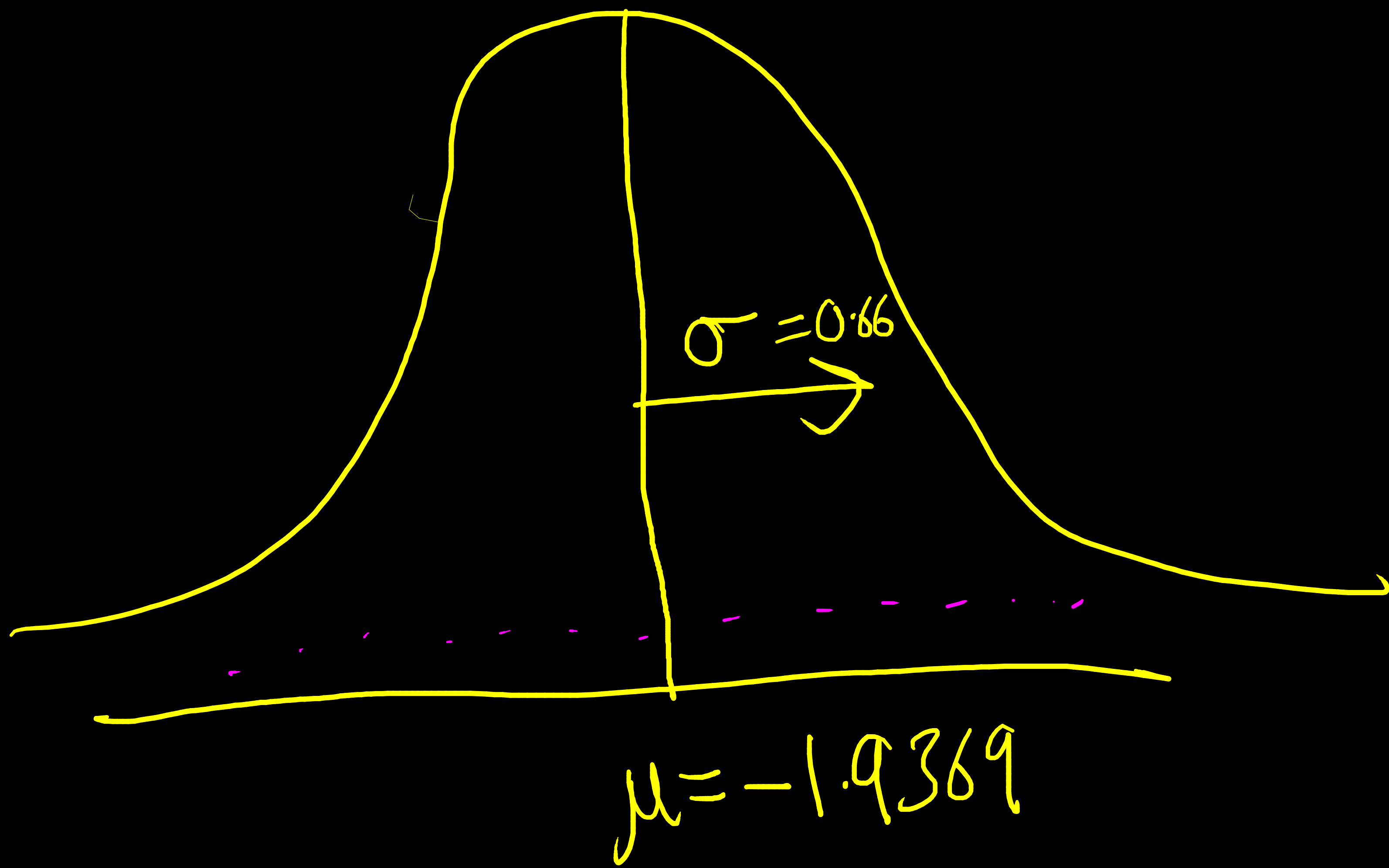
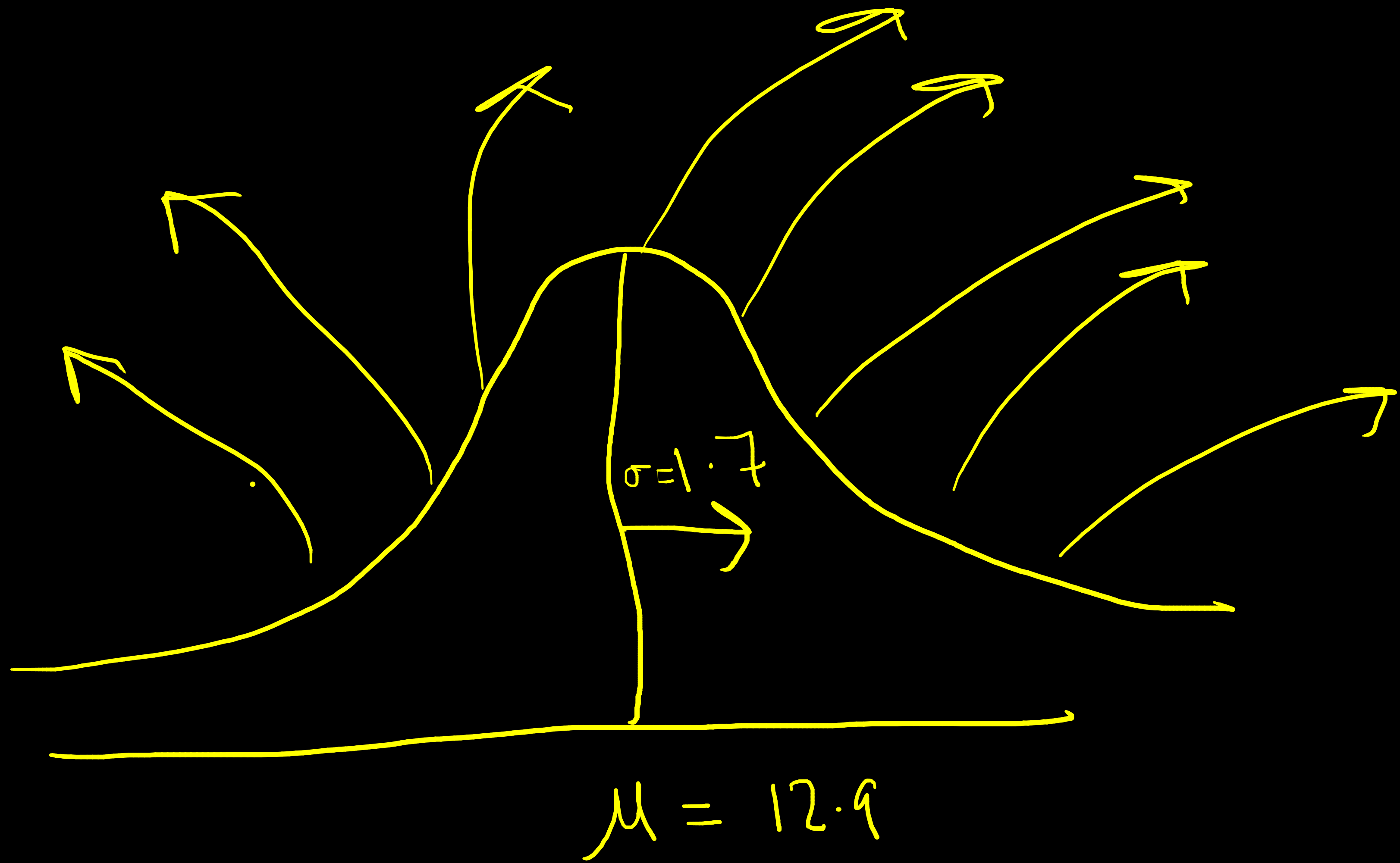
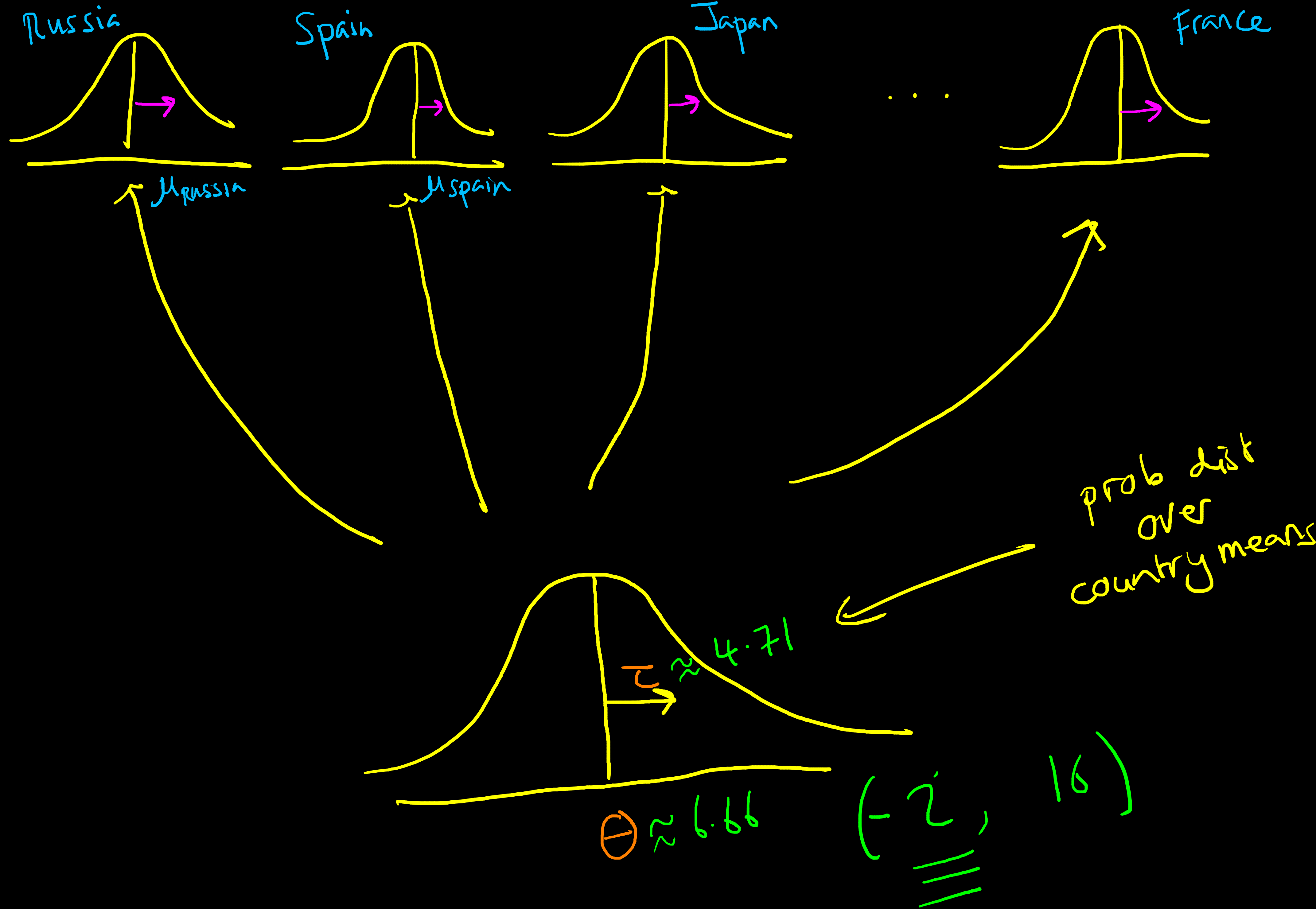

σ = 0.66
μ = -1.9369


σ=1·7
μ = 12·9


Russia
Spain
Japan
. . .
France
μrussia
μspain
prob dist over country means
τ ≈ 4.71
θ ≈ 6.66
(-2, 16)

for j in 1 ... J

for i in 1 ... n_j

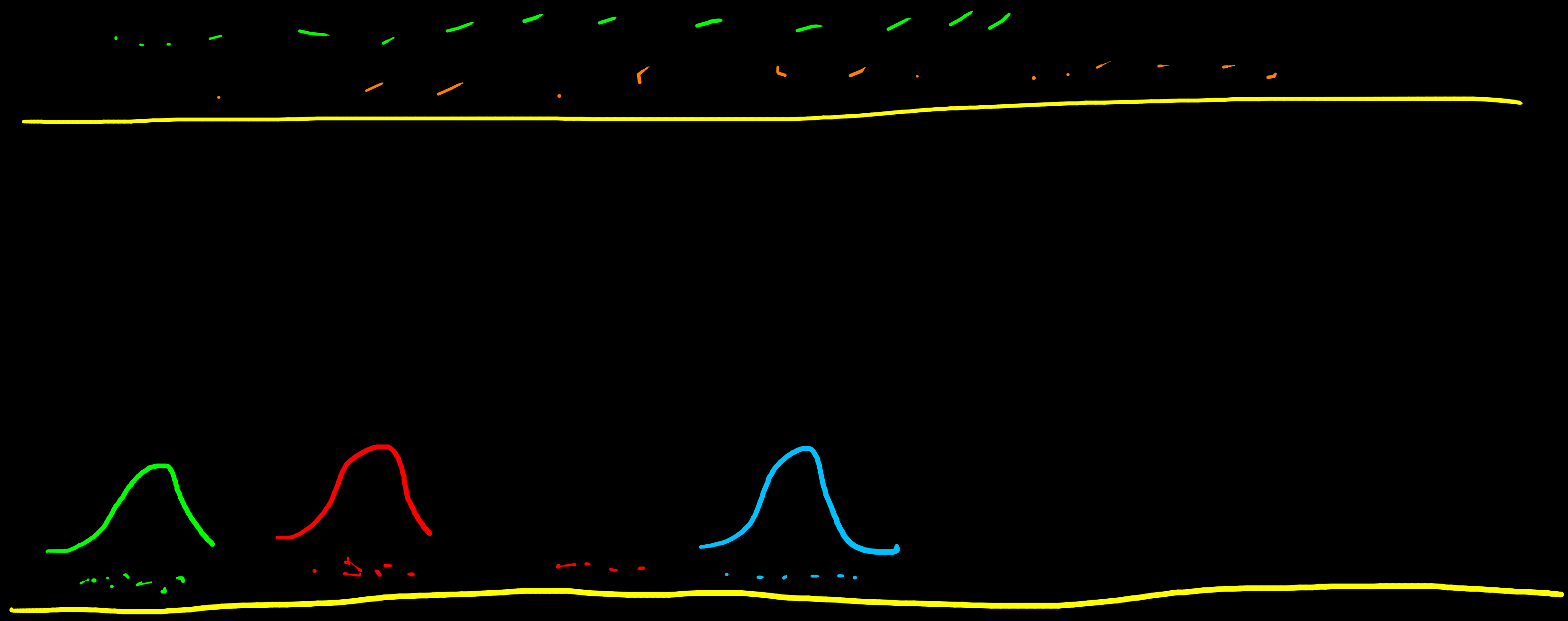
$y_{ji} \sim N(\mu_j, \sigma^2)$

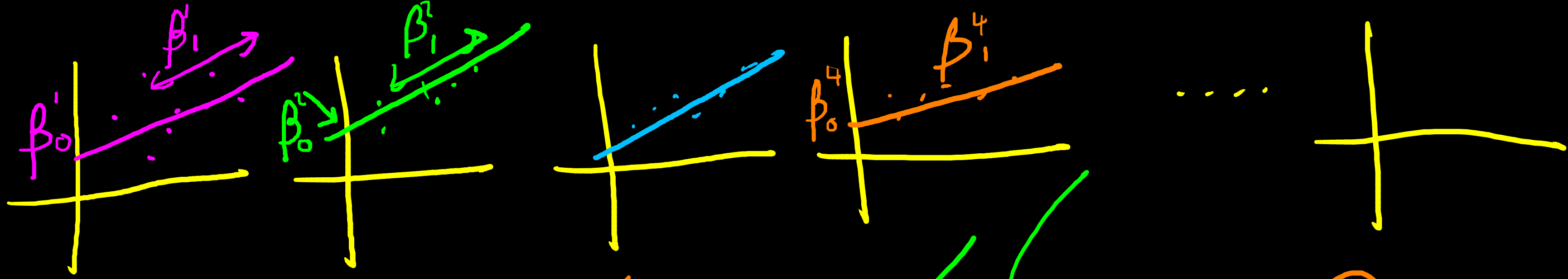
$\mu_j \sim N(\theta, \tau^2)$

$n_j = 12$

$Z_j = \mu_j - \theta$

$$\begin{cases} y_{ji} = \theta + Z_j + \varepsilon_{ji} \\ Z_j \sim N(0, \tau^2) \;;\; \varepsilon_{ji} \sim N(0, \sigma^2) \end{cases}$$

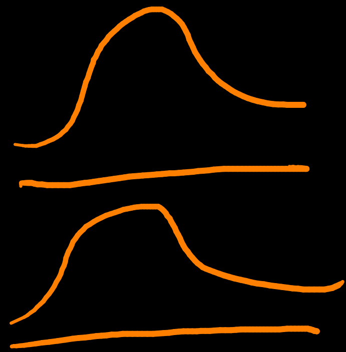




for j in 1... J

$$\beta_0^j \sim N(b_0, \tau_0^2)$$

$$\beta_1^j \sim N(b_1, \tau_1^2)$$



for i in 1...n

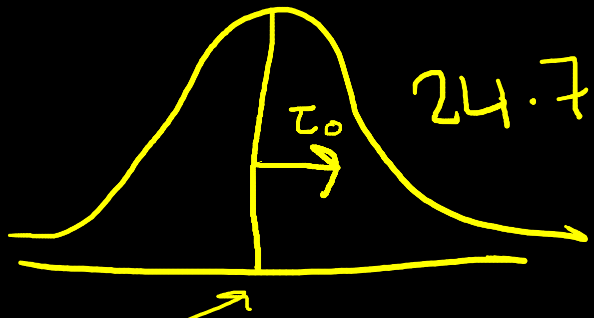
$$y_i \sim N(\mu_i, \sigma^2)$$

$$\mu_i = \beta_0 + \beta_1 x_i$$

$$\mu = \beta_0 + \beta_1 x$$

$$= \beta_0 \times 1 + \beta_1 \times x$$

intercepts

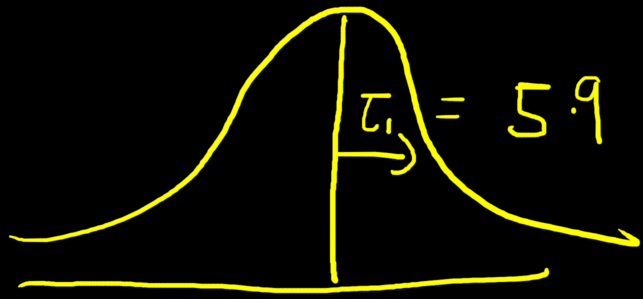


24.741

$b_0 = 251.4$

95% $\approx$ (200 , 300)
interval
over subject
specific
intercepts

slope

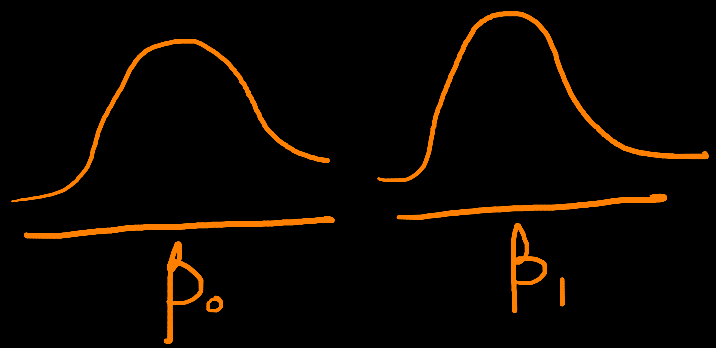


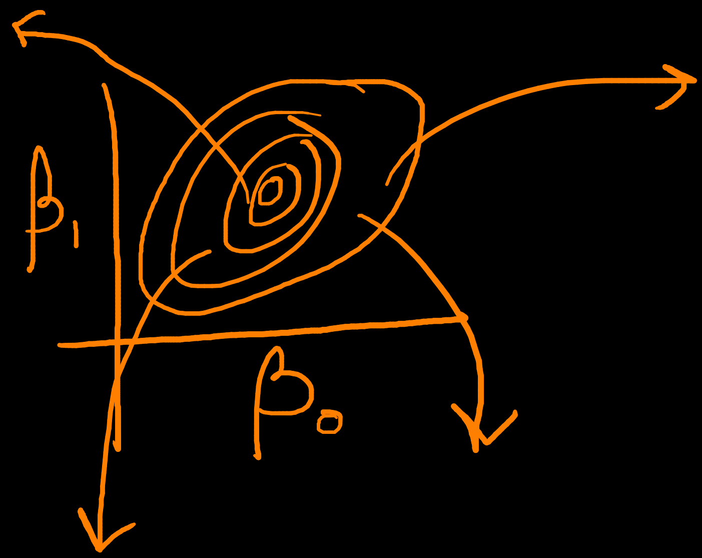
$= 5.9$

$b_1 = 10.47$

$10 - 12 , 10 + 12$

$(-2 , 22)$





$$\begin{bmatrix}\beta_0\\ \beta_1\end{bmatrix} \sim N\left(\begin{bmatrix} b_0 \\ b_1\end{bmatrix}, \Sigma\right)$$

$$\Sigma = \begin{bmatrix}\sigma_0^2 & \sigma_0\sigma_1\rho \\ \sigma_0\sigma_1\rho & \sigma_1^2\end{bmatrix}$$

$x_1 \quad x_2 \quad \cdots \quad x_n$

MLE μ: $\bar{x}$

$$s = \sqrt{\frac{(x_i - \bar{x})^2}{n-1}}$$

MLE σ: s

REML σ: $n-1$

M_0 , LL_0 , D_0

M_1 , LL_1 , D_1

$\boxed{D_0 - D_1}$ LRT

$= -2 (LL_0 - LL_1)$

$= -2 \log \left(\frac{\text{Likelihood } M_1}{\text{Likelihood } M_0} \right)$

$D_0 = -2 \, LL_0$

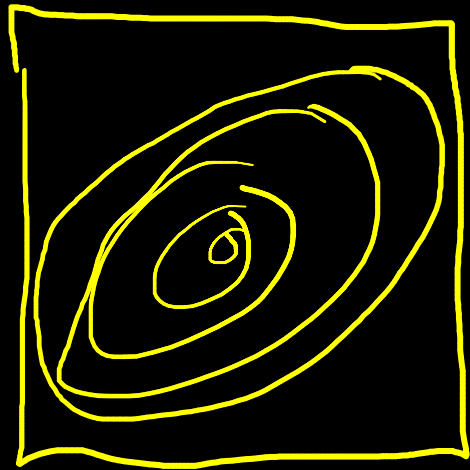
$LL_0 = \log \text{Likelihood } M_0$

$LL_1 = \log \text{Likelihood } M_1$

$\log \left(\frac{\text{likelihood } M_0}{\text{likelihood } M_1} \right)$

$= \log \text{like } M_0 - \log \text{likelihood } M_1$

$$\begin{bmatrix}\beta_0^j \\ \beta_1^j\end{bmatrix} = \vec{\beta}_j$$



for i in 1 ... n

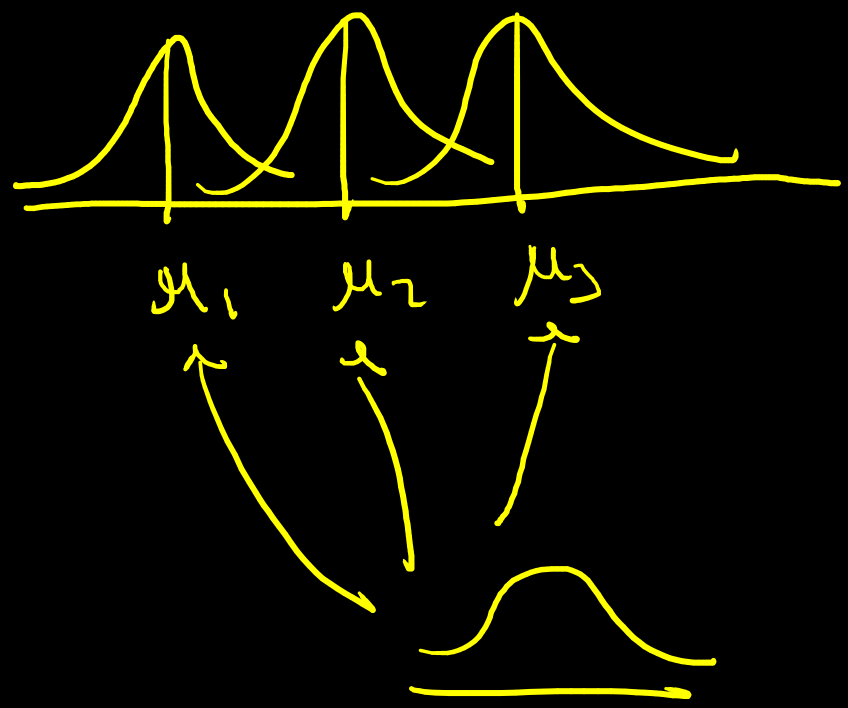
$$y_i \sim N(\beta_0^{[s_i]} + \beta_1^{[s_i]} x_i, \sigma^2)$$

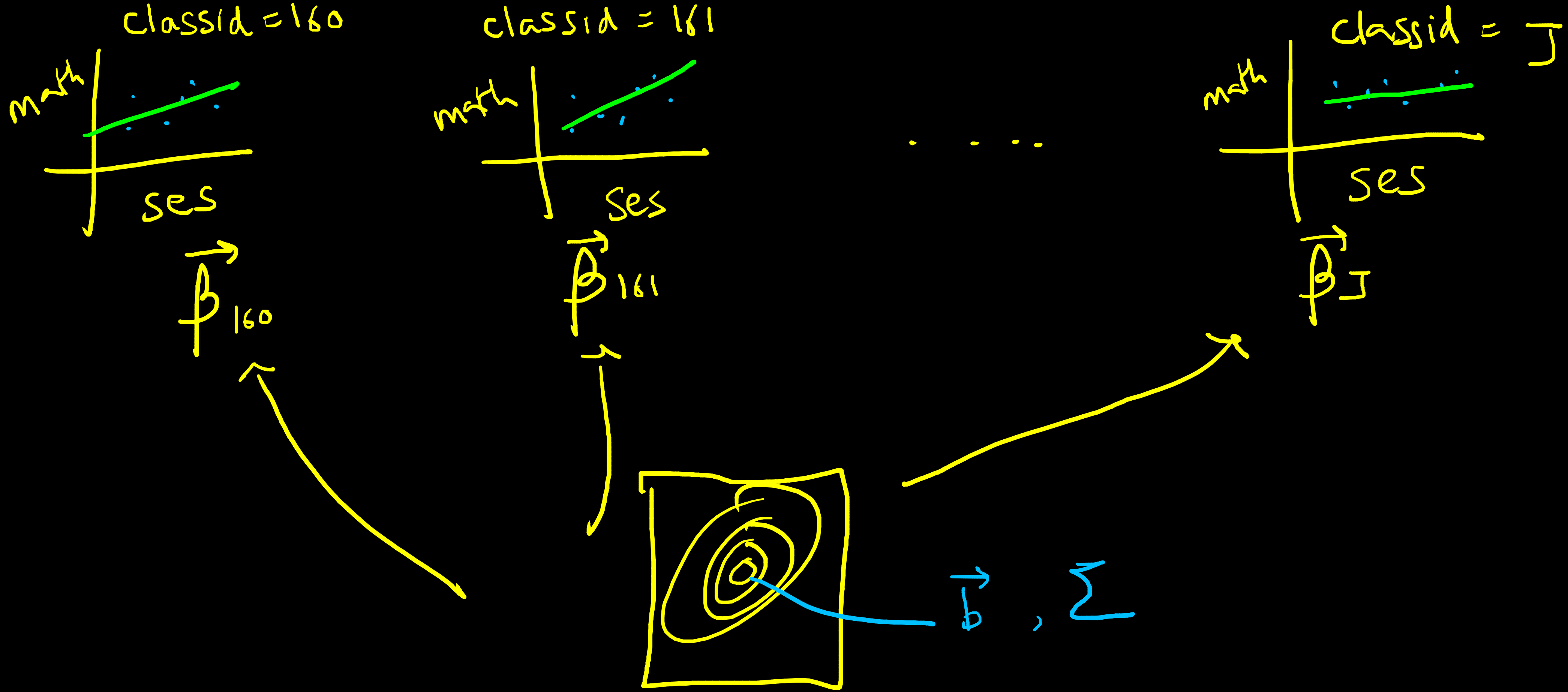
$$\vec{\beta}_j \sim N(\vec{b}, \Sigma)$$

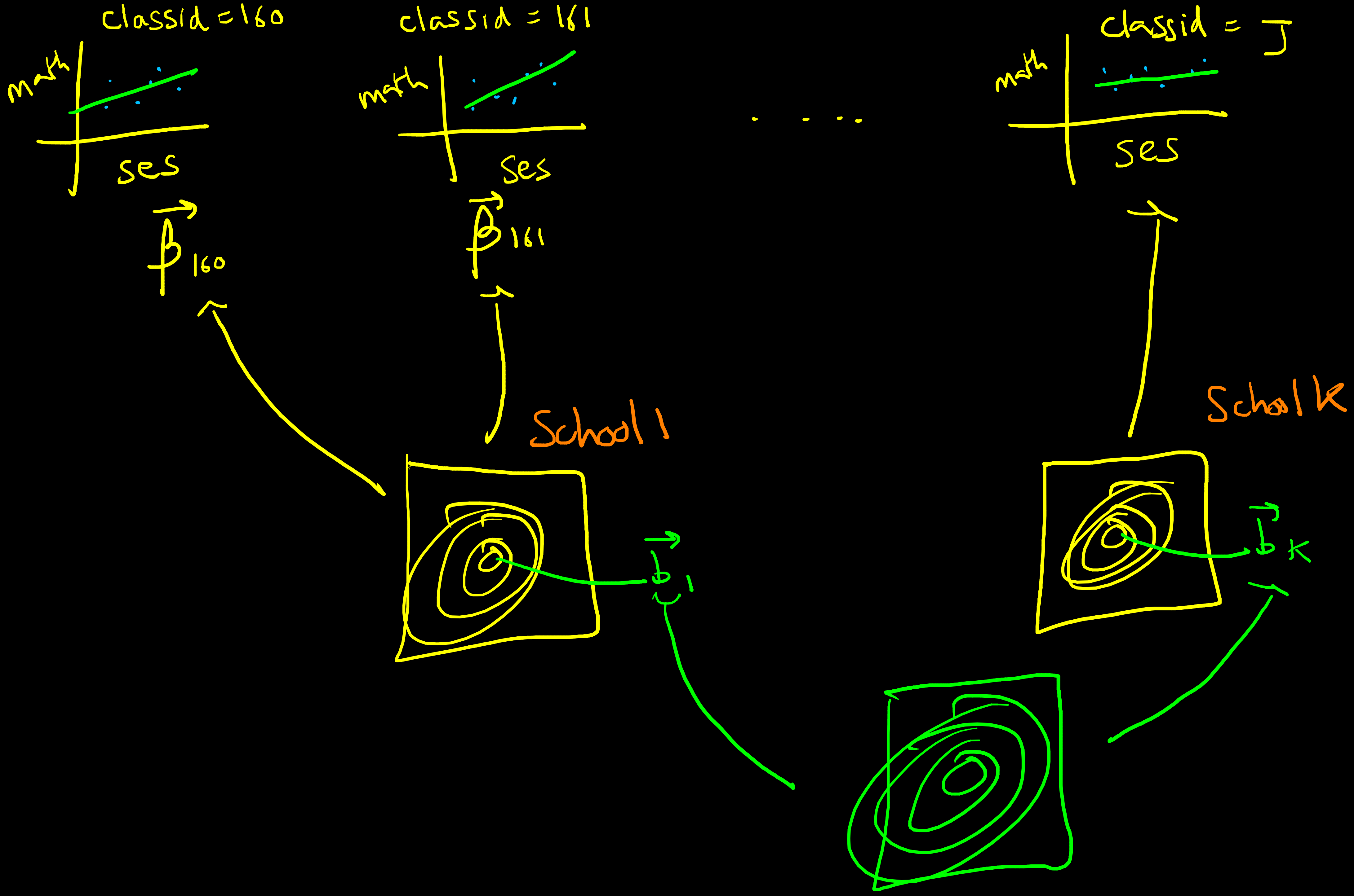
$$\vec{\beta}_j = \vec{b} + \vec{Z}_j, \quad \vec{Z}_j \sim N(0, \Sigma)$$

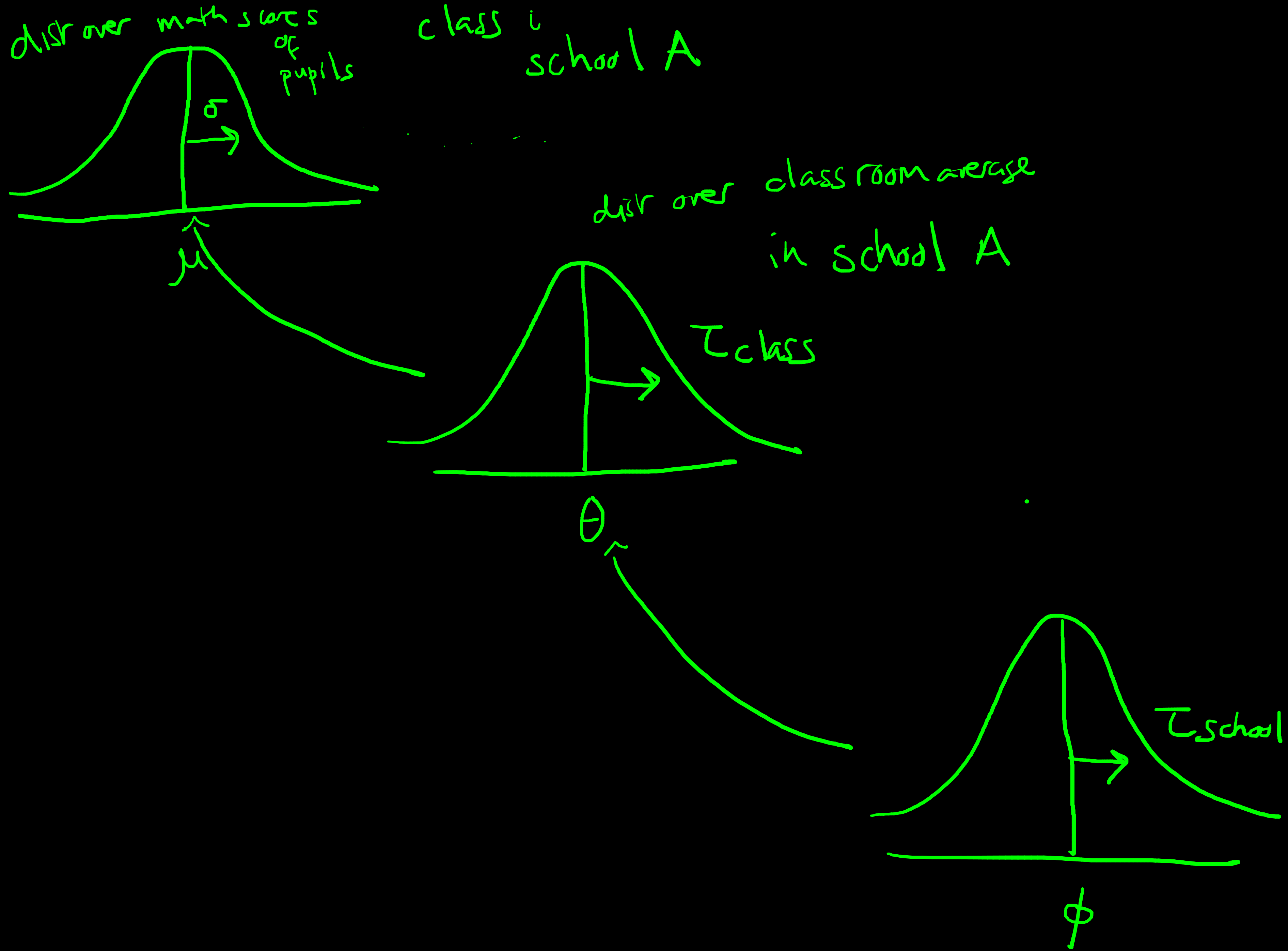
$\rightarrow X \sim N(\mu, \sigma^2)$

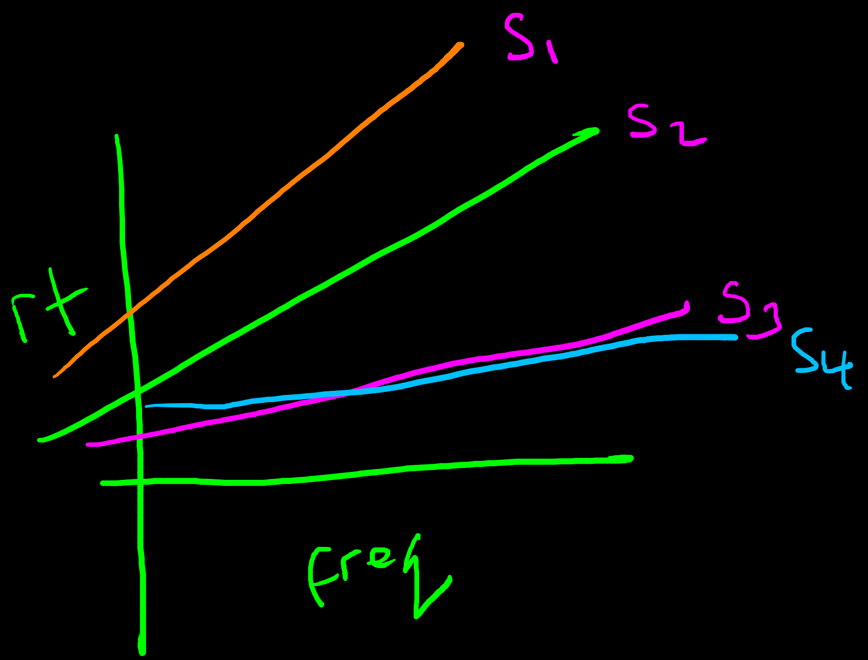
$\rightarrow X = \mu + \varepsilon \;, \; \varepsilon \sim N(0, \sigma^2)$

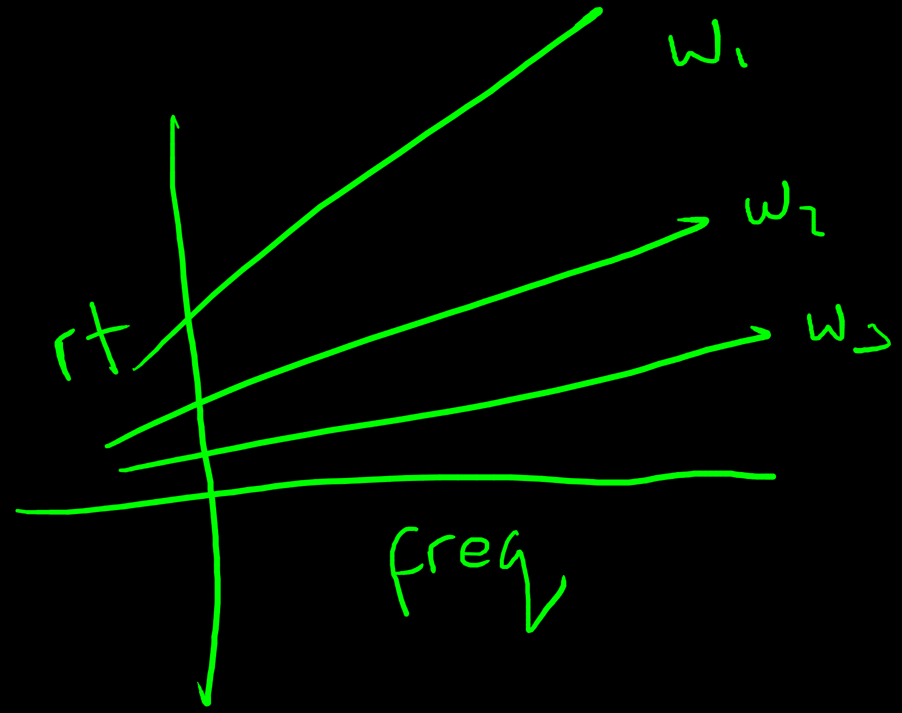

μ_1
μ_2
μ_3


classid = 160
classid = 161
classid = J
math
ses
$\vec{\beta}_{160}$
$\vec{\beta}_{161}$
$\vec{\beta}_J$
$\vec{b}$, Σ


classid = 160
classid = 161
classid = J
math
ses
$\vec{\beta}_{160}$
$\vec{\beta}_{161}$
School 1
School K
$\vec{b}_1$
$\vec{b}_K$


distr over math scores of pupils
class i
school A
σ
μ
distr over classroom average
in school A
τclass
θ
τschool
ϕ


S1
S2
S3
S4
rt
freq


w1
w2
w3
rt
freq