

Generalized Additive (Mixed) Models: In a Nutshell

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Generalized Additive Models (GAMs) in a nutshell

- ▶ In linear models and generalized linear models, the parameters¹ are linear functions of predictors.
- ▶ In generalized additive models, these linear functions are replaced or complemented by smooth nonlinear functions.
- ▶ These smooth nonlinear functions are² weighted sums of *basis function*.

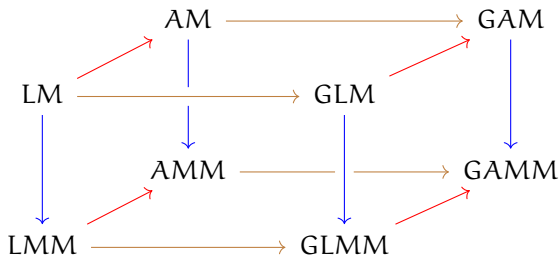
¹Or transformations of parameters.

²In practice, nearly always.

Generalized Additive Mixed Models (GAMMs) in a nutshell

- ▶ Generalized additive mixed models (GAMMs) are the GAM counterpart of linear or generalized linear mixed models.
- ▶ In linear or generalized linear mixed models, the intercepts or slopes randomly by grouping variables.
- ▶ In GAMMs, the linear functions or nonlinear functions vary randomly by grouping variables.

From LMs to GLMs to GAMs etc



- ▶ non-normal distributions and link functions
- ▶ parameters vary randomly
- ▶ smooth nonlinear functions

Normal linear models

- ▶ We often write (simple) linear regression as follows:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

where ϵ_i is normally distributed, i.e. $\epsilon_i \sim N(0, \sigma^2)$.

- ▶ This means

$$y_i \sim N(\mu_i, \sigma^2),$$

$$\mu_i = \beta_0 + \beta_1 x_i.$$

- ▶ In other words, in linear regression, we assume the outcome variable is normally distributed around a mean that varies as a linear function of the predictor variables.

Generalized linear models

- ▶ In generalized linear models, the outcome variable is not assumed to be continuous or normally distributed.
- ▶ For example, it could be binary, or ordinal, or categorical (polychotomous).
- ▶ If the outcome is binary, we could extend the normal linear model as follows:

$$y_i \sim \text{Bernoulli}(\theta_i),$$
$$\text{logit}(\theta) = \beta_0 + \beta_1 x_i.$$

where *logit* is a nonlinear *link* function.

- ▶ In other words, in logistic regression, we assume the outcome variable is Bernoulli random variable and a transformation of its mean varies as a linear function of the predictor variables.

Mixed models

- ▶ A linear mixed effects model can be written as follows:

$$y_i \sim N(\mu_i, \sigma^2),$$
$$\mu_i = \underbrace{b_0 + b_1 x_i}_{\text{fixed effects}} + \underbrace{\zeta_{[s_i]0} + \zeta_{[s_i]1} x_i}_{\text{random effects}},$$

where the ζ are random coefficients that vary by s_i .

- ▶ It is equivalent to a non-multilevel model (the *fixed effects* models) plus a normally distributed random variation to the intercept and slope for each s_i (the *random effects*).

Additive models

- In a normal additive model, we replace the linear function with a smooth nonlinear (s) one:

$$y_i \sim N(\mu_i, \sigma^2),$$
$$\mu_i = s(x_i).$$

In practice, this smooth nonlinear function is a weight sum of nonlinear basis functions:

$$s(x_i) = \sum_{k=1}^K \beta_k \phi_k(x_i),$$

where each ϕ_k is usually a *spline* function.