Example 5.2: Latent growth curve models

Part 1: Growth curves only

```
library(foreign)
library(lavaan)
```

```
## This is lavaan 0.6-1
```

```
## lavaan is BETA software! Please report any bugs.
```

```
crime.data <- read.spss("nypa95_listwise.sav", to.data.frame = TRUE)
summary(crime.data)
```

```
##       ST2            STPLC2          POPDEN90           POV12590
##  Min.   :36.00   Min.   :3600199   Min.   :0.001401   Min.   : 0.7382
##  1st Qu.:36.00   1st Qu.:3654634   1st Qu.:0.227116   1st Qu.: 6.0320
##  Median :42.00   Median :4219160   Median :0.745823   Median :10.7491
##  Mean   :39.74   Mean   :4018959   Mean   :0.982708   Mean   :12.4804
##  3rd Qu.:42.00   3rd Qu.:4254940   3rd Qu.:1.392138   3rd Qu.:17.2683
##  Max.   :42.00   Max.   :4287272   Max.   :9.151260   Max.   :50.9599
##                                    NA's   :3          NA's   :3
##     JANFEB95         MARAPR95         MAYJUN95        JLYAUG95
##  Min.   : 2.415   Min.   : 2.473   Min.   :2.415   Min.   :2.370
##  1st Qu.: 4.664   1st Qu.: 4.842   1st Qu.:4.953   1st Qu.:5.047
##  Median : 5.215   Median : 5.371   Median :5.449   Median :5.580
##  Mean   : 5.166   Mean   : 5.317   Mean   :5.401   Mean   :5.515
##  3rd Qu.: 5.706   3rd Qu.: 5.804   3rd Qu.:5.901   3rd Qu.:6.060
##  Max.   :10.910   Max.   :10.260   Max.   :9.508   Max.   :9.567
##
##     CPDEN90            CPV12590             PA            CPAPOV90
##  Min.   :-0.88835   Min.   :-13.417   Min.   :0.0000   Min.   :-27.037
##  1st Qu.:-0.66263   1st Qu.: -8.123   1st Qu.:0.0000   1st Qu.:  0.000
##  Median :-0.14392   Median : -3.406   Median :1.0000   Median :  0.000
##  Mean   : 0.09296   Mean   : -1.675   Mean   :0.6241   Mean   :  1.380
##  3rd Qu.: 0.50239   3rd Qu.:  3.113   3rd Qu.:1.0000   3rd Qu.:  2.587
##  Max.   : 8.26151   Max.   : 36.805   Max.   :1.0000   Max.   : 90.813
##  NA's   :3          NA's   :3                          NA's   :3
```

```
crime.data <- na.omit(crime.data)
```

The dataset consists of crime-rates in 952 communities in states of New York and Pennsylvania. Crime rates were measured in four equidistant time periods: JANFEB95, MARAPR95, MAYJUN95,JLYAUG95. Three communities had some missing data and were removed prior to the analysis.

Before we start the analysis, take a look at the means and variances of the crime rate variables. Do you expect to see an in- or decrease in crime rates over time? Do you expect to see a lot of interindividual variation? Are there potential predictor variables that could explain initial levels and change in crime rates? (These are the 'questions to ask before creating a LGCM')

Let's fit a LGCM. First, we will re-label the four timepoints for convenience:

```
names(crime.data)[5:8] <- c("Time1", "Time2", "Time3", "Time4")

crime.model1 <- '
  # specify latent intercept:
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
```

```
  # set variance of latent intercept to zero:
  i ~~ 0*i
  # restrict residual variances to be equal at each timepoint:
  Time1 ~~ r*Time1
  Time2 ~~ r*Time2
  Time3 ~~ r*Time3
  Time4 ~~ r*Time4
'
crime.fit1 <- growth(crime.model1, data = crime.data)
summary(crime.fit1, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  19 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                    3443.747
##   Degrees of freedom                                12
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.000    0.000
##     Time2             1.000                               0.000    0.000
##     Time3             1.000                               0.000    0.000
##     Time4             1.000                               0.000    0.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                               0.000    0.000
##    .Time2             0.000                               0.000    0.000
##    .Time3             0.000                               0.000    0.000
##    .Time4             0.000                               0.000    0.000
##     i                 5.351    0.013  413.817    0.000      Inf      Inf
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     i                 0.000                                 NaN      NaN
##    .Time1      (r)    0.637    0.015   43.635    0.000    0.637    1.000
##    .Time2      (r)    0.637    0.015   43.635    0.000    0.637    1.000
##    .Time3      (r)    0.637    0.015   43.635    0.000    0.637    1.000
##    .Time4      (r)    0.637    0.015   43.635    0.000    0.637    1.000
```

```
indices <- c("chisq", "df", "pvalue", "cfi", "srmr", "rmsea", "aic")
fitmeasures(crime.fit1, indices)
```

```
##    chisq       df    pvalue      cfi      srmr     rmsea      aic
```

```
## 3443.747   12.000    0.000    0.000    0.522    0.548 9091.662
```

Note that any variation between communities or over time in this model is assumed to be residual error. There is no growth in this model, because there is only an intercept. The intercept has zero variance, so does not differ between observations (communities). This model does not fit well, according to all fit indices. Let's give the latent intercept a freely estimated variance, to allow for variation in crime rates between the different communities:

```
crime.model2 <- '
  # specify latent intercept with a freely estimated mean and variance
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # restrict residual variances to be equal at each timepoint
  Time1 ~~ r*Time1
  Time2 ~~ r*Time2
  Time3 ~~ r*Time3
  Time4 ~~ r*Time4
'
crime.fit2 <- growth(crime.model2, data = crime.data)
summary(crime.fit2, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  18 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                     563.978
##   Degrees of freedom                                11
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.693    0.868
##     Time2             1.000                               0.693    0.868
##     Time3             1.000                               0.693    0.868
##     Time4             1.000                               0.693    0.868
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                               0.000    0.000
##    .Time2             0.000                               0.000    0.000
##    .Time3             0.000                               0.000    0.000
##    .Time4             0.000                               0.000    0.000
##     i                 5.351    0.023  229.122    0.000    7.723    7.723
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1      (r)    0.157    0.004   37.789    0.000    0.157    0.246
##    .Time2      (r)    0.157    0.004   37.789    0.000    0.157    0.246
```

```
##    .Time3      (r)     0.157    0.004   37.789    0.000    0.157    0.246
##    .Time4      (r)     0.157    0.004   37.789    0.000    0.157    0.246
##     i                  0.480    0.024   20.153    0.000    1.000    1.000
```

```
fitmeasures(crime.fit2, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##  563.978   11.000    0.000    0.834    0.093    0.230 6213.892
```

By allowing for variation in crime rates between neighbourhood, the model fit is already much improved (but still not good). Also, the residual variances of each measurement occasion have decreased a lot, so this model already better explains the observed crime data. Of course, we were interested in growth (in- or decreases in crime), so let's introduce a latent slope in the model:

```
crime.model3 <- '
  # specify latent intercept:
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # specify latent slope:
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
  ## s ~  0*1
  # note that beaujean included last line in his syntax
  # but mean of latent slope should be freely estimated

  # set variance of latent slope to 0:
  s ~~ 0*s
  # beaujean did not put line above in syntax, but slope
  # should have zero variance
  s ~~ 0*i # slope is a constant, so should not correlate with intercept

  # residual variances:
  Time1 ~~ r*Time1
  Time2 ~~ r*Time2
  Time3 ~~ r*Time3
  Time4 ~~ r*Time4
'
crime.fit3 <- growth(crime.model3, data = crime.data)
summary(crime.fit3, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  25 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                     146.038
##   Degrees of freedom                                10
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
```

```
##     Time1             1.000                               0.697    0.884
##     Time2             1.000                               0.697    0.884
##     Time3             1.000                               0.697    0.884
##     Time4             1.000                               0.697    0.884
##   s =~
##     Time1             0.000                               0.000    0.000
##     Time2             1.000                               0.000    0.000
##     Time3             2.000                               0.000    0.000
##     Time4             3.000                               0.000    0.000
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i ~~
##     s                 0.000                                 NaN      NaN
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                               0.000    0.000
##    .Time2             0.000                               0.000    0.000
##    .Time3             0.000                               0.000    0.000
##    .Time4             0.000                               0.000    0.000
##     i                 5.181    0.025  209.892    0.000    7.437    7.437
##     s                 0.113    0.005   21.215    0.000      Inf      Inf
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     s                 0.000                                 NaN      NaN
##    .Time1      (r)    0.135    0.004   37.789    0.000    0.135    0.218
##    .Time2      (r)    0.135    0.004   37.789    0.000    0.135    0.218
##    .Time3      (r)    0.135    0.004   37.789    0.000    0.135    0.218
##    .Time4      (r)    0.135    0.004   37.789    0.000    0.135    0.218
##     i                 0.485    0.024   20.382    0.000    1.000    1.000
```

```r
fitmeasures(crime.fit3, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##  146.038   10.000    0.000    0.959    0.034    0.120 5797.952
```

Again, fit has improved a lot. Crime rates increase, on average: the mean of the latent slope is .113. Again, residual variances decreased compared to the earlier model. Let's allow the change in crime rates to differ between neighbourhoods, by freely estimating the variance of the latent slope:

```r
crime.model4 <- '
  # define latent intercept:
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # define latent slope:
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
  # residual variances:
  Time1 ~~ r*Time1
  Time2 ~~ r*Time2
  Time3 ~~ r*Time3
  Time4 ~~ r*Time4
'
crime.fit4 <- growth(crime.model4, data = crime.data)
summary(crime.fit4, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  47 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      24.540
##   Degrees of freedom                                 8
##   P-value (Chi-square)                           0.002
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.722    0.912
##     Time2             1.000                               0.722    0.933
##     Time3             1.000                               0.722    0.927
##     Time4             1.000                               0.722    0.897
##   s =~
##     Time1             0.000                               0.000    0.000
##     Time2             1.000                               0.132    0.171
##     Time3             2.000                               0.265    0.340
##     Time4             3.000                               0.397    0.493
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i ~~
##     s                -0.023    0.005   -4.273    0.000   -0.239   -0.239
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                               0.000    0.000
##    .Time2             0.000                               0.000    0.000
##    .Time3             0.000                               0.000    0.000
##    .Time4             0.000                               0.000    0.000
##     i                 5.181    0.025  207.075    0.000    7.173    7.173
##     s                 0.113    0.006   17.735    0.000    0.855    0.855
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1      (r)    0.106    0.003   30.854    0.000    0.106    0.169
##    .Time2      (r)    0.106    0.003   30.854    0.000    0.106    0.177
##    .Time3      (r)    0.106    0.003   30.854    0.000    0.106    0.175
##    .Time4      (r)    0.106    0.003   30.854    0.000    0.106    0.164
##     i                 0.522    0.027   19.024    0.000    1.000    1.000
##     s                 0.017    0.002    9.190    0.000    1.000    1.000
```

```r
fitmeasures(crime.fit4, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##   24.540    8.000    0.002    0.995    0.019    0.047 5680.455
```

Again, we see an improvement in model fit. The model fits well, according to all fit indices. In addition, the residual variances have decreased again compated to the earlier model, indicating that the crime rates are better explained by this model.

Short cut through growth curve models 1 through 4

Note that fitting models 1, 2 and 3 above is not really necessary: we may also directly fit model 4 and check whether the mean and variance of the latent intercept and the mean and variance of the latent slope are substantial and signficiant. If not, they can be omitted from the model.

If the variance of the latent intercept is not significant, there are no interindividual differences on the first measurement occasion ('baseline'); in the example, that would mean that all neighbourhoods have the same starting crime rate. If the mean of the latent slope is not signficiant, there is no change over time, on average; that is, no in- or decrease on average. If the variance of the latent slope is not significant, there are no intra-individual differences in growth.

Equality of residual variances

In the fourth model, all residual variances are restricted to be equal across timepoints. This makes sense from a substantial point of view: If we use the same measure at each timepoint, we can expect the measurement error to be the same at each timepoint. But let's see whether a model with different residual variances fits better:

```
crime.model5 <- '
  # define latent intercept:
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # define latent slope:
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
'
crime.fit5 <- growth(crime.model5, data = crime.data)
summary(crime.fit5, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  49 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                       8.277
##   Degrees of freedom                                 5
##   P-value (Chi-square)                           0.142
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                                0.719    0.907
##     Time2             1.000                                0.719    0.926
##     Time3             1.000                                0.719    0.943
##     Time4             1.000                                0.719    0.881
```

```
##   s =~
##     Time1             0.000                                     0.000    0.000
##     Time2             1.000                                     0.120    0.155
##     Time3             2.000                                     0.241    0.315
##     Time4             3.000                                     0.361    0.442
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i ~~
##     s                -0.020    0.006   -3.401    0.001   -0.227   -0.227
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                                0.000    0.000
##    .Time2             0.000                                0.000    0.000
##    .Time3             0.000                                0.000    0.000
##    .Time4             0.000                                0.000    0.000
##     i                 5.181    0.025  207.046    0.000    7.201    7.201
##     s                 0.113    0.006   17.824    0.000    0.941    0.941
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.111    0.010   10.665    0.000    0.111    0.177
##    .Time2             0.111    0.007   15.997    0.000    0.111    0.184
##    .Time3             0.085    0.006   14.095    0.000    0.085    0.147
##    .Time4             0.138    0.011   12.531    0.000    0.138    0.206
##     i                 0.518    0.028   18.608    0.000    1.000    1.000
##     s                 0.014    0.002    6.404    0.000    1.000    1.000
```

```r
fitmeasures(crime.fit5, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##    8.277    5.000    0.142    0.999    0.011    0.026 5670.192
```

This is the best-fitting model, according to all fit indices.

We see that at each timepoint, about $.9^2 = .81$, (.9 being a rough estimate of the average standardized loading of the latent intercept) 81% of the variance in crime rates can be attributed to differences between communities. The growth in every two months explains about $.16^2 = .0256$, (.16 being a rough estimate of the two-monthly increase in standardized loadings of the latent slope) 2.6% of variance in observed crime rates.

Part 2: LGCM with covariates

With the earlier model, we could evaluate whether there were differences between observations and over time. Of course, it would be much more interesting to find out if there are predictors of the latent intercept and slope: whether other variables can explain (variation in) baseline levels and growth.

We add the variables state and poverty, and their interaction to the best-fitting crime rate growth model we found earlier. We start with adding PA (a dummy coded variable indicating whether the community was in Pennsylvania), to see if state is a predictor of crime rates:

```r
crime.model6 <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope
```

```r
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
  # regression
  i ~ PA
  s ~ PA
'
crime.fit6 <- growth(crime.model6, data = crime.data)
summary(crime.fit6, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  50 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                       9.726
##   Degrees of freedom                                 7
##   P-value (Chi-square)                           0.205
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.720    0.908
##     Time2             1.000                               0.720    0.926
##     Time3             1.000                               0.720    0.943
##     Time4             1.000                               0.720    0.881
##   s =~
##     Time1             0.000                               0.000    0.000
##     Time2             1.000                               0.121    0.155
##     Time3             2.000                               0.241    0.316
##     Time4             3.000                               0.362    0.443
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i ~
##     PA               -0.251    0.051   -4.925    0.000   -0.349   -0.169
##   s ~
##     PA               -0.047    0.013   -3.614    0.000   -0.390   -0.189
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .i ~~
##    .s                -0.023    0.006   -3.945    0.000   -0.269   -0.269
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                               0.000    0.000
##    .Time2             0.000                               0.000    0.000
##    .Time3             0.000                               0.000    0.000
##    .Time4             0.000                               0.000    0.000
```

```
##    .i                 5.337    0.040  132.638    0.000    7.416    7.416
##    .s                 0.143    0.010   13.876    0.000    1.182    1.182
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.110    0.010   10.636    0.000    0.110    0.176
##    .Time2             0.111    0.007   15.995    0.000    0.111    0.184
##    .Time3             0.086    0.006   14.221    0.000    0.086    0.147
##    .Time4             0.137    0.011   12.606    0.000    0.137    0.206
##    .i                 0.503    0.027   18.532    0.000    0.971    0.971
##    .s                 0.014    0.002    6.265    0.000    0.964    0.964
```

```
fitmeasures(crime.fit6, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##    9.726    7.000    0.205    0.999    0.010    0.020 5615.847
```

The model fits the data well. The negative regression coefficients indicate that in Pennsylvania, there is less crime at baseline, and less increase in crime rate over time. Note also that models 5 and 6 are not nested, so difference in model fit cannot be statistically tested. Let's also add poverty (CPV12590, representing the percentage of the population living in poverty):

```
crime.model7 <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
  # regressions
  i ~ PA + CPV12590
  s ~ PA + CPV12590
'
crime.fit7 <- growth(crime.model7, data = crime.data)
summary(crime.fit7, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  59 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      10.208
##   Degrees of freedom                                 9
##   P-value (Chi-square)                           0.334
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.720    0.907
##     Time2             1.000                               0.720    0.926
##     Time3             1.000                               0.720    0.943
##     Time4             1.000                               0.720    0.881
```

```
##   s =~
##     Time1             0.000                                      0.000    0.000
##     Time2             1.000                                      0.120    0.155
##     Time3             2.000                                      0.241    0.315
##     Time4             3.000                                      0.361    0.442
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i ~
##     PA               -0.246    0.049   -5.071    0.000   -0.342   -0.166
##     CPV12590          0.029    0.003    9.933    0.000    0.041    0.325
##   s ~
##     PA               -0.047    0.013   -3.645    0.000   -0.393   -0.191
##     CPV12590         -0.002    0.001   -2.207    0.027   -0.014   -0.115
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .i ~~
##    .s                -0.019    0.005   -3.508    0.000   -0.245   -0.245
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                                0.000    0.000
##    .Time2             0.000                                0.000    0.000
##    .Time3             0.000                                0.000    0.000
##    .Time4             0.000                                0.000    0.000
##    .i                 5.383    0.039  139.537    0.000    7.482    7.482
##    .s                 0.140    0.010   13.543    0.000    1.162    1.162
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.111    0.010   10.774    0.000    0.111    0.176
##    .Time2             0.111    0.007   16.048    0.000    0.111    0.183
##    .Time3             0.086    0.006   14.246    0.000    0.086    0.147
##    .Time4             0.137    0.011   12.622    0.000    0.137    0.206
##    .i                 0.448    0.025   18.158    0.000    0.866    0.866
##    .s                 0.014    0.002    6.184    0.000    0.951    0.951
```

```r
fitmeasures(crime.fit7, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##   10.208    9.000    0.334    1.000    0.009    0.012 5523.179
```

The model fits well. We see that higher poverty rates result in higher average crime rates (positive effect on intercept) and in a decline of crime rates (negative effect on slope, but this seems to be a small effect).

Let's assess the interaction effect of state and povery on crime rates:

```r
# create stateXpoverty interaction variable (PA is a dummy, CPV12590 is already centered):
crime.data$STATExPOV <- crime.data$PA * crime.data$CPV12590

crime.model8 <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
```

```
  # regression
  s ~ PA + CPV12590 + STATExPOV
  i ~ PA + CPV12590 + STATExPOV
'
crime.fit8 <- growth(crime.model8, data = crime.data)
summary(crime.fit8, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  69 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                      11.711
##   Degrees of freedom                                11
##   P-value (Chi-square)                           0.386
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.720    0.908
##     Time2             1.000                               0.720    0.926
##     Time3             1.000                               0.720    0.942
##     Time4             1.000                               0.720    0.881
##   s =~
##     Time1             0.000                               0.000    0.000
##     Time2             1.000                               0.120    0.155
##     Time3             2.000                               0.241    0.316
##     Time4             3.000                               0.361    0.443
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   s ~
##     PA               -0.049    0.013   -3.734    0.000   -0.411   -0.199
##     CPV12590         -0.001    0.001   -0.626    0.531   -0.007   -0.056
##     STATExPOV        -0.001    0.002   -0.801    0.423   -0.011   -0.073
##   i ~
##     PA               -0.266    0.049   -5.373    0.000   -0.369   -0.179
##     CPV12590          0.037    0.005    7.360    0.000    0.052    0.414
##     STATExPOV        -0.012    0.006   -1.952    0.051   -0.017   -0.111
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .i ~~
##    .s                -0.020    0.005   -3.564    0.000   -0.249   -0.249
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                               0.000    0.000
```

```
##    .Time2             0.000                               0.000    0.000
##    .Time3             0.000                               0.000    0.000
##    .Time4             0.000                               0.000    0.000
##    .i                 5.396    0.039  138.207    0.000    7.499    7.499
##    .s                 0.141    0.010   13.494    0.000    1.172    1.172
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.111    0.010   10.766    0.000    0.111    0.176
##    .Time2             0.111    0.007   16.046    0.000    0.111    0.183
##    .Time3             0.086    0.006   14.274    0.000    0.086    0.148
##    .Time4             0.137    0.011   12.601    0.000    0.137    0.205
##    .i                 0.446    0.025   18.145    0.000    0.862    0.862
##    .s                 0.014    0.002    6.191    0.000    0.949    0.949
```

```
fitmeasures(crime.fit8, indices)
```

```
##    chisq       df   pvalue      cfi     srmr    rmsea      aic
##   11.711   11.000    0.386    1.000    0.008    0.008 5520.533
```

The interaction is not significant. As the sample size is not small (N = 952), that gives us reason to conclude that there is no interaction effect of poverty and state on crime rates. This means that the effect of poverty does is not different for communities within and outside of Pennsylvania. We keep model 7 as the best model for crime rate growth, and stick to the interpretation supplied there.

Alternative specifications of LGCMs

As a researcher, you have to decide on the effect (or scale) of time on the outcome variable, in LGCMs. In the example, we have assumed the difference between the first and second timepoint to be equal to the difference between the second and third timepoint, for example. And we assumed the first timepoint to be the 'starting point'. The researcher decided on this, by definining the slope. Other options would be:

Code the intercept to not be the first timepoint:

```
model <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope
  s =~ -3*Time1 + -2*Time2 + -1*Time3 + 0*Time4
'
```

Two units as the time between data collection periods:

```
model <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope
  s =~ 0*Time1 + 2*Time2 + 4*Time3 + 6*Time4
'
```

This will get the exact same estimates, but the mean of the latent slope will be divided by 2, and the variance would be divided by $2^2 = 4$.

Non-equidistant timpoints:

For example, we can have the first three timepoints one month apart, and the fourth timepoint 4 months later:

```
model <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope
  s =~ 0*Time1 + 1*Time2 + 2*Time3 + 6*Time4
'
```

Quadratic growth

Up to now, we have assumed the growth to be linear. But the effect of time may be expected to increase quadratically. Then, we need to define a second latent slope, which loadings are the square of the first latent slope's loadings:

```
model.quad <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4
  # slope 1
  s1 =~ 0*Time1 + 1*Time2 + 2*Time3 + 3*Time4
  # slope 2
  s2 =~ 0*Time1 + 1*Time2 + 4*Time3 + 9*Time4
'
crime.fit.quad <- growth(model.quad, data = crime.data)
summary(crime.fit.quad, standardized = TRUE)
```

```
## lavaan (0.6-1) converged normally after  66 iterations
##
##   Number of observations                           952
##
##   Estimator                                         ML
##   Model Fit Test Statistic                       4.778
##   Degrees of freedom                                 1
##   P-value (Chi-square)                           0.029
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard Errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i =~
##     Time1             1.000                               0.723    0.909
##     Time2             1.000                               0.723    0.935
##     Time3             1.000                               0.723    0.948
##     Time4             1.000                               0.723    0.883
##   s1 =~
##     Time1             0.000                               0.000    0.000
##     Time2             1.000                               0.142    0.184
##     Time3             2.000                               0.285    0.374
##     Time4             3.000                               0.427    0.522
```

```
##   s2 =~
##     Time1             0.000                                   0.000    0.000
##     Time2             1.000                                   0.039    0.051
##     Time3             4.000                                   0.156    0.205
##     Time4             9.000                                   0.352    0.429
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   i ~~
##     s1               -0.027    0.040   -0.675    0.500   -0.260   -0.260
##     s2                0.002    0.010    0.253    0.800    0.088    0.088
##   s1 ~~
##     s2               -0.003    0.009   -0.341    0.733   -0.537   -0.537
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.000                                0.000    0.000
##    .Time2             0.000                                0.000    0.000
##    .Time3             0.000                                0.000    0.000
##    .Time4             0.000                                0.000    0.000
##     i                 5.172    0.026  201.593    0.000    7.155    7.155
##     s1                0.139    0.017    8.195    0.000    0.976    0.976
##     s2               -0.009    0.005   -1.623    0.105   -0.225   -0.225
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .Time1             0.110    0.037    3.015    0.003    0.110    0.174
##    .Time2             0.108    0.012    8.760    0.000    0.108    0.180
##    .Time3             0.088    0.012    7.358    0.000    0.088    0.151
##    .Time4             0.119    0.038    3.167    0.002    0.119    0.178
##     i                 0.523    0.043   12.039    0.000    1.000    1.000
##     s1                0.020    0.040    0.504    0.614    1.000    1.000
##     s2                0.002    0.002    0.657    0.511    1.000    1.000
```

The mean and variance of the second (quadratic) slope are not signficiant, indicating that there is no quadratic growth, in addition to linear growth, of crime rates.

Piecewise linear slopes

We could have a specific theory about the growth. For example, that it increases linearly up to a specific point in time, and starts to decrease linearly after that point. Then, we would need to specify two latent slopes (one for the increase and one for the decrese):

```
model.piece <- '
  # intercept
  i =~ 1*Time1 + 1*Time2 + 1*Time3 + 1*Time4 + 1*Time5 + 1*Time6
  # slope 1
  s1 =~ -1*Time1 + -2*Time2 + -1*Time3 + 0*Time4 + 0*Time5 + 0*Time6
  # slope 2
  s2 =~ 0*Time1 + 0*Time2 + 0*Time3 + 0*Time4 + 1*Time5 + 2*Time6
'
```