Generalized Additive (Mixed) Models: In a Nutshell

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Generalized Additive Models (GAMs) in a nutshell

- In linear models and generalized linear models, the parameters[1] are linear functions of predictors.
- In generalized additive models, these linear functions are replaced or complemented by smooth nonlinear functions.
- These smooth nonlinear functions are[2] weighted sums of *basis functions*.

[1] Or transformations of parameters.

[2] In practice, nearly always.

Generalized Additive Mixed Models (GAMMs) in a nutshell

- Generalized additive mixed models (GAMMs) are the GAM counterpart of linear or generalized linear mixed models.
- In linear or generalized linear mixed models, the intercepts or slopes randomly by grouping variables.
- In GAMMs, the linear functions or nonlinear functions vary randomly by grouping variables.

From LMs to GLMs to GAMs etc

$$
\begin{tikzcd}
 & AM \arrow[rr] \arrow[dd] & & GAM \arrow[dd] \\
LM \arrow[ur] \arrow[rr] \arrow[dd] & & GLM \arrow[ur] \arrow[dd] & \\
 & AMM \arrow[rr] & & GAMM \\
LMM \arrow[ur] \arrow[rr] & & GLMM \arrow[ur] &
\end{tikzcd}
$$

- non-normal distributions and link functions
- parameters vary randomly
- smooth nonlinear functions

Normal linear models

- We often write (simple) linear regression as follows:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

  where ϵ_i is normally distributed, i.e. $\epsilon_i \sim N(0, \sigma^2)$.
- This means

$$y_i \sim N(\mu_i, \sigma^2),$$
$$\mu_i = \beta_0 + \beta_1 x_i.$$

- In other words, in linear regression, we assume the outcome variable is normally distributed around a mean that varies as a linear function of the predictor variables.

Generalized linear models

- In generalized linear models, the outcome variable is not assumed to be continuous or normally distributed.
- For example, it could be binary, or ordinal, or categorical (polychotomous).
- If the outcome is binary, we could extend the normal linear model as follows:

$$\begin{aligned} y_i &\sim \text{Bernoulli}(\theta_i), \\ \text{logit}(\theta) &= \beta_0 + \beta_1 x_i. \end{aligned}$$

  where *logit* is a nonlinear *link* function.
- In other words, in logistic regression, we assume the outcome variable is Bernoulli random variable and a transformation of its mean varies as a linear function of the predictor variables.

Mixed models

- A linear mixed effects model can be written as follows:

$$\begin{aligned} y_i &\sim N(\mu_i, \sigma^2), \\ \mu_i &= \underbrace{b_0 + b_1 x_i}_{\text{fixed effects}} + \underbrace{\zeta_{[s_i]0} + \zeta_{[s_i]1} x_i}_{\text{random effects}}, \end{aligned}$$

  where the ζ are random coefficients that vary by s_i.
- It is equivalent to a non-multilevel model (the *fixed effects* models) plus a normally distributed random variation to the intercept and slope for each s_i (the *random effects*).

Additive models

- In a normal additive model, we replace the linear function with a smooth nonlinear (s) one:

$$
\begin{aligned}
y_i &\sim N(\mu_i, \sigma^2), \\
\mu_i &= s(x_i).
\end{aligned}
$$

In practice, this smooth nonlinear function is a weight sum of nonlinear basis functions:

$$s(x_i) = \sum_{k=1}^{K} \beta_k \phi_k(x_i),$$

where each ϕ_k is usually a *spline* function.