Answers to exercises ordered categorical indicator variables

Exercise 6.1

Item wordings:

- 1. The woman decides on her own that she does not wish to have a child.
- 2. The couple agree that they do not wish to have a child.
- 3. The woman is not married and does not wish to marry the man.
- 4. The couple cannot afford any more children.

```
library("ltm")
names(Abortion) <- c(paste0("I", 1:4))</pre>
```

a) Find the proportion who endorsed each item (i.e., the mean score).

```
colMeans(Abortion)
```

```
## I1 I2 I3 I4
## 0.4379947 0.5936675 0.6358839 0.6174142
```

Item I3 is the most-endorsed (easiest) item, item I1 is the least endorsed (most difficult) item.

b) Fit a CFA for binary responses using the CFA function, assuming a single latent variable underlies the item responses.

```
library("lavaan")
model <- '
   lib_ab_views =~ I1 + I2 + I3 + I4
'
fit.abo <- cfa(model, data = Abortion, ordered = paste0("I", 1:4))
summary(fit.abo, standardized = TRUE, fit.measures = TRUE)</pre>
```

```
## lavaan 0.6.15 ended normally after 13 iterations
##
##
     Estimator
                                                       DWLS
##
     Optimization method
                                                     NLMINB
##
     Number of model parameters
##
     Number of observations
                                                        379
##
## Model Test User Model:
##
                                                   Standard
                                                                  Scaled
                                                      7.291
     Test Statistic
                                                                  12.647
##
```

```
##
     Degrees of freedom
                                                     0.026
##
     P-value (Chi-square)
                                                                  0.002
     Scaling correction factor
                                                                  0.587
##
##
                                                                  0.234
     Shift parameter
##
       simple second-order correction
##
## Model Test Baseline Model:
##
##
     Test statistic
                                                  4919.480
                                                               3905.848
##
     Degrees of freedom
                                                         6
                                                                      6
##
     P-value
                                                     0.000
                                                                  0.000
                                                                  1.260
##
     Scaling correction factor
##
## User Model versus Baseline Model:
##
##
     Comparative Fit Index (CFI)
                                                     0.999
                                                                  0.997
##
     Tucker-Lewis Index (TLI)
                                                     0.997
                                                                  0.992
##
##
    Robust Comparative Fit Index (CFI)
                                                                  0.944
     Robust Tucker-Lewis Index (TLI)
##
                                                                  0.831
##
## Root Mean Square Error of Approximation:
##
##
     RMSEA
                                                     0.084
                                                                  0.119
##
     90 Percent confidence interval - lower
                                                     0.025
                                                                  0.062
     90 Percent confidence interval - upper
                                                     0.153
                                                                  0.185
##
     P-value H_O: RMSEA <= 0.050
                                                     0.145
                                                                  0.025
     P-value H_0: RMSEA >= 0.080
                                                     0.614
##
                                                                  0.880
##
     Robust RMSEA
                                                                  0.377
##
##
     90 Percent confidence interval - lower
                                                                  0.179
##
     90 Percent confidence interval - upper
                                                                  0.611
     P-value H_O: Robust RMSEA <= 0.050
##
                                                                  0.007
##
     P-value H_0: Robust RMSEA >= 0.080
                                                                  0.990
## Standardized Root Mean Square Residual:
##
##
     SRMR
                                                     0.029
                                                                  0.029
##
## Parameter Estimates:
##
##
     Standard errors
                                                Robust.sem
     Information
##
                                                  Expected
##
     Information saturated (h1) model
                                              Unstructured
## Latent Variables:
                      Estimate Std.Err z-value P(>|z|)
##
                                                             Std.lv Std.all
##
     lib_ab_views =~
##
       Ι1
                         1.000
                                                              0.921
                                                                        0.921
##
       12
                         1.020
                                   0.035
                                                     0.000
                                                              0.940
                                                                        0.940
                                           29.205
##
       13
                         1.046
                                   0.032
                                           32.997
                                                     0.000
                                                               0.964
                                                                        0.964
                                  0.034
##
       14
                         0.982
                                           28.553
                                                     0.000
                                                              0.905
                                                                        0.905
##
## Intercepts:
```

.I1 0.000 0.00 ## .I2 0.000 0.00 ## .I3 0.000 0.00 ## .I4 0.000 0.00 ## Thresholds: ## Thresholds: ## Estimate Std.Err z-value P(> z) Std.lv Std.al ## I1 t1 0.156 0.065 2.410 0.016 0.156 0.15 ## I2 t1 -0.237 0.065 -3.639 0.000 -0.237 -0.23 ## I3 t1 -0.347 0.066 -5.273 0.000 -0.347 -0.34 ## I4 t1 -0.299 0.066 -4.559 0.000 -0.299 -0.29 ## ## Wariances: ## Wariances: ## Estimate Std.Err z-value P(> z) Std.lv Std.al ## J1 t1 0.151 0.151 0.151 ## .I1 0.151 0.151 0.151
.I3
.I4 0.000 0.000 ## lib_ab_views 0.000 0.000 ## ## Thresholds: ## Til t1 0.156 0.065 2.410 0.016 0.156 0.15 ## I2 t1 -0.237 0.065 -3.639 0.000 -0.237 -0.23 ## I3 t1 -0.347 0.066 -5.273 0.000 -0.347 -0.34 ## I4 t1 -0.299 0.066 -4.559 0.000 -0.299 -0.29 ## ## Variances: ## ## Variances: ## = Estimate Std.Err z-value P(> z) Std.lv Std.al ## .I1 0.151
<pre>## lib_ab_views 0.000 ## ## Thresholds: ## Thresholds: ## Il t1 0.156 0.065 2.410 0.016 0.156 0.15 ## I2 t1 -0.237 0.065 -3.639 0.000 -0.237 -0.23 ## 3 t1 -0.347 0.066 -5.273 0.000 -0.347 -0.34 ## 14 t1 -0.299 0.066 -4.559 0.000 -0.299 -0.29 ## ## Wariances: ## Variances: ## Usinances: ## I1</pre>
Thresholds: ## Thresholds: ## Estimate Std.Err z-value P(> z) Std.lv Std.al ## Il t1 0.156 0.065 2.410 0.016 0.156 0.15 ## I2 t1 -0.237 0.065 -3.639 0.000 -0.237 -0.23 ## I3 t1 -0.347 0.066 -5.273 0.000 -0.347 -0.34 ## I4 t1 -0.299 0.066 -4.559 0.000 -0.299 -0.29 ## ## Variances: ## Variances: ## Estimate Std.Err z-value P(> z) Std.lv Std.al ## .I1 0.151
Thresholds:
Estimate Std.Err z-value P(> z) Std.lv Std.al ## I1 t1 0.156 0.065 2.410 0.016 0.156 0.15 ## I2 t1 -0.237 0.065 -3.639 0.000 -0.237 -0.23 ## I3 t1 -0.347 0.066 -5.273 0.000 -0.347 -0.34 ## I4 t1 -0.299 0.066 -4.559 0.000 -0.299 -0.29 ## ## Variances: ## Variances: ## Estimate Std.Err z-value P(> z) Std.lv Std.al ## .I1 0.151
I1 t1 0.156 0.065 2.410 0.016 0.156 0.15 ## I2 t1 -0.237 0.065 -3.639 0.000 -0.237 -0.23 ## I3 t1 -0.347 0.066 -5.273 0.000 -0.347 -0.34 ## I4 t1 -0.299 0.066 -4.559 0.000 -0.299 -0.29 ## ## Variances: ## Estimate Std.Err z-value P(> z) Std.lv Std.al ## .I1 0.151
I2 t1
<pre>## I3 t1</pre>
<pre>## I4 t1</pre>
<pre>## ## Variances: ##</pre>
<pre>## Variances: ##</pre>
Estimate Std.Err z-value P(> z) Std.lv Std.al ## .I1 0.151 0.151
.I1 0.151 0.151 0.15
.I2 0.117 0.11 0.11
.I3 0.071 0.071 0.07
.I4 0.182 0.182 0.182
lib_ab_views 0.849 0.040 21.276 0.000 1.000 1.00
##
Scales y*:
Estimate Std.Err z-value P(> z) Std.lv Std.al
I1 1.000 1.00
I2 1.000 1.00
I3 1.000 1.00
I4 1.000 1.00

c) The robust χ^2 value are significant, which is to be expected with a sample size of 379. The robust CFI indicates good model fit, as does the SRMR. The robust RMSEA indicates that the model does not fit well, and the p-value of the close fit test indicates that close fit should be rejected.

Looking at the standardized loadings, all are significant and substantial. All loadings have similar values. The variance of the latent trait is significant.

All in all, I would conclude that model fit seems acceptable.

- d) If you would have to create a 1-item abortion attitude test, I would use Item 3, because it has the highest discrimination parameter.
- e) If the 1-item test has to be used to find persons with extremely liberal views on abortion, I would select the item with the highest threshold (difficulty): Item 1. Persons agreeing with this statement have relatively the most liberal views on abortion.

Additional Exercise: HADS

```
library("foreign")
HADS <- read.spss("HADS.sav", use.value.labels = TRUE, to.data.frame = TRUE)
summary(HADS)</pre>
```

```
geslacht
   Respondentnummer
                     leeftijd
                                                          HADS1
                                een man :217
##
  Min.
        :500002 Min. :18.00
                                                 bijna nooit: 43
  1st Qu.:500162 1st Qu.:35.00
                                 een vrouw :285
                                                 soms
## Median :500333 Median :43.00
                                                            :202
                                                 vaak
        :500335 Mean
   Mean
                        :42.84
                                                 bijna altijd: 97
##
   3rd Qu.:500512 3rd Qu.:51.00
## Max. :500689 Max. :80.00
                                              HADS4
                                                                HADS5
##
           HADS2
                             HADS3
                                                        bijna nooit :179
## bijna nooit :214 bijna nooit : 75
                                      bijna altijd: 31
##
   soms
              :151
                     soms
                                :175
                                      vaak
                                                 : 81
                                                        soms
                                                                   :170
   vaak
              :103 vaak
                                :180
                                      soms
                                                 :219
                                                        vaak
                                                                   :116
##
  bijna altijd: 34 bijna altijd: 72
                                      bijna nooit :171
                                                        bijna altijd: 37
##
##
##
           HADS6
                             HADS7
##
   bijna nooit : 67
                     bijna nooit :199
##
              :204
   soms
                     soms
                                :187
## vaak
              :167
                     vaak
                                :101
## bijna altijd: 64
                    bijna altijd: 15
##
##
```

a) Fit a graded response model to the data:

```
## lavaan 0.6.15 ended normally after 18 iterations
##
##
                                                       DWLS
     Estimator
     Optimization method
                                                     NLMINB
##
##
     Number of model parameters
                                                         28
##
                                                        502
##
     Number of observations
##
## Model Test User Model:
##
                                                   Standard
                                                                  Scaled
     Test Statistic
                                                     94.652
                                                                 171.090
##
##
    Degrees of freedom
                                                          14
                                                                      14
##
     P-value (Chi-square)
                                                      0.000
                                                                   0.000
##
     Scaling correction factor
                                                                   0.559
##
     Shift parameter
                                                                   1.733
##
       simple second-order correction
##
## Parameter Estimates:
##
##
     Standard errors
                                                 Robust.sem
##
     Information
                                                   Expected
```

##	Information satu	rated (h1)	model	Unst	ructured		
##							
	Latent Variables:	Patrimet .	O+ 1 E		D(> I= I)	O+ 1 1	O+ 1 - 11
##		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	anx =~	1 000				0.020	0.000
##	HADS1	1.000	0 022	00 001	0 000	0.832	0.832
##	HADS2	0.961	0.033	29.081	0.000	0.799	0.799
## ##	HADS3 HADS4	0.962	0.033	29.428 18.089	0.000	0.800 0.629	0.800 0.629
##	HADS5	0.756 0.737	0.042	18.153	0.000	0.629	0.629
##	HADS6	0.737	0.041	25.691	0.000	0.730	0.013
##	HADS7	0.878	0.034	27.160	0.000	0.759	0.759
##	пары	0.912	0.034	27.100	0.000	0.759	0.759
##	Intercenta						
##	Intercepts:	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	.HADS1	0.000	Stu.EII	Z varue	F(> Z)	0.000	0.000
##	.HADS2	0.000				0.000	0.000
##	.HADS3	0.000				0.000	0.000
##	.HADS4	0.000				0.000	0.000
##	.HADS5	0.000				0.000	0.000
##	.HADS6	0.000				0.000	0.000
##	.HADS7	0.000				0.000	0.000
##	anx	0.000				0.000	0.000
##	diix	0.000				0.000	0.000
##	Thresholds:						
##	iniconoras.	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	HADS1 t1	-1.368	0.080	-17.124	0.000	-1.368	-1.368
##	HADS1 t2	-0.242	0.057	-4.276	0.000	-0.242	-0.242
##	HADS1 t3	0.866	0.064	13.462	0.000	0.866	0.866
##	HADS2 t1	-0.186	0.056	-3.298	0.001	-0.186	-0.186
##	HADS2 t2	0.604	0.060	10.089	0.000	0.604	0.604
##	HADS2 t3	1.493	0.086	17.407	0.000	1.493	1.493
##	HADS3 t1	-1.039	0.068	-15.170	0.000	-1.039	-1.039
##	HADS3 t2	-0.005	0.056	-0.089	0.929	-0.005	-0.005
##	HADS3 t3	1.065	0.069	15.388	0.000	1.065	1.065
##	HADS4 t1	-1.540	0.088	-17.450	0.000	-1.540	-1.540
##	HADS4 t2	-0.762	0.062	-12.224	0.000	-0.762	-0.762
##	HADS4 t3	0.411	0.058	7.113	0.000	0.411	0.411
##	HADS5 t1	-0.368	0.057	-6.406	0.000	-0.368	-0.368
##	HADS5 t2	0.511	0.059	8.696	0.000	0.511	0.511
##	HADS5 t3	1.449	0.084	17.336	0.000	1.449	1.449
##	HADS6 t1	-1.110	0.071	-15.740	0.000	-1.110	-1.110
##	HADS6 t2	0.100	0.056	1.783	0.075	0.100	0.100
##	HADS6 t3	1.138	0.071	15.944	0.000	1.138	1.138
##	HADS7 t1	-0.263	0.057	-4.632	0.000	-0.263	-0.263
##	HADS7 t2	0.735	0.062	11.887	0.000	0.735	0.735
##	HADS7 t3	1.883	0.112	16.784	0.000	1.883	1.883
##							
##	Variances:						
##		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	.HADS1	0.308				0.308	0.308
##	.HADS2	0.361				0.361	0.361
##	.HADS3	0.360				0.360	0.360
##	.HADS4	0.604				0.604	0.604

```
##
      .HADS5
                           0.624
                                                                   0.624
                                                                             0.624
##
      .HADS6
                           0.467
                                                                             0.467
                                                                   0.467
##
      .HADS7
                           0.424
                                                                   0.424
                                                                             0.424
##
                           0.692
                                     0.033
                                             21.088
                                                         0.000
                                                                   1.000
                                                                             1.000
       anx
##
## Scales y*:
##
                                   Std.Err z-value P(>|z|)
                                                                          Std.all
                        Estimate
                                                                  Std.lv
                           1.000
##
       HADS1
                                                                   1.000
                                                                             1.000
##
       HADS2
                           1.000
                                                                   1.000
                                                                             1.000
                           1.000
##
       HADS3
                                                                   1.000
                                                                             1.000
##
       HADS4
                           1.000
                                                                   1.000
                                                                             1.000
##
       HADS5
                           1.000
                                                                             1.000
                                                                   1.000
##
       HADS6
                           1,000
                                                                   1.000
                                                                             1.000
       HADS7
                           1.000
                                                                   1.000
                                                                             1.000
##
```

- b) HADS4 seems to be the easiest item, because it has the lowest thresholds for all categories.
- c) With 'easiest', we mean that for this item, lower latent trait (anxiety) levels are needed to endorse a higher response category.
- d) Yes, all category thresholds are ordered similarly across items; they go from low to high.
- e) To fit a partial credit model to the data, we pre-multiple the indicators using the same label:

```
HADS.PCM.mod <- '
anx =~ 1*HADS1 + 1*HADS2 + 1*HADS3 + 1*HADS4 + 1*HADS5 + 1*HADS6 + 1*HADS7
'
HADS.PCM.fit <- cfa(HADS.PCM.mod, data = HADS, ordered = paste("HADS", 1:7, sep=""))
summary(HADS.PCM.fit, standardized = TRUE)</pre>
```

```
## lavaan 0.6.15 ended normally after 3 iterations
##
##
     Estimator
                                                        DWLS
##
     Optimization method
                                                      NLMINB
##
     Number of model parameters
                                                          22
##
##
     Number of observations
                                                         502
##
## Model Test User Model:
##
                                                   Standard
                                                                  Scaled
##
     Test Statistic
                                                     192.056
                                                                  206.433
##
     Degrees of freedom
                                                          20
                                                                       20
     P-value (Chi-square)
                                                       0.000
                                                                   0.000
##
##
     Scaling correction factor
                                                                   0.950
                                                                   4.277
##
     Shift parameter
##
       simple second-order correction
##
## Parameter Estimates:
##
##
                                                 Robust.sem
     Standard errors
##
     Information
                                                    Expected
     Information saturated (h1) model
##
                                               Unstructured
##
## Latent Variables:
```

##			Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	anx =~			2041222		- (1-1)	204.2.	504.411
##	HADS1	(1)	1.000				0.750	0.750
##	HADS2	(1)	1.000				0.750	0.750
##	HADS3	(1)	1.000				0.750	0.750
##	HADS4	(1)	1.000				0.750	0.750
##	HADS5	(1)	1.000				0.750	0.750
##	HADS6	(1)	1.000				0.750	0.750
##	HADS7	(1)	1.000				0.750	0.750
##	1111201	(1)	1.000				0.100	01100
##	Intercepts:							
##	interespent.		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	.HADS1		0.000	2041222		- (1-1)	0.000	0.000
##	.HADS2		0.000				0.000	0.000
##	.HADS3		0.000				0.000	0.000
##	.HADS4		0.000				0.000	0.000
##	.HADS5		0.000				0.000	0.000
##	.HADS6		0.000				0.000	0.000
##	.HADS7		0.000				0.000	0.000
##	anx		0.000				0.000	0.000
##	ann.		0.000				0.000	0.000
##	Thresholds:							
##	IIII obiio i do .		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	HADS1 t1		-1.368	0.080	-17.124	0.000	-1.368	-1.368
##	HADS1 t2		-0.242	0.057	-4.276	0.000	-0.242	-0.242
##	HADS1 t3		0.866	0.064	13.462	0.000	0.866	0.866
##	HADS2 t1		-0.186	0.056	-3.298	0.001	-0.186	-0.186
##	HADS2 t2		0.604	0.060	10.089	0.000	0.604	0.604
##	HADS2 t3		1.493	0.086	17.407	0.000	1.493	1.493
##	HADS3 t1		-1.039	0.068	-15.170	0.000	-1.039	-1.039
##	HADS3 t2		-0.005	0.056	-0.089	0.929	-0.005	-0.005
##	HADS3 t3		1.065	0.069	15.388	0.000	1.065	1.065
##	HADS4 t1		-1.540	0.088	-17.450	0.000	-1.540	-1.540
##	HADS4 t2		-0.762	0.062	-12.224	0.000	-0.762	-0.762
##	HADS4 t3		0.411	0.058	7.113	0.000	0.411	0.411
##	HADS5 t1		-0.368	0.057	-6.406	0.000	-0.368	-0.368
##	HADS5 t2		0.511	0.059	8.696	0.000	0.511	0.511
##	HADS5 t3		1.449	0.084	17.336	0.000	1.449	1.449
##	HADS6 t1		-1.110	0.071	-15.740	0.000	-1.110	-1.110
##	HADS6 t2		0.100	0.056	1.783	0.075	0.100	0.100
##	HADS6 t3		1.138	0.071	15.944	0.000	1.138	1.138
##	HADS7 t1		-0.263	0.057	-4.632	0.000	-0.263	-0.263
##	HADS7 t2		0.735	0.062	11.887	0.000	0.735	0.735
##	HADS7 t3		1.883	0.112	16.784	0.000	1.883	1.883
##								
##	Variances:							
##			Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
##	.HADS1		0.438				0.438	0.438
##	.HADS2		0.438				0.438	0.438
##	.HADS3		0.438				0.438	0.438
##	.HADS4		0.438				0.438	0.438
##	.HADS5		0.438				0.438	0.438
##	.HADS6		0.438				0.438	0.438
##	.HADS7		0.438				0.438	0.438

```
##
                            0.562
                                      0.019
                                               30.186
                                                          0.000
                                                                    1.000
                                                                               1.000
       anx
##
##
   Scales y*:
                                                       P(>|z|)
##
                        Estimate
                                   Std.Err z-value
                                                                   Std.lv
                                                                            Std.all
##
       HADS1
                            1.000
                                                                    1.000
                                                                               1.000
                            1.000
##
       HADS2
                                                                    1.000
                                                                               1.000
##
       HADS3
                            1.000
                                                                    1.000
                                                                               1.000
##
       HADS4
                            1.000
                                                                    1.000
                                                                               1.000
##
       HADS5
                            1.000
                                                                    1.000
                                                                               1.000
##
       HADS6
                            1.000
                                                                    1.000
                                                                               1.000
##
       HADS7
                            1.000
                                                                    1.000
                                                                               1.000
```

Note that again we see Item 4 is the easiest item, with the lowest thresholds.

f) The standardized loadings in the GRM differ only somewhat between items, with the largest difference being around .2. So we could prefer the PCM for that reason: Because there are no substantial differences in item loadings. But if we want to be able to distinguish between items that discriminate more or less, we could prefer the GRM. Looking at the standard errors of the item loadings, they are statistically significantly different between some of the items (e.g., items 1, 2 and 3 have very similar loadings, but these significantly differ from the loadings of items 4 and 5).

We test the difference in fit and inspect model fit indices:

```
fitinds <- c("chisq.robust", "df", "pvalue.robust", "cfi.robust",</pre>
             "rmsea.robust", "srmr")
fitMeasures(HADS.GRM.fit, fitinds)
                  cfi.robust rmsea.robust
##
             df
                                                    srmr
##
         14.000
                        0.882
                                     0.176
                                                   0.066
fitMeasures(HADS.PCM.fit, fitinds)
##
             df
                  cfi.robust rmsea.robust
                                                    srmr
##
         20.000
                        0.832
                                     0.176
                                                   0.097
lavTestLRT(HADS.PCM.fit, HADS.GRM.fit)
##
## Scaled Chi-Squared Difference Test (method = "satorra.2000")
##
## lavaan NOTE:
##
       The "Chisq" column contains standard test statistics, not the
##
       robust test that should be reported per model. A robust difference
##
       test is a function of two standard (not robust) statistics.
##
                Df AIC BIC
                              Chisq Chisq diff Df diff Pr(>Chisq)
##
## HADS.GRM.fit 14
                             94.652
                                                        1.213e-12 ***
## HADS.PCM.fit 20
                            192.056
                                        67.696
                                                      6
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
```

There is a significant difference in fit, so from a statistical point of view we should prefer the GRM, which is more complex. This is also indicated by the CFI values. However, if we use the RMSEA as the main criterion for model selection, PCM and GRM are equally preferable (note that RMSEA has a (strong) preference for more parsimoneous models).

```
library("ltm")
GRM.IRT <- grm(HADS[ , 4:10])
coef(GRM.IRT)</pre>
```

Extra: Would we reach the same conclusion using ML estimation?

```
##
        Extrmt1 Extrmt2 Extrmt3 Dscrmn
## HADS1 -1.668 -0.269
                          1.025 2.610
## HADS2 -0.236
                  0.718
                          1.863 2.264
## HADS3 -1.254
                 0.002
                          1.268 2.533
## HADS4 -2.540 -1.168
                          0.655 1.365
## HADS5 -0.598
                 0.786
                          2.335 1.338
## HADS6 -1.511
                  0.154
                          1.541 1.870
## HADS7
        -0.332
                  0.923
                          2.485 2.062
PCM.IRT <- grm(HADS[ , 4:10], constrained = TRUE)
anova (PCM.IRT, GRM.IRT)
##
##
   Likelihood Ratio Table
              AIC
                      BIC log.Lik
                                     LRT df p.value
## PCM.IRT 7579.43 7672.24 -3767.72
## GRM.IRT 7532.45 7650.57 -3738.22 58.99 6 <0.001
```

The ML-estimated GRM indicates highest discriminatory power for HADS item 1, followed by HADS items 3, 2 and 7. This is similar to what we found using DWLS estimation. Also, item 4 seems most easy, both with ML and DWLS estimation.

The likelihood ratio test, AIC and BIC all indicate that the GRM fits the data better than the PCM.

h)

First, we convert the HADS items to numeric:

```
HADS2 <- sapply(HADS[ , 4:10], as.numeric)
```

Then we fit a CFA to the numeric items. We can use the same model specification as for the GRM, and have to specify the type of estimator used:

```
HADS.ML.fit <- cfa(HADS.GRM.mod, data = HADS2, meanstructure = TRUE)
parameterestimates(HADS.ML.fit, standardized = TRUE)[, c(1:5, 7, 11)]
```

```
##
                               se pvalue std.all
        lhs op
                 rhs
                       est
        anx =~ HADS1 1.000 0.000
## 1
                                      NA
                                           0.772
## 2
        anx = ~ HADS2 0.982 0.064
                                           0.702
                                           0.761
## 3
        anx =~ HADS3 1.029 0.062
                                       0
## 4
        anx = ~ HADS4 0.713 0.059
                                           0.558
                                       0
## 5
        anx =~ HADS5 0.774 0.065
                                       0
                                           0.558
## 6
        anx = ~ HADS6 0.865 0.060
                                           0.666
## 7
        anx =~ HADS7 0.831 0.057
                                           0.672
```

```
HADS1 ~~ HADS1 0.309 0.026
                                             0.403
## 9
      HADS2 ~~ HADS2 0.454 0.034
                                        0
                                             0.507
## 10 HADS3 ~~ HADS3 0.351 0.028
                                             0.420
## 11 HADS4 ~~ HADS4 0.514 0.035
                                        0
                                             0.689
## 12 HADS5 ~~ HADS5 0.608 0.041
                                         0
                                             0.689
## 13 HADS6 ~~ HADS6 0.428 0.031
                                             0.556
                                        0
## 14 HADS7 ~~ HADS7 0.383 0.028
                                         0
                                             0.548
## 15
        anx ~~
                  anx 0.457 0.047
                                        0
                                             1.000
## 16 HADS1 ~1
                      2.703 0.039
                                        0
                                             3.088
## 17 HADS2 ~1
                      1.914 0.042
                                         0
                                             2.023
## 18 HADS3 ~1
                      2.496 0.041
                                             2.730
                                         0
## 19 HADS4 ~1
                      3.056 0.039
                                         0
                                             3.538
## 20 HADS5 ~1
                      2.022 0.042
                                        0
                                             2.153
                      2.454 0.039
## 21 HADS6 ~1
                                         0
                                             2.797
## 22 HADS7 ~1
                      1.865 0.037
                                        0
                                             2.230
## 23
                      0.000 0.000
                                       NA
                                             0.000
        anx ~1
```

The standardized loadings indicate that item 1 is the best indicator, followed by item 3, 2 and then 7. So in that respect, treating the items a continuous or ordered does not really seem to make a difference.

The item intercepts indicate that item 4 is the easiest item. Item intercepts are the expected value of the item score, when the LV has a value of 0. So, the higher the item intercept, the higher the item score given the same latent trait value.

The standardized loadings are a bit lower in the model where we treat the indicators as continuous. The residual variance are higher in the model where we treat the indicators as continuous. This is in line with the very first observation we made in Example 6.2: Pearson correlations (assuming continuous variables) are lower than tetra- and polychoric correlations (which assume ordered categorical variables, which arise from an underlying continuous latent variable).

```
fitinds2 <- c("chisq", "df", "pvalue", "cfi", "rmsea", "srmr")</pre>
fitmeasures(HADS.ML.fit, fitinds2)
##
     chisq
                                 cfi
                 df
                     pvalue
                                        rmsea
                                                  srmr
## 149.170
             14.000
                      0.000
                               0.898
                                        0.139
                                                 0.053
fitMeasures(HADS.GRM.fit, fitinds)
```

```
## df cfi.robust rmsea.robust srmr
## 14.000 0.882 0.176 0.066
```

The model fit does differ quite a bit between the models, which is to be expected.

g+h) Now we treat the items as interval-scale variables and use robust ML:

```
HADS.MLR.fit <- cfa(HADS.GRM.mod, data = HADS2, estimator = "MLR", meanstructure = TRUE) parameterestimates(HADS.MLR.fit, standardized = TRUE)[, c(1:5, 7, 11)]
```

```
##
                                se pvalue std.all
        lhs op
                  rhs
                         est
        anx = ~ HADS1 1.000 0.000
## 1
                                        NA
                                             0.772
## 2
        anx = ~ HADS2 0.982 0.079
                                         0
                                             0.702
## 3
        anx =~ HADS3 1.029 0.065
                                         0
                                             0.761
## 4
        anx = ~ HADS4 0.713 0.058
                                             0.558
                                         0
```

```
## 5
        anx = ~ HADS5 0.774 0.065
                                            0.558
## 6
        anx = ~ HADS6 0.865 0.050
                                            0.666
                                        0
        anx = ~ HADS7 0.831 0.067
                                            0.672
## 7
      HADS1 ~~ HADS1 0.309 0.031
## 8
                                        0
                                            0.403
      HADS2 ~~ HADS2 0.454 0.039
                                            0.507
## 10 HADS3 ~~ HADS3 0.351 0.035
                                        0
                                            0.420
## 11 HADS4 ~~ HADS4 0.514 0.037
                                            0.689
## 12 HADS5 ~~ HADS5 0.608 0.040
                                        0
                                            0.689
## 13 HADS6 ~~ HADS6 0.428 0.039
                                        0
                                            0.556
## 14 HADS7 ~~ HADS7 0.383 0.030
                                            0.548
        anx ~~
                  anx 0.457 0.045
                                            1.000
## 16 HADS1 ~1
                      2.703 0.039
                                            3.088
                                        0
## 17 HADS2 ~1
                      1.914 0.042
                                        0
                                            2.023
## 18 HADS3 ~1
                      2.496 0.041
                                            2.730
## 19 HADS4 ~1
                      3.056 0.039
                                        0
                                            3.538
## 20 HADS5 ~1
                      2.022 0.042
                                        0
                                            2.153
## 21 HADS6 ~1
                      2.454 0.039
                                        0
                                            2.797
## 22 HADS7 ~1
                      1.865 0.037
                                            2.230
## 23
                      0.000 0.000
                                            0.000
        anx ~1
                                       NA
```

We get a similar ranking of item loadings, and similar ranking of difficulties. Note however, that in the ML-estimated model, higher item intercepts correspond to lower item thresholds in the DWLS-estimated model.

Some authorative references on whether we can treat ordered-categorical items as interval-scale:

Dolan, C. V. (1994). Factor analysis of variables with 2, 3, 5 and 7 response categories: A comparison of categorical variable estimators using simulated data. *British Journal of Mathematical and Statistical Psychology*, 47(2), 309-326.

DiStefano, C. (2002). The impact of categorization with confirmatory factor analysis. *Structural Equation Modeling*, 9(3), 327-346.

Rhemtulla, M., Brosseau-Liard, P. E., & Savalei, V. (2012). When can categorical variables be treated as continuous? A comparison of robust continuous and categorical SEM estimation methods under suboptimal conditions. *Psychological Methods*, 17(3), 354.