A gentle introduction to the case for Bayesian data analysis

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Abstract

After a long period of controversy, the use of Bayesian data analysis (BDA) in psychology has reached a tipping point. Psychology - as with many other sciences - is beginning to accept that Bayesian methods offer an important and useful set of tools for answering certain questions and dealing with complex, messy real-world data. One implication is that in future, all psychologists - even those expert in and happily using classical, frequentist methods - need to have a basic Bayesian grounding in order to keep abreast of a literature where BDA is commonplace. Furthermore, it may become increasingly difficult to publish in the top empirical journals without adding some Bayesian methods to one's toolbox.

Thanks to ...

Mark Andrews

Jens Roeser

ESRC (Advanced Training Initiative)

Overview

1. The turn to Bayes in Psychology
2. So ... how does Bayesian data analysis work?
3. A simple illustrative example: *predicting the US presidential election*
4. Another simple example: *finding evidence to support the null hypothesis*
5. Using Bayesian tools: *inference in multilevel models*
6. Complex Bayesian models: *cognitive processes in language production*
7. Some amateur futurology

Psychology is embracing Bayesian data analysis (again)

On taking up the post of editor of the *Journal of Experimental Psychology: General* - arguably the most prestigious outlet for experimental work in psychology - the editor announced that the journal

> *will seek to attract work in specialized areas critical to the development of our science, for example [...] the adoption of Bayesian methods in data analysis.*

[Gauthier, 2012]

What is wrong with classical (frequentist) methods?

Leaving aside philosophical problems (e.g., that inference is based on unobserved data) here are a few of the things that have made me reconsider frequentist methods ...

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- relevant external information enters a frequentist analysis in an *ad hoc* way

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3. A probability model that *combines* the data and the prior information (the posterior)

Normal distribution example

Does a single exposure to an US flag influence voting intention?

Carter et al. (2011) report a priming study in which a single exposure to a US flag changes voting intentions (making them more Republican)

They report an 'effect size' of over a quarter of a standard deviation:

$$\hat{\mu}_E - \hat{\mu}_C = 0.142, \quad t_{181} = 2.02, \quad p = .04 \quad d = 0.298$$

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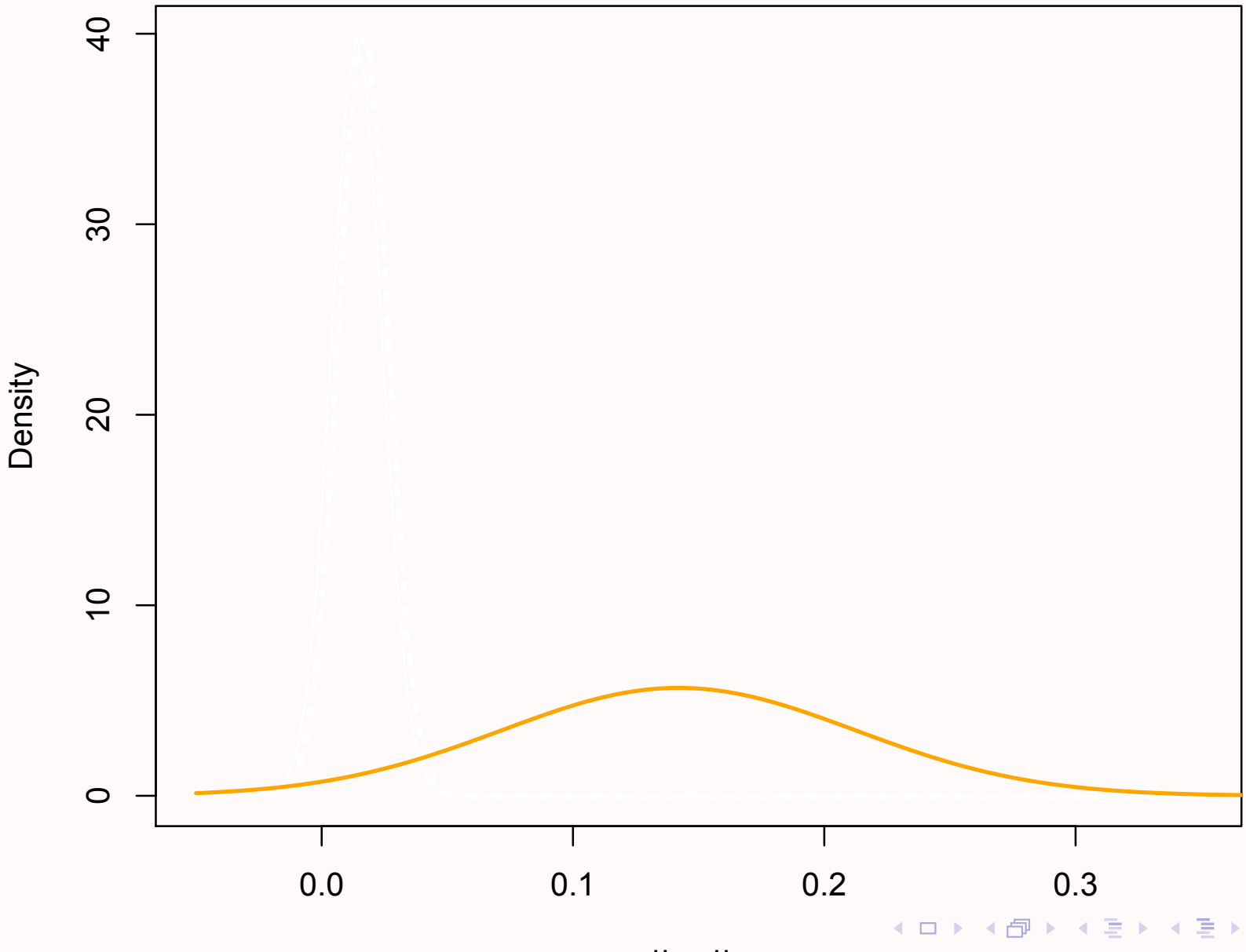
... this seems unrealistically large e.g., being associated with a 10.7% swing

The likelihood

The likelihood is a mathematical function that models the probability of the observed data as a function of a parameter (e.g., a population mean) or a set of parameters

The observed mean is 0.142 with a standard error of 0.07

Assuming a normal distribution for the effect, the likelihood would look like this ...

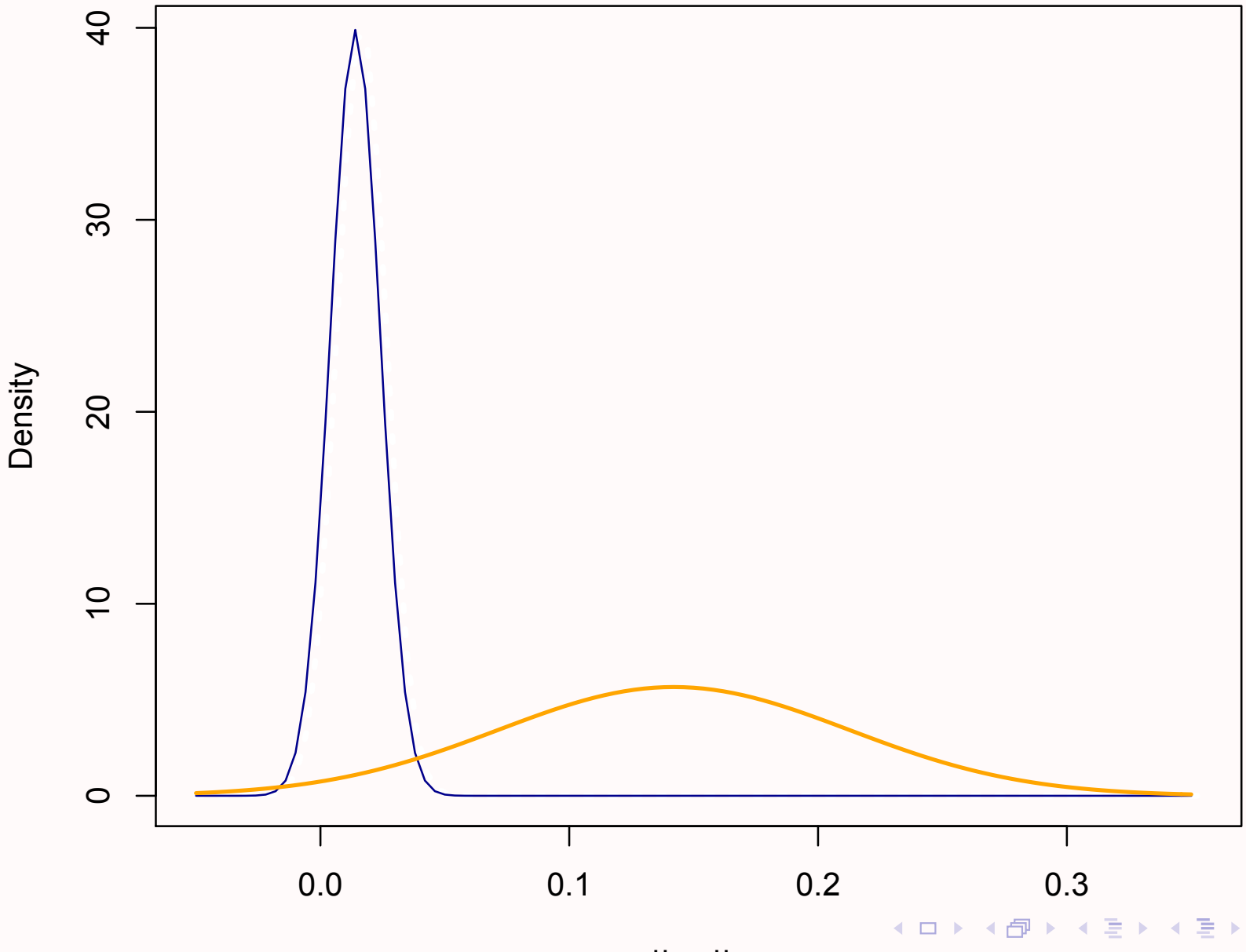

Density
0
10
20
30
40
0.0
0.1
0.2
0.3

The prior

The prior is a probability distribution that reflects relevant information about a parameter (e.g., a population mean)

e.g.,
$M = 0.014$ with corresponds to roughly a 1% swing
$SD = 0.01$ implies 95% chance of a swing from -0.5% to 2.5%

Again, assuming a normal distribution for the prior ...


Density
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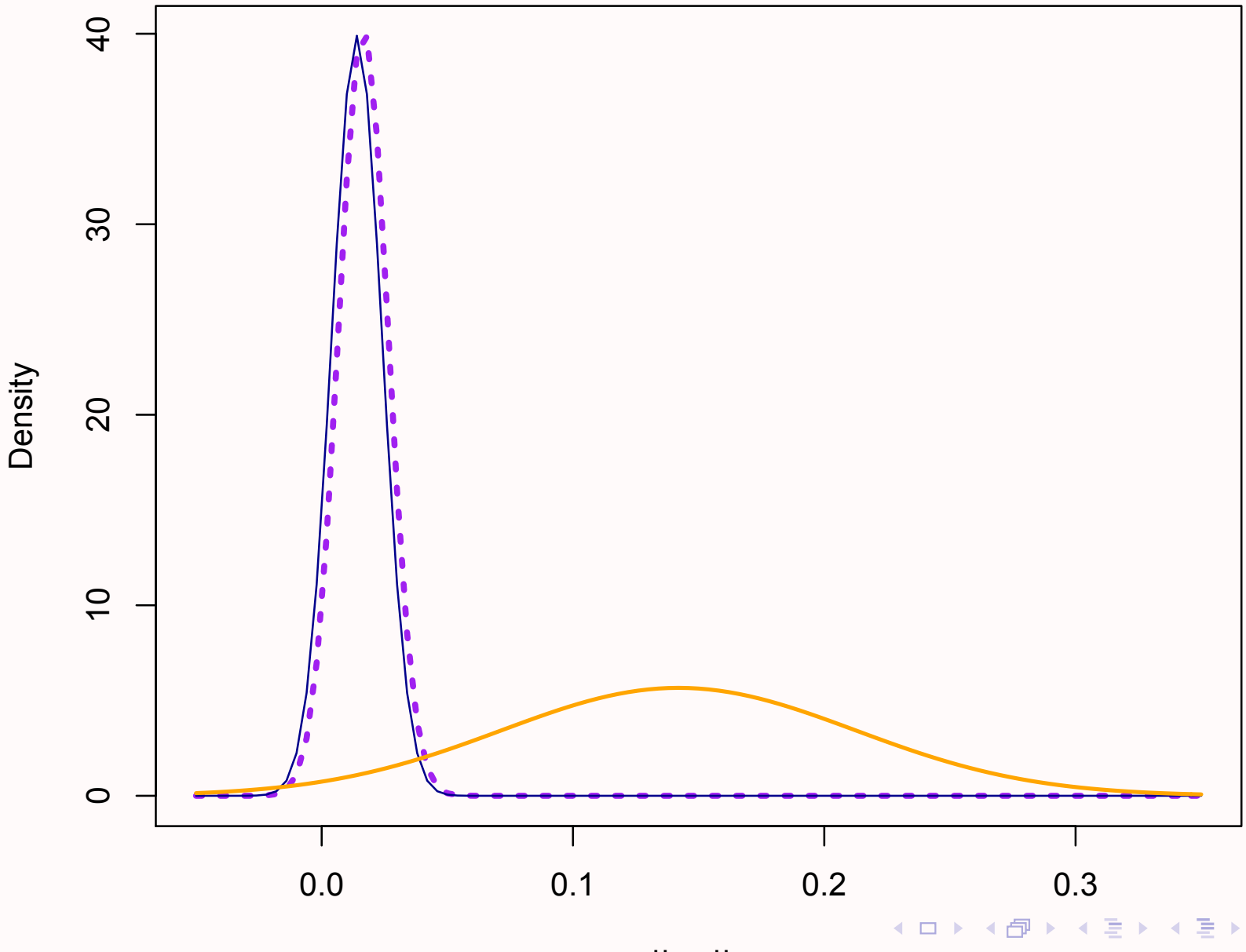
Obtaining the posterior using Bayes' theorem ...

$$\overbrace{\mathrm{P}(\theta|\mathcal{D},\Omega)}^{\text{Posterior}} = \frac{\overbrace{\mathrm{P}(\mathcal{D}|\theta)}^{\text{Likelihood}}\overbrace{\mathrm{P}(\theta|\Omega)}^{\text{Prior}}}{\int \mathrm{P}(\mathcal{D}|\theta)\mathrm{P}(\theta|\Omega)d\theta}$$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Frequentist confidence interval

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or using default prior on effect size $BF_{10} = 1.05$

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- More flexibility in modelling complex, messy (real world) data sets
- Greater emphasis on transparency in modelling

A simple illustrative example: *predicting the US presidential election*

- Guam 'straw poll' predicted result every result since 1984
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- So 8 out 8 elections predicted the result correctly
- Naive estimate: that $P(\text{Cinton victory}) = 1$

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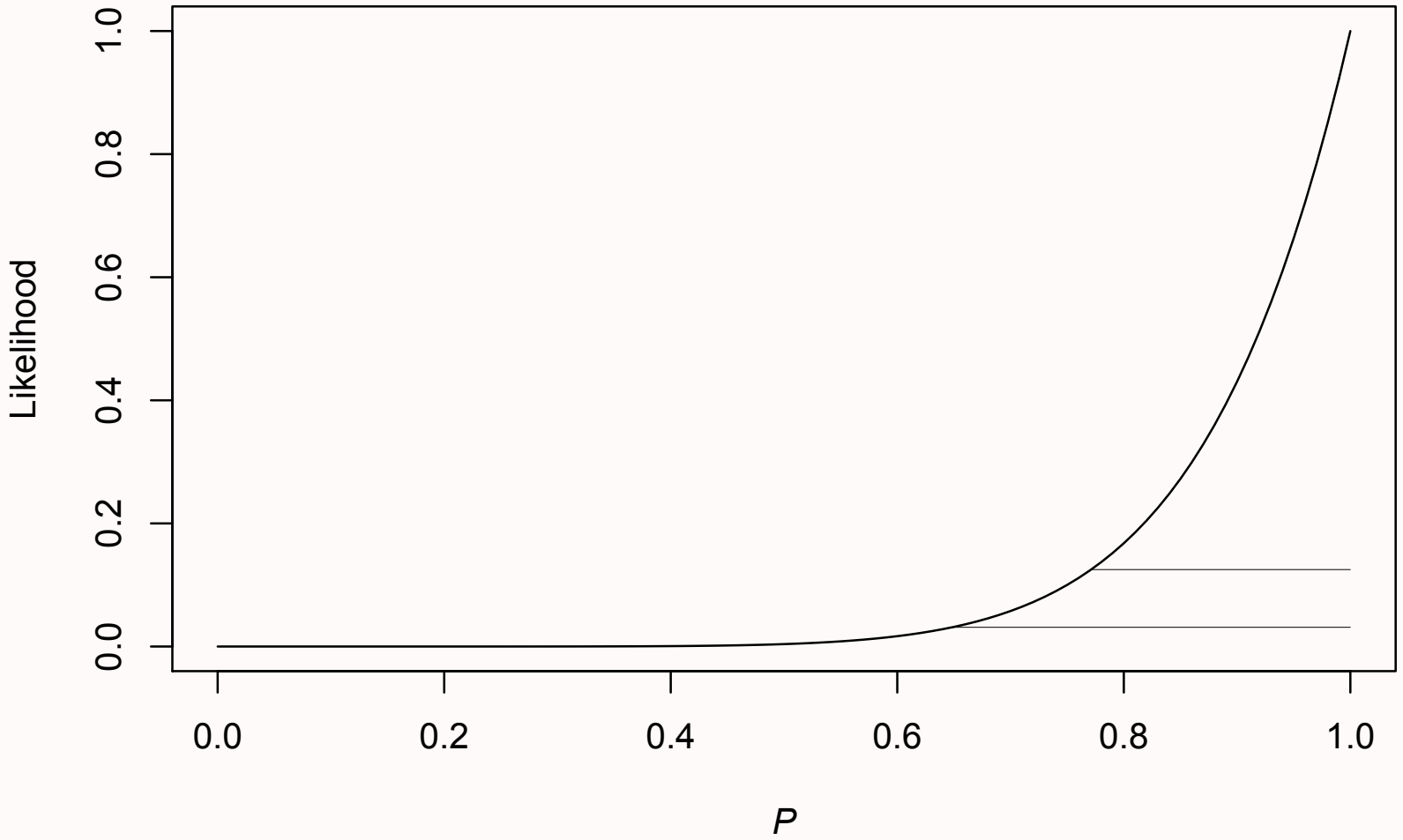
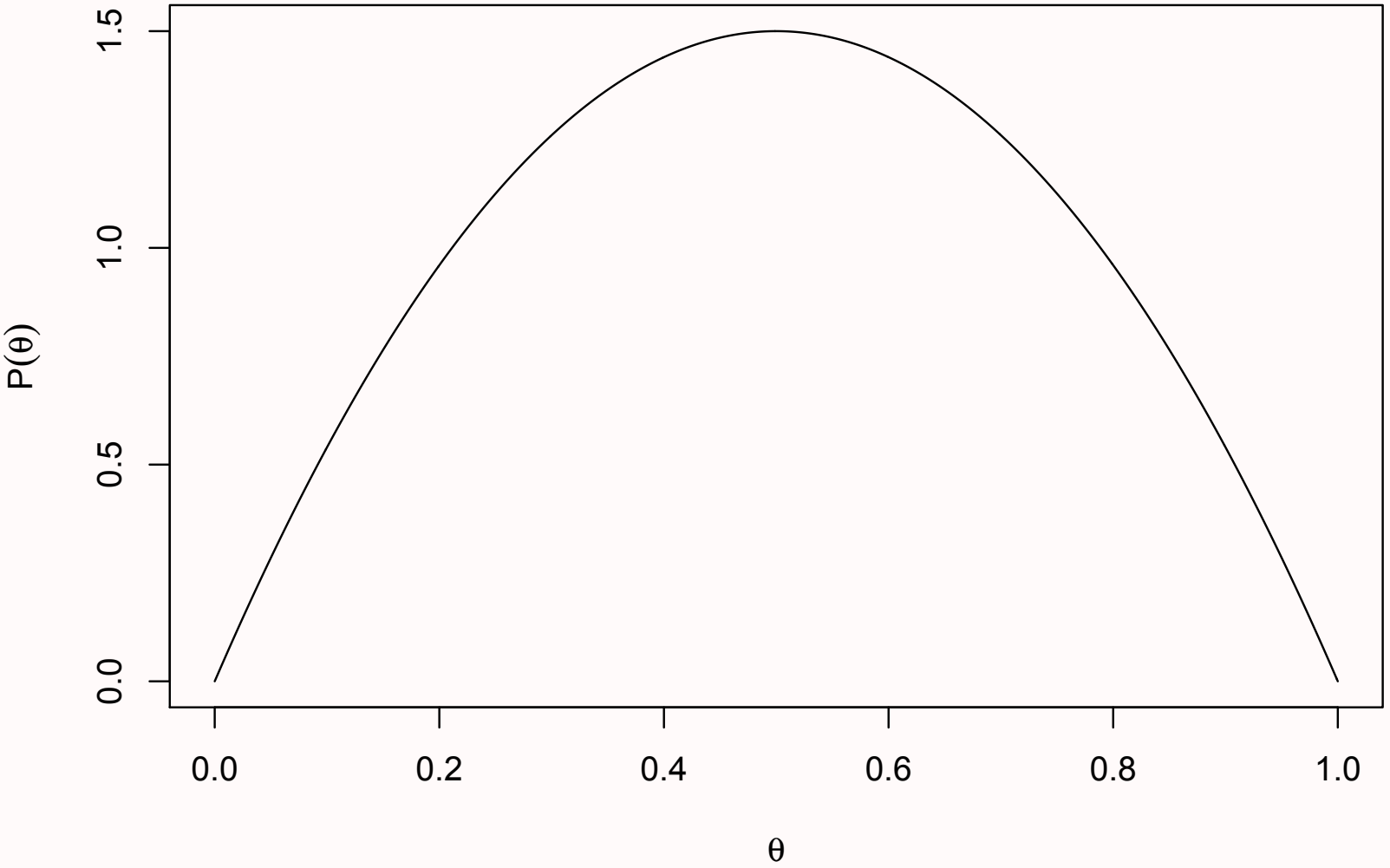
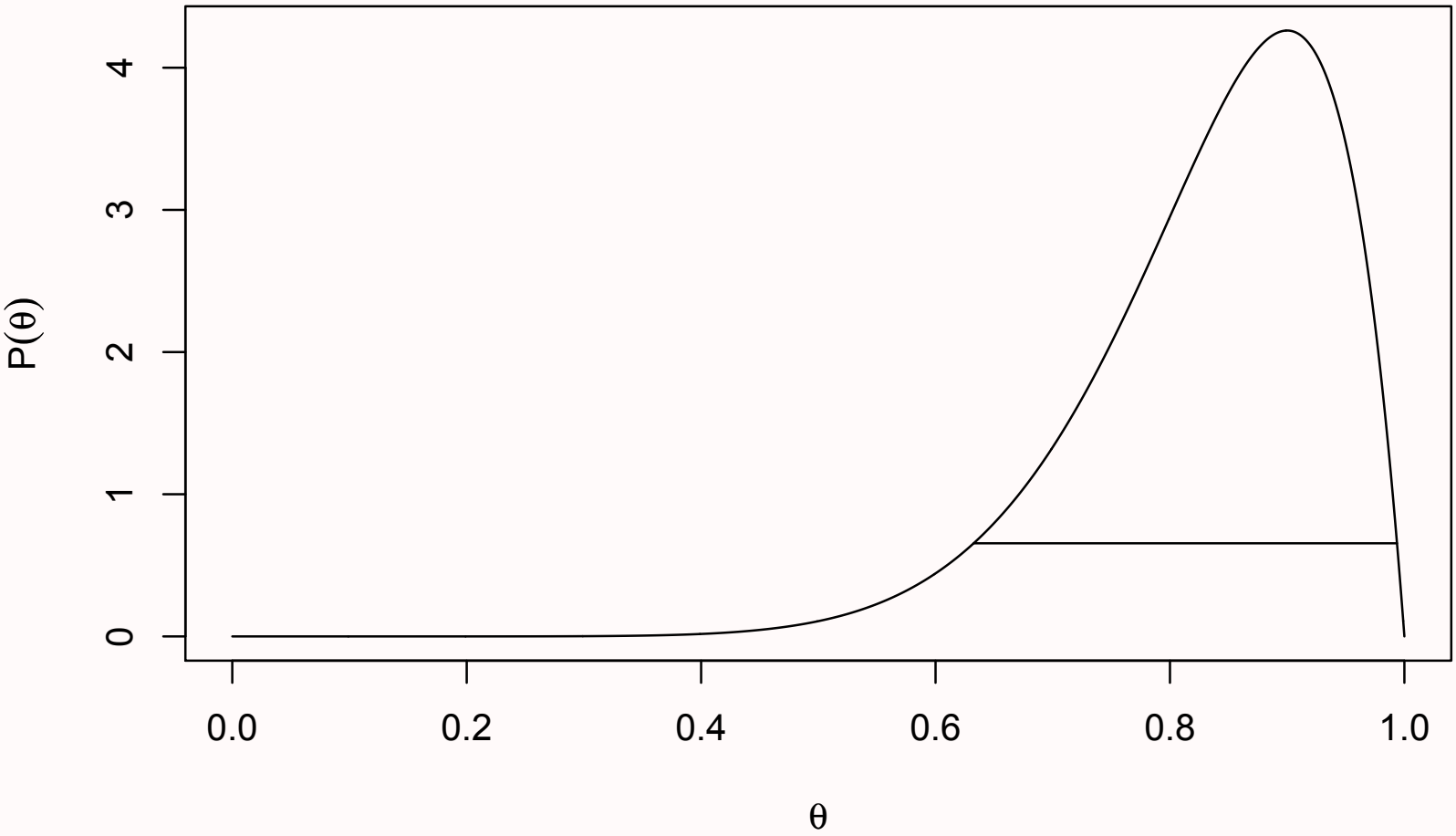
- *Beta*(α, β)
- Combined with likelihood for a binomial proportion



Likelihood for 8 out of 8 wins



Prior with $Beta(2,2)$ distribution



Posterior distributed as $Beta(10, 2)$

Comparing the Bayesian and frequentist solutions:

```
> binom.bayes(8,8, prior.shape1= 2, prior.shape2 = 2)
  method x n shape1 shape2      mean     lower upper  sig
1  bayes 8 8     10      2 0.8333333 0.6356405     1 0.05

> binconf(8,8,method='asymptotic')
 PointEst Lower Upper
        1     1     1

> binconf(8,8,method='wilson')
 PointEst     Lower Upper
        1 0.6755924     1

> binconf(10,12,method='asymptotic')
  PointEst     Lower    Upper
 0.8333333 0.6224748 1.044192
```

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- Avoids *ad hoc* fixes

Another simple example: *finding evidence to support the null hypothesis*

- Bayesian interval estimate
- Krushke's Bayesian alternative to the t test (BEST package in R)

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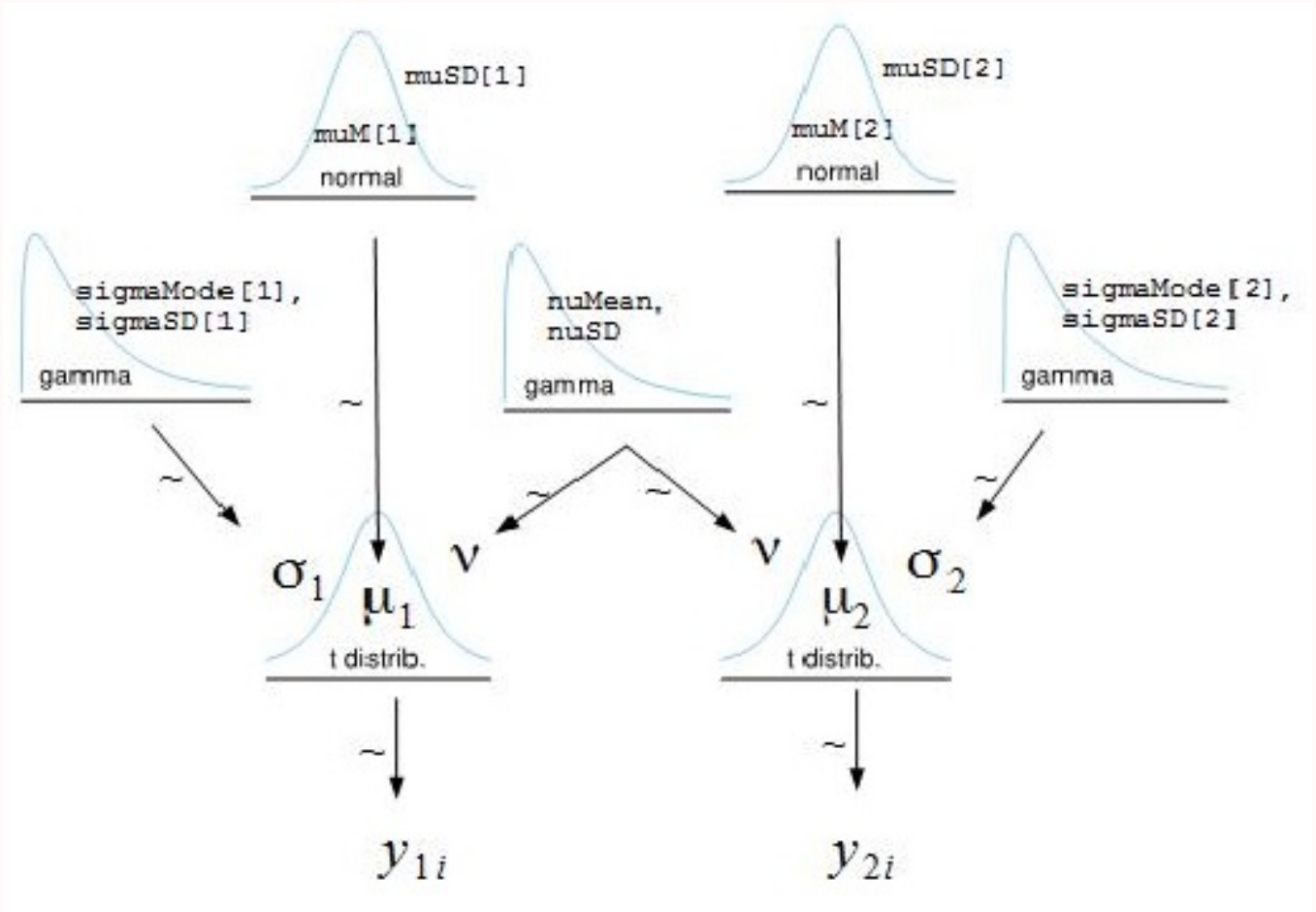
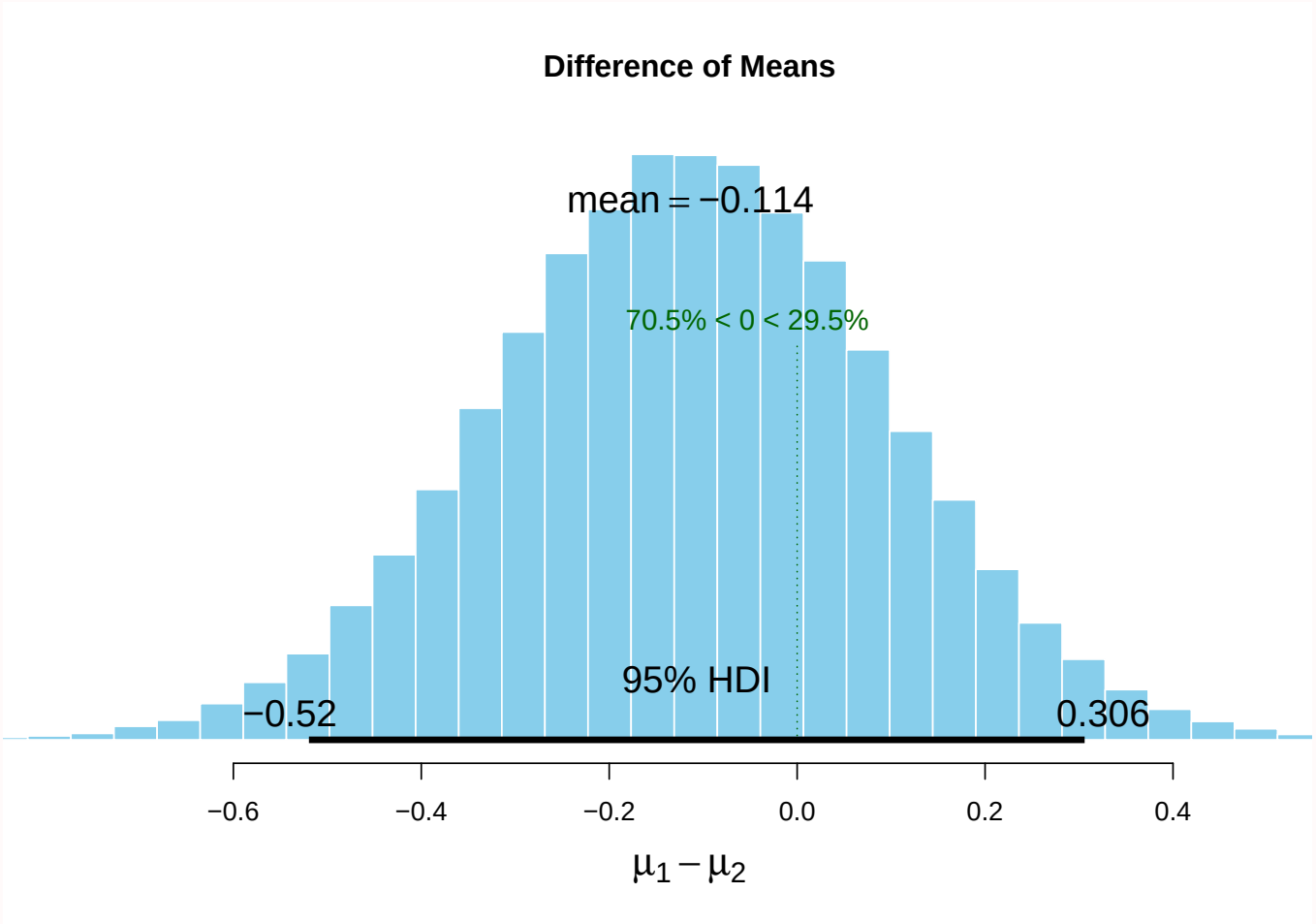
Krushke's robust Bayesian alternative to the t test

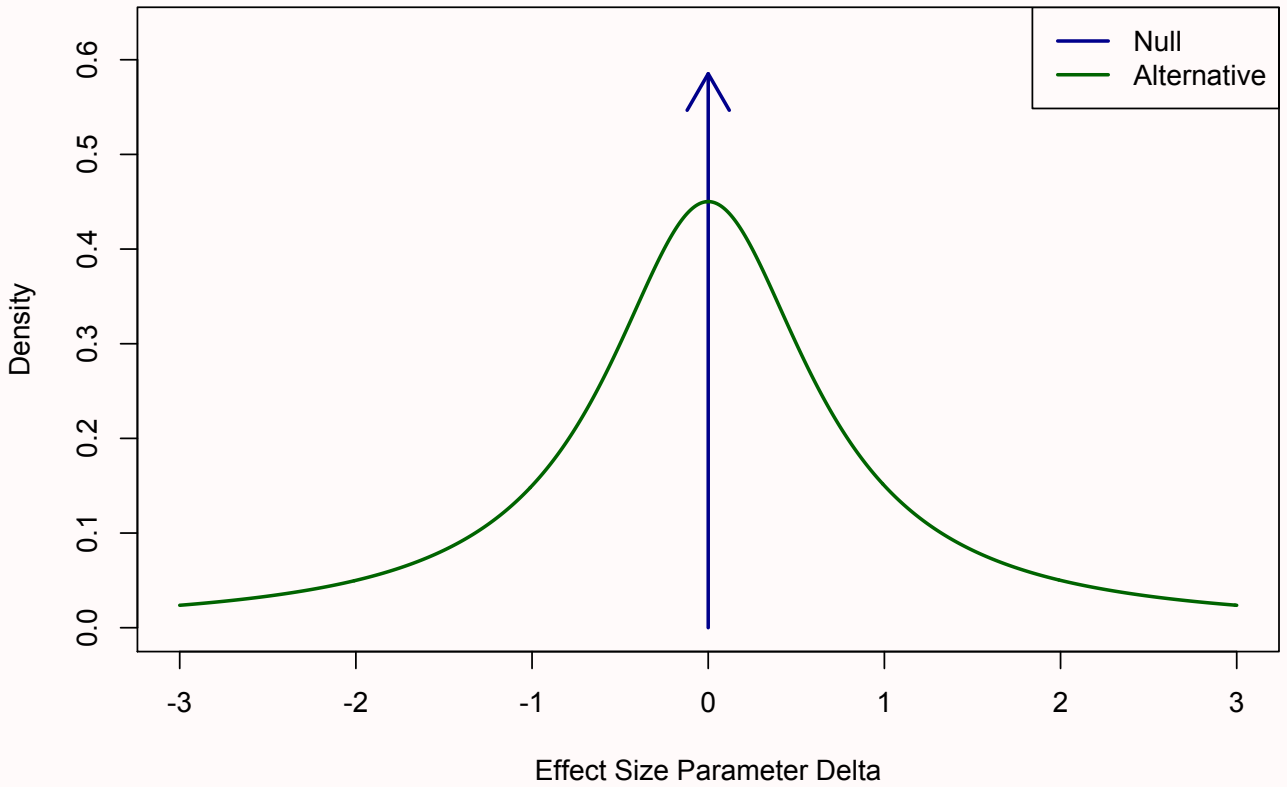


Flag priming replication from ManyLabs

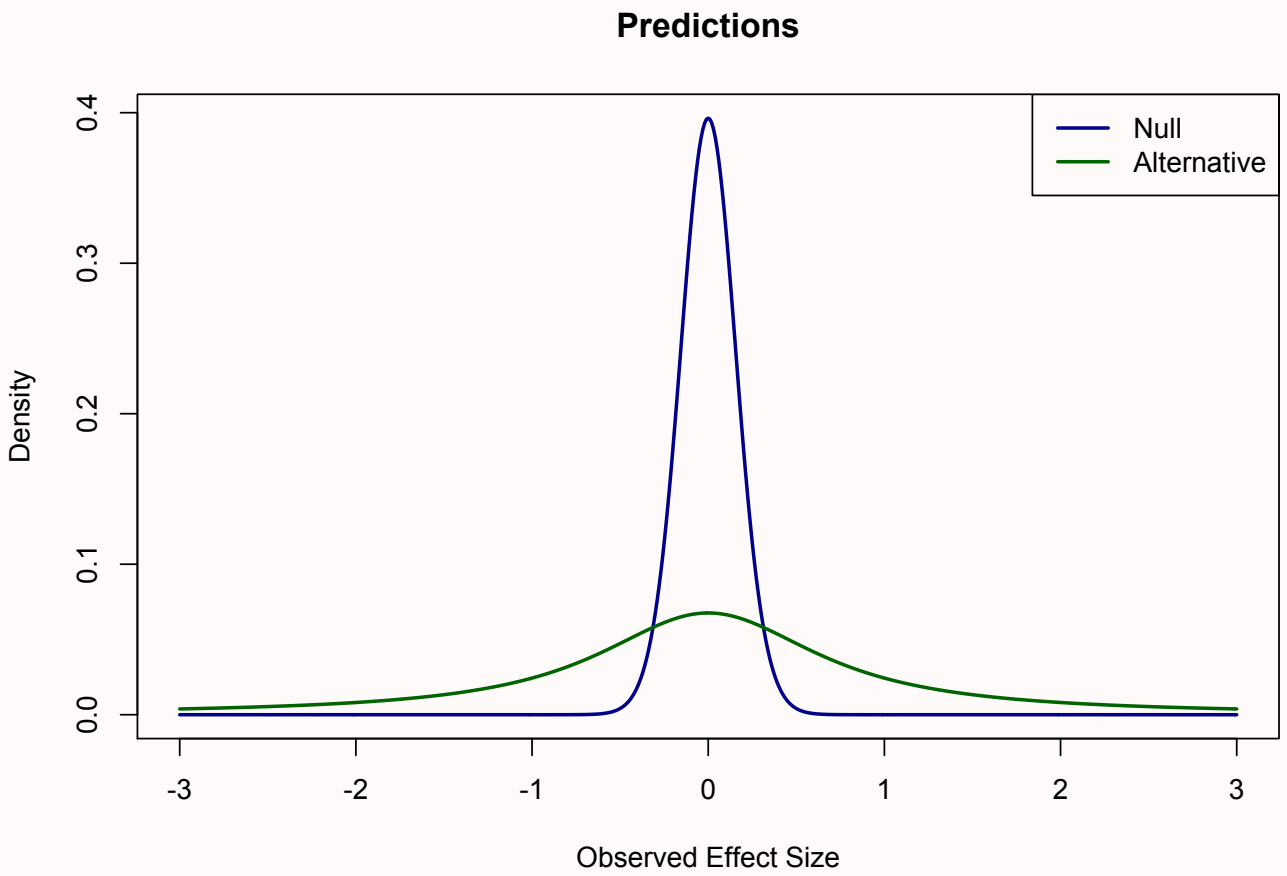


Highest posterior density interval for difference

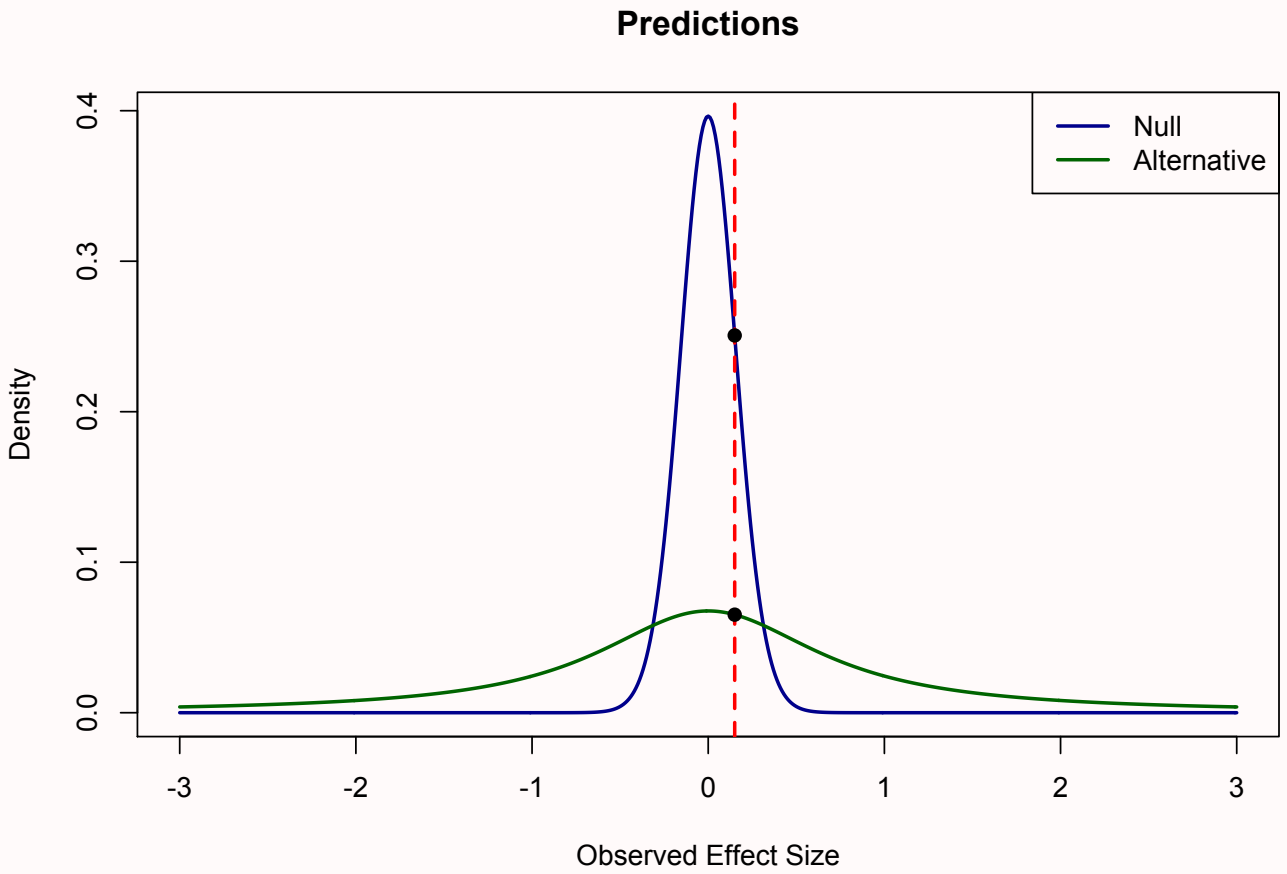
The models being compared



The predictions from each model



The Bayes factor for the observed effect size



Bayes factor with default JZS prior

```
> library(BayesFactor)
>
> ttestBF(formula=flagdv ~ flagGroup, data=flag.data)

Bayes factor analysis
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[1] Alt., r=0.707 : 0.2825635 +/-0.02%

Against denominator:
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The Bayes Factor is $1/0.283 = 3.54$ in favour of H_0

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... with MCMC methods being the preferred approach (being generally both safe and versatile)

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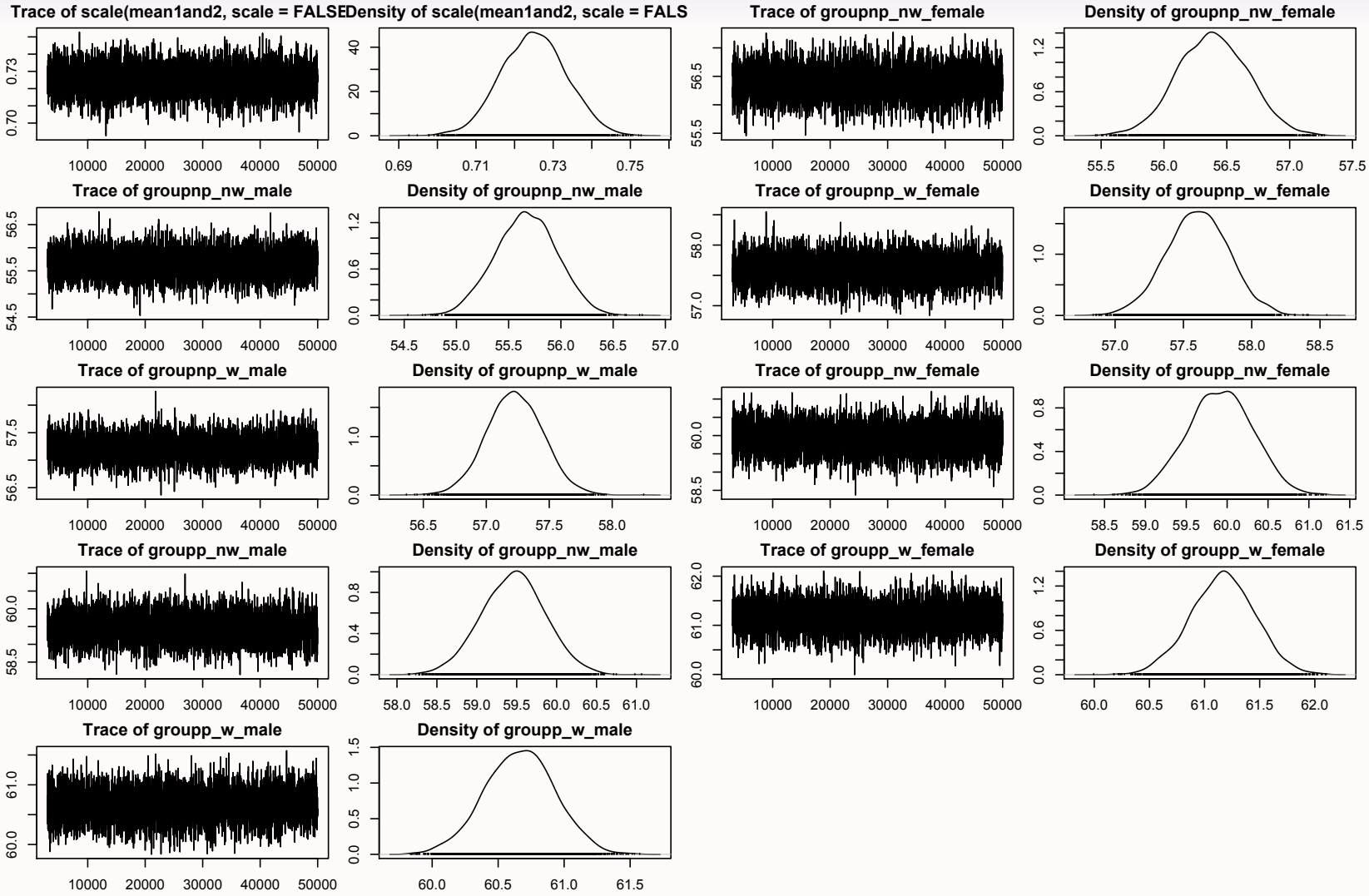
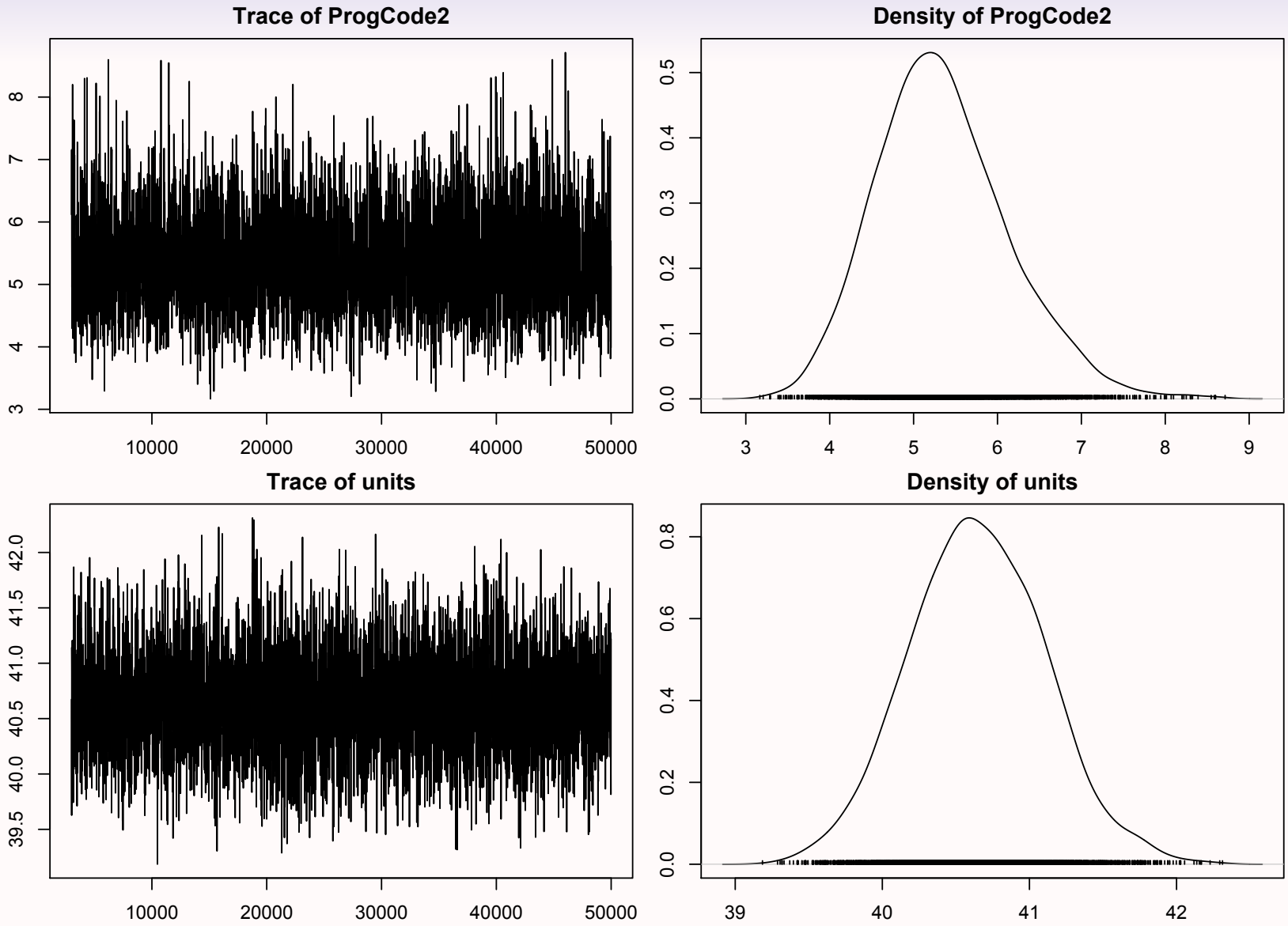
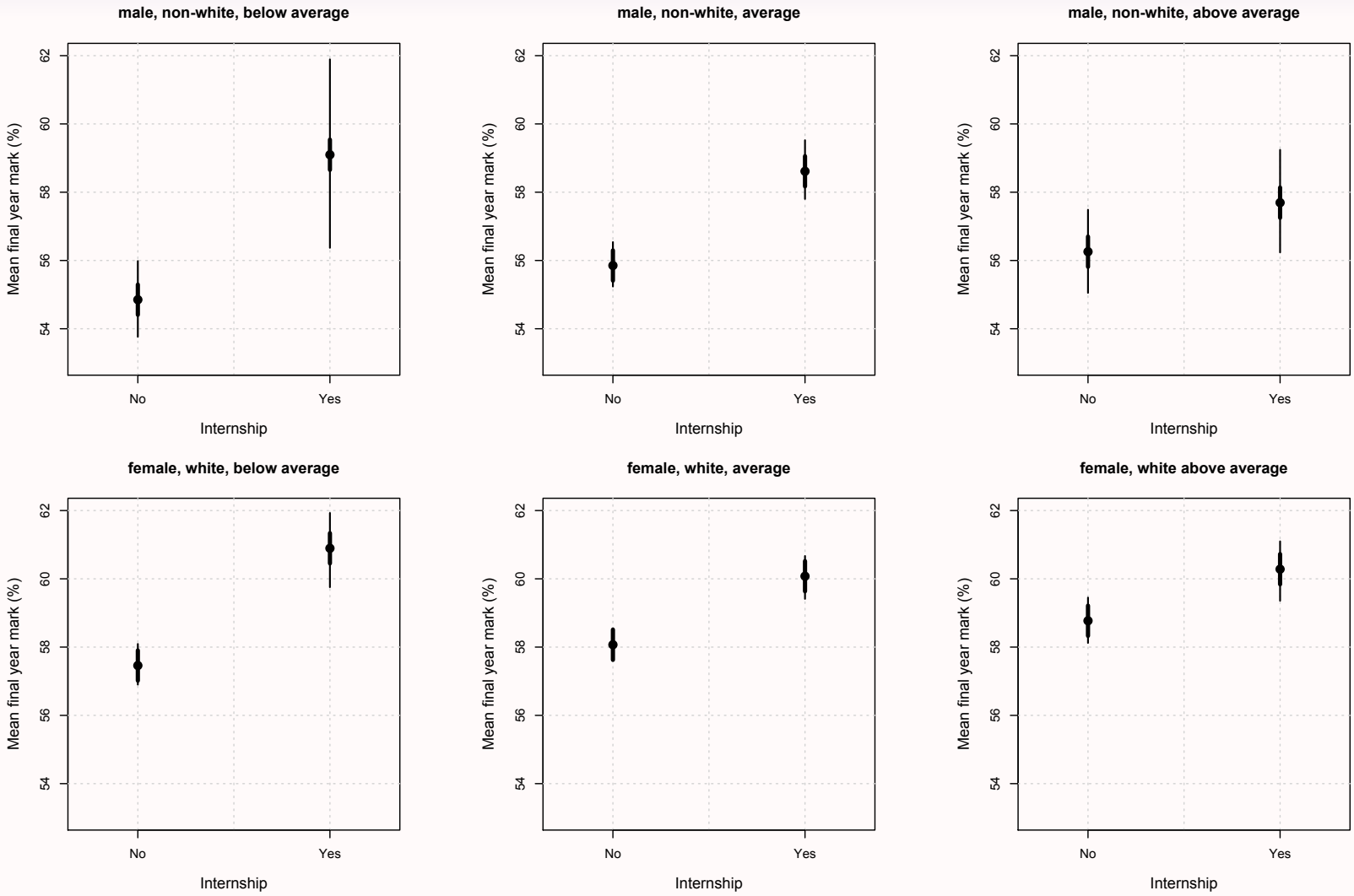
(... after controlling for year 1 and year 2 mean mark)



Trace of MCMC chains for fixed effects



Trace of MCMC chains for random effects



HPD intervals

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Complex Bayesian models: *cognitive processes in language production*

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- . . . in both spoken and written sentence production?

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- First noun phrase: simple vs. complex
- Codability: high vs. low codability of N2

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i. **Peter** moved up and the *N2* and Tania moved down.
ii. **Peter and the *N2*** moved up and Tania moved down.

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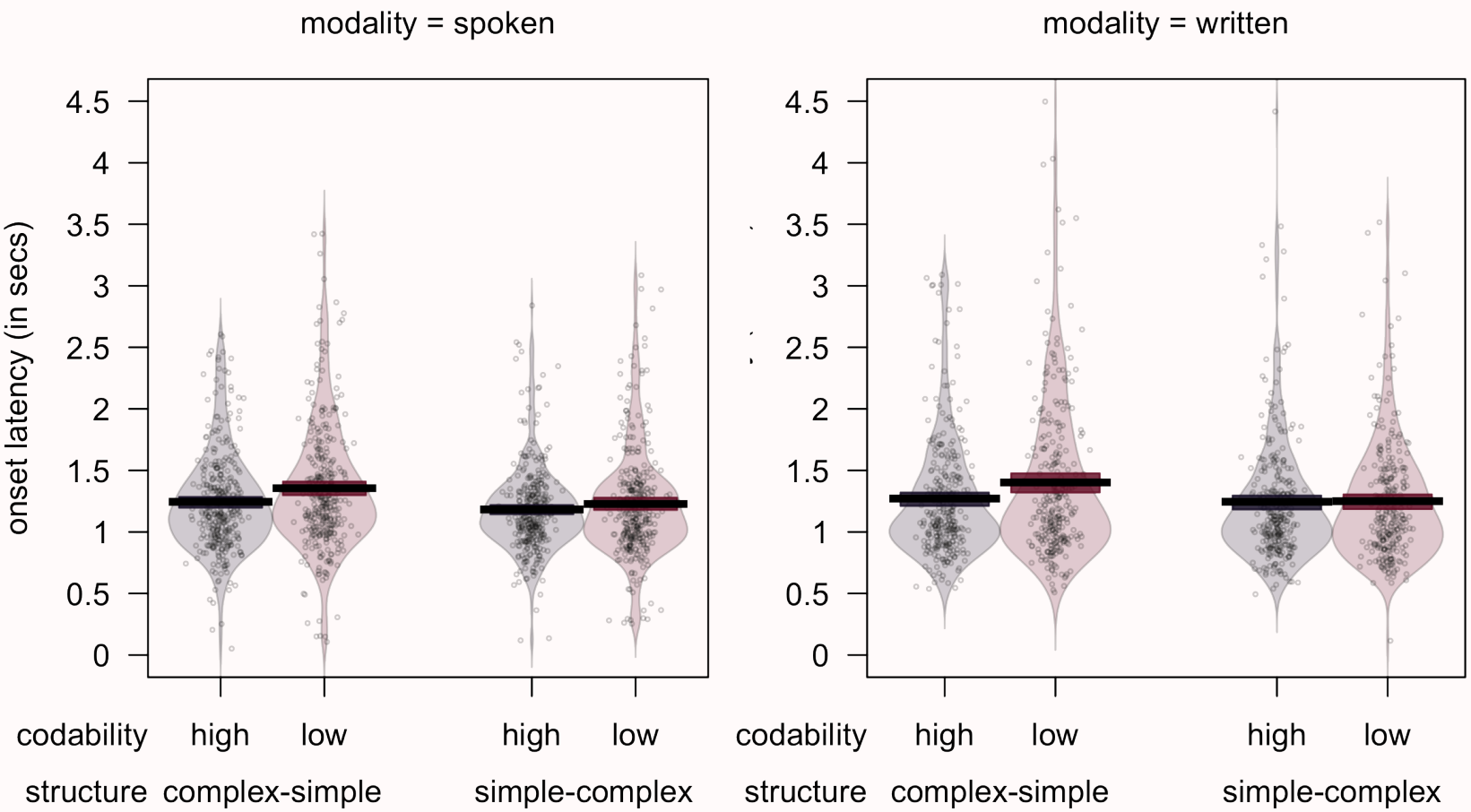
Wider range of alternative models (e.g., mixture models)

Setting up the model using `stan_lmer()` from `rstanarm`

```
stan_lmer(onset ~ struc + coda + moda + stru_coda + stru_mod  + coda_mod + coda_stru_mod
               + (struc + coda + moda + stru_coda + stru_mod + coda_mod + coda_stru_mod | sub
               + (struc + coda + moda + stru_coda + stru_mod + coda_mod + coda_stru_mod | iter
               prior_intercept = student_t(df = 7, 0),
               prior = student_t(df = 1, 0),
               prior_covariance = decov(regularization = 2),
               data = d,
               chains = 4,
               iter = 2000,
               cores = 4,
               adapt_delta = .97)
```

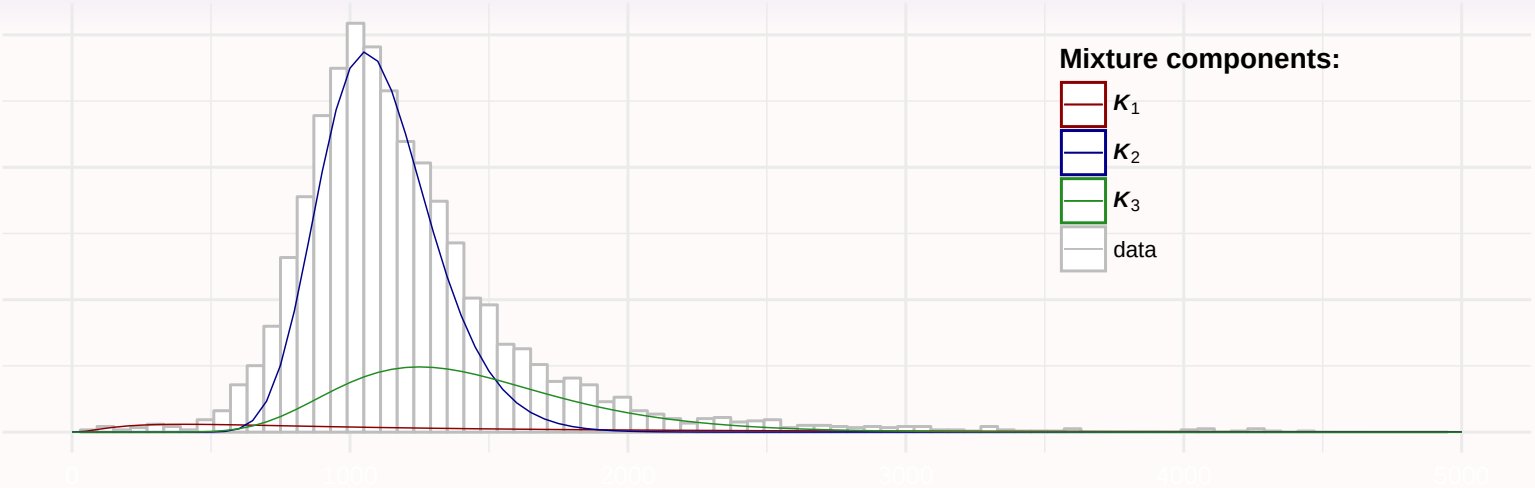


Onset data with HPD intervals

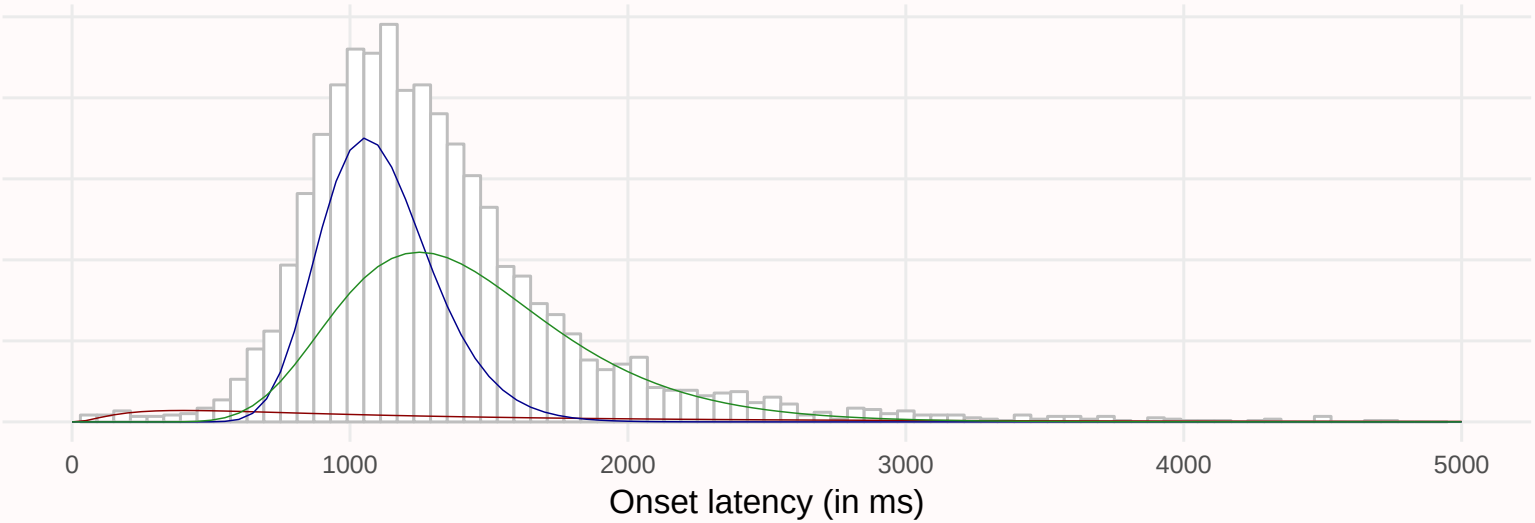
Parameter estimates with HDP intervals

	Median	2.5%	97.5%	P<0
(Intercept)	1273.89	1209.32	1337.22	0.00
struc	45.12	23.44	67.00	0.00
coda	36.11	10.71	62.35	0.00
moda	22.96	-32.17	78.18	0.20
stru_coda	23.55	3.44	43.72	0.01
stru_mod	0.59	-20.92	22.37	0.48
coda_mod	-3.80	-24.53	17.02	0.64
coda_stru_mod	7.80	-11.49	27.12	0.21
sigma	413.79	402.18	426.69	0.00

Simple NP



Conjoined NP



Mixture model exploring components of onset latency

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- ... but it is worth being open to Bayesian ideas
- New software (almost) eliminates the practical obstacles

What the future holds ...

Increasing acceptance of alternatives to frequentist methods

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Bayesian methods commonplace (and dominating) in some fields

Greater emphasis on data sharing and publication of code

Bayesian statistics is difficult in the sense that thinking is difficult.

Donald A. Berry