The "Boxes and Bulbs" Problem

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The problem

The “Boxes and Bulbs” problem[1] is as follows:

> *Box A has 10 lightbulbs, of which 4 are defective. Box B has 6 lightbulbs, of which 1 is defective. Box C has 8 lightbulbs, of which 3 are defective.*
>
> *I) If we randomly choose a box, and then randomly choose a lightbulb from that box, what is the probability that we will choose a non-defective bulb? II) If we do choose a nondefective bulb, what is the probability it came from Box C?*

[1]This problem is taken from *Schaum's Outlines: Probability* (2nd ed., 2000), pages 87-88.

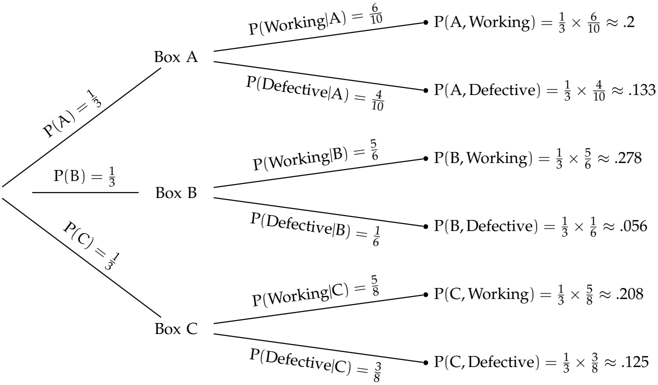
The solution

First, note that the solution requires the calculation of two probabilities:

$$\mathrm{P}(\mathrm{Bulb} = \mathrm{Working}) \quad \text{and} \quad \mathrm{P}(\mathrm{Box} = \mathrm{C} | \mathrm{Bulb} = \mathrm{Working}).$$

These are the answers to Part I and Part II, respectively. Second, if you correctly work out the first probability, i.e. $\mathrm{P}(\mathrm{Bulb} = \mathrm{Working})$, then will have all the information necessary to obtain the second probability, i.e. $\mathrm{P}(\mathrm{Box} = \mathrm{C} | \mathrm{Bulb} = \mathrm{Working})$.

The generative model



- This tree structure gives you all the information you need to answer the questions. Note that on the right are given all the joint probabilities, which can be arranged to a joint probability table as follows:

	Working	Defective
Box A	.2	.133
Box B	.278	.056
Box C	.208	.125

- Now it is simple to see the overall probability of choosing a working bulb. It is probability of choosing Box A and choosing a working bulb *or* choosing Box B and choosing a working bulb *or* choosing Box C and choosing a working bulb. This is the sum of the first column, which is $.2 + .278 + .208 \approx .686$.
- Likewise, to get the probability of choosing Box C *given* that we have chosen a working bulb, we see what proportion of the total probability of choosing a working bulb, i.e. .686 is from when we choose Box C and a working bulb, i.e. .208. In other words, it is $.208/.686 \approx .304$.

Using probability rules

- The first question asks us to calculate $\mathrm{P}(\mathrm{Bulb} = \mathrm{Working})$. We know that this is equal to

$$\begin{aligned}\mathrm{P}(\mathrm{Working}) &= \mathrm{P}(\mathrm{Working}, A) + \mathrm{P}(\mathrm{Working}, \mathrm{B}) + \mathrm{P}(\mathrm{Working}, \mathrm{C}),\\ &= .2 + .278 + .208.\\ &\approx .686.\end{aligned}$$

- When asked for $\mathrm{P}(\mathrm{C}|\mathrm{Working})$ we use the rule of conditional probability being the joint probability divided by the marginal probability, i.e.

$$\begin{aligned}\mathrm{P}(\mathrm{C}|\mathrm{Working}) &= \frac{\mathrm{P}(\mathrm{Working}, \mathrm{C})}{\mathrm{P}(\mathrm{Working})},\\ &= \frac{.208}{.686},\\ &\approx .304.\end{aligned}$$