Piecewise linear models

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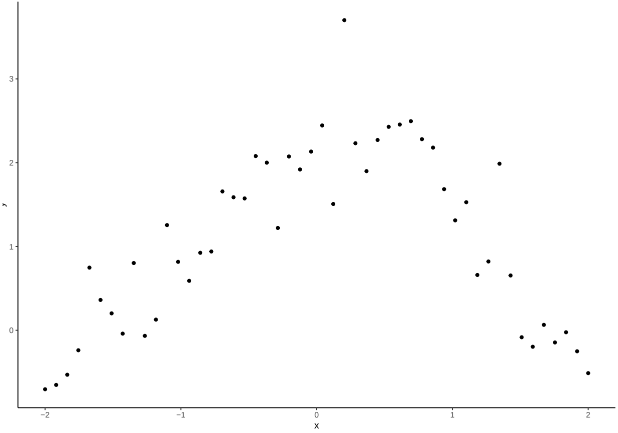
A piecewise linear model

- Our observed data is $\{(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)\}$.
- Our model is
$$y_i \sim N(\mu_i, \sigma^2)$$
where
$$\mu_i = \begin{cases} \alpha_1 + \beta_1 x_i, & \text{if } x_i \leqslant x_{\text{crit}} \\ \alpha_2 + \beta_2 x_i, & \text{if } x_i > x_{\text{crit}}, \end{cases}$$
where $\alpha_1, \beta_1, \alpha_2, \beta_2, x_{\text{crit}}$ are unknown.

Example: A "tent" map

If we require $\beta_1 > 0$ and $\beta_2 < 0$, then for any value of α_1, we can ensure that the two lines intersect at $x = x_{\text{crit}}$, by setting $\alpha_2 = \alpha_1 + x_{\text{crit}}(\beta_1 - \beta_2)$. The resulting function resembles a, possibly uneven, tent with its sharp peak at $x = x_{\text{crit}}$.



Fitting the "tent" map with `nls`

```
M_tent <- nls(y ~ (x <= x_c) * (a_1 + b_1 * x) +
                (x > x_c) * ((a_1 + x_c * (b_1 - b_2)) + b_2 * x),
                start = list(a_1 = 0, b_1 = 2, b_2 = -2, x_c = 0),
                data = tent_df)
```

Note that our α_2 parameter is constrained to be equal to $\alpha_1 + x_{crit}(\beta_1 - \beta_2)$.

Fitting the "tent" map with `nls`