .p23. == .p56.  2.017  1   0.156
## 2 .p24. == .p57. 92.989  1   0.000
## 3 .p25. == .p58. 32.190  1   0.000
## 4 .p26. == .p59.  8.430  1   0.004
## 5 .p27. == .p60.  8.430  1   0.004
## 6 .p28. == .p61.  3.358  1   0.067
## 7 .p29. == .p62.  1.242  1   0.265
## 8 .p30. == .p63.  1.607  1   0.205
```

```r
pars <- parameterestimates(WISC.scal.fit)
pars[pars$label %in% c(".p24.", ".p25.", ".p26.", ".p27."),]
```

```
##          lhs op rhs block group label    est    se      z pvalue ci.lower
## 24     Vocab ~1         1     1 .p24. 19.608 0.400 48.964      0   18.823
## 25     Compr ~1         1     1 .p25.  7.617 0.185 41.143      0    7.254
## 26 PictCompl ~1         1     1 .p26.  7.792 0.152 51.209      0    7.494
## 27  BlockDes ~1         1     1 .p27.  5.713 0.414 13.809      0    4.902
## 57     Vocab ~1         2     2 .p24. 19.608 0.400 48.964      0   18.823
## 58     Compr ~1         2     2 .p25.  7.617 0.185 41.143      0    7.254
## 59 PictCompl ~1         2     2 .p26.  7.792 0.152 51.209      0    7.494
## 60  BlockDes ~1         2     2 .p27.  5.713 0.414 13.809      0    4.902
##    ci.upper
## 24   20.392
## 25    7.980
## 26    8.090
## 27    6.524
## 57   20.392
## 58    7.980
## 59    8.090
## 60    6.524
```

Vocabulary, Comprehension, Picture Completion and subtests have different intercepts.

```r
WISC.scal.fit2 <- cfa(WISC.mod, sample.cov = WISC.cov.list,
                     sample.mean = WISC.mean.list,
                     sample.nobs = WISC.n.list, meanstructure = TRUE,
                     group.equal = "intercepts",
                     group.partial = c("Compr ~ 1","Vocab ~ 1", "PictCompl ~ 1", "BlockDes ~ 1"))
fitMeasures(WISC.scal.fit2, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic
##    64.711    35.000     0.002     0.967     0.065     0.040 17618.258
```

```r
lavTestLRT(WISC.scal.fit2, WISC.conf.fit)
```

```
## Chi Square Difference Test
##
```

```
##                 Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## WISC.conf.fit  34 17618 17833 62.187
## WISC.scal.fit2 35 17618 17830 64.711     2.5238       1     0.1121
```

Lifting these three equality restrictions still yields acceptable model fit. We continue with testing equality of measurement error variances:

```
WISC.uni.fit <- cfa(WISC.mod, sample.cov = WISC.cov.list,
                    sample.mean = WISC.mean.list,
                    sample.nobs = WISC.n.list, meanstructure = TRUE,
                    group.equal = c("intercepts", "residuals"),
                    group.partial = c("Compr ~ 1","Vocab ~ 1", "PictCompl ~ 1", "BlockDes ~ 1"))
```

```
## Warning in lav_object_post_check(object): lavaan WARNING: some estimated lv
## variances are negative
```

```
fitMeasures(WISC.uni.fit, fit.indices)
```

```
##     chisq        df    pvalue       cfi     rmsea      srmr       aic
##   197.745    43.000     0.000     0.827     0.134     0.106 17735.292
```

```
lavTestLRT(WISC.uni.fit, WISC.scal.fit2)
```

```
## Chi Square Difference Test
##
##                 Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## WISC.scal.fit2 35 17618 17830  64.711
## WISC.uni.fit   43 17735 17915 197.745     133.03       8  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
lavTestScore(WISC.uni.fit)
```

```
## $test
##
## total score test:
##
##    test     X2 df p.value
## 1 score 96.059 12       0
##
## $uni
##
## univariate score tests:
##
##      lhs op   rhs     X2 df p.value
## 1    .p9. == .p42. 17.767  1   0.000
## 2   .p10. == .p43.  0.043  1   0.836
## 3   .p11. == .p44. 40.795  1   0.000
## 4   .p12. == .p45.  0.016  1   0.899
## 5   .p13. == .p46.  0.568  1   0.451
## 6   .p14. == .p47. 15.323  1   0.000
## 7   .p15. == .p48. 30.594  1   0.000
## 8   .p16. == .p49.  1.367  1   0.242
## 9   .p23. == .p56.  0.000  1   1.000
## 10  .p28. == .p61.  7.162  1   0.007
## 11  .p29. == .p62.  4.135  1   0.042
## 12  .p30. == .p63.  3.297  1   0.069
```

We get a warning about negative LV variances, which indicates a problem with the model. Obviously, restricting all measurement error variances to be equal is not a good idea. Let us stick with the configural invariant model, and look at the differences in parameter estimates to get an idea of the differences between the two versions of the WISC. Five out of eight equality restrictions on reisdual variances have modification indices > 5.

```
pars <- parameterestimates(WISC.conf.fit, standardized = TRUE)
col_names <- c("lhs", "op", "rhs", "group", "est", "se", "pvalue", "std.all")
pars[pars$op == "~~", colnames(pars) %in% col_names]
```

```
##          lhs op       rhs group    est     se pvalue std.all
## 9      Simil ~~     Simil     1  3.602  0.419  0.000   0.650
## 10     Vocab ~~     Vocab     1 14.834  2.454  0.000   0.406
## 11     Compr ~~     Compr     1  4.226  0.518  0.000   0.587
## 12 PictCompl ~~ PictCompl     1  3.293  0.451  0.000   0.696
## 13  BlockDes ~~  BlockDes     1 26.304  3.282  0.000   0.742
## 14     Arith ~~     Arith     1  1.202  0.179  0.000   0.537
## 15   DigSpan ~~   DigSpan     1  2.090  0.272  0.000   0.614
## 16       Cod ~~       Cod     1 81.310  8.768  0.000   0.827
## 17        VC ~~        VC     1  1.939  0.472  0.000   1.000
## 18        PR ~~        PR     1  1.436  0.458  0.002   1.000
## 19        WM ~~        WM     1  1.037  0.231  0.000   1.000
## 20        VC ~~        PR     1  1.513  0.310  0.000   0.907
## 21        VC ~~        WM     1  1.187  0.224  0.000   0.837
## 22        PR ~~        WM     1  1.000  0.211  0.000   0.820
## 42     Simil ~~     Simil     2  7.014  1.237  0.000   0.261
## 43     Vocab ~~     Vocab     2 12.323  1.991  0.000   0.290
## 44     Compr ~~     Compr     2 11.823  1.394  0.000   0.489
## 45 PictCompl ~~ PictCompl     2 18.276  2.247  0.000   0.642
## 46  BlockDes ~~  BlockDes     2 42.184  7.143  0.000   0.484
## 47     Arith ~~     Arith     2  2.580  1.642  0.116   0.154
## 48   DigSpan ~~   DigSpan     2  5.127  0.578  0.000   0.696
## 49       Cod ~~       Cod     2 98.930 10.059  0.000   0.912
## 50        VC ~~        VC     2 19.891  2.790  0.000   1.000
## 51        PR ~~        PR     2 10.204  2.546  0.000   1.000
## 52        WM ~~        WM     2 14.146  2.316  0.000   1.000
## 53        VC ~~        PR     2 10.806  1.934  0.000   0.758
## 54        VC ~~        WM     2 10.853  1.637  0.000   0.647
## 55        PR ~~        WM     2  9.022  1.594  0.000   0.751
```

We see that the residual variances for Similarities, Vocabulary, Picture Completion, Block Design and Arithmetic are lower in the second group (WISC-IV) thin in the first group (WISC). The residual variances for Digit Span and Coding are larger for the WISC-IV than for the WISC.

We also see stronger correlations between the latent factors in the WISC, than in the WISC-IV.

```
pars[pars$op == "=~", colnames(pars) %in% col_names]
```

```
##    lhs op       rhs group   est    se pvalue std.all
## 1   VC =~     Simil     1 1.000 0.000     NA   0.592
## 2   VC =~     Vocab     1 3.346 0.457      0   0.771
## 3   VC =~     Compr     1 1.238 0.185      0   0.643
## 4   PR =~ PictCompl     1 1.000 0.000     NA   0.551
## 5   PR =~  BlockDes     1 2.526 0.498      0   0.508
## 6   WM =~     Arith     1 1.000 0.000     NA   0.681
## 7   WM =~   DigSpan     1 1.126 0.171      0   0.621
```

```
## 8    WM =~        Cod     1 4.049 0.847      0    0.416
## 34   VC =~      Simil     2 1.000 0.000     NA    0.860
## 35   VC =~      Vocab     2 1.233 0.094      0    0.843
## 36   VC =~      Compr     2 0.788 0.072      0    0.715
## 37   PR =~ PictCompl     2 1.000 0.000     NA    0.599
## 38   PR =~  BlockDes     2 2.100 0.309      0    0.718
## 39   WM =~      Arith     2 1.000 0.000     NA    0.920
## 40   WM =~    DigSpan     2 0.397 0.062      0    0.551
## 41   WM =~        Cod     2 0.820 0.216      0    0.296
```

We see a similar (but reversed) pattern for the loadings: Standardized loadings are lower for Digit Span and Coding for the WISC-IV than for the WISC. At the same time, standardized loadings are higher for Similarities, Vocabulary, Picture Completion, Block Design and Arithmetic for the WISC-IV than for the WISC.

```
pars[pars$op == "~1", colnames(pars) %in% col_names]
```

```
##           lhs op rhs group   est    se pvalue std.all
## 23      Simil ~1         1  5.67 0.166      0   2.409
## 24      Vocab ~1         1 21.50 0.427      0   3.557
## 25      Compr ~1         1  7.83 0.190      0   2.918
## 26 PictCompl ~1          1  8.00 0.154      0   3.679
## 27   BlockDes ~1         1  6.50 0.421      0   1.092
## 28      Arith ~1         1  5.50 0.106      0   3.676
## 29    DigSpan ~1         1  7.67 0.130      0   4.156
## 30        Cod ~1         1 34.83 0.701      0   3.513
## 31         VC ~1         1  0.00 0.000     NA   0.000
## 32         PR ~1         1  0.00 0.000     NA   0.000
## 33         WM ~1         1  0.00 0.000     NA   0.000
## 56      Simil ~1         2 11.83 0.367      0   2.281
## 57      Vocab ~1         2 21.67 0.461      0   3.322
## 58      Compr ~1         2 15.17 0.348      0   3.085
## 59 PictCompl ~1          2 17.83 0.377      0   3.341
## 60   BlockDes ~1         2 18.67 0.660      0   2.000
## 61      Arith ~1         2 15.00 0.289      0   3.668
## 62    DigSpan ~1         2 12.17 0.192      0   4.485
## 63        Cod ~1         2 45.83 0.736      0   4.401
## 64         VC ~1         2  0.00 0.000     NA   0.000
## 65         PR ~1         2  0.00 0.000     NA   0.000
## 66         WM ~1         2  0.00 0.000     NA   0.000
```

We see that the subscale intercepts are higher for all subtests of the WISC-IV than of the WISC. Thus, the subtests of the WISC-IV may be easier. Such a difference could for example be accounted for by the scoring rules used to compute IQ scores based on the subtest (but we would require much larger samples to do that).