One Model May Not Fit All: Subgroup Detection Using Model-Based Recursive Partitioning

Marjolein Fokkema[1] and Mirka Henninger[2] & Carolin Strobl[2]

[1]Leiden University

[2]Universität Zürich

Abstract

Model-based recursive partitioning (MOB, Zeileis, Hothorn, & Hornik, 2008) is a flexible framework for detecting subgroups of persons showing different effects in a wide range of parametric models. It provides a versatile tool for detecting and explaining heterogeneity of intervention effects. In this tutorial paper, we provide an introduction to the general MOB framework. In two specific case studies, we show how MOB-based methods can be used to detect and explain heterogeneity in two widely-used frameworks in educational studies: mixed-effects and item response theory (IRT) models. In the first case study, we show how GLMM trees (Fokkema, Smits, Zeileis, Hothorn, & Kelderman, 2018) can be used to detect subgroups with different parameters in mixed-effects models. We apply GLMM trees to longitudinal data from a study on the effects of Head Start, to identify subgroups of families where children show comparatively larger or smaller gains in performance. In a second case study, we show how Rasch trees (Strobl, Kopf, & Zeileis, 2015a) can be used to detect subgroups with different item parameters in IRT models, i.e. differential item functioning (DIF). We show how a recently developed stopping criterion (Henninger, Debelak, & Strobl, 2023) can be used to guide subgroup detection based on DIF effect sizes.

Introduction

Model-based recursive partitioning (MOB; Zeileis et al., 2008) is a semi-parametric approach for detecting differences in the parameters of a statistical model between groups of persons. It generalizes the principle of recursive partitioning, that is also used in classification and regression trees (CART; Breiman, Friedman, Olshen, & Stone, 1984). In CART, the aim is to detect groups of persons, defined by (combinations of) covariate values, that differ in their mean on a response variable. For example, if we want to predict math exam grades using student characteristics assessed in a previous year, we could find that a subgroup of students who are highly motivated and have good reading skills show higher average math exam grades. MOB generalizes this idea: It also identifies groups of persons defined by (combinations of) covariate values, but the groups can differ in a wider range of parameters of a statistical model, instead of only the mean. An example could be that students who are highly motivated and have good reading skills show higher slopes in a regression model relating the time spent studying to math exam grades.

The framework of MOB is very flexible and it can be applied to a wide range of statistical models, such as linear and generalized linear regression (Kopf, Augustin, & Strobl, 2013; Zeileis et al., 2008) or models for paired-comparison data (Strobl, Wickelmaier, & Zeileis, 2011; Wiedermann, Frick, & Merkle, 2021). In this article, we will highlight two specific MOB methods that we consider particularly relevant for research in school psychology: MOB for mixed-effects models (Fokkema et al., 2018) and MOB for Rasch measurement and Item Response Theory models (IRT, Henninger, Debelak, & Strobl, 2023; Komboz, Strobl, & Zeileis, 2017; Strobl, Kopf, & Zeileis, 2015b). Mixed-effects models become relevant whenever data are collected in repeated measures or nested data structures. For example, when children are tested at several time points, the time points are nested in children; when children from different classes from different schools participate in a study, children are nested in classes, which are in turn nested in schools. In application example 1 we will illustrate how MOB can be used to detect subgroup-specific intervention effects while taking into account the nested data structure. In this example, the focus is on detecting subgroups with different parameters of a *regression* model. We thus assume that the psychological test score(s) of interest have already been validated.

For application example 2, we go back in the research process to the point where a new psychological test has been administered to a validation sample. Here, we focus on detecting subgroups with different parameters of a *measurement* model. Validity assessment requires that we make sure that test results are comparable, for example between children of different genders. If certain items show different measurement parameters between groups, this may put certain groups at a relative disadvantage, and these items are said to violate measurement invariance or to exhibit differential item functioning (DIF). In order to test for DIF in the framework of Item Response Theory or Rasch modelling (Anthony, DiPerna, & Lei, 2016; Debelak, Strobl, & Zeigenfuse, 2022; Maller, 1997), the item parameters are compared between groups of persons. This can be done in a way that allows to detect DIF, while accounting for possible true group differences in ability. Traditional approaches for testing DIF require the groups to be pre-specified in order to test for DIF. In application example 2 we show how MOB flexibly allows to detect groups with different item parameters in a data-driven way.

In the following section, we first give a short introduction into the algorithm and statistical concepts behind MOB. Readers interested in learning more about its predecessor method, classification and regression trees, are referred to the introduction by Strobl, Malley, and Tutz (2009).

The MOB Algorithm

The main rationale of MOB is that one global model may not fit all observations in a dataset equally well. In many studies, additional covariates may be available. It may then be possible to uncover subgroups defined by these covariates, and obtain better-fitting models in each of those subgroups (Zeileis et al., 2008).

We illustrate the idea using a very simple, simulated toy dataset, comprising 250 observations and four variables: A continuous response variable y, and three covariates, x_1, x_2 and x_3, as possible partitioning variables. To keep the example simple, we apply MOB to a very basic global model, which comprises only an intercept. It would also be possible to use, e.g., a linear regression model or a logistic regression model as the global model. Figure 1, left, shows the distribution of y in a boxplot, with the global intercept indicated as a triangle. Obviously, the global intercept does not describe all observations equally well, there is quite some unexplained variation around it.

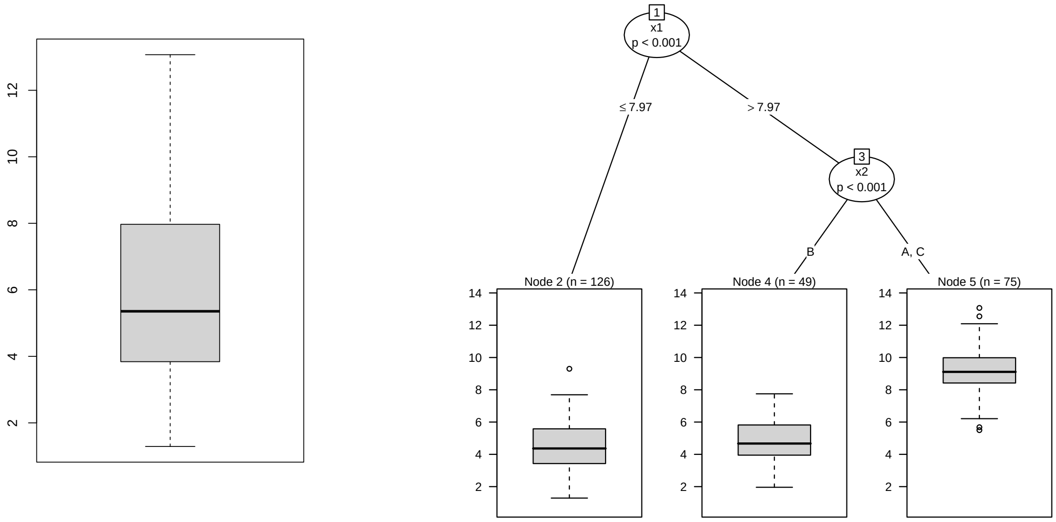
To detect possible subgroups with different values for the parameters, MOB cycles iteratively through the following steps:

1. The model parameters are first estimated jointly for all persons in the current node, starting with the root node containing the full sample.

2. Structural change in the model parameters is assessed with respect to each available covariate.

Figure 1

Left: Univariate distribution of the response variable. Right: Tree with group-specific distributions of the response variable in the terminal nodes.



3. If there is significant structural change, the observations in the current node are split using the covariate associated with the strongest change.

4. Steps 1–3 are repeated recursively in each resulting node until there is no more significant structural change (or the groups becomes too small).

We applied MOB to the observations in the left panel of Figure 1, specifying x_1, x_2 and x_3 as potential partitioning variables. The resulting tree is shown in the right panel of Figure 1. In the root node, which contains all observations (step 1), structural change tests were performed for each of the three covariates (step 2). The tests revealed that x_1, a continuous covariate, was most strongly associated with instabilities in the intercept and x_1 was thus selected first for splitting (step 3). The p value resulting from the structural change test for the first split in x_1 is depicted in the root node. Next, the cutpoint for x_1 was selected so that the two resulting subgroups exhibit the strongest parameter differences, while the observations within the subgroup are as similar as possible. In the right daughter node, additional significant instability was detected with respect to x_2, a categorical covariate. Again, the cutpoint in x_2 was selected so that the two resulting subgroups exhibit strongest parameter differences. In the left daughter node (node 2), no further splits were created, because none of the three covariates were significantly associated with any further instability in this subgroup (step 4). The same held for nodes 4 and 5, so that the third covariate, x_3, was never selected for splitting. The subgroup-specific distributions of the response variable are presented in the end nodes at the bottom of the tree. The fact that the tree in Figure 1 displays more than one end node confirms that one global model for all observations does not appropriately capture the pattern in the data. The tree also shows shows that, out of the three covariates that were presented to the algorithm, only two were actually selected for splitting. This automatic variable selection is an important characteristic of classification and regression tree and MOB algorithms.

There are three further important characteristics of tree and MOB algorithms that we would like to mention here: The first characteristic is the type of variable and cutpoint selection an algorithm employs. Traditional classification and regression tree algorithms, like those of Breiman et al. (1984) and Quinlan (1993), performed variable and cutpoint selection in one step, which leads to an undesirable behavior called variable selection bias. That is, the traditional algorithms prefer variables offering more possible cutpoints in the selection process – regardless of their true information content. An algorithm that still has this problem is, for example, the `rpart` algorithm in R, based on the original CART algorithm by Breiman et al. (1984). More modern algorithms for classification and regression trees have solved this problem and offer unbiased variable selection, such as `QUEST` (Loh & Shih, 1997), which is available in SPSS, and `ctree` (Hothorn, Hornik, & Zeileis, 2006), which is available in `R` in packages `party` and `partykit` (Hothorn & Zeileis, 2015). The latter forms the basis for all MOB approaches presented in this paper. For more details on the statistical theory behind unbiased classification and regression trees and MOB, see Hothorn et al. (2006); Strobl et al. (2015a, 2009).

The second characteristic relates to the way a tree or MOB algorithm stops splitting: Modern algorithms for classification and regression trees and MOB use a criterion of statistical significance to stop splitting. Once there are no more covariates that show a significant structural change in any node, splitting is halted. In this way, the algorithm selects only those partitioning variables that are relevant for distinguishing the groups (i.e., it performs automatic variable selection, as illustrated in the right panel of Figure 1). Moreover, the trees will not grow as large as possible, but splitting is stopped when no more significant structural change is detected. While traditional tree algorithms like those of Breiman et al. (1984) and Quinlan (1993) grew very large trees and then cut them back (so called *pruning*), modern tree and MOB algorithms employ significance tests as stopping criteria (and some can also use effect size measures, as we will see in application example 2). This allows for stopping tree growing as soon as no significant differences can be detected anymore. Other stopping criteria are based on the number of persons in the end nodes. These criteria ensure that the sample sizes in the end nodes are large enough to estimate the statistical model in each end node. Note that it is thus possible that no split will be implemented, when none of the specified partitioning variables show significant structural change.

we should discuss this in the application examples; there are some defaults but for models with many parameters it might make sense to increase them

The third important characteristic of classification and regression trees as well as MOB is that the entire structure identified by the trees does not have to be pre-specified by the researcher in a confirmatory manner, but is learned from the data in an exploratory manner. This is a key feature of the MOB approach that makes it very flexible and sets it apart from purely parametric approaches, where only those main effects and interactions that are explicitly included in the specification of the model are considered. While there are phases in psychological and educational research where it is very important to specify hypotheses a-priori and test them in a confirmatory manner, in early stages of research exploratory methods are an important addition to the statistical toolbox for researchers. Still, an important challenge for the researcher remains: To specify the parametric model of interest, to specify the set of possible partitioning variables and to choose the settings of the MOB algorithm. The current paper aims to provide guidance.

Next, we discuss two specific methods that make use of the general MOB framework introduced above. The methods allow for partitioning two types of statistical models that are particularly relevant for school psychology research: Mixed-effects models for repeated measures or nested data structures and measurement models for validating psychological and educational tests.

Using MOB for Subgroup Detection in Mixed-Effects Models

Mixed-effects models contain two types of effects: Fixed and random effects. Fixed effects are typically used to capture population-averaged effects, while random effects are used to capture inter-individual variation deviating from these fixed effects. In many studies, researchers are specifically interested in testing hypotheses relating to the population-averaged effects, while the random effects are included in the model to properly account for inter-individual variation, and correlations between observations within the same unit (Raudenbush & Bryk, 2002).

GLMM trees combine MOB and mixed-effects models and were introduced by Fokkema et al. (2018). While the 'standard' MOB trees (Zeileis et al., 2008) allow for subgroup detection in fixed-effects GLMs, GLMM trees additionally estimate and account for random effects and can thus be used for partitioning mixed-effects regression models. Because researchers' interests in multilevel models commonly focus on the fixed effects (of time or treatment, for example), the GLMM tree algorithm only targets structural change in the fixed-effects parameters. The random-effects parameters can be specified as usual and are assumed constant; that is, they are estimated using all observations in the dataset. Structural change is thus assessed with respect to the fixed-effects parameters, while dependency between observations within the same units is accounted for by the random effects. The resulting subgroups will differ in their estimates for the fixed-effects coefficients only. As such, GLMM trees allow for detecting subgroups in multilevel models that differ with respect to any (set of) fixed-effects parameters of interest. For example, users may be interested in detecting subgroups with different means (Fokkema, Edbrooke-Childs, & Wolpert, 2021), but also differential effects of treatment (Fokkema et al., 2018) or differential growth over time (Fokkema & Zeileis, 2023), to name but a few examples.

The `R` functions for fitting GLMM trees are the `lmertree` and `glmertree` functions from package `glmertree`. The usage of this function will be illustrated in application example 1. Further mathematical and computational details about the GLMM tree model are also described in Fokkema et al. (2018) and Fokkema and Zeileis (2023).

Using MOB for Detecting DIF in Measurement Models

Validity assessment of scores on psychological or educational tests requires researchers to assess whether the same construct is measured in the same way for different groups. In the framework of IRT and Rasch measurement, test items are typically investigated with respect to item misfit, multidimensionality and other violations of the measurement model (cf., for example Debelak et al., 2022, for an introduction). A particularly crucial assumption for the comparability of test scores between groups is measurement invariance. Items that violate measurement invariance by showing different measurement properties for different groups of participants display DIF.

MOB can be used to detect DIF by means of testing whether the item parameters of the measurement model exhibit significant instability with respect to (combinations of) covariates. For the Rasch measurement model, the `R` function for conducting the MOB analysis is the `raschtree` function from package `psychotree` (Strobl et al., 2015a). The usage of this function will be illustrated in application example 2. We will see that, just like for GLMM trees, we need to specify which variables are part of the parametric model. In the case of the Rasch model, this will be the test items. Moreover, we need to specify, which covariates are made available to the MOB algorithm for selecting relevant splitting variables and cutpoints.

If one joint Rasch model holds for the entire sample, that is, if there is no DIF, a

Rasch tree should show no splits. However, in certain settings in educational research, such as large scale assessments, very large sample sizes are available for testing for DIF. Large sample sizes are good for detecting even small effects or model violations with a high statistical power. The same holds for the statistical tests used for detecting parameter change in the MOB algorithm, so that in larger samples even very small parameter differences can be detected. However, in DIF detection this may mean that even very small DIF effects, that in practice can be considered ignorable, will be detected if only the sample is big enough. As we will illustrate in the first part of application example 2, for a large sample of university students who have taken a quiz to test their general knowledge, this can lead to very large trees, that are hard to interpret and contain splits that would not be considered relevant by measurement experts. Therefore, an extension of Rasch trees has been suggested by Henninger, Debelak, and Strobl (2023) based on the Mantel-Haenszel effect size measure for DIF. Holland and Thayer (1985) have suggested an intuitive classification of DIF effect sizes based in the Mantel-Haenszel statistic, that is being widely used in educational testing. In this classification, category A stands for negligible DIF (small effect size or not statistically significant), B for medium DIF (neither A nor C), and C for large DIF (large effect size and statistically significant). Henninger, Debelak, and Strobl (2023) have incorporated this classification as an additional stopping criterion for Rasch trees, so that the user can decide, for example, that a split should only be conducted if the detected DIF is of category B or C, while negligible DIF of category A should be ignored.

As we will show in application example 2, for large sample sizes this can be very helpful because it results in shorter trees that are easier to interpret and contain only splits corresponding to DIF effect sizes considered relevant in practice. Together with a purification step (see Henninger, Debelak, & Strobl, 2023, and application example 2 for details), the Mantel-Haenszel classification can also be used for highlighting those items that show DIF with respect to certain groups of persons graphically. This can help generate hypothesis about the sources of DIF, as we will ilustrate in application example 2, and can also aid the decision how to proceed with the DIF items. For example, items that show DIF between different language groups may often be improved by means of making sure that in all translations the meaning is as similar as possible, and/or that the words employed in the translations for the different languages are equally frequently used. In other situations, sources of DIF might be harder to eliminate, so that often DIF items may be excluded from a test. Either way, the measurement model needs to be refitted and its assumptions checked again after the final set of items has been decided upon and, in the case of modified items, administered again.

Another important aspect to keep in mind is that DIF can be caused by one or more items measuring a secondary dimension in addition to the dimension that is intended to be measured by the test. This would be the case in instruments intended to measure math aptitude containing pure algebra problems as well as story problems. Students whose native language is not the same as the test language can have a disadvantage in answering the story problems, for example when they contain seldomly used words. These items may then show DIF between native and non-native speakers. When encountering this, the test developers will have to decide whether the items with story problems should be excluded from the test, whether they can be improved, for example by using more frequently used words, or whether the test should be considered two- rather than one-dimensional. For a discussion of the connection between DIF and multidimensionality, see also Ackerman (1992); Strobl, Kopf, Kohler, von Oertzen, and Zeileis (2021).

We will now illustrate how to use the `R` packages `glmertree` and `psychotree` to conduct the MOB analyses.

Application Example 1: Subgroup Detection in Mixed-Effects Models

Dataset

To illustrate the use of GLMM trees, we analyze a dataset from Deming (2009), who evaluated long-term benefits of participation in Head Start. Head Start is a federally funded nationwide pre-school program for children from low-income families in the United States. Participation in Head Start takes place from ages 3 through 5. Deming (2009) compared performance of siblings who differed in their participation in the program using data from the National Longitudinal Survey of Youth (NLSY; REF) .

Marjolein, add reference

The sample consists of 273 families with at least two siblings, where at least one sibling participated in Head Start and at least one sibling did not participate in Head Start or any other preschool program. Data from children in those families who participated in another preschool program were excluded. The family structure allows siblings who did not participate in Head Start to serve as a natural control to assess the effects of participating in Head Start. The outcome variable comprises repeated assessments on the Peabody Picture Vocabulary Test (PPVT; REF) , we will thus model PPVT trajectories over time. There were an average number of 2.14 PPVT scores per child, for 29% of the children there was only a single PPVT score available.

Marjolein: Add reference

Our dataset contains five family characteristics: The mother's score on the Armed Forced Qualification Test (AFTQ), adjusted for age; the families' income (averaged over the years for which data was available); race (Black, Hispanic or White); mother's years of completed education; mother's body height . Note that the latter variable is one that should be completely irrelevant for predicting performance on a vocabulary test; it is included here to illustrate that the GLMM tree algorithm can fruitfully distinguish signal from noise variables. The dataset analyzed here only comprises data from families and children for whom complete data was available.

Marjolein: Add scale for measurement

We load the data and inspect the first rows:

```r
HS_dat <- readRDS("HS_dat.Rda")
head(HS_dat, 3)
```

```
       AFTQ     Race   Income Mom_height Mom_edu_yrs ChildID MotherID Program
1  3.478122 Hispanic 37731.07        502          12   20502      205      HS
2  3.478122 Hispanic 37731.07        502          12   20501      205    None
3 15.964368    Black 16119.13        504          10   22403      224    None
  PPVT Age Age_orig
1   18   4        4
```

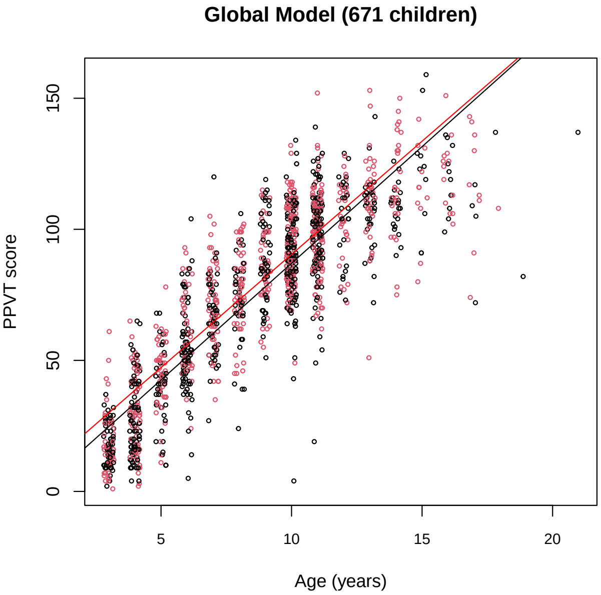
```
2   48   7          7
3   69   7          7
```

We inspect the complete dataset by plotting PPVT scores against age, separated by program participation: None (black) versus Head Start (red). To show the global effect of age and Head Start participation, we first fit a mixed-effects model comprising their main and interaction effects, using package `lme4`. This would be the model fitted in the root node of the GLMM tree. To account for the correlation between repeated assessments on the same child, and between siblings with the same mother, we specify a random intercept for children, nested within mothers:

```
library("lme4")
lmm <- lmer(PPVT ~ Program*Age + (1|MotherID/ChildID), data = HS_dat)
```

The results are presented in Figure 2, which shows that children participating in Head Start show slightly higher performance than their non-participating siblings and that this difference is slightly diminished but persistent over time. This result agrees with the findings of Deming (2009). Figure 2 already suggests that the effect of age on performance is very strong, compared to which the effects of intervention (Head Start) are existent but relatively small, which is typical for young children growing up to adults. Note that the `R` code for creating the figure is omitted here, because we want to focus on fitting and interpreting GLMM trees. Code for exact replication of the results presented here is provided in the Supplementary Materials.

Figure 2



Linear mixed effects model tree

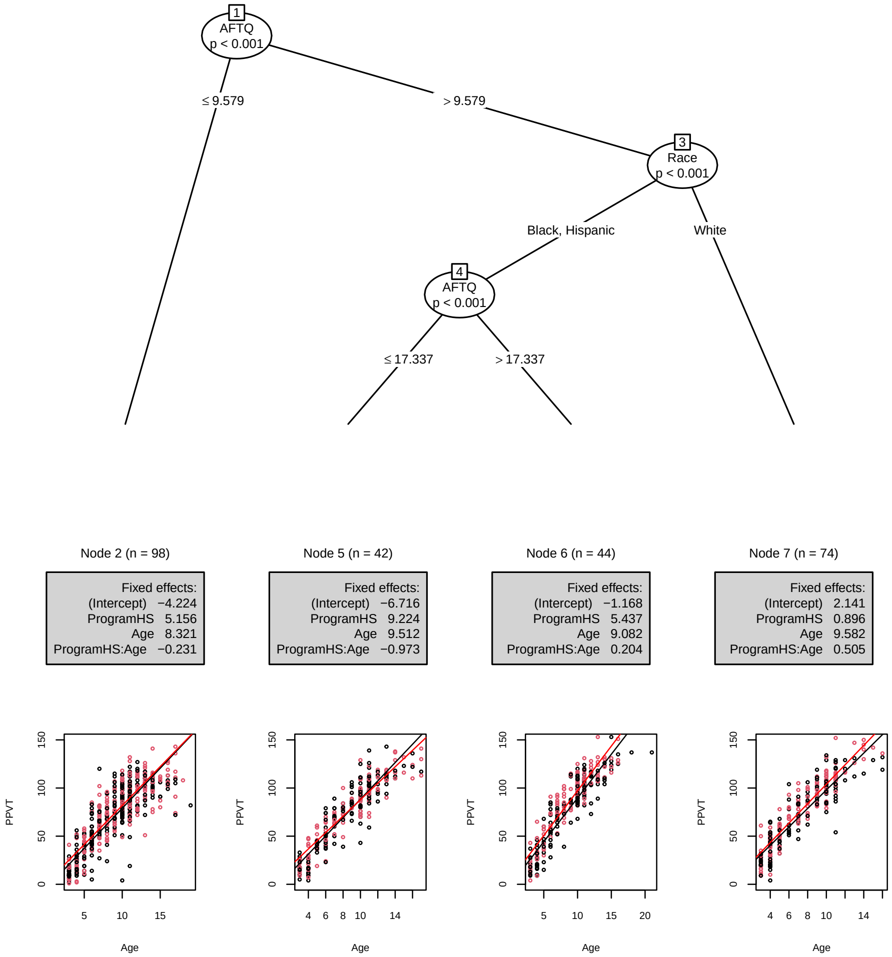
Next, we test whether the intercepts and slopes of the two regression lines differ as a function of the partitioning variables, using function `lmertree` from R package **glmertree**:

```
library("glmertree")
HS_tree <- lmertree(PPVT ~ Program*Age | (1|MotherID/ChildID) | AFTQ + Race +
                                Income + Mom_edu_yrs + Mom_height,
                              data = HS_dat, cluster = MotherID, minsize = 250)
```

With the first argument, we specified the model `formula`, which has three parts separated by vertical bars: The left part (`PPVT ~ Program*Age`) specifies the response variable, followed by a tilde (~) and the fixed-effects predictors of relevance. The middle part (`1|MotherID/ChildID`) specified the random effect: Repeated PPVT assessments are nested within children, which are nested within mothers. The right part (`AFTQ + Race + Income + Mom_edu_yrs + Mom_height`) specified the partitioning variables: covariates that may possibly affect the values of the fixed-effects parameters.

With the second argument, we specified the dataset which contain the variables. With the `cluster` argument, we specified that the partitioning variables are measured at the level of the mothers. As a result, parameter stability tests will be performed on the appropriate level (Fokkema & Zeileis, 2023). Not specifying the `cluster` argument would

Figure 3



result in use of the default observation-level parameter stability tests, which would yield inflated type-I error rates. That is, it could result in detection of spurious subgroups. Finally, because we want to retain large enough subgroups, we specified that the minimum number of observations in a terminal node should be 250.

Next, we plot the tree. With multiple fixed-effects predictors of interest, the default plots may become too crowded or difficult to interpret. We therefore specify `type = "simple"` to facilitate interpretation, and using the `nodesize_level` argument, we specified that the sample size printed above every terminal node should count the number of children, not the number of individual observations:

```
plot(HS_tree, type = "simple", nodesize_level = 2)
```

The resulting tree is presented in Figure 3. Below each terminal node, we plotted

Table 1

Predicted PPVT scores at different ages.

Node	Program	PPVT at age 6	PPVT at age 18
2	None	16.16	31.08
2	HS	20.75	35.26
5	None	16.58	33.64
5	HS	23.42	38.74
6	None	21.08	37.36
6	HS	27.02	43.67
7	None	25.61	42.80
7	HS	27.75	45.84

the observations and the two regression curves given by the coefficients in that node. The first split was made based on the AFTQ variable, which represents the mother's score on the Armed Forced Qualification Test, adjusted for the age at which they completed the test. The group with higher mother's AFTQ scores is further split based on race. The Black and Hispanic group is further split based on income.

To aid interpretation of the coefficients in Figure 3, Table 1 provides predicted PPVT scores for each of the groups and programs at ages 6 and 18. All nodes show a modest benefit of Head Start participation at age 6, about 3-4 points on the PPVT. This benefit remains the same over time for the group with lower mother's AFTQ scores. The benefit increases over time for White children with higher mother's AFTQ scores. Interestingly, the benefit decreases over time for Black and Hispanic children with higher mother's AFTQ scores. These results correspond to the conclusions of Deming (2009).

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Marjolein: With different specification of the GLMM tree (no square root, parameter stability tests at mom level, random intercepts of children nested within mothers) the results have changed, still need to adjust de-

Application Example 2: Subgroup Detection in Rasch Models

Discussion

In this tutorial paper we have outlined the rationale of the MOB algorithm and how it can be used for detecting parameter differences in mixed-effects and Rasch models. The main advantage of MOB is that it can detect groups of persons with different model parameters in a data-driven manner. This makes it more flexible for detecting differences that were not hypothesized by the researcher. For example in DIF analysis, it is often the case that obvious sources of DIF have already been avoided by the content experts during item creation. Any remaining DIF is unexpected. Therefore, any DIF detection approach that relies on the researchers correctly specifying the exact groups of persons that exhibit DIF can miss DIF if it is associated with other (combinations of) covariates or other cutpoints than the ones investigated. In this sense, the more exploratory approach of Rasch trees has higher statistical power to detect DIF in previously unknown groups (Strobl et al., 2015b). The same argument holds for parameter differences in mixed-effects models, where the substantial hypotheses formulated a priori are typically about global intervention effects, not about possible subgroup-specific differences.

However, researchers should be aware that a covariate-based approach like MOB will only be able to detect parameter differences if the relevant covariates were recorded in the study and supplied to the algorithm. If no or not many potentially relevant covariates are available, an alternative framework for detecting previously unknown groups of persons with different model parameters is mixture modelling (see, e.g., De Ayala & Santiago, 2017; Frick, Strobl, & Zeileis, 2015, in the context of Rasch modelling). Mixture modelling aims at identifying latent classes of persons with different properties. It can also be combined with observed covariates. A comparison of MOB and mixture modelling is given by Frick, Strobl, and Zeiles (2014).

Another important caveat is that those covariates that are selected for splitting by the MOB algorithm are not necessarily causal for the observed parameter differences. They might also be proxies for other, unobserved covariates. For example, if DIF in the item parameters of a test on reading ability is found between different cities or districts, a variety of factors could be causally driving these effects, such as different compositions of native languages or sociodemographics.

Finally, as with any exploratory method, MOB should be considered as a tool for generating hypothesis and not as a confirmatory technique. While the MOB algorithm, as it is implemented in the `R` packages presented in this tutorial, has been constructed with care and uses statistical significance tests (and in the case of Mantel-Haenszel trees also effect sizes) to avoid spurious splits, for confirmatory testing of hypothesis it is recommended to use fresh data or resampling techniquest like cross validation.

I (Caro) have added a few more thoughts for the discussion above - Marjolein maybe this is already sufficient in length? please feel free to use anything from above or below and get back to me if you add/keep points where we might disagree

Limitations of MOB

I (Caro) would not make subsections within discussion

Due to the exploratory nature of MOB, it does not allow for hypothesis tests

I (Caro) think this will sound confusing because earlier in the paper we explain that MOB is indeed based on significance tests, and due to the fact that the stopping is based on the significance test and (at least in party and psychotree) we use Bonferroni for adjusting against the number of covariates, it is not like we are doing completely uncontrolled multiple testing - in the Raschtree paper we write about this: Moreover, it is important to note that our model-based recursive partitioning algorithm is not affected by an inflation of chance due to its recursive nature. Indeed, several statistical tests are successively conducted in a Rasch tree—but each test is conducted only if the previous test yielded a significant result. In this sense, the recursive approach forms a closed testing procedure, which does not lead to an inflation of chance as is well known from the literature on multiple comparisons (Marcus, Peritz, & Gabriel, 1976; Hochberg & Tamhane, 1987). For the Rasch tree, this means that the postulated significance level holds for the entire tree, not only for each individual split. This ensures that DIF is not erroneously detected as an artifact of the recursive nature of the algorithm. – another, related, limitation that I would agree with and might be what you also had in mind is that the tree will not necessarily find the globally best partition, because every split is based on the previous split choices, here is what we wrote about this in the tree intro PsychMeth paper: Thus, variable selection in a single tree is affected by order effects similar to those present in stepwise variable selection approaches for parametric regression (that is also instable against random variation of the learning data, as pointed out by Austin and Tu 2004). In both recursive partitioning and stepwise regression, the approach of adding one locally optimal variable at a time does not necessarily (or rather hardly ever) lead to the globally best model over all possible combinations of variables. – should we write something similar here?

, as there is no valid way to account for the exploratory searching of the subgroups. Thus, even if detected subgroups are substantively meaningful or differences are large, if researchers want to ascertain statistical significance of the subgroup differences, this should be done on new data using confirmatory techniques.

Marjolein write a little more here? e.g. such as cross validation.

sample size dependence, similar ideas like MH trees also possible for other MOB?

possibly Mirka add something about visual interpretation? because effect sizes in mixed effects models are not very clear

The Rasch tree method has been extended to include an effect size measure that can stop the tree from growing if the effect size is non-substantial, but also supports researchers in interpreting the tree's results with respect to whether DIF effects are negligible, medium,

Write about future work.

or large. At the same time, effect sizes are less straightforward to calculate and interpret in linear mixed models, and therewith in the linear mixed effects model tree. A remedy to this issue can be interpretation techniques, such as partial dependence plots or individual conditional expectations plots. These interpretation techniques support researchers in gauging the size of the effect of predictor variables visually by depicting the predicted value of the machine learning method as a function of the value of the predictor variable(s). A comprehensive introduction, tutorial, and discussion into interpretation techniques for machine learning methods is given by Molnar (2019) and Henninger, Debelak, Rothacher, and Strobl (2023).

This is my (Mirka) suggestion for the interpretation techniques. Please feel free to add, edit, delete, however you prefer!

Mention that the exclusion of random effects from partitioning can both be advantage and disadvantage. In future work, use of parameter stability tests for random-effects parameters of Ting and Ed will be explored.

Mention that transformations chosen in growth curve modeling may yield different effects. In future work, recursive partitioning of GAMs will be explored to obviate

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Add DOIs.

Appendix

Items of the culture scale for application example 2

1. Which painter created this painting? – Andy Warhol.
2. What do these four buildings have in common? – All four were designed by the same architects.
3. Roman numbers: What is the meaning of CLVI? – 156.
4. What was the German movie with the most viewers since 1990? – Der Schuh des Manitu.
5. In which TV series was the US president portrayed by an African American actor for a long time? – 24.
6. What is the name of the bestselling novel by Daniel Kehlmann? – Die Vermessung der Welt (Measuring The World).
7. Which city is the setting for the novel 'Buddenbrooks'? – Lübeck.
8. In which city is this building located? – Paris.
9. Which one of the following operas is not by Mozart? – Aida.