In [1]:

```
from math import pi,sqrt
import matplotlib.pyplot as plt
import numpy as np
from IPython.display import display, Image
display(Image(filename="RD_formulae.png"));display(Image(filename="press_fit_pressure.png"));display(Image(filename="RD_strain.png"));
```

Thick cylinders & discs

Thick cylinder

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

Rotating disc

$$\sigma_r = A - \frac{B}{r^2} - \frac{(3+\nu)}{8}\rho\omega^2 r^2$$

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{(1+3\nu)}{8}\rho\omega^2 r^2$$

Rotating disc with thermal gradient

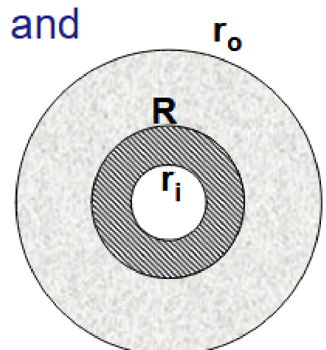
$$\sigma_r = A - \frac{B}{r^2} - \frac{(3+\nu)}{8}\rho\omega^2 r^2 - \frac{E\alpha}{r^2}\int Tr\, dr$$

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{(1+3\nu)}{8}\rho\omega^2 r^2 + \frac{E\alpha}{r^2}\int Tr\, dr - E\alpha T$$

If the shaft is solid, r_i = 0 and

$$p = \frac{E\delta_r}{2R}\left[1 - \frac{R^2}{r_o^2}\right]$$

The press fit has no axial pressure, so σ_l = 0, and it is a biaxial stress condition.



The circumferential strain $\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu\,\sigma_r}{E}$

which equals the radial strain (because C = 2πr).

Because the radial change $\delta = R\,\varepsilon_r$, we get the increase in Inner Radius of the outer member (hub):

In [2]:

```
Ro = 0.32 / 2 #m
Ri = 0.03 #m

rho = 7830 #kg/m3
v = .288

pressure = 0e6 #MPa
w = 6500 * 2*pi / 60 #rpm
E = 210e9 #GPa
```

Inner shaft

In [3]:

```
B_shaft = 0 #solid shaft
A_shaft =  pressure + (1/8)*(3+v)*rho*(w**2)*(Ri**2)
print(f"A = {A_shaft:.2f}, B = {B_shaft:.2f}")

r = np.linspace(1e-9,Ri,100)
stress_radial = np.array(A_shaft - B_shaft / r**2 - (1/8)*(3+v)*rho*(w**2)*r**2)
stress_hoop = np.array(A_shaft + B_shaft / r**2 - (1/8)*(1+3*v)*rho*(w**2)*r**2)
stress_axial = np.zeros(len(r))
stress = np.array([stress_radial, stress_hoop, stress_axial])
vm_stress = (1/sqrt(2)) * np.sqrt( (stress[0]-stress[1])**2 + (stress[1]-
stress[2])**2 + (stress[2]-stress[0])**2 )
print(f"Von mises stress outer {vm_stress[-1]/1e6:.2f}MPa")
print(f"Radial stress outer {stress_radial[-1]/1e6:.2f}MPa")
print(f"Hoop stress outer {stress_hoop[-1]/1e6:.2f}MPa")

delta_r_shaft = (Ri / E) * (stress_hoop[-1] - v * (stress_radial[-1] +
stress_axial[-1]))
print(f"radial change {delta_r_shaft*1e6:.2f} microns")
```

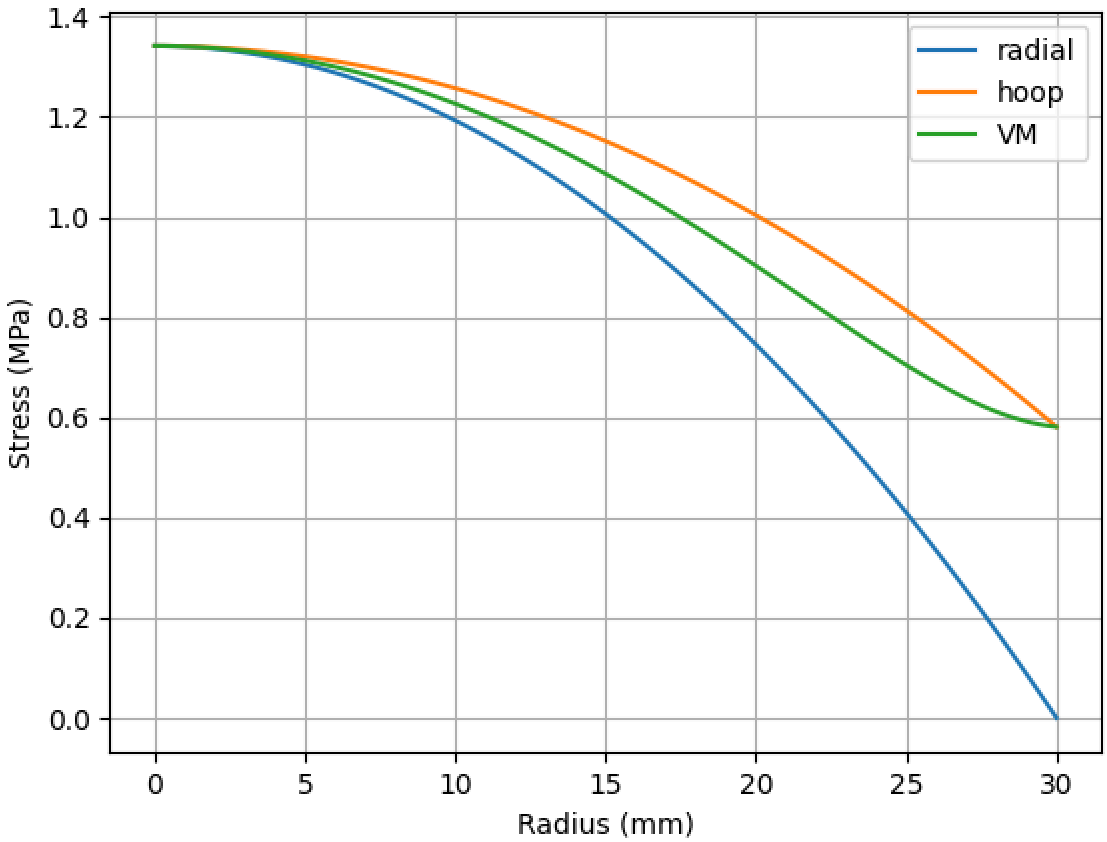
```
A = 1341930.56, B = 0.00
Von mises stress outer 0.58MPa
Radial stress outer 0.00MPa
Hoop stress outer 0.58MPa
radial change 0.08 microns
```

In [4]:

```
fig, ax = plt.subplots()
ax.plot(r*1e3,1e-6*stress_radial, label = 'radial')
ax.plot(r*1e3,1e-6*stress_hoop, label = 'hoop')
ax.plot(r*1e3,1e-6*vm_stress, label = 'VM')
ax.set_xlabel("Radius (mm)")
ax.set_ylabel("Stress (MPa)")
ax.grid()
ax.legend()
```

Out[4]:

```
<matplotlib.legend.Legend at 0x2117e4f6320>
```



Outer flywheel

In [5]:

```
B_flywheel = (pressure - (1/8)*(3+v)*rho*(w**2)*(Ro**2-Ri**2)) / ( (1/Ro**2) -
(1/Ri**2) )
A_flywheel = B_flywheel / Ro**2 + (1/8)*(3+v)*rho*(w**2)*(Ro**2)
print(f"A = {A_flywheel:.2f}, B = {B_flywheel:.2f}")

r = np.linspace(Ri,Ro,100)
stress_radial = np.array(A_flywheel - B_flywheel / r**2 - (1/8)*(3+v)*rho*
(w**2)*r**2)
stress_hoop = np.array(A_flywheel + B_flywheel / r**2 - (1/8)*(1+3*v)*rho*
(w**2)*r**2)
stress_axial = np.zeros(len(r))
stress = np.array([stress_radial, stress_hoop, stress_axial])
vm_stress = (1/sqrt(2)) * np.sqrt( (stress[0]-stress[1])**2 + (stress[1]-
stress[2])**2 + (stress[2]-stress[0])**2 )
print(f"Von mises stress inner {vm_stress[0]/1e6:.2f}MPa")
print(f"Radial stress inner {stress_radial[0]/1e6:.2f}MPa")
print(f"Hoop stress inner {stress_hoop[0]/1e6:.2f}MPa")

delta_r_flywheel = (Ri / E) * (stress_hoop[0] - v * (stress_radial[0] +
stress_axial[0]))
pressure_delta = (E * (delta_r_flywheel-delta_r_shaft) / (2*Ri)) * (1 -
Ri**2/Ro**2)
print(f"radial change {delta_r_flywheel*1e6:.2f} microns")
print(f"pressure delta {pressure_delta/1e6:.2f} MPa")
```

```
A = 39512399.76, B = 34353.42
Von mises stress inner 76.92MPa
Radial stress inner 0.00MPa
Hoop stress inner 76.92MPa
radial change 10.99 microns
pressure delta 36.83 MPa
```

In [6]:
```python
r = np.linspace(Ri,Ro,100)
fig, ax = plt.subplots()
ax.plot(r*1e3,1e-6*stress_radial, label = 'radial')
ax.plot(r*1e3,1e-6*stress_hoop, label = 'hoop')
ax.plot(r*1e3,1e-6*vm_stress, label = 'VM')
ax.set_xlabel("Radius (mm)")
ax.set_ylabel("Stress (MPa)")
ax.grid()
ax.legend()
```

Out[6]: `<matplotlib.legend.Legend at 0x2117e45faf0>`