Exercises Basic CFAs

Additional exercise 1 (model parameter matrices)

a) For the standardized-latent-variable identification approach, think about how the describe how the β, Λ, Ψ and Θ matrices will differ from that of the marker identification approach, and how they will be similar. Use function `inspect()` with argument `what = "est"` to check.

b) For example 3.3 part III, write out the β, Λ, Ψ and Θ matrices. That is, write out the matrices for the two-factor model with correlated factors; the two-factor model in which one factor is regressed on the other; the one-factor model with correlated errors between two subtests.

Exercise 3.1

Umstattd-Meyer, Janke, and Beaujean (2013) measured poor psychosocial health as a single factor model using three items from a depression questionnaire and a measure of social activity. The covariance matrix (which contains four additional variables not used in the current exercise) is given below ($N = 6053$). (This matrix will be used for both Exercise 3.1 and 3.2):

```
Health.cov2 <- lav_matrix_lower2full(c(
   0.77,
   0.38, 0.65,
   0.39, 0.39, 0.62,
  -0.25,-0.32,-0.27, 6.09,
   0.31, 0.29, 0.26,-0.36, 7.67,
   0.24, 0.25, 0.19,-0.18, 0.51, 1.69,
  -3.16,-3.56,-2.63, 6.09,-3.12,-4.58,204.79,
  -0.92,-0.88,-0.72, 0.88,-1.49,-1.41, 16.53, 7.24
))
rownames(Health.cov2) <- colnames(Health.cov2) <- c("Dep1","Dep2","Dep3", "SocAct",
                                                     "Falls", "ChronCond", "PhysAct",
                                                     "PersMob")
```

a) Fit a single-factor model with the first four indicators. Use both the marker variable and standardized latent variable identification approaches. For the marker variable method, use `Dep1` as the marker variable. Check whether the resulting χ^2 and df are identical between the two methods.

b) Looking at the standardized loadings, do you think it is a good idea to combine the three depression indicators and Social Activity as a measure of poor psychosocial health?

Exercise 3.2

The model for Exercise 3.1 was part of a larger SEM, shown in Figure 3.11 of the Beaujean book.

a) Fit the SEM model in Figure 3.11 (see Beaujean book) to the data. Use the marker variable identification approach only. Evaluate and describe model fit. Make sure to add `fit.measures = TRUE` and `standardized = TRUE` to your call to `summary()` to obtain fit indices and standardized parameter estimates. Include both parameter estimates (e.g., significance and standardized values of loadings) as well as model fit indices (e.g., $\chi^2(df)$, RMSEA, CFI, SRMR) in your evaluation of model fit.

b) Are both psychosocial and physical health predictive of personal mobility? Describe their effects.

Additional exercise 2 (reflective vs. formative latent variables)

Below, code to construct a covariance matrix is provided. The sample size was $N = 500$. The models to be fitted to these data consist of three latent variables:

- Variables $X1$, $X2$ and $X3$ are indicators of a latent variable Stress.
- Variables $Y1$, $Y2$ and $Y3$ are indicators of a reflective latent variable Satisfaction.
- Variables $Y4$, $Y5$ and $Y6$ are indicators of a reflective latent variable Optimism.
- Satisfaction and Optimism should be regressed on Stress.

a) Fit a model to the data where Stress is a formative latent variable. (Hint: A formative latent variable is defined using `<~` instead of `=~`).

b) Fit a model to the data where Stress is a reflective latent variable.

c) Compare the values of the fit indices and the standardized loadings between the reflective and formative model. Based on these values, would you prefer the formative or reflective model?

```
## Input covariances:
cormat <- lav_matrix_lower2full(c(
  1.000,
  0.700,  1.000,
  0.713,  0.636,  1.000,
  0.079,  0.066,  0.076,  1.000,
  0.088,  0.058,  0.070,  0.681,  1.000,
  0.084,  0.056,  0.074,  0.712,  0.633,  1.000,
  0.279,  0.248,  0.240,  0.177,  0.155,  0.170,  1.000,
  0.250,  0.214,  0.222,  0.157,  0.143,  0.152,  0.373,  1.000,
  0.280,  0.236,  0.251,  0.173,  0.178,  0.171,  0.448,  0.344,  1.000
))

## Input standard deviations:
sds = c(2.5, 2.1, 3.0, 4.1, 3.9, 4.4, 1.2, 1.0, 1.2)

## Reconstruct covariance matrix from correlations and sds:
covmat <- diag(sds) %*% cormat %*% diag(sds)

## Assign row and column names:
rownames(covmat) <- colnames(covmat) <- c("Y1", "Y2", "Y3", "Y4", "Y5", "Y6",
                                          "X1", "X2", "X3")
```

Additional exercise 3 (ML vs. robust ML; model modifications)

Load the Holzinger and Swineford (1939) dataset, included in the lavaan package:

```
data(HolzingerSwineford1939)
```

This is a classic dataset with several scores on mental ability subtests, of 7th- and 8th-grade children. We use first six subtests (x1 - x6).

a) Fit a single factor model using robust ML estimation. Use function `summary()` to inspect the fitted model. Make sure to add `standardized = TRUE` and `fit.measures = TRUE` in your call to `summary()`. Does the model fit well (evaluate parameter estimates as well as model fit indices, evaluate the latter using the Hu & Bentler (1999) criteria)?

b) Compare the values of standard and robust chi-square values, CFI, RMSEA and SRMR.

c) Inspect parameter estimates, modification indices and residuals to create a better fitting model.

d) Does your new model fit well, according to the Hu & Bentler (1999) criteria?