Miscellaneous Problems

Dealing with missing data

We will analyse the Holzinger Swineford data included in the lavaan package.

```
library("lavaan")
summary(HolzingerSwineford1939)
```

```
##
           id
                           sex
                                            ageyr
                                                          agemo
##
    Min.
              1.0
                             :1.000
                                                             : 0.000
                     Min.
                                       Min.
                                               :11
                                                     Min.
    1st Qu.: 82.0
                     1st Qu.:1.000
                                       1st Qu.:12
                                                     1st Qu.: 2.000
##
    Median :163.0
                     Median :2.000
                                       Median:13
                                                     Median : 5.000
##
    Mean
            :176.6
                     Mean
                             :1.515
                                       Mean
                                               :13
                                                     Mean
                                                             : 5.375
##
    3rd Qu.:272.0
                     3rd Qu.:2.000
                                       3rd Qu.:14
                                                      3rd Qu.: 8.000
##
    Max.
            :351.0
                     Max.
                             :2.000
                                       Max.
                                               :16
                                                     Max.
                                                             :11.000
##
                            grade
##
             school
                                                x1
                                                                  x2
##
    Grant-White: 145
                       Min.
                               :7.000
                                         Min.
                                                 :0.6667
                                                            Min.
                                                                    :2.250
##
    Pasteur
                :156
                        1st Qu.:7.000
                                         1st Qu.:4.1667
                                                            1st Qu.:5.250
                       Median :7.000
                                                            Median :6.000
##
                                         Median :5.0000
##
                       Mean
                               :7.477
                                         Mean
                                                 :4.9358
                                                                    :6.088
                                                            Mean
##
                        3rd Qu.:8.000
                                         3rd Qu.:5.6667
                                                            3rd Qu.:6.750
##
                       Max.
                               :8.000
                                         Max.
                                                 :8.5000
                                                            Max.
                                                                    :9.250
##
                        NA's
                               :1
##
           xЗ
                                              x5
                                                               x6
                            x4
##
    Min.
            :0.250
                     Min.
                             :0.000
                                       Min.
                                               :1.000
                                                         Min.
                                                                 :0.1429
                     1st Qu.:2.333
                                                         1st Qu.:1.4286
##
    1st Qu.:1.375
                                       1st Qu.:3.500
##
    Median :2.125
                     Median :3.000
                                       Median :4.500
                                                         Median :2.0000
##
    Mean
            :2.250
                     Mean
                             :3.061
                                       Mean
                                               :4.341
                                                         Mean
                                                                 :2.1856
##
    3rd Qu.:3.125
                     3rd Qu.:3.667
                                       3rd Qu.:5.250
                                                         3rd Qu.:2.7143
            :4.500
                             :6.333
                                               :7.000
##
    Max.
                     Max.
                                                                 :6.1429
                                       Max.
                                                         Max.
##
##
           x7
                            x8
                                               x9
##
    Min.
            :1.304
                     Min.
                             : 3.050
                                        Min.
                                                :2.778
##
    1st Qu.:3.478
                     1st Qu.: 4.850
                                        1st Qu.:4.750
##
    Median :4.087
                     Median : 5.500
                                        Median :5.417
            :4.186
                                                :5.374
##
    Mean
                     Mean
                             : 5.527
                                        Mean
##
    3rd Qu.:4.913
                     3rd Qu.: 6.100
                                        3rd Qu.:6.083
##
    Max.
            :7.435
                     Max.
                             :10.000
                                        Max.
                                                :9.250
##
```

We will fit a three-factor CFA model to the x variables in the dataset:

```
HS.model <- '
visual =~ x1 + x2 + x3
textual =~ x4 + x5 + x6
speed =~ x7 + x8 + x9
visual ~ 0*1
```

```
CD_fit <- cfa(HS.model, data = HolzingerSwineford1939, meanstructure = TRUE)
#summary(CD_fit, standardized = TRUE)
fit.inds <- c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr", "aic", "bic")
fitmeasures(CD_fit, fit.inds)</pre>
```

Benchmark: Complete data

```
##
      chisq
                    df
                         pvalue
                                      cfi
                                                                              bic
                                              rmsea
                                                         srmr
                                                                    aic
##
     85.306
               24.000
                          0.000
                                    0.931
                                              0.092
                                                        0.060 7535.490 7646.703
```

Generate missingness We introduce some missing data. The values will be missing completely at random, with a probability of .3 for any value being missing:

```
HSMiss <- HolzingerSwineford1939[,paste("x", 1:9, sep="")]</pre>
set.seed(42)
randomMiss <- rbinom(prod(dim(HSMiss)), 1, 0.20)</pre>
randomMiss <- matrix(as.logical(randomMiss), nrow=nrow(HSMiss))</pre>
HSMiss[randomMiss] <- NA</pre>
head(HSMiss)
##
           x1
                x2
                       x3
                                 x4
                                      x5
                                                 x6
                                                          x7
                                                               x8
                                                                         x9
## 1
           NA 7.75 0.375 2.333333
                                      NA 1.2857143 3.391304
                                                                NA
                                                                         NA
## 2
           NA 5.25 2.125 1.666667
                                      NA 1.2857143 3.782609 6.25 7.916667
## 3 4.500000 5.25 1.875
                                 NA 1.75 0.4285714
                                                          NA 3.90
## 4
           NA
                NA 3.000 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
## 5 4.833333
                NA 0.875 2.666667 4.00 2.5714286 3.695652
## 6 5.333333 5.00 2.250 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
LD_fit <- cfa(HS.model, data = HSMiss, meanstructure = TRUE)
```

Listwise deletion approach

##

22.454

24.000

0.552

1.000

```
## Warning in lav_object_post_check(object): lavaan WARNING: covariance matrix of latent variables
##
                   is not positive definite;
##
                   use lavInspect(fit, "cov.lv") to investigate.
lavInspect(LD_fit, "cov.lv")
##
           visual textul speed
## visual
          0.503
## textual 0.774 1.365
## speed
           0.152 0.061 0.118
#summary(LD_fit, standardized = TRUE)
fitmeasures(LD_fit, fit.inds)
##
      chisq
                  df
                       pvalue
                                    cfi
                                           rmsea
                                                     srmr
                                                               aic
                                                                        bic
```

Multiple imputation approach We now impute the data using package mice. We use generate five imputed datasets and use the predictive mean matching method, which is (a.f.a.i.k.) the current state of the art in missing data imputation:

0.000

0.076 1298.143 1355.503

```
library("mice")
m <- 5</pre>
```

```
set.seed(42)
imp_data <- mice(HSMiss, m = m, method = "pmm")</pre>
##
##
    iter imp variable
##
     1
         1
            x1
                x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                               x9
##
         2
            x1
                 x2
                         x4
                                  x6
                                      x7
                                          x8
                                               x9
     1
                     x3
                             x5
##
         3
            x1
                 x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                               x9
##
         4
                     xЗ
                                      x7
                x2
                         x4
                             x5
                                 x6
                                          x8
                                              x9
     1
            x1
##
     1
         5
            x1
                 x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          8x
                                               x9
##
     2
         1
            x1
                x2
                     x3
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                               x9
##
     2
         2
                x2
                     xЗ
            x1
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                              x9
##
     2
         3
                x2
                         x4
                     xЗ
                                  x6
                                          x8
                                               x9
            x1
                             x5
                                      x7
##
     2
         4
                x2
                     xЗ
                         x4
                                  x6
                                      x7
                                          8x
                                               x9
            x1
                             x5
         5
                             x5
##
     2
                x2
                                  x6
                                               x9
            x1
                     xЗ
                         x4
                                      x7
                                          8x
##
     3
         1
            x1
                x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          8x
                                              x9
##
     3
         2
                x2
                     xЗ
                                  x6
                                              x9
            x1
                         x4
                             x5
                                      x7
                                          x8
     3
##
         3
            x1
                x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                               x9
##
     3
         4
                                               x9
            x1
                 x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          8x
                                          x8
##
     3
         5
                x2
                     xЗ
                         x4
                                  x6
                                              x9
            x1
                             x5
                                      x7
##
     4
         1
            x1
                 x2
                     x3
                         x4
                             x5
                                  x6
                                      x7
                                          8x
                                               x9
##
     4
         2
            x1
                x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                               x9
         3
                                               x9
##
     4
            x1
                 x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          x8
##
     4
         4
                x2
                     xЗ
                                  x6
                                              x9
            x1
                         x4
                             x5
                                      x7
                                          8x
##
     4
         5
            x1
                 x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          8x
                                               x9
                                              x9
##
                         x4
     5
         1
            x1
                x2
                     xЗ
                             x5
                                  x6
                                      x7
                                          x8
##
         2
                x2
                     xЗ
                         x4
                                          x8
                                               x9
            x1
                             x5
                                  x6
                                      x7
         3 x1
##
     5
                x2
                     xЗ
                         x4
                             x5
                                  x6
                                      x7
                                          x8
                                              x9
##
     5
         4
                 x2
                     xЗ
                         x4
                                  x6
                                      x7
                                          x8
                                               x9
            x1
                             x5
##
     5
         5
                x2
                         x4
                                              x9
            x1
                     xЗ
                             x5
                                  x6
                                          x8
                                      x7
We extract the imputed datasets using function complete() and save them in a list:
data_list <- list()</pre>
for (i in 1:m) data list[[i]] <- complete(imp data, action = i)</pre>
lapply(data_list, head)
## [[1]]
##
           x1
                 x2
                       xЗ
                                 x4
                                      x5
                                                 x6
                                                           x7
                                                                x8
                                                                         x9
## 1 3.833333 7.75 0.375 2.333333 4.75 1.2857143 3.391304 5.35 3.777778
## 2 5.333333 5.25 2.125 1.666667 3.25 1.2857143 3.782609 6.25 7.916667
## 3 4.500000 5.25 1.875 1.333333 1.75 0.4285714 3.173913 3.90 3.611111
## 4 5.333333 5.25 3.000 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
## 5 4.833333 6.25 0.875 2.666667 4.00 2.5714286 3.695652 6.20 5.916667
## 6 5.333333 5.00 2.250 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
##
## [[2]]
##
                 x2
                       xЗ
                                      x5
                                                                x8
                                                                         x9
           x1
                                 x4
                                                 x6
                                                           x7
## 1 3.833333 7.75 0.375 2.333333 4.50 1.2857143 3.391304 5.00 4.833333
## 2 4.833333 5.25 2.125 1.666667 4.25 1.2857143 3.782609 6.25 7.916667
## 3 4.500000 5.25 1.875 2.000000 1.75 0.4285714 3.043478 3.90 3.472222
## 4 4.166667 8.00 3.000 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
## 5 4.833333 6.00 0.875 2.666667 4.00 2.5714286 3.695652 4.85 5.916667
## 6 5.333333 5.00 2.250 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
##
```

```
## 2 4.666667 5.25 2.125 1.666667 4.00 1.2857143 3.782609 6.25 7.916667
## 3 4.500000 5.25 1.875 1.666667 1.75 0.4285714 1.869565 3.90 3.472222
## 4 3.166667 5.00 3.000 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
## 5 4.833333 6.25 0.875 2.666667 4.00 2.5714286 3.695652 6.95 5.916667
## 6 5.333333 5.00 2.250 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
##
## [[4]]
##
                x2
                      xЗ
                                x4
                                     x5
                                               x6
                                                         x7
                                                              x8
                                                                       x9
           x1
## 1 4.000000 7.75 0.375 2.333333 3.00 1.2857143 3.391304 3.90 3.333333
## 2 4.666667 5.25 2.125 1.666667 3.00 1.2857143 3.782609 6.25 7.916667
## 3 4.500000 5.25 1.875 1.666667 1.75 0.4285714 2.434783 3.90 4.833333
## 4 6.000000 5.75 3.000 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
## 5 4.833333 6.50 0.875 2.666667 4.00 2.5714286 3.695652 5.45 5.916667
## 6 5.333333 5.00 2.250 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
##
## [[5]]
##
           x1
                x2
                      xЗ
                                x4
                                     x5
                                               x6
                                                         x7
                                                                       x9
## 1 4.833333 7.75 0.375 2.333333 5.25 1.2857143 3.391304 3.80 4.777778
## 2 5.833333 5.25 2.125 1.666667 3.25 1.2857143 3.782609 6.25 7.916667
## 3 4.500000 5.25 1.875 2.000000 1.75 0.4285714 2.652174 3.90 3.472222
## 4 5.833333 7.75 3.000 2.666667 4.50 2.4285714 3.000000 5.30 4.861111
## 5 4.833333 5.75 0.875 2.666667 4.00 2.5714286 3.695652 5.85 5.916667
## 6 5.333333 5.00 2.250 1.000000 3.00 0.8571429 4.347826 6.65 7.500000
We see that the missing values have been imputed with different values in every dataset.
Now we use the cfa.mi() function to fit a CFA model on the imputed data:
library("semTools")
MI_fit <- cfa.mi(HS.model, data_list, meanstructure = TRUE)
summ_MI_fit <- summary(MI_fit)</pre>
## lavaan.mi object based on 5 imputed data sets.
## See class?lavaan.mi help page for available methods.
##
## Convergence information:
## The model converged on 5 imputed data sets
##
## Rubin's (1987) rules were used to pool point and SE estimates across 5 imputed data sets, and to cal
tmp <- fitmeasures(MI_fit)</pre>
round(tmp[fit.inds], digits = 3L)
##
                  df
                       pvalue
                                    cfi
                                                                aic
                                                                         bic
      chisa
                                           rmsea
                                                      srmr
##
     39.988
              24.000
                        0.021
                                  0.965
                                           0.047
                                                     0.051 7522.817 7634.030
FIML_fit <- cfa(HS.model, data = HSMiss, missing = "fiml")</pre>
#summary(FIML_fit)
fitmeasures(LD_fit, fit.inds)
```

Full information Maximum Likelihood (FIML)

[[3]]

x2

x1

x3

x5

1 3.500000 7.75 0.375 2.333333 4.00 1.2857143 3.391304 6.05 3.277778

x6

x4

x8

_x7

x9

```
## chisq df pvalue cfi rmsea srmr aic bic
## 22.454 24.000 0.552 1.000 0.000 0.076 1298.143 1355.503
```

Comparison of methods

We compare parameter estimates and standard errors between the complete dataset, listwise deletion, multiple imputation and FIML:

```
##
                     rhs LD.est LD.se MI.est MI.se FIML.est FIML.se CD.est CD.se
          lhs op
## 2
       visual =~
                          0.660 0.226 0.471 0.113
                                                        0.482
                                                                0.113
                                                                      0.554 0.100
## 3
                          0.558 0.236
                                                        0.633
                                                                       0.729 0.109
       visual =~
                      xЗ
                                        0.648 0.117
                                                                0.115
## 5
      textual =~
                      x5
                          0.965 0.131
                                        1.137 0.085
                                                        1.154
                                                                0.083
                                                                       1.113 0.065
                      x6
                                                        0.951
                                                                0.069
## 6
      textual =~
                          0.764 0.111
                                        0.968 0.072
                                                                       0.926 0.055
## 8
        speed =~
                      8x
                          1.289 0.603
                                        1.090 0.222
                                                        1.122
                                                                0.209
                                                                       1.180 0.165
## 9
        speed =~
                      x9
                          2.796 1.465
                                        1.419 0.303
                                                        1.586
                                                                0.410
                                                                       1.082 0.151
## 11
           x1 ~~
                          1.317 0.304
                                        0.476 0.142
                                                        0.477
                                                                0.140
                                                                       0.549 0.114
                      x1
## 12
           x2 ~~
                      x2 0.771 0.168
                                        1.191 0.128
                                                        1.179
                                                                0.119 1.134 0.102
## 13
           x3 ~~
                          1.059 0.216
                                        0.824 0.103
                                                        0.852
                                                                0.100 0.844 0.091
                      xЗ
## 14
           x4 ~~
                      x4
                          0.144 0.111
                                        0.329 0.054
                                                        0.346
                                                                0.056 0.371 0.048
## 15
           x5 ~~
                      x5
                          0.696 0.175
                                        0.481 0.073
                                                        0.447
                                                                0.071
                                                                       0.446 0.058
## 16
           x6 ~~
                      x6
                          0.537 0.127
                                        0.343 0.053
                                                        0.341
                                                                0.051
                                                                       0.356 0.043
## 17
           x7 ~~
                      x7
                          0.683 0.145
                                        0.878 0.106
                                                        0.855
                                                                0.107
                                                                       0.799 0.081
                                                                0.098 0.488 0.074
## 18
           x8 ~~
                          0.891 0.195
                                        0.632 0.089
                                                        0.658
                      8x
## 19
           x9 ~~
                       x9
                          0.055 0.369
                                        0.509 0.110
                                                        0.429
                                                                0.141
                                                                       0.566 0.071
## 20
                          0.503 0.293
                                        0.900 0.187
                                                        0.898
                                                                0.178
                                                                      0.809 0.145
       visual ~~
                  visual
## 21 textual ~~ textual
                          1.365 0.319
                                        0.889 0.126
                                                        0.902
                                                                0.114
                                                                       0.979 0.112
## 22
        speed ~~
                   speed
                           0.118 0.097
                                        0.307 0.098
                                                        0.262
                                                                0.097
                                                                       0.384 0.086
## 23
                          0.774 0.248
                                        0.456 0.091
                                                        0.467
                                                                0.084
                                                                       0.408 0.074
       visual ~~ textual
## 24
       visual ~~
                   speed
                          0.152 0.100
                                        0.292 0.074
                                                        0.274
                                                                0.063
                                                                       0.262 0.056
## 25 textual ~~
                   speed
                          0.061 0.069
                                        0.171 0.057
                                                        0.157
                                                                0.049 0.173 0.049
## 26
           x1 ~1
                           4.877 0.191
                                        4.953 0.084
                                                        4.949
                                                                0.073 4.936 0.067
## 27
           x2 ~1
                           5.895 0.141
                                        6.121 0.084
                                                        6.136
                                                                0.075 6.088 0.068
## 28
           x3 ~1
                           2.038 0.156
                                        2.207 0.078
                                                        2.212
                                                                0.069
                                                                       2.250 0.065
## 29
           x4 ~1
                           2.747 0.174
                                        3.005 0.079
                                                        3.014
                                                                0.068
                                                                       3.061 0.067
## 30
           x5 ~1
                           4.165 0.198
                                        4.331 0.091
                                                        4.319
                                                                0.077
                                                                       4.341 0.074
## 31
                                                                0.065
           x6 ~1
                           2.186 0.163
                                        2.163 0.078
                                                        2.167
                                                                       2.186 0.063
## 32
           x7 ~1
                           4.383 0.127
                                        4.176 0.078
                                                        4.176
                                                                0.068
                                                                       4.186 0.063
## 33
           x8 ~1
                           5.707 0.147
                                        5.519 0.071
                                                        5.501
                                                                0.064
                                                                       5.527 0.058
                           5.424 0.140
                                        5.382 0.075
                                                        5.361
                                                                0.066 5.374 0.058
           x9 ~1
```

Those are a lot of numbers to compare, let's create some plots:

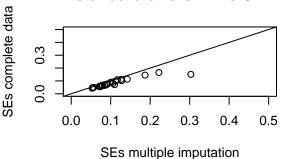
```
par(mfrow = c(2, 2))
plot(comp_data$LD.se, comp_data$CD.se, xlim = c(0, 0.8), ylim = c(0, 0.8),
```

```
main = "standard errors LD vs CD",
     xlab = "SEs listwise deletion",
     ylab = "SEs complete data")
abline(0, 1)
plot(comp_data$MI.se, comp_data$CD.se, xlim = c(0, 0.5), ylim = c(0, 0.5),
     main = "standard errors MI vs CD",
     ylab = "SEs complete data",
     xlab = "SEs multiple imputation")
abline(0, 1)
plot(comp_data\$FIML.se, comp_data\$CD.se, xlim = c(0, 0.5), ylim = c(0, 0.5),
     main = "standard errors FIML vs CD",
     ylab = "SEs complete data",
     xlab = "SEs full information ML")
abline(0, 1)
```

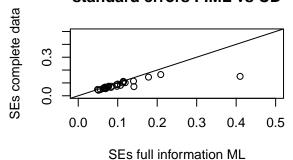
standard errors LD vs CD

SEs complete data 0.8 4.0 0.0 **2000 300 300 300** 0.0 0.2 0.4 0.6 0.8 SEs listwise deletion

standard errors MI vs CD



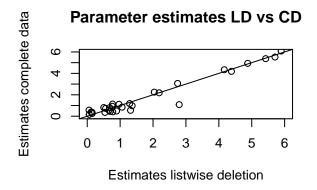
standard errors FIML vs CD

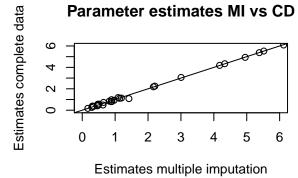


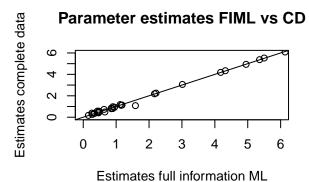
Listwise deletion yields much larger standard errors than we would obtain if we had the complete data. The standard errors obtained with multiply imputed data are much closer to those obtained with the complete data. The MI standard errors tend to be somewhat higher, but this is what should happen, as wel did not use the full dataset with MI. The bottom-left plot indicates a similar pattern for FIML: standard errors are only somewhat larger than when analysing complete data.

```
par(mfrow = c(2, 2))
plot(comp_data$LD.est, comp_data$CD.est, xlim = c(0, 6), ylim = c(0, 6),
     main = "Parameter estimates LD vs CD",
     xlab = "Estimates listwise deletion",
     ylab = "Estimates complete data")
```

```
abline(0, 1)
plot(comp_data$MI.est, comp_data$CD.est, xlim = c(0, 6), ylim = c(0, 6),
    main = "Parameter estimates MI vs CD",
    ylab = "Estimates complete data",
    xlab = "Estimates multiple imputation")
abline(0, 1)
plot(comp_data$FIML.est, comp_data$CD.est, xlim = c(0, 6), ylim = c(0, 6),
    main = "Parameter estimates FIML vs CD",
    ylab = "Estimates complete data",
    xlab = "Estimates full information ML")
abline(0, 1)
```







The parameter estimates with listwise deletion vary much more from the parameter estimates than would have been obtained with the complete data. The parameter estimates with MI and FIML resemble those obtained with the complete data much more closer.

Parameters relating to exogenous variables

In many SEM analyses, parameters relating to exogenous variables will often not be provided. Often, exogenous variables will be considered fixed. As a result, their (co)variances are fixed to their sample (co)variances, instead of being estimated as parameters in the model. For the model fit (χ^2 and df), this does not make a difference. But sometimes you may want to inspect the variation or associations between the exogenous variables.

```
HS_data <- HolzingerSwineford1939
HS_data$age <- with(HS_data, ageyr + agemo/12)</pre>
HS_data$sex <- HS_data$sex - 1 # to make it 0-1 coded
HS.model2 <- '
  visual = x1 + x2 + x3
 textual = \sim x4 + x5 + x6
  visual + textual ~ sex + age
HS_mod1 <- cfa(HS.model2, data = HS_data, estimator = "MLR")</pre>
summary(HS_mod1, standardized = TRUE)
## lavaan 0.6-6 ended normally after 30 iterations
##
##
     Estimator
                                                          ML
     Optimization method
                                                      NLMINB
##
     Number of free parameters
##
                                                          17
##
                                                         301
##
     Number of observations
##
## Model Test User Model:
                                                     Standard
                                                                    Robust
##
##
     Test Statistic
                                                       35.619
                                                                    35.485
##
     Degrees of freedom
                                                           16
                                                                        16
##
     P-value (Chi-square)
                                                        0.003
                                                                     0.003
##
     Scaling correction factor
                                                                     1.004
##
          Yuan-Bentler correction (Mplus variant)
##
## Parameter Estimates:
##
##
     Standard errors
                                                    Sandwich
##
     Information bread
                                                    Observed
     Observed information based on
                                                     Hessian
##
##
## Latent Variables:
##
                       Estimate Std.Err z-value P(>|z|)
                                                               Std.lv Std.all
##
     visual =~
                          1.000
                                                                0.850
                                                                          0.729
##
       x1
##
       x2
                          0.635
                                    0.163
                                             3.890
                                                       0.000
                                                                0.540
                                                                          0.459
##
                          0.804
                                    0.174
                                             4.610
                                                       0.000
                                                                0.683
                                                                          0.605
       хЗ
##
     textual =~
##
       x4
                          1.000
                                                                0.993
                                                                          0.855
##
       x5
                          1.110
                                    0.067
                                            16.632
                                                       0.000
                                                                 1.102
                                                                          0.856
##
       x6
                          0.919
                                    0.061
                                            14.952
                                                       0.000
                                                                 0.912
                                                                          0.834
##
## Regressions:
                       Estimate Std.Err z-value P(>|z|)
                                                               Std.lv Std.all
##
##
     visual ~
```

```
##
                          -0.329
                                     0.123
                                              -2.676
                                                        0.007
                                                                 -0.387
                                                                           -0.194
       sex
                                                                 -0.045
##
                          -0.038
                                     0.064
                                              -0.593
                                                        0.553
                                                                           -0.045
       age
##
     textual ~
                                    0.122
                                              0.624
##
                           0.076
                                                        0.533
                                                                  0.077
                                                                            0.038
       sex
##
       age
                          -0.236
                                     0.057
                                              -4.129
                                                        0.000
                                                                 -0.237
                                                                           -0.241
##
##
  Covariances:
                                  Std.Err z-value P(>|z|)
##
                       Estimate
                                                                 Std.lv
                                                                          Std.all
##
    .visual ~~
##
                           0.384
                                     0.105
                                                        0.000
                                                                            0.479
      .textual
                                               3.652
                                                                  0.479
##
## Variances:
##
                       Estimate Std.Err z-value
                                                      P(>|z|)
                                                                 Std.lv
                                                                          Std.all
##
                                               3.714
                                                        0.000
                                                                  0.636
                                                                            0.468
      .x1
                           0.636
                                     0.171
##
                           1.091
                                     0.110
                                              9.957
                                                        0.000
                                                                  1.091
                                                                            0.789
      .x2
##
      .x3
                           0.808
                                     0.111
                                              7.294
                                                        0.000
                                                                  0.808
                                                                            0.634
##
      .x4
                           0.364
                                     0.050
                                              7.257
                                                        0.000
                                                                  0.364
                                                                            0.270
##
      .x5
                           0.445
                                     0.058
                                              7.606
                                                        0.000
                                                                  0.445
                                                                            0.268
##
                           0.364
                                     0.048
                                              7.559
                                                        0.000
                                                                  0.364
                                                                            0.304
      .x6
##
      .visual
                           0.695
                                     0.192
                                               3.613
                                                        0.000
                                                                  0.963
                                                                            0.963
##
      .textual
                           0.925
                                     0.112
                                              8.235
                                                        0.000
                                                                  0.937
                                                                            0.937
```

We see that the (co)variances of the exogenous variables (sex and age) are not estimated in the model. As a results, we cannot inspect their association. To include them in the model as model parameters, we have to additionally specify fixed.x = FALSE in the call to cfa():

```
HS_mod2 <- cfa(HS.model2, data = HS_data, estimator = "MLR", fixed.x = FALSE)
summary(HS_mod2, standardized = TRUE)</pre>
```

```
## lavaan 0.6-6 ended normally after 32 iterations
##
##
     Estimator
                                                          ML
                                                     NLMINB
##
     Optimization method
##
     Number of free parameters
                                                          20
##
##
     Number of observations
                                                        301
##
## Model Test User Model:
##
                                                    Standard
                                                                   Robust
                                                      35.619
                                                                   35.485
##
     Test Statistic
##
     Degrees of freedom
                                                           16
                                                                       16
##
     P-value (Chi-square)
                                                       0.003
                                                                    0.003
##
     Scaling correction factor
                                                                    1.004
##
          Yuan-Bentler correction (Mplus variant)
##
## Parameter Estimates:
##
##
     Standard errors
                                                   Sandwich
     Information bread
##
                                                   Observed
     Observed information based on
##
                                                    Hessian
##
## Latent Variables:
##
                       Estimate Std.Err z-value P(>|z|)
                                                               Std.lv Std.all
##
     visual =~
##
       x1
                          1.000
                                                                0.850
                                                                         0.729
```

##	x2	0.635	0.163	3.890	0.000	0.540	0.459
##	x3	0.804	0.174	4.610	0.000	0.683	0.605
##	textual =~						
##	x4	1.000				0.993	0.855
##	x5	1.110	0.067	16.632	0.000	1.102	0.856
##	x6	0.919	0.061	14.952	0.000	0.912	0.834
##							
##	Regressions:						
##		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	visual ~						
##	sex	-0.329	0.123	-2.676	0.007	-0.387	-0.194
##	age	-0.038	0.064	-0.593	0.553	-0.045	-0.045
##	textual ~						
##	sex	0.076	0.122	0.624	0.533	0.077	0.038
##	age	-0.236	0.057	-4.129	0.000	-0.237	-0.241
##							
##	Covariances:						
##		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	.visual ~~						
##	.textual	0.384	0.105	3.652	0.000	0.479	0.479
##	sex ~~						
##	age	-0.081	0.029	-2.791	0.005	-0.081	-0.160
##							
##	Variances:						
##		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	.x1	0.636	0.171	3.714	0.000	0.636	0.468
##	.x2	1.091	0.110	9.957	0.000	1.091	0.789
##	.x3	0.808	0.111	7.294	0.000	0.808	0.634
##	.x4	0.364	0.050	7.257	0.000	0.364	0.270
##	.x5	0.445	0.058	7.606	0.000	0.445	0.268
##	.x6	0.364	0.048	7.559	0.000	0.364	0.304
##	.visual	0.695	0.192	3.613	0.000	0.963	0.963
##	.textual	0.925	0.112	8.235	0.000	0.937	0.937
##	sex	0.250	0.001	289.990	0.000	0.250	1.000
##	age	1.035	0.087	11.907	0.000	1.035	1.000