# Longitudinal CFA examples

## Additional exercise 2

Get the dataset 'HDRS.long.csv' from BlackBoard. To load it in **R**, type:

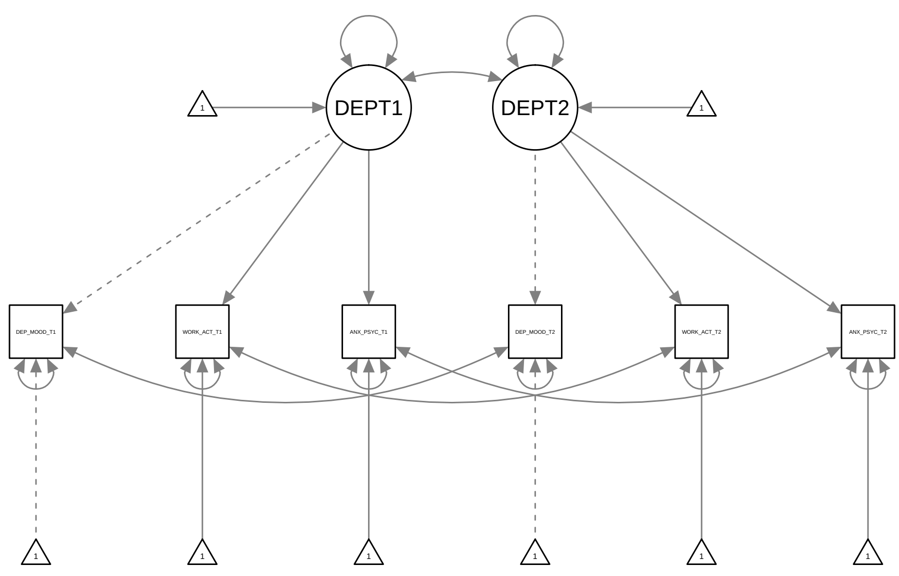
```
dataset <- read.table("HDRS.long.csv")
```

The dataset consists of item scores on three items of the Hamilton Depression Rating Scale, assessing Depressed Mood, Work Activity and Anxious Psychological symptoms. Fit the following (configurally invariant) model to the data:

```
HDRSmod1 <- '
  ## Define latent variables:
  DEPT1 =~ DEP_MOOD_T1 + WORK_ACT_T1 + ANX_PSYC_T1
  DEPT2 =~ DEP_MOOD_T2 + WORK_ACT_T2 + ANX_PSYC_T2

  ## Define associations over time:
  DEP_MOOD_T1 ~~ DEP_MOOD_T2
  WORK_ACT_T1 ~~ WORK_ACT_T2
  ANX_PSYC_T1 ~~ ANX_PSYC_T2

  ## Set latent factor means free, fix first intercepts of first item:
  DEPT1 ~ NA*1
  DEPT2 ~ NA*1
  DEP_MOOD_T1 ~ 0*1
  DEP_MOOD_T2 ~ 0*1
'
```



a) How does this model account for dependencies of variables over time?

b) Evaluate whether this configurally invariant model fits well. Make sure to inspect parameter estimates as well as fit indices.

c) Test whether factor loadings, item intercepts and residual variances are equal over time. You will have to use parameter labels to apply equality restrictions over time (that is, you cannot use the `group.equal` argument, because this is not a multigroup analysis).

d) Select the best fitting invariance model and interpret the values of the latent variable means and (co)variances. What do the values tell you about inter-individual differences and change in depression over time?

# Additional exercise 2

```
dataset <- read.table("HDRS.long.csv")
```

We fit the configurally invariant model to the data:

```
HDRSmod1 <- '
  ## Define latent variables:
  DEPT1 =~ DEP_MOOD_T1 + WORK_ACT_T1 + ANX_PSYC_T1
  DEPT2 =~ DEP_MOOD_T2 + WORK_ACT_T2 + ANX_PSYC_T2

  ## Define associations over time:
  DEP_MOOD_T1 ~~ DEP_MOOD_T2
  WORK_ACT_T1 ~~ WORK_ACT_T2
  ANX_PSYC_T1 ~~ ANX_PSYC_T2

  ## Set latent factor means free, fix first intercepts of first item:
  DEPT1 ~ NA*1
  DEPT2 ~ NA*1
  DEP_MOOD_T1 ~ 0*1
  DEP_MOOD_T2 ~ 0*1
'
HDRSfit1 <- cfa(HDRSmod1, data = dataset, estimator = "MLR")
summary(HDRSfit1, standardized = TRUE)
```

```
## lavaan 0.6-6 ended normally after 42 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         22
##
##   Number of observations                           153
##
## Model Test User Model:
##                                               Standard      Robust
##   Test Statistic                                 3.599       3.574
##   Degrees of freedom                                 5           5
##   P-value (Chi-square)                           0.608       0.612
##   Scaling correction factor                                  1.007
##        Yuan-Bentler correction (Mplus variant)
##
## Parameter Estimates:
```

```
##
##   Standard errors                              Sandwich
##   Information bread                            Observed
##   Observed information based on                 Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   DEPT1 =~
##     DEP_MOOD_T1       1.000                               0.709    0.793
##     WORK_ACT_T1       0.956    0.227    4.209    0.000    0.678    0.612
##     ANX_PSYC_T1       0.666    0.184    3.615    0.000    0.473    0.509
##   DEPT2 =~
##     DEP_MOOD_T2       1.000                               0.633    0.727
##     WORK_ACT_T2       1.148    0.210    5.457    0.000    0.727    0.684
##     ANX_PSYC_T2       1.000    0.174    5.761    0.000    0.633    0.664
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .DEP_MOOD_T1 ~~
##    .DEP_MOOD_T2       0.055    0.057    0.975    0.330    0.055    0.170
##  .WORK_ACT_T1 ~~
##    .WORK_ACT_T2       0.114    0.076    1.499    0.134    0.114    0.168
##  .ANX_PSYC_T1 ~~
##    .ANX_PSYC_T2       0.155    0.075    2.085    0.037    0.155    0.272
##   DEPT1 ~~
##     DEPT2             0.226    0.080    2.811    0.005    0.504    0.504
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     DEPT1             1.484    0.072   20.534    0.000    2.091    2.091
##     DEPT2             0.817    0.071   11.562    0.000    1.291    1.291
##    .DEP_MOOD_T1       0.000                               0.000    0.000
##    .DEP_MOOD_T2       0.000                               0.000    0.000
##    .WORK_ACT_T1       0.393    0.351    1.120    0.263    0.393    0.354
##    .ANX_PSYC_T1       0.365    0.271    1.347    0.178    0.365    0.392
##    .WORK_ACT_T2      -0.069    0.170   -0.407    0.684   -0.069   -0.065
##    .ANX_PSYC_T2       0.156    0.133    1.175    0.240    0.156    0.164
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .DEP_MOOD_T1       0.297    0.110    2.706    0.007    0.297    0.371
##    .WORK_ACT_T1       0.768    0.130    5.905    0.000    0.768    0.626
##    .ANX_PSYC_T1       0.640    0.095    6.711    0.000    0.640    0.741
##    .DEP_MOOD_T2       0.358    0.090    3.966    0.000    0.358    0.472
##    .WORK_ACT_T2       0.600    0.110    5.454    0.000    0.600    0.531
##    .ANX_PSYC_T2       0.509    0.091    5.579    0.000    0.509    0.559
##     DEPT1             0.503    0.137    3.661    0.000    1.000    1.000
##     DEPT2             0.401    0.118    3.399    0.001    1.000    1.000
```

a) Correlations between observed variables over time are accounted for in the model by the covariance beteen the common factors (`DEPT1` and `DEPT2`), and the three covariances between measurement errors of the observed variables over time.

b) All loadings are significant and all standardized loadings are > .5, indicating that the indicators indeed measure a common depression factor. The correlations between the Depression LVs is .504 and significant, indicating that depression levels over time correlate substantially.

Let's inspect the model fit indices:

```
indices <- c("chisq.robuscaled", "df", "pvalue.scaled", "cfi.robust",
             "rmsea.robust", "srmr", "aic")
fitmeasures(HDRSfit1, indices)
```

```
##            df pvalue.scaled    cfi.robust  rmsea.robust          srmr
##         5.000         0.612         1.000         0.000         0.023
##           aic
##      2373.904
```

All fit indices indicate excellent model fit.

c) We restrict the factor loadings to equality over time using parameter labels:

```
HDRSmod2 <- '
  DEPT1 =~ l1*DEP_MOOD_T1 + l2*WORK_ACT_T1 + l3*ANX_PSYC_T1
  DEPT2 =~ l1*DEP_MOOD_T2 + l2*WORK_ACT_T2 + l3*ANX_PSYC_T2
  DEP_MOOD_T1 ~~ DEP_MOOD_T2
  WORK_ACT_T1 ~~ WORK_ACT_T2
  ANX_PSYC_T1 ~~ ANX_PSYC_T2
  DEPT1 ~ NA*1
  DEPT2 ~ NA*1
  DEP_MOOD_T1 ~ 0*1
  DEP_MOOD_T2 ~ 0*1
'
HDRSfit2 <- cfa(HDRSmod2, data = dataset, estimator = "MLR")
fitmeasures(HDRSfit2, indices)
```

```
##            df pvalue.scaled    cfi.robust  rmsea.robust          srmr
##         7.000         0.517         1.000         0.000         0.037
##           aic
##      2372.416
```

```
lavTestLRT(HDRSfit1, HDRSfit2)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##          Df    AIC    BIC Chisq Chisq diff Df diff Pr(>Chisq)
## HDRSfit1  5 2373.9 2440.6 3.599
## HDRSfit2  7 2372.4 2433.0 6.111     2.6885       2     0.2607
```

The factor loadings appear to be equal across timepoints.

We continue by restricting the item intercepts to equality over time, again using parameter labels:

```
HDRSmod3 <- '
  DEPT1 =~ l1*DEP_MOOD_T1 + l2*WORK_ACT_T1 + l3*ANX_PSYC_T1
  DEPT2 =~ l1*DEP_MOOD_T2 + l2*WORK_ACT_T2 + l3*ANX_PSYC_T2
  DEP_MOOD_T1 ~~ DEP_MOOD_T2
  WORK_ACT_T1 ~~ WORK_ACT_T2
  ANX_PSYC_T1 ~~ ANX_PSYC_T2
```

```
  DEPT1 ~ NA*1
  DEPT2 ~ NA*1
  DEP_MOOD_T1 ~ 0*1
  DEP_MOOD_T2 ~ 0*1
  WORK_ACT_T1 ~ i2*1
  ANX_PSYC_T1 ~ i3*1
  WORK_ACT_T2 ~ i2*1
  ANX_PSYC_T2 ~ i3*1
'
HDRSfit3 <- cfa(HDRSmod3, data = dataset, estimator = "MLR")
fitmeasures(HDRSfit3, indices)
```

```
##            df pvalue.scaled    cfi.robust  rmsea.robust          srmr
##         9.000         0.141         0.977         0.058         0.056
##           aic
##      2376.113
```

```
lavTestLRT(HDRSfit3, HDRSfit2)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##          Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HDRSfit2  7 2372.4 2433.0  6.111
## HDRSfit3  9 2376.1 2430.7 13.808     6.6947       2    0.03518 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The item intercepts do not appear equal across timepoints, according to the $\Delta\chi^2$ and $\Delta CFI$. Let's go back to the metric invariance model to inspect the difference in item intercepts:

```
par <- parameterestimates(HDRSfit2)
par[par$op == "~1", 1:8]
```

```
##            lhs op rhs label    est    se      z pvalue
## 10       DEPT1 ~1           1.484 0.072 20.534  0.000
## 11       DEPT2 ~1           0.817 0.071 11.562  0.000
## 12 DEP_MOOD_T1 ~1           0.000 0.000     NA     NA
## 13 DEP_MOOD_T2 ~1           0.000 0.000     NA     NA
## 23 WORK_ACT_T1 ~1           0.192 0.268  0.715  0.474
## 24 ANX_PSYC_T1 ~1           0.073 0.192  0.379  0.704
## 25 WORK_ACT_T2 ~1          -0.022 0.150 -0.148  0.883
## 26 ANX_PSYC_T2 ~1           0.269 0.109  2.461  0.014
```

The item intercepts seems to be higher for `WORK_ACT` at T1 than at T2, and higher for `ANX_PSY` at T2 than at T1.

```
lavTestScore(HDRSfit3)
```

```
## Warning in lavTestScore(HDRSfit3): lavaan WARNING: se is not `standard'; not
## implemented yet; falling back to ordinary score test
```

```
## $test
##
```

```
## total score test:
##
##    test    X2 df p.value
## 1 score 9.676  4   0.046
##
## $uni
##
## univariate score tests:
##
##     lhs op   rhs    X2 df p.value
## 1  .p2. ==  .p5. 4.416  1   0.036
## 2  .p3. ==  .p6. 7.261  1   0.007
## 3 .p14. == .p16. 5.271  1   0.022
## 4 .p15. == .p17. 5.601  1   0.018
```

```
par <- parameterestimates(HDRSfit3, standardized = TRUE)
par[par$op == "~1", c(1:8, 11)]
```

```
##            lhs op rhs label    est    se      z pvalue std.lv
## 10       DEPT1 ~1           1.488 0.072 20.661  0.000  2.382
## 11       DEPT2 ~1           0.813 0.067 12.118  0.000  1.266
## 12 DEP_MOOD_T1 ~1           0.000 0.000     NA     NA  0.000
## 13 DEP_MOOD_T2 ~1           0.000 0.000     NA     NA  0.000
## 14 WORK_ACT_T1 ~1        i2 -0.102 0.178 -0.571  0.568 -0.102
## 15 ANX_PSYC_T1 ~1        i3  0.300 0.101  2.972  0.003  0.300
## 16 WORK_ACT_T2 ~1        i2 -0.102 0.178 -0.571  0.568 -0.102
## 17 ANX_PSYC_T2 ~1        i3  0.300 0.101  2.972  0.003  0.300
```

The score tests yield similar values for both intercepts. Note that these score tests reflect the expected change in the ML $\chi^2$ value, not the robust ML $\chi^2$; we also receive a warning about that in our output.

Based on these results, both equality restrictions on item intercepts appear equally problematic, so we lift the equality restriction from both item intercepts. We continue testing the equality of residual variances over time:

```
HDRSmod4 <- '
  DEPT1 =~ l1*DEP_MOOD_T1 + l2*WORK_ACT_T1 + l3*ANX_PSYC_T1
  DEPT2 =~ l1*DEP_MOOD_T2 + l2*WORK_ACT_T2 + l3*ANX_PSYC_T2
  DEP_MOOD_T1 ~~ DEP_MOOD_T2
  WORK_ACT_T1 ~~ WORK_ACT_T2
  ANX_PSYC_T1 ~~ ANX_PSYC_T2
  DEPT1 ~ NA*1
  DEPT2 ~ NA*1
  DEP_MOOD_T1 ~ 0*1
  DEP_MOOD_T2 ~ 0*1
  DEP_MOOD_T1 ~~ u1*DEP_MOOD_T1
  WORK_ACT_T1 ~~ u2*WORK_ACT_T1
  ANX_PSYC_T1 ~~ u3*ANX_PSYC_T1
  DEP_MOOD_T2 ~~ u1*DEP_MOOD_T2
  WORK_ACT_T2 ~~ u2*WORK_ACT_T2
  ANX_PSYC_T2 ~~ u3*ANX_PSYC_T2
'
HDRSfit4 <- cfa(HDRSmod4, data = dataset, estimator = "MLR")
fitmeasures(HDRSfit4, indices)
```

```
##           df pvalue.scaled    cfi.robust  rmsea.robust          srmr
##       10.000         0.642         1.000         0.000         0.040
##          aic
##     2368.009
```

```r
lavTestLRT(HDRSfit4, HDRSfit2)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##          Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## HDRSfit2  7 2372.4 2433.0 6.1110
## HDRSfit4 10 2368.0 2419.5 7.7039     1.6525       3     0.6475
```

The restriction on item's residual variances are tenable.

d)

```r
par <- parameterestimates(HDRSfit4, standardized = TRUE)
par[par$lhs %in% c("DEPT1", "DEPT2"), c(1:8, 11)]
```

```
##      lhs op         rhs label   est    se      z pvalue std.lv
## 1  DEPT1 =~ DEP_MOOD_T1    l1 1.000 0.000     NA     NA  0.650
## 2  DEPT1 =~ WORK_ACT_T1    l2 1.092 0.172  6.362  0.000  0.710
## 3  DEPT1 =~ ANX_PSYC_T1    l3 0.854 0.124  6.863  0.000  0.555
## 4  DEPT2 =~ DEP_MOOD_T2    l1 1.000 0.000     NA     NA  0.661
## 5  DEPT2 =~ WORK_ACT_T2    l2 1.092 0.172  6.362  0.000  0.722
## 6  DEPT2 =~ ANX_PSYC_T2    l3 0.854 0.124  6.863  0.000  0.564
## 10 DEPT1 ~1                   1.484 0.072 20.534  0.000  2.283
## 11 DEPT2 ~1                   0.817 0.071 11.562  0.000  1.236
## 20 DEPT1 ~~       DEPT1       0.422 0.094  4.491  0.000  1.000
## 21 DEPT2 ~~       DEPT2       0.437 0.109  3.994  0.000  1.000
## 22 DEPT1 ~~       DEPT2       0.210 0.070  2.987  0.003  0.489
```

The latent variable means indicate a substantial decrease of depression over time. Looking at the standard errors, the difference is statistically significant. The difference in LV means is about .65, which is more or less equal to the standard deviations of the LVs at both timepoints. Thus, in terms of effect sizes, this is a strong decrease in depression levels over time.

The standardized latent variable covariance is positive, statistically significant, and indicates a strong association between depression levels at both timepoints.

Thus, on average, depression levels decrease over time. Those with higher (lower) depression levels at T1 will have higher (lower) depression levels at T2.

## Additional exercise 3: Types and treatment for depression

Get the file 'depression.txt' from BlackBoard and read it into R:

```r
data <- read.table("depression.txt")
head(data)
```

```
##   dep1_T1 dep2_T1 dep3_T1 dep4_T1 dep1_T2 dep2_T2 dep3_T2 dep4_T2       type
## 1      15      13       5      12       8       7      13      11    chronic
```

```
## 2        2       5      10       7       2       3       5       6    chronic
## 3       17       5      23      16       7       5       5       4 first-time
## 4       10       1       8      15      16       8      10      18    chronic
## 5       15       3       9      12      10       6       7       8    chronic
## 6        7       7       7      12      18       6       8       8    chronic
##   treat
## 1     0
## 2     1
## 3     1
## 4     0
## 5     1
## 6     0
```

The data consists of 400 observations from depressed patients receiving treatment. The data contains 4 indicators for depression (dep1 through dep4), measured at two occasions (T1 and T2). Also, there is a variable `type`, which is an indicator for the depression subtype (chronic versus first-time depression), and a variable `treat` which is a dummy indicator for treatment (0 for treatment as usual, 1 for a new treatment, which combines cognitive-behavioral therapy with anti-depressant medication).

A summary of the model to be fitted to the data is provided in Figure 1. Fit this model to the data, using a multigroup model in lavaan. That is, add `group = 'type'` when applying the `cfa()` function to the data.

You may assume the latent means are equal between the groups. Answer the following research questions by consecutively applying equality restrictions to the `loadings`, `intercepts`, `residuals`, `lv.variances` and `regressions`, using the `group.equal` argument:

a) Is the measurement model underlying the depression indicators equal between the chronic and first-time depression groups?
b) Are the latent variances at T1 and T2 equal between the two groups?
c) Is the effect of treatment equal between the chronic and first-time depression groups?

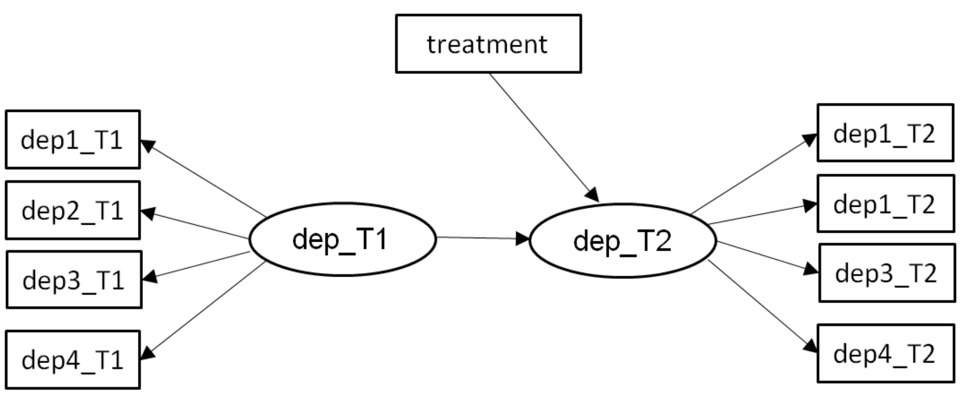



Figure 1: Path diagram for additional exercise 3

# Additional exercise 3

We load the data:

```r
data <- read.table("depression.txt")
```

We define the model:

```r
mod <- '
  ## Define latent variables:
  dep_T1 =~ dep1_T1 + dep2_T1 + dep3_T1 + dep4_T1
  dep_T2 =~ dep1_T2 + dep2_T2 + dep3_T2 + dep4_T2

  ## Define regressions:
  dep_T2 ~ dep_T1 + treat

  ## Allow for correlated measurement errors between time points:
  dep1_T1 ~~ dep1_T2
  dep2_T1 ~~ dep2_T2
  dep3_T1 ~~ dep3_T2
  dep4_T1 ~~ dep4_T2

  ## Use marker-variable identification for mean structure:
  dep1_T1 ~ 0*1
  dep1_T2 ~ 0*1
  dep_T1 ~ NA*1
  dep_T2 ~ NA*1
'
```

We fit a configural invariant model to the data:

```r
fit1 <- cfa(mod, data = data, group = "type", estimator = "MLR")
summary(fit1, standardized = TRUE, fit.measures = TRUE)
```

```
## lavaan 0.6-6 ended normally after 240 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         60
##
##   Number of observations per group:
##     chronic                                        195
##     first-time                                     205
##
## Model Test User Model:
##                                               Standard      Robust
##   Test Statistic                                75.801      78.957
##   Degrees of freedom                                44          44
##   P-value (Chi-square)                           0.002       0.001
##   Scaling correction factor                                  0.960
##        Yuan-Bentler correction (Mplus variant)
##   Test statistic for each group:
##     chronic                                     43.130      44.926
##     first-time                                  32.671      34.031
##
## Model Test Baseline Model:
##
##   Test statistic                              1212.764    1210.074
##   Degrees of freedom                                72          72
```

```
##   P-value                                        0.000       0.000
##   Scaling correction factor                                 1.002
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.972       0.969
##   Tucker-Lewis Index (TLI)                       0.954       0.950
##
##   Robust Comparative Fit Index (CFI)                         0.971
##   Robust Tucker-Lewis Index (TLI)                            0.952
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -9190.577   -9190.577
##   Scaling correction factor                                 1.049
##       for the MLR correction
##   Loglikelihood unrestricted model (H1)      -9152.677   -9152.677
##   Scaling correction factor                                 1.011
##       for the MLR correction
##
##   Akaike (AIC)                               18501.154   18501.154
##   Bayesian (BIC)                             18740.642   18740.642
##   Sample-size adjusted Bayesian (BIC)        18550.258   18550.258
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.060       0.063
##   90 Percent confidence interval - lower         0.036       0.039
##   90 Percent confidence interval - upper         0.083       0.086
##   P-value RMSEA <= 0.05                          0.221       0.167
##
##   Robust RMSEA                                              0.062
##   90 Percent confidence interval - lower                     0.039
##   90 Percent confidence interval - upper                     0.083
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.047       0.047
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
##
## Group 1 [chronic]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dep_T1 =~
##     dep1_T1           1.000                               2.619    0.602
##     dep2_T1           0.769    0.201    3.830    0.000    2.016    0.484
##     dep3_T1           0.533    0.243    2.196    0.028    1.396    0.282
```

```
##     dep4_T1           0.831    0.185    4.487    0.000    2.176    0.490
##   dep_T2 =~
##     dep1_T2           1.000                               2.649    0.528
##     dep2_T2           0.744    0.213    3.495    0.000    1.971    0.412
##     dep3_T2           1.124    0.237    4.747    0.000    2.978    0.684
##     dep4_T2           0.888    0.207    4.296    0.000    2.351    0.496
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dep_T2 ~
##     dep_T1            0.795    0.234    3.394    0.001    0.786    0.786
##     treat            -2.402    0.469   -5.119    0.000   -0.907   -0.453
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .dep1_T1 ~~
##    .dep1_T2           3.946    1.743    2.264    0.024    3.946    0.267
##  .dep2_T1 ~~
##    .dep2_T2           2.332    1.399    1.667    0.095    2.332    0.147
##  .dep3_T1 ~~
##    .dep3_T2           4.483    1.408    3.184    0.001    4.483    0.297
##  .dep4_T1 ~~
##    .dep4_T2           5.393    1.719    3.138    0.002    5.393    0.339
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .dep1_T1           0.000                               0.000    0.000
##    .dep1_T2           0.000                               0.000    0.000
##     dep_T1            9.795    0.313   31.300    0.000    3.739    3.739
##    .dep_T2            2.436    2.311    1.054    0.292    0.920    0.920
##    .dep2_T1           1.586    1.969    0.805    0.421    1.586    0.381
##    .dep3_T1           4.491    2.426    1.851    0.064    4.491    0.907
##    .dep4_T1           1.687    1.797    0.939    0.348    1.687    0.380
##    .dep2_T2           1.819    1.957    0.930    0.352    1.819    0.380
##    .dep3_T2          -1.685    2.173   -0.775    0.438   -1.685   -0.387
##    .dep4_T2           0.848    1.890    0.449    0.653    0.848    0.179
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .dep1_T1          12.075    2.225    5.427    0.000   12.075    0.638
##    .dep2_T1          13.262    1.683    7.881    0.000   13.262    0.765
##    .dep3_T1          22.588    2.475    9.127    0.000   22.588    0.921
##    .dep4_T1          15.007    2.077    7.224    0.000   15.007    0.760
##    .dep1_T2          18.119    2.139    8.470    0.000   18.119    0.721
##    .dep2_T2          18.986    2.217    8.564    0.000   18.986    0.830
##    .dep3_T2          10.084    1.981    5.091    0.000   10.084    0.532
##    .dep4_T2          16.902    2.352    7.187    0.000   16.902    0.754
##     dep_T1            6.861    2.515    2.728    0.006    1.000    1.000
##    .dep_T2            1.247    1.274    0.979    0.328    0.178    0.178
##
##
## Group 2 [first-time]:
##
## Latent Variables:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dep_T1 =~
##     dep1_T1           1.000                               3.791    0.660
##     dep2_T1           1.171    0.161    7.256    0.000    4.440    0.723
##     dep3_T1           1.003    0.121    8.267    0.000    3.803    0.727
##     dep4_T1           1.137    0.124    9.202    0.000    4.311    0.738
##   dep_T2 =~
##     dep1_T2           1.000                               3.643    0.662
##     dep2_T2           1.035    0.145    7.146    0.000    3.770    0.678
##     dep3_T2           1.273    0.142    8.942    0.000    4.638    0.808
##     dep4_T2           1.280    0.126   10.127    0.000    4.662    0.786
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dep_T2 ~
##     dep_T1            0.896    0.088   10.163    0.000    0.932    0.932
##     treat            -1.883    0.341   -5.519    0.000   -0.517   -0.258
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .dep1_T1 ~~
##    .dep1_T2           7.615    1.461    5.211    0.000    7.615    0.427
##  .dep2_T1 ~~
##    .dep2_T2           5.457    1.515    3.601    0.000    5.457    0.315
##  .dep3_T1 ~~
##    .dep3_T2           2.214    1.465    1.511    0.131    2.214    0.182
##  .dep4_T1 ~~
##    .dep4_T2           5.207    1.607    3.240    0.001    5.207    0.361
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .dep1_T1           0.000                               0.000    0.000
##    .dep1_T2           0.000                               0.000    0.000
##     dep_T1            9.498    0.399   23.809    0.000    2.506    2.506
##    .dep_T2           -2.572    0.842   -3.054    0.002   -0.706   -0.706
##    .dep2_T1          -1.758    1.566   -1.122    0.262   -1.758   -0.286
##    .dep3_T1           0.473    1.187    0.398    0.690    0.473    0.090
##    .dep4_T1          -0.973    1.210   -0.804    0.422   -0.973   -0.167
##    .dep2_T2          -0.050    0.859   -0.058    0.954   -0.050   -0.009
##    .dep3_T2          -2.013    0.796   -2.531    0.011   -2.013   -0.351
##    .dep4_T2          -1.632    0.722   -2.259    0.024   -1.632   -0.275
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .dep1_T1          18.656    1.772   10.530    0.000   18.656    0.565
##    .dep2_T1          17.973    2.113    8.505    0.000   17.973    0.477
##    .dep3_T1          12.878    1.726    7.462    0.000   12.878    0.471
##    .dep4_T1          15.532    2.033    7.641    0.000   15.532    0.455
##    .dep1_T2          17.010    1.790    9.505    0.000   17.010    0.562
##    .dep2_T2          16.741    1.801    9.297    0.000   16.741    0.541
##    .dep3_T2          11.432    2.011    5.685    0.000   11.432    0.347
##    .dep4_T2          13.408    1.929    6.951    0.000   13.408    0.382
##     dep_T1           14.369    2.905    4.946    0.000    1.000    1.000
##    .dep_T2            0.855    0.580    1.473    0.141    0.064    0.064
```

We see that all loadings are substantial and significant, with exception of `dep3_T1` in group 1. The fit appears adequate, or even good.

Furthermore, we could inspect and interpret some of the other parameter estimates: The association between depression at T1 and T2 is strong and positive as expected, in both groups. The new treatment appears to have a negative effect on depression in both groups (also as expected). Looking at the intercepts, the mean depression levels appear similar at T1 in the two groups, but the intercept of depression at T2 is positive in the first group and negative in the second. This could indicate that the two groups have similar mean levels of depression before treatment, but not after treatment.

a) We test the equality of the measurement models in the two groups:

```
fit2 <- cfa(mod, data = data, group = "type", estimator = "MLR",
            group.equal = "loadings")
fit.indices <- c("chisq", "df", "pvalue", "cfi", "rmsea", "rmsea.ci.lower",
                 "rmsea.ci.upper", "srmr", "aic", "bic")
fitMeasures(fit2, fit.indices)
```

```
##          chisq             df         pvalue            cfi          rmsea
##         81.670         50.000          0.003          0.972          0.056
## rmsea.ci.lower rmsea.ci.upper           srmr            aic            bic
##          0.033          0.078          0.051      18495.024      18710.563
```

```
lavTestLRT(fit1, fit2)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit1 44 18501 18741 75.801
## fit2 50 18495 18711 81.671      4.974       6     0.5472
```

```
fit3 <- cfa(mod, data = data, group = "type", estimator = "MLR",
            group.equal = c("loadings", "intercepts"))
fitMeasures(fit3, fit.indices)
```

```
##          chisq             df         pvalue            cfi          rmsea
##         84.004         56.000          0.009          0.975          0.050
## rmsea.ci.lower rmsea.ci.upper           srmr            aic            bic
##          0.026          0.071          0.053      18485.358      18676.948
```

```
lavTestLRT(fit2, fit3)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit2 50 18495 18711 81.671
## fit3 56 18485 18677 84.004     2.2674       6     0.8935
```

```r
fit4 <- cfa(mod, data = data, group = "type", estimator = "MLR",
            group.equal = c("loadings", "intercepts", "residuals"))
fitMeasures(fit4, fit.indices)
```

```
##          chisq             df         pvalue            cfi          rmsea
##        103.643         64.000          0.001          0.965          0.056
## rmsea.ci.lower rmsea.ci.upper           srmr            aic            bic
##          0.035          0.075          0.059      18488.996      18648.655
```

```r
lavTestLRT(fit3, fit4)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit3 56 18485 18677  84.004
## fit4 64 18489 18649 103.643     19.697       8    0.01155 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Equality of loadings and intercepts appears tenable. Only the restriction on residual variances yields a significant change in $\Delta\chi^2$ and $\Delta CFI$. Let's assess which constraint(s) appear problematic:

```r
lavTestScore(fit4)
```

```
## Warning in lavTestScore(fit4): lavaan WARNING: se is not `standard'; not
## implemented yet; falling back to ordinary score test
```

```
## $test
##
## total score test:
##
##    test     X2 df p.value
## 1 score 25.253 20   0.192
##
## $uni
##
## univariate score tests:
##
##      lhs op   rhs     X2 df p.value
## 1   .p2. == .p38.  0.515  1   0.473
## 2   .p3. == .p39.  0.176  1   0.675
## 3   .p4. == .p40.  0.114  1   0.735
## 4   .p6. == .p42.  0.000  1   0.983
## 5   .p7. == .p43.  0.197  1   0.657
## 6   .p8. == .p44.  0.341  1   0.559
## 7  .p19. == .p55.  2.847  1   0.092
## 8  .p20. == .p56.  4.000  1   0.046
## 9  .p21. == .p57. 10.541  1   0.001
## 10 .p22. == .p58.  0.050  1   0.823
## 11 .p23. == .p59.  0.765  1   0.382
## 12 .p24. == .p60.  0.385  1   0.535
```

```
## 13 .p25. == .p61.  0.294  1   0.587
## 14 .p26. == .p62.  1.036  1   0.309
## 15 .p30. == .p66.  0.249  1   0.618
## 16 .p31. == .p67.  0.246  1   0.620
## 17 .p32. == .p68.  0.080  1   0.778
## 18 .p33. == .p69.  0.000  1   0.999
## 19 .p34. == .p70.  0.262  1   0.609
## 20 .p35. == .p71.  0.094  1   0.759
```

```
par <- parameterestimates(fit4)
par[par$label == ".p21.", 1:8]
```

```
##        lhs op     rhs block group label    est    se
## 21 dep3_T1 ~~ dep3_T1     1     1 .p21. 17.642 1.529
## 57 dep3_T1 ~~ dep3_T1     2     2 .p21. 17.642 1.529
```

The equality restriction on the residual variances of the dep3 items for T1 seems most problematic. Let's release that restriction:

```
fit5 <- cfa(mod, data = data, group = "type", estimator = "MLR",
            group.equal = c("loadings", "intercepts", "residuals"),
            group.partial = "dep3_T1 ~~ dep3_T1")
fitMeasures(fit5, fit.indices)
```

```
##          chisq             df         pvalue            cfi          rmsea
##         92.683         63.000          0.009          0.974          0.049
## rmsea.ci.lower rmsea.ci.upper           srmr            aic            bic
##          0.025          0.069          0.055      18480.036      18643.686
```

```
lavTestLRT(fit3, fit5)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit3 56 18485 18677 84.004
## fit5 63 18480 18644 92.683     8.9509       7     0.2562
```

Partial measurement invariance seems tenable between the two groups.

b) We test for equality of latent variances between groups:

```
fit6 <- cfa(mod, data = data, group = "type", estimator= "MLR",
            group.equal = c("loadings", "intercepts", "residuals",
                            "lv.variances"),
            group.partial =  "dep3_T1 ~~ dep3_T1")
fitMeasures(fit6, fit.indices)
```

```
##          chisq             df         pvalue            cfi          rmsea
##        140.527         65.000          0.000          0.934          0.076
## rmsea.ci.lower rmsea.ci.upper           srmr            aic            bic
##          0.059          0.094          0.176      18523.880      18679.547
```

```
lavTestLRT(fit5, fit6)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df   AIC   BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit5 63 18480 18644  92.683
## fit6 65 18524 18680 140.527     45.998       2  1.027e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Restricting the variances of the LVs yields a significant deterioration of model fit. Let's inspect the differences in variances from the best-fitting model up till now (`fit5`):

```
par <- parameterestimates(fit5, standardized = TRUE)
par[par$op == "~~", c(1:3, 5:7, 14)]
```

```
##        lhs op     rhs group label    est std.all
## 11 dep1_T1 ~~ dep1_T2     1         5.154   0.300
## 12 dep2_T1 ~~ dep2_T2     1         1.930   0.117
## 13 dep3_T1 ~~ dep3_T2     1         4.108   0.265
## 14 dep4_T1 ~~ dep4_T2     1         4.391   0.287
## 19 dep1_T1 ~~ dep1_T1     1 .p19. 16.698   0.832
## 20 dep2_T1 ~~ dep2_T1     1 .p20. 15.667   0.790
## 21 dep3_T1 ~~ dep3_T1     1       21.881   0.879
## 22 dep4_T1 ~~ dep4_T1     1 .p22. 15.713   0.803
## 23 dep1_T2 ~~ dep1_T2     1 .p23. 17.703   0.759
## 24 dep2_T2 ~~ dep2_T2     1 .p24. 17.330   0.773
## 25 dep3_T2 ~~ dep3_T2     1 .p25. 10.978   0.581
## 26 dep4_T2 ~~ dep4_T2     1 .p26. 14.929   0.670
## 27  dep_T1 ~~  dep_T1     1         3.366   1.000
## 28  dep_T2 ~~  dep_T2     1         0.223   0.040
## 29   treat ~~   treat     1         0.249   1.000
## 47 dep1_T1 ~~ dep1_T2     2         7.190   0.418
## 48 dep2_T1 ~~ dep2_T2     2         4.939   0.300
## 49 dep3_T1 ~~ dep3_T2     2         2.027   0.171
## 50 dep4_T1 ~~ dep4_T2     2         5.910   0.386
## 55 dep1_T1 ~~ dep1_T1     2 .p19. 16.698   0.511
## 56 dep2_T1 ~~ dep2_T1     2 .p20. 15.667   0.443
## 57 dep3_T1 ~~ dep3_T1     2       12.812   0.472
## 58 dep4_T1 ~~ dep4_T1     2 .p22. 15.713   0.463
## 59 dep1_T2 ~~ dep1_T2     2 .p23. 17.703   0.537
## 60 dep2_T2 ~~ dep2_T2     2 .p24. 17.330   0.556
## 61 dep3_T2 ~~ dep3_T2     2 .p25. 10.978   0.338
## 62 dep4_T2 ~~ dep4_T2     2 .p26. 14.929   0.428
## 63  dep_T1 ~~  dep_T1     2        15.955   1.000
## 64  dep_T2 ~~  dep_T2     2         1.024   0.067
## 65   treat ~~   treat     2         0.249   1.000
```

We see higher variances of the LVs in the second (first-time) group than in the first (chronic) group. It appears that at both measurement occasions, the first-time depressed patients differ more strongly amongst each other in levels of depression than chronically depressed patients. Also, we see that the residual variance of item 3 at T1 is higher in the first (chronic) than in the second (first-time) group.

As the equality restriction on variances of the LVs does not appear tenable, we release the restriction in further models.

c) We test whether the treatment effect is equal between the two groups by restricting the regression parameters to equality:

```
fit7 <- cfa(mod, data = data, group = "type", estimator= "MLR",
            group.equal = c("loadings", "intercepts", "residuals",
                            "regressions"),
            group.partial =  "dep3_T1 ~~ dep3_T1")
```

```
## Warning in lav_model_estimate(lavmodel = lavmodel, lavpartable = lavpartable, :
## lavaan WARNING: the optimizer warns that a solution has NOT been found!
```

That is a serious warning. Somehow the model I defined yields an optimization problem that is not easy to solve.

There are two possible courses of action now:

*Approach 1: Check out the parameter estimates of the best-fitting succesfully fitted model (and draw conclusions based on those)*

We can inspect the regression estimates in the best-fitting model we obtained thus far (from `fit5`):

```
par[par$op == "~", c(1:3, 5:9, 14)]
```

```
##       lhs op    rhs group label    est    se      z std.all
## 9  dep_T2  ~ dep_T1     1        1.126 0.246  4.570   0.872
## 10 dep_T2  ~  treat     1       -2.126 0.336 -6.333  -0.447
## 45 dep_T2  ~ dep_T1     2        0.911 0.077 11.774   0.932
## 46 dep_T2  ~  treat     2       -1.997 0.337 -5.935  -0.255
```

The direction of the effect is the same in the two groups: the higher `dep_T1`, the higher `dep_T2`; and the new treatment yields lower `dep_T2` than the old treatment (treatment as usual). This is also in line with the results we obtained in the very first model (`fit1`).

Looking at the point estimates, the effect of treatment seems larger in the first (chronic) group than in the second (first-time) group. However, the difference between the treatment effects in the two groups is smaller than the standard errors of the parameter estimates. Based on that, it seems best to retain the null hypothesis that the effect of treatment is the same in both groups.

*Approach 2: Check out parameter estimates of the unsuccesfully fitted model (and see if we can respecify the model to fit it succesfully)*

If we would inspect the parameterestimates of the fitted model, we would see that parameter estimation for the `dep2` item at T1 seems to be caught in an improbable area:

```
parameterestimates(fit7)[,1:8]
```

The parameter estimates reveal weird value for the loading of item `dep2_T1`. We know that the loading of the item should be $> 0$. Supplying this as a restriction to the model-fitting function may solve the problem:

```
mod2 <- '
  ## Define latent variables:
  dep_T1 =~ dep1_T1 + c(a,a)*dep2_T1 + dep3_T1 + dep4_T1
  dep_T2 =~ dep1_T2 + dep2_T2 + dep3_T2 + dep4_T2

  ## Define regressions:
  dep_T2 ~ dep_T1 + treat
```

```
  ## Allow for correlated measurement errors between time points:
  dep1_T1 ~~ dep1_T2
  dep2_T1 ~~ dep2_T2
  dep3_T1 ~~ dep3_T2
  dep4_T1 ~~ dep4_T2

  ## Use marker-variable identification for mean structure:
  dep1_T1 ~ 0*1
  dep1_T2 ~ 0*1
  dep_T1 ~ NA*1
  dep_T2 ~ NA*1

  ## apply restriction to the value of the loading for item dep2_T1
  a > 0
'
fit8 <- cfa(mod2, data = data, group = "type", estimator= "MLR",
            group.equal = c("loadings", "intercepts", "residuals",
                            "regressions"),
            group.partial =  "dep3_T1 ~~ dep3_T1")
```

The additional restriction seems to have solved the problem. Earlier, the optimizer (`NLMINB`) was probably caught in a local minimum.

Now we can proceed by testing the fit between the models:

```
lavTestLRT(fit8, fit5)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit5 63 18480 18644 92.683
## fit8 65 18478 18633 94.372      1.317       2     0.5176
```

Restricting the regressions to be equal between the two groups did not significantly decrease model fit. Therefore, we can retain the null hypothesis of equal effects of treatment and pre-treatment depression in the two groups.

Note that both of our last two approaches yielded the same conclusion: the effect of the new treatment does not seem to differ between the two groups. Also note that the effect of the new treatment, compared to treatment as usual, was significant in both groups.

# Additional exercise 2: CFA + LGCM

Get file `data.csv` from blackboard, and load it into R as follows:

```
data <- read.csv('data.csv')
summary(data)
```

The data are from a clinical trial, assessing the effects of a treatment for depression:

Depression has been assessed before (t1), during (t2) and after (t3) treatment. Depression was measured using three continuous indicators (X1, X2 and X3) at each occasion.

The dataset contains a variable called `treatment`, which takes a value of 0 for the control group, and 1 for the experimental group.

a) Ignore the treatment variable for now. Fit a factor model with a single common factor for each timepoint to the data. Describe the fitted model: Does the model fit the data well? Are the factor loadings more or less equal over time? Do the values of the LVs change over time (in- or decrease)? Are the values of the LVs correlated over time?

Note: As we expect a change in depression over time, make sure you use marker identification for the depression LVs.

b) Now add a latent slope and intercept to the model, to explain growth and stability of the LVs over time.

Note: Using the `sem()` function may be more appropriate if you combine CFA and LGCM. Note that for the LGCM part, the intercepts of the indicator variables for the latent intercept and slope need to be zero. Note that for the CFA part, the intercepts of the indicator variables for the common factors need to be freely estimated (with exception of the intercepts of the marker indicator variables).

c) Now also include the covariate, the indicator for treatment, in your model. Describe the effect of the intervention on depression levels: Do the treatment groups differ in their initial levels of depression? Do the treatment groups differ in their in- or decreases in depression? In other words: does the treatment appear to be effective?

## Additional exercise 2: CFA + LGCM

```
data <- read.csv('data.csv')
```

The data are from a clinical trial, assessing the effects of a treatment for depression:

Depression has been assessed before (t1), during (t2) and after (t3) treatment

Depression was measured using three indicators (X1, X2 and X3) at each occasion

The dataset contains a variable called 'treatment', which takes a value of 0 for the control group, and 1 for the experimental group.

a) We fit a factor model with a single factor for each timepoint:

```
mod0 <- '
  dept1 =~ X1t1 + X2t1 + X3t1
  dept2 =~ X1t2 + X2t2 + X3t2
  dept3 =~ X1t3 + X2t3 + X3t3
  ## Freely estimate latent means, so latent depression levels should be
  ## allowed to vary over time:
  X1t1 ~ 0*1
  X1t2 ~ 0*1
  X1t3 ~ 0*1
  dept1 ~ NA*1
  dept2 ~ NA*1
  dept3 ~ NA*1
'
fit0 <- cfa(model = mod0, data = data, meanstructure = TRUE, estimator = "MLR")
indices <- c("chisq.scaled", "df", "pvalue", "cfi.robust", "rmsea.robust",
             "srmr.robust", "srmr")
fitmeasures(fit0, indices)
```

```
## chisq.scaled           df       pvalue   cfi.robust rmsea.robust         srmr
```

```
##       35.657       24.000        0.075        0.997        0.043        0.016
```

The $\chi^2$ value is non-significant, so the model seems to fit the data well. CFI, RMSEA, SRMR indicate good model fit.

```r
summary(fit0, standardized = TRUE)
```

```
## lavaan 0.6-6 ended normally after 75 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         30
##
##   Number of observations                           250
##
## Model Test User Model:
##                                               Standard      Robust
##   Test Statistic                                34.547      35.657
##   Degrees of freedom                                24          24
##   P-value (Chi-square)                           0.075       0.059
##   Scaling correction factor                                  0.969
##        Yuan-Bentler correction (Mplus variant)
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dept1 =~
##     X1t1              1.000                               1.028    0.896
##     X2t1              1.056    0.047   22.332    0.000    1.085    0.906
##     X3t1              1.142    0.053   21.422    0.000    1.173    0.922
##   dept2 =~
##     X1t2              1.000                               1.675    0.965
##     X2t2              0.986    0.026   37.237    0.000    1.651    0.959
##     X3t2              0.994    0.027   37.212    0.000    1.665    0.948
##   dept3 =~
##     X1t3              1.000                               2.731    0.987
##     X2t3              0.995    0.017   59.501    0.000    2.717    0.984
##     X3t3              0.987    0.016   60.257    0.000    2.697    0.981
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dept1 ~~
##     dept2             0.913    0.131    6.950    0.000    0.531    0.531
##     dept3             1.021    0.184    5.556    0.000    0.364    0.364
##   dept2 ~~
##     dept3             3.884    0.367   10.583    0.000    0.849    0.849
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .X1t1              0.000                               0.000    0.000
```

```
##    .X1t2              0.000                                 0.000    0.000
##    .X1t3              0.000                                 0.000    0.000
##     dept1             0.008    0.073    0.108    0.914    0.008    0.008
##     dept2            -0.591    0.110   -5.386    0.000   -0.353   -0.353
##     dept3            -1.288    0.175   -7.358    0.000   -0.471   -0.471
##    .X2t1             -0.049    0.047   -1.054    0.292   -0.049   -0.041
##    .X3t1             -0.058    0.048   -1.218    0.223   -0.058   -0.046
##    .X2t2             -0.011    0.045   -0.235    0.814   -0.011   -0.006
##    .X3t2             -0.084    0.049   -1.700    0.089   -0.084   -0.048
##    .X2t3              0.020    0.044    0.455    0.649    0.020    0.007
##    .X3t3              0.011    0.050    0.223    0.823    0.011    0.004
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .X1t1              0.261    0.035    7.446    0.000    0.261    0.198
##    .X2t1              0.257    0.032    7.985    0.000    0.257    0.179
##    .X3t1              0.243    0.040    5.996    0.000    0.243    0.150
##    .X1t2              0.208    0.032    6.573    0.000    0.208    0.069
##    .X2t2              0.238    0.031    7.639    0.000    0.238    0.080
##    .X3t2              0.309    0.036    8.579    0.000    0.309    0.100
##    .X1t3              0.197    0.029    6.768    0.000    0.197    0.026
##    .X2t3              0.238    0.029    8.135    0.000    0.238    0.031
##    .X3t3              0.278    0.034    8.247    0.000    0.278    0.037
##     dept1             1.056    0.107    9.860    0.000    1.000    1.000
##     dept2             2.805    0.271   10.335    0.000    1.000    1.000
##     dept3             7.458    0.577   12.915    0.000    1.000    1.000
```

The factor loadings are substantial and appear to increase, very slightly, over time. Depression levels seems to decrease over time, indicated by the intercepts (means) of the LVs. Also, the LVs are substantially correlated over time.

b) We add a latent slope and intercept:

```
mod1 <- '
  dept1 =~ X1t1 + X2t1 + X3t1
  dept2 =~ X1t2 + X2t2 + X3t2
  dept3 =~ X1t3 + X2t3 + X3t3
  ## Freely estimate latent means, so latent depression levels should be
  ## allowed to vary over time:
  X1t1 ~ 0*1
  X1t2 ~ 0*1
  X1t3 ~ 0*1

  ## Add latent intercept and slope:
  latint =~ 1*dept1 + 1*dept2 + 1*dept3
  latslop =~ 1*dept2 + 2*dept3
  ## Freely estimate latent intercept and slope means:
  latint ~ NA*1
  latslop ~ NA*1
'
fit1 <- sem(mod1, data = data, estimator = "MLR")
fitmeasures(fit1, indices)
```

```
## chisq.scaled           df       pvalue   cfi.robust rmsea.robust         srmr
##       36.239       25.000        0.085        0.997        0.042        0.017
```

```r
anova(fit0, fit1)
```

```
## Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
##
## lavaan NOTE:
##     The "Chisq" column contains standard test statistics, not the
##     robust test that should be reported per model. A robust difference
##     test is a function of two standard (not robust) statistics.
##
##      Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit0 24 5622.5 5728.2 34.547
## fit1 25 5621.1 5723.2 35.154    0.60783       1     0.4356
```

Adding the latent intercept and slope yields a well-fitting model. As this and the earlier model involve the exact same set of observed variables, we can also do a $\Delta\chi^2$ test. The difference in model fit is not signficant and we therefore prefer the most parsimonious model (`fit1`).

```r
summary(fit1, standardized = TRUE)
```

```
## lavaan 0.6-6 ended normally after 80 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         29
##
##   Number of observations                           250
##
## Model Test User Model:
##                                               Standard      Robust
##   Test Statistic                                35.154      36.239
##   Degrees of freedom                                25          25
##   P-value (Chi-square)                           0.085       0.068
##   Scaling correction factor                                  0.970
##        Yuan-Bentler correction (Mplus variant)
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dept1 =~
##     X1t1              1.000                               1.028    0.896
##     X2t1              1.056    0.047   22.329    0.000    1.085    0.906
##     X3t1              1.142    0.053   21.418    0.000    1.173    0.922
##   dept2 =~
##     X1t2              1.000                               1.675    0.965
##     X2t2              0.986    0.026   37.230    0.000    1.651    0.959
##     X3t2              0.994    0.027   37.205    0.000    1.665    0.948
##   dept3 =~
##     X1t3              1.000                               2.731    0.987
##     X2t3              0.995    0.017   59.502    0.000    2.717    0.984
##     X3t3              0.987    0.016   60.261    0.000    2.697    0.981
```

```
##   latint =~
##     dept1             1.000                               0.873    0.873
##     dept2             1.000                               0.536    0.536
##     dept3             1.000                               0.329    0.329
##   latslop =~
##     dept2             1.000                               0.701    0.701
##     dept3             2.000                               0.859    0.859
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   latint ~~
##     latslop           0.108    0.097    1.108    0.268    0.102    0.102
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .X1t1              0.000                               0.000    0.000
##    .X1t2              0.000                               0.000    0.000
##    .X1t3              0.000                               0.000    0.000
##     latint            0.021    0.072    0.287    0.774    0.023    0.023
##     latslop          -0.643    0.085   -7.548    0.000   -0.548   -0.548
##    .X2t1             -0.056    0.046   -1.228    0.220   -0.056   -0.047
##    .X3t1             -0.066    0.047   -1.393    0.164   -0.066   -0.052
##    .X2t2             -0.000    0.044   -0.007    0.994   -0.000   -0.000
##    .X3t2             -0.074    0.047   -1.550    0.121   -0.074   -0.042
##    .X2t3              0.015    0.044    0.350    0.726    0.015    0.006
##    .X3t3              0.006    0.049    0.128    0.898    0.006    0.002
##    .dept1             0.000                               0.000    0.000
##    .dept2             0.000                               0.000    0.000
##    .dept3             0.000                               0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .X1t1              0.261    0.035    7.447    0.000    0.261    0.198
##    .X2t1              0.257    0.032    7.984    0.000    0.257    0.179
##    .X3t1              0.243    0.040    5.995    0.000    0.243    0.150
##    .X1t2              0.208    0.032    6.585    0.000    0.208    0.069
##    .X2t2              0.238    0.031    7.637    0.000    0.238    0.080
##    .X3t2              0.309    0.036    8.577    0.000    0.309    0.100
##    .X1t3              0.197    0.029    6.768    0.000    0.197    0.026
##    .X2t3              0.238    0.029    8.134    0.000    0.238    0.031
##    .X3t3              0.278    0.034    8.247    0.000    0.278    0.037
##    .dept1             0.250    0.126    1.995    0.046    0.237    0.237
##    .dept2             0.408    0.095    4.312    0.000    0.145    0.145
##    .dept3             0.713    0.335    2.130    0.033    0.096    0.096
##     latint            0.805    0.141    5.730    0.000    1.000    1.000
##     latslop           1.377    0.158    8.739    0.000    1.000    1.000
```

The mean (intercept) of the latent intercept is close to zero and non-significant, reflecting the mean of depression at the start of treatment. The mean (intercept) of the latent slope is negative and significant, indicating that depression levels decrease over time. The residual variance indicate that there is not a lot of item variance that is not explained by the depression LVs, and neither is there a lot of variance in the depression LVS that is not explained by the growth model (i.e., latent intercept and slope).

c) We add the treatment indicator:

```r
mod2 <- '
  dept1 =~ X1t1 + X2t1 + X3t1
  dept2 =~ X1t2 + X2t2 + X3t2
  dept3 =~ X1t3 + X2t3 + X3t3
  ## Freely estimate latent means, so latent depression levels should be
  ## allowed to vary over time:
  X1t1 ~ 0*1
  X1t2 ~ 0*1
  X1t3 ~ 0*1

  ## Add latent intercept and slope:
  latint =~ 1*dept1 + 1*dept2 + 1*dept3
  latslop =~ 1*dept2 + 2*dept3
  ## Freely estimate latent intercept and slope means:
  latint ~ NA*1
  latslop ~ NA*1

  ## Add treatment effect:
  latint ~ treatment
  latslop ~ treatment
'
fit2 <- sem(mod2, data = data, estimator = "MLR")
fitmeasures(fit2, indices)
```

```
## chisq.scaled           df       pvalue   cfi.robust rmsea.robust         srmr
##       42.564       32.000        0.118        0.997        0.036        0.017
```

```r
summary(fit2, standardized = TRUE)
```

```
## lavaan 0.6-6 ended normally after 69 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         31
##
##   Number of observations                           250
##
## Model Test User Model:
##                                               Standard      Robust
##   Test Statistic                                41.638      42.564
##   Degrees of freedom                                32          32
##   P-value (Chi-square)                           0.118       0.100
##   Scaling correction factor                                  0.978
##        Yuan-Bentler correction (Mplus variant)
##
## Parameter Estimates:
##
##   Standard errors                             Sandwich
##   Information bread                           Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   dept1 =~
```

```
##     X1t1              1.000                               1.028    0.896
##     X2t1              1.055    0.047   22.363    0.000    1.084    0.906
##     X3t1              1.142    0.053   21.388    0.000    1.174    0.922
##   dept2 =~
##     X1t2              1.000                               1.667    0.965
##     X2t2              0.987    0.027   37.226    0.000    1.646    0.959
##     X3t2              0.995    0.027   37.280    0.000    1.659    0.948
##   dept3 =~
##     X1t3              1.000                               2.739    0.987
##     X2t3              0.995    0.017   59.668    0.000    2.724    0.984
##     X3t3              0.987    0.016   60.537    0.000    2.703    0.982
##   latint =~
##     dept1             1.000                               0.872    0.872
##     dept2             1.000                               0.538    0.538
##     dept3             1.000                               0.327    0.327
##   latslop =~
##     dept2             1.000                               0.701    0.701
##     dept3             2.000                               0.853    0.853
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   latint ~
##     treatment        -0.006    0.131   -0.048    0.961   -0.007   -0.004
##   latslop ~
##     treatment        -0.826    0.152   -5.444    0.000   -0.708   -0.353
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .latint ~~
##    .latslop           0.112    0.095    1.176    0.240    0.114    0.114
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .X1t1              0.000                               0.000    0.000
##    .X1t2              0.000                               0.000    0.000
##    .X1t3              0.000                               0.000    0.000
##    .latint            0.024    0.093    0.257    0.797    0.027    0.027
##    .latslop          -0.209    0.104   -2.007    0.045   -0.179   -0.179
##    .X2t1             -0.056    0.046   -1.227    0.220   -0.056   -0.047
##    .X3t1             -0.066    0.047   -1.392    0.164   -0.066   -0.051
##    .X2t2              0.001    0.044    0.016    0.987    0.001    0.000
##    .X3t2             -0.073    0.047   -1.532    0.126   -0.073   -0.042
##    .X2t3              0.015    0.044    0.335    0.738    0.015    0.005
##    .X3t3              0.006    0.049    0.113    0.910    0.006    0.002
##    .dept1             0.000                               0.000    0.000
##    .dept2             0.000                               0.000    0.000
##    .dept3             0.000                               0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .X1t1              0.260    0.035    7.415    0.000    0.260    0.198
##    .X2t1              0.258    0.032    8.043    0.000    0.258    0.180
##    .X3t1              0.242    0.041    5.958    0.000    0.242    0.150
##    .X1t2              0.208    0.032    6.573    0.000    0.208    0.070
```

```
##     .X2t2                 0.237    0.031    7.609    0.000    0.237    0.080
##     .X3t2                 0.311    0.036    8.580    0.000    0.311    0.101
##     .X1t3                 0.198    0.029    6.788    0.000    0.198    0.026
##     .X2t3                 0.238    0.029    8.127    0.000    0.238    0.031
##     .X3t3                 0.277    0.034    8.281    0.000    0.277    0.037
##     .dept1                0.253    0.125    2.024    0.043    0.240    0.240
##     .dept2                0.386    0.092    4.180    0.000    0.139    0.139
##     .dept3                0.793    0.322    2.461    0.014    0.106    0.106
##     .latint               0.803    0.140    5.730    0.000    1.000    1.000
##     .latslop              1.193    0.144    8.308    0.000    0.875    0.875
```

The model still fits well. Note that we added an observed variable to the model, so we cannot compare model fit with a $\Delta\chi^2$ test.

The effect of treatment on the latent intercept is nearly zero and non-significant, indicating that the treatment groups do not differ in their initial levels of depression. This is a good sign: When we randomize patients to two treatment groups, the groups should not have different means at baseline.

The effect of treatment on the latent slope is significant and negative, indicating that treatment causes a decrease in depression over time. The intercept of the latent slope is negative, indicating that on average, depression levels also decrease for the control group, but this decrease is small.

Conclusion: treatment effectively reduces depression levels. The non-significant effect of treatment on the slope indicates the groups are similar at baseline. The significant negative effect of treatment on the slope indicates that the treatment group shows a stronger decrease in depression levels over time.