

Blouch Validation Results

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1 Introduction

I evaluated the model’s performance on data I simulated using the Model Validation Code SBR2.R script available in the Validation Code folder in the blouch-project project github.com. All data was simulated on a randomly sampled set of 50 tip species from the 10KTrees primate phylogeny (Arnold et al. 2010). This data were then analyzed using the requisite *Blouch* model, and parameter values were compared to the true parameter values.

For each model I tested data simulated with short, medium, and long half lives. Each model accounted for measurement error. Measurement error was added to the X and Y variables by simulating from a random normal distribution with $\mu = 0$ and $\sigma = 0.01$. All runs were using two chains were 2000 iterations per chain.

Results show that all models are generally able to recover the true parameter values, though at the longest half life (0.75) posterior half-lives are biased downwards - this is because one prior was used for all simulations. Given an empirical analysis, wider priors on the half-life might be explored first, then focusing on the peak to produce more precise estimates.

Note that for the multilevel models, results may include divergent transitions for the centered versions, which can lead to unexplored regions of the posterior distribution. As discussed in Grabowski (in revision), in such cases the non-centered version of the model may remedy these issues.

2 Simulated Regimes

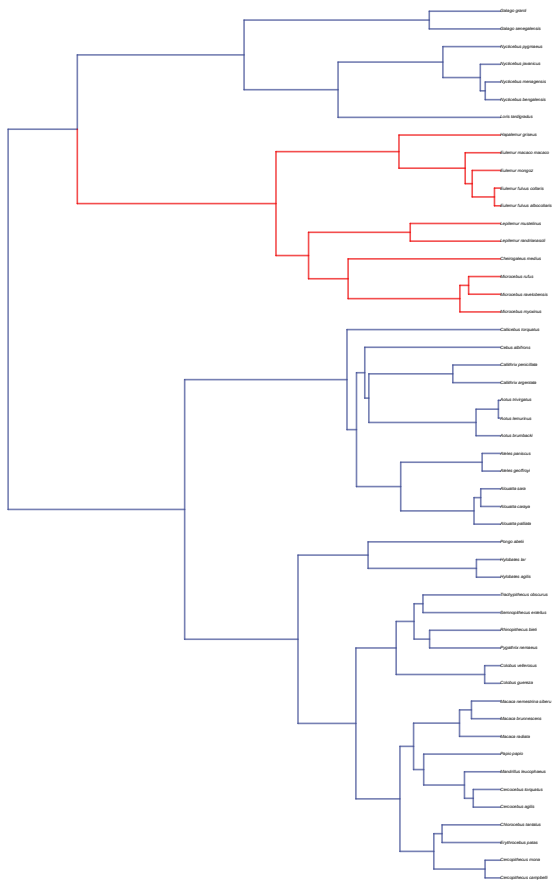


Figure 1: Simulated regimes for Multi-Optima Models with two regimes.

3 Direct Effect Models

3.0.1 $hl = 0.1$

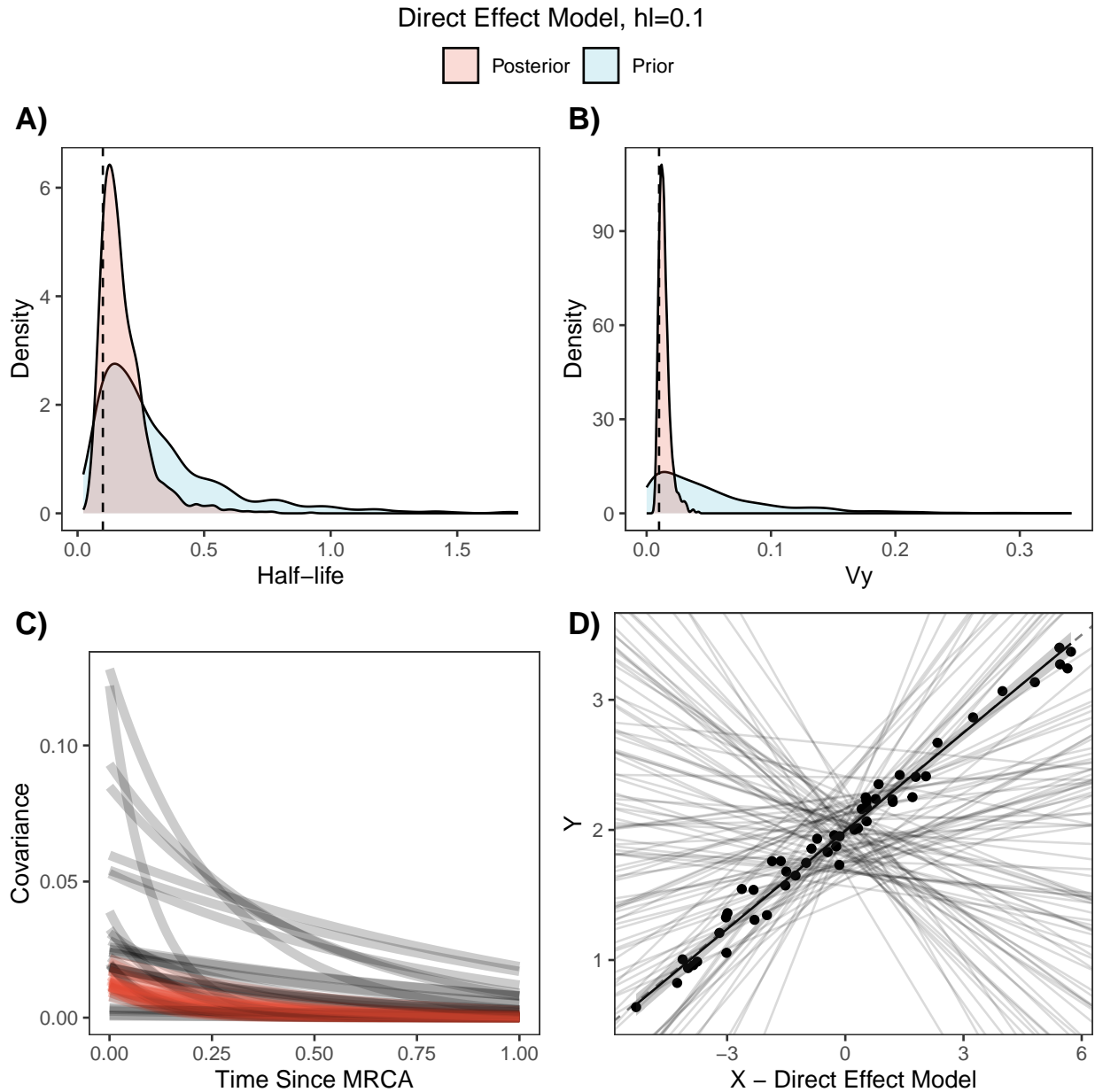


Figure 2: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior prediction, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D) are true values of the parameter.

3.0.2 $hl = 0.25$

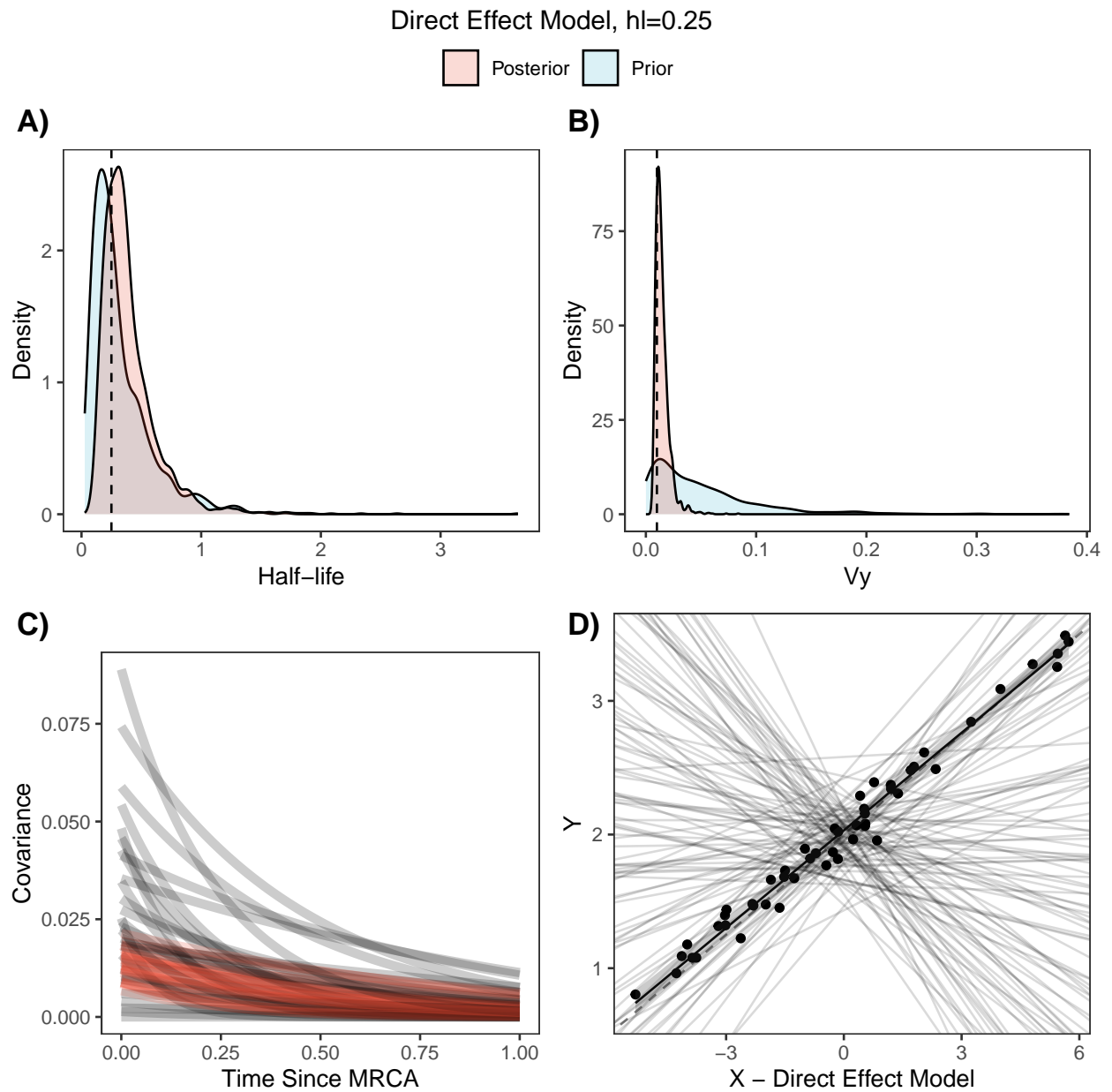


Figure 3: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior prediction, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D) are true values of the parameter.

3.0.3 $hl = 0.75$

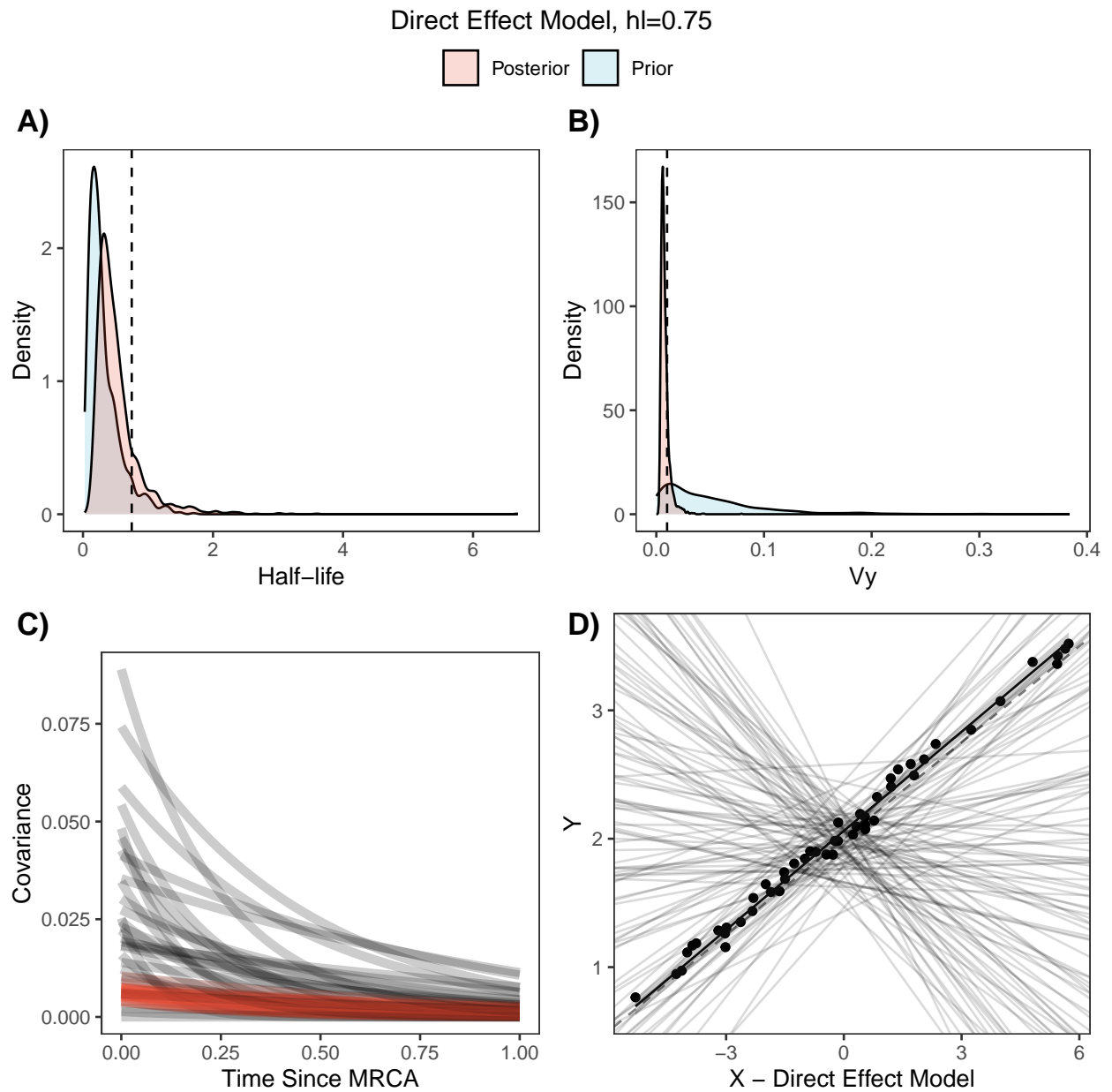


Figure 4: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior prediction, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D) are true values of the parameter.

4 Adaptation Models

4.0.1 $hl = 0.1$

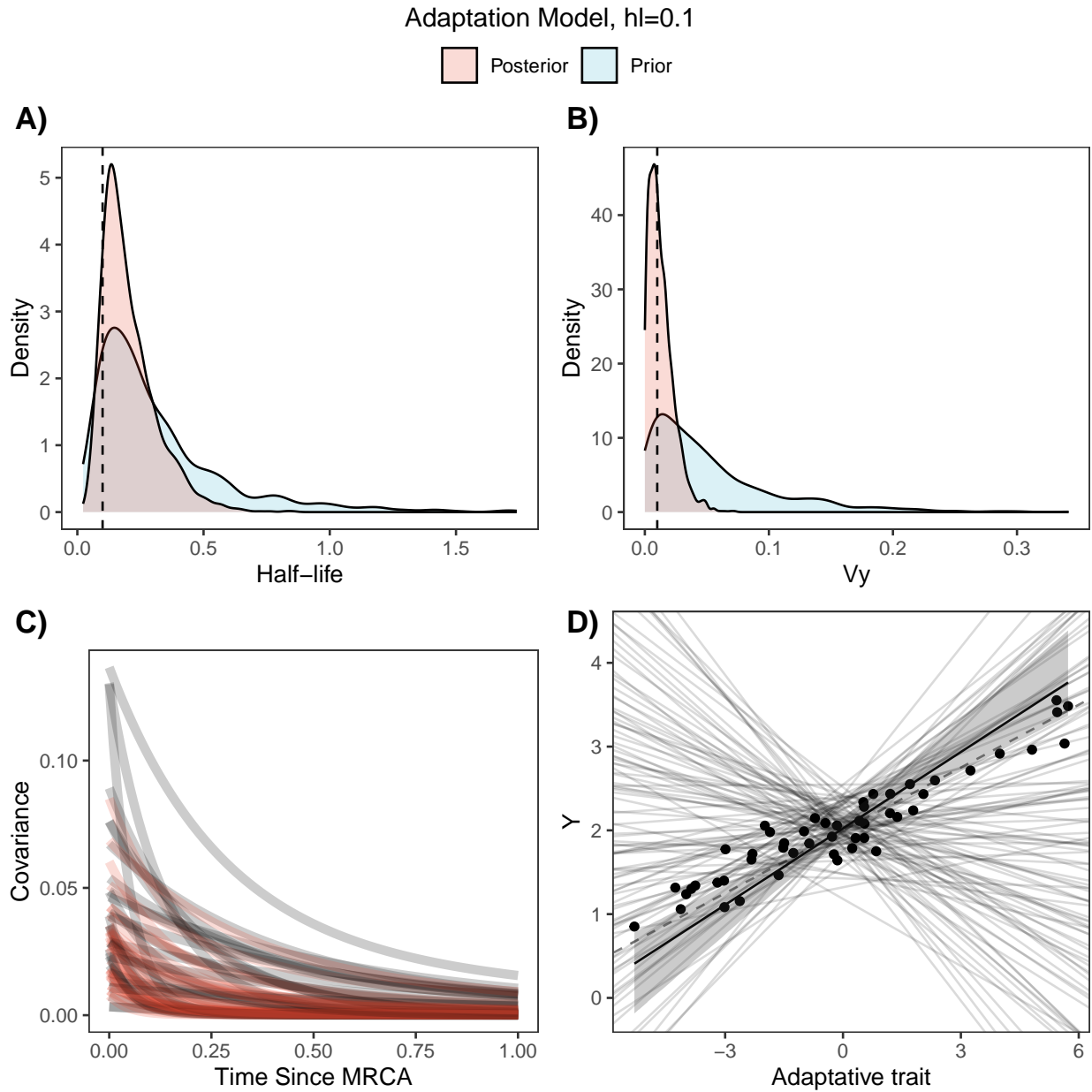


Figure 5: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior prediction, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D) are true values of the parameter.

4.0.2 $hl = 0.25$

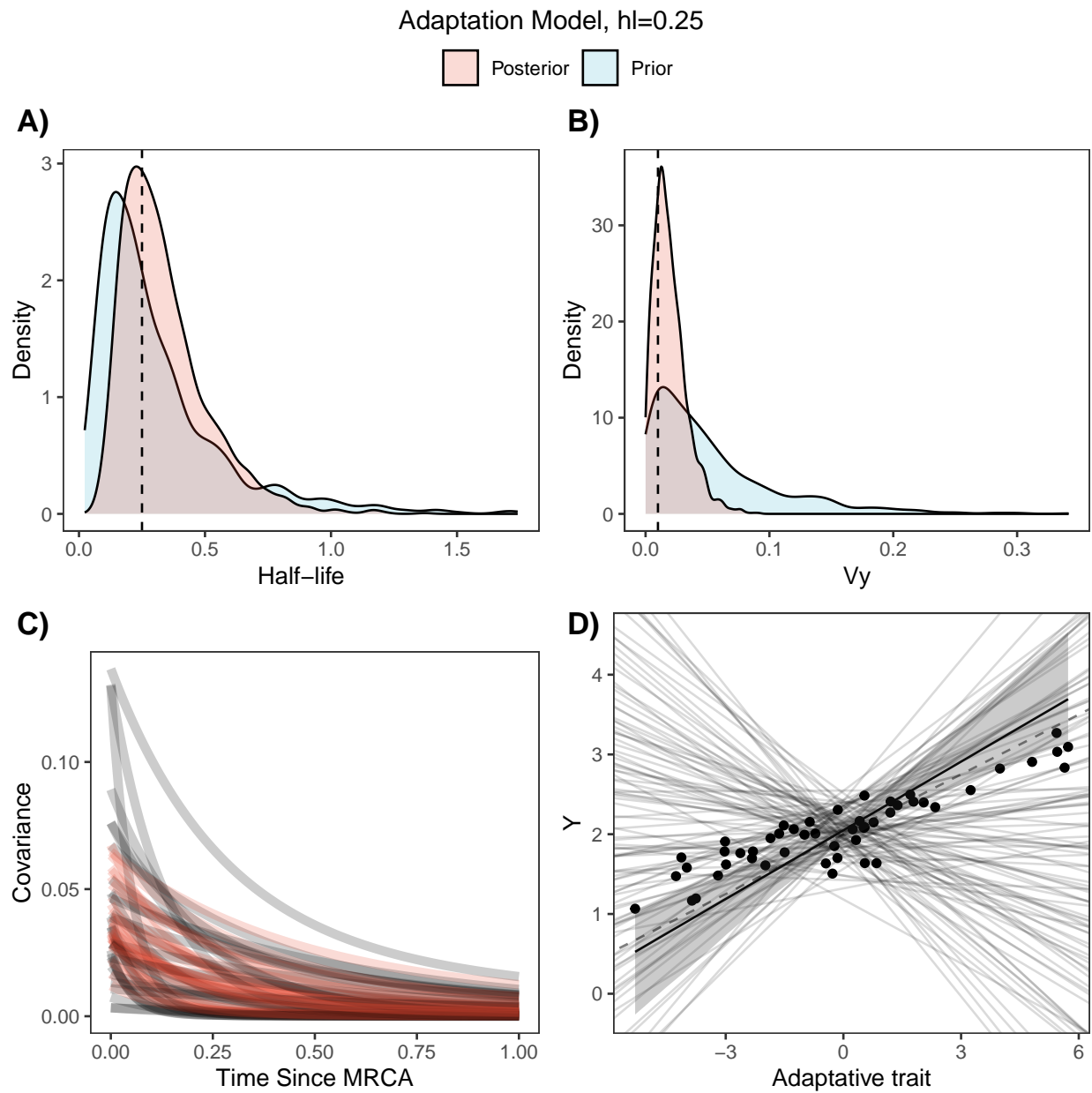


Figure 6: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior prediction, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D) are true values of the parameter.

4.0.3 $hl = 0.75$

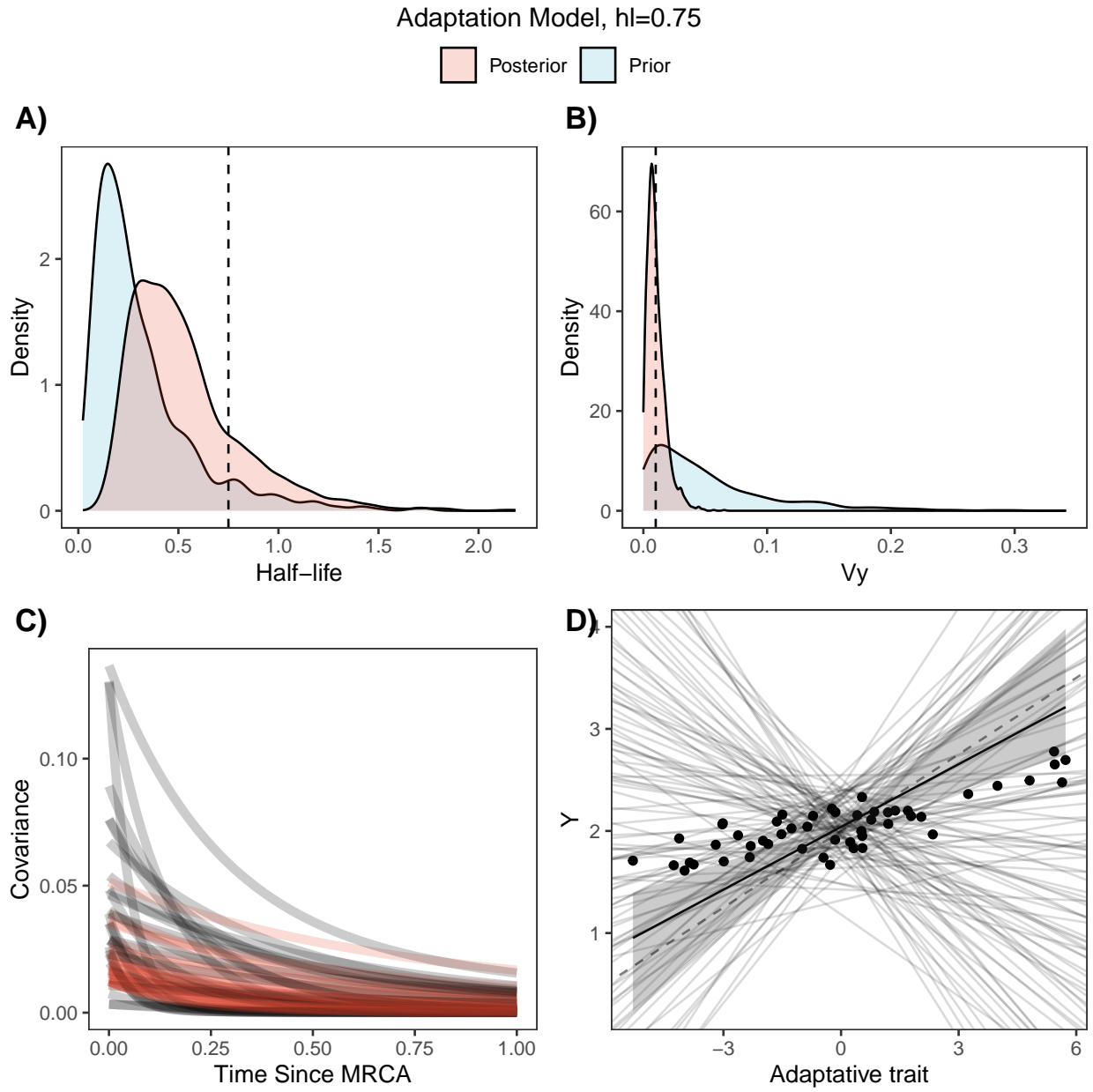


Figure 7: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior prediction, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D) are true values of the parameter.

5 Direct Effect Adaptation Models

5.0.1 $hl = 0.1$

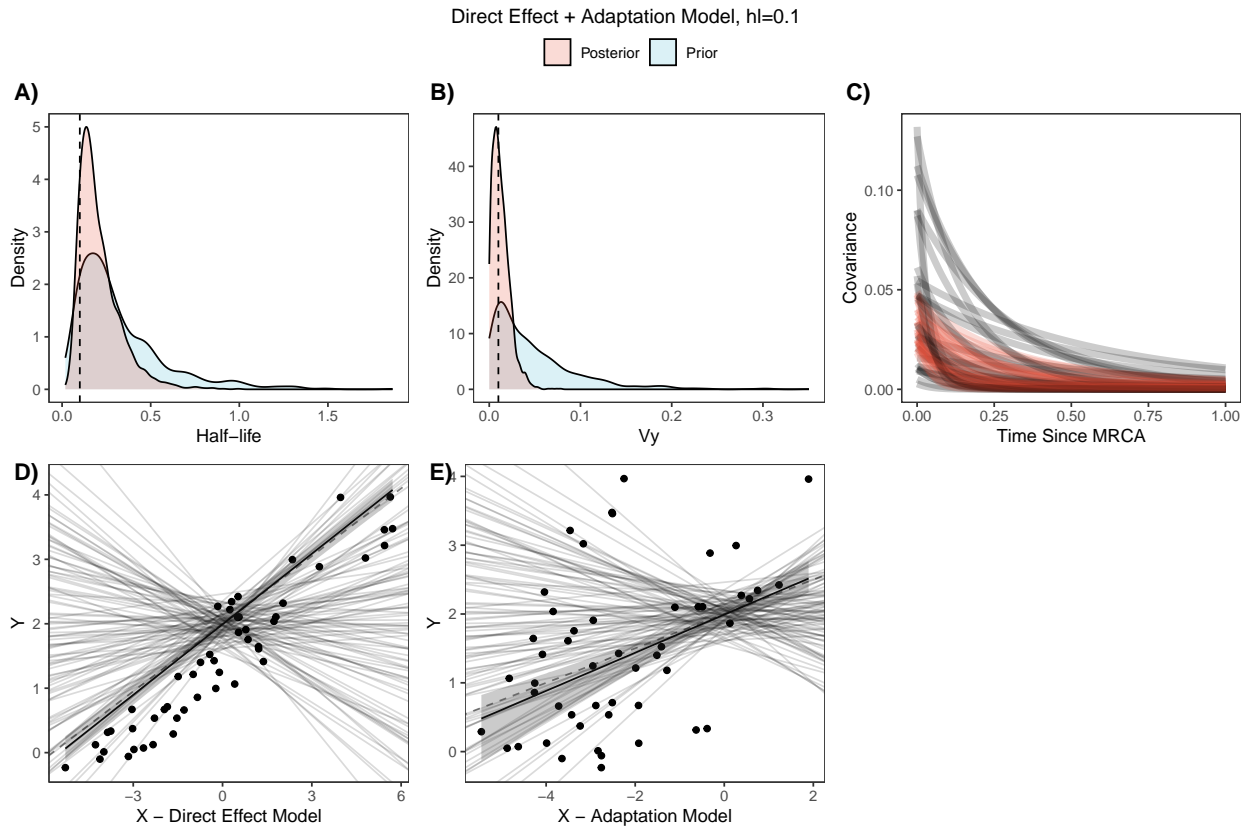


Figure 8: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

5.0.2 $hl = 0.25$

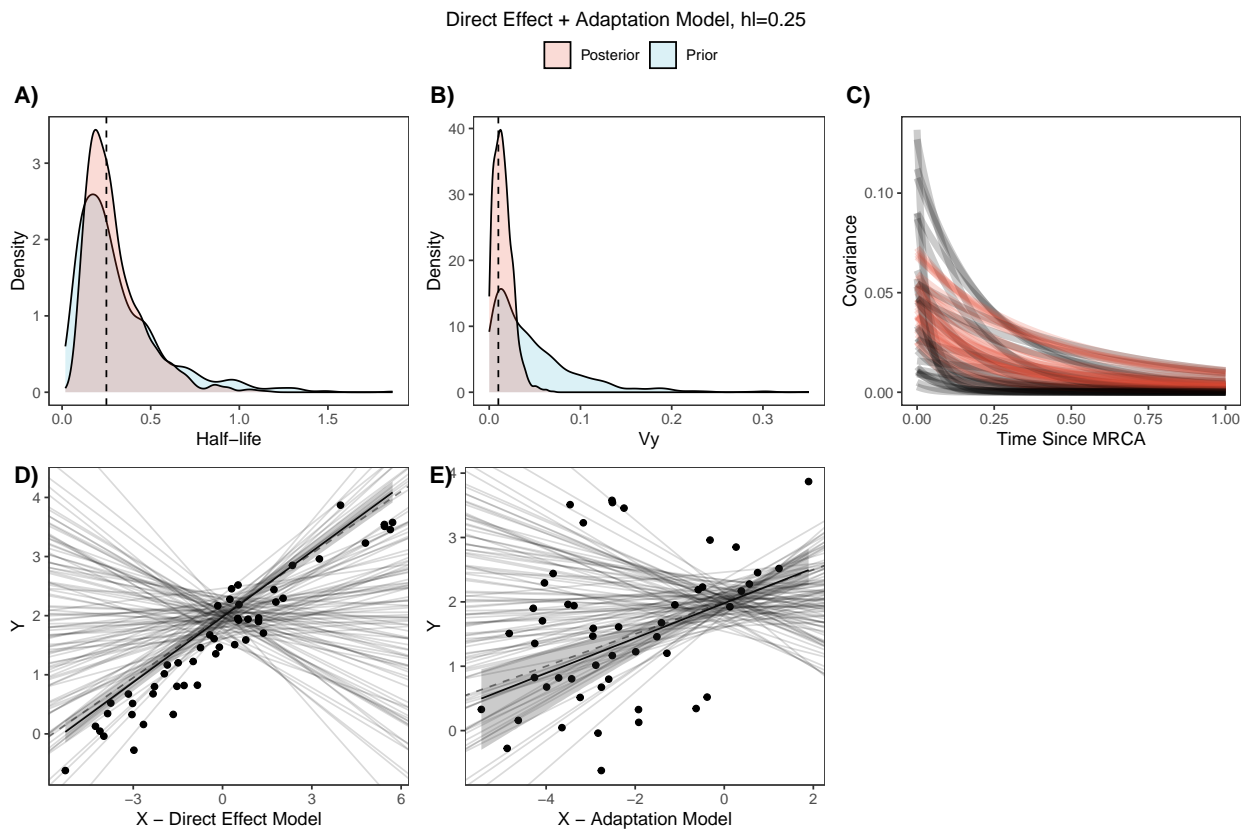


Figure 9: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

5.0.3 $hl = 0.75$

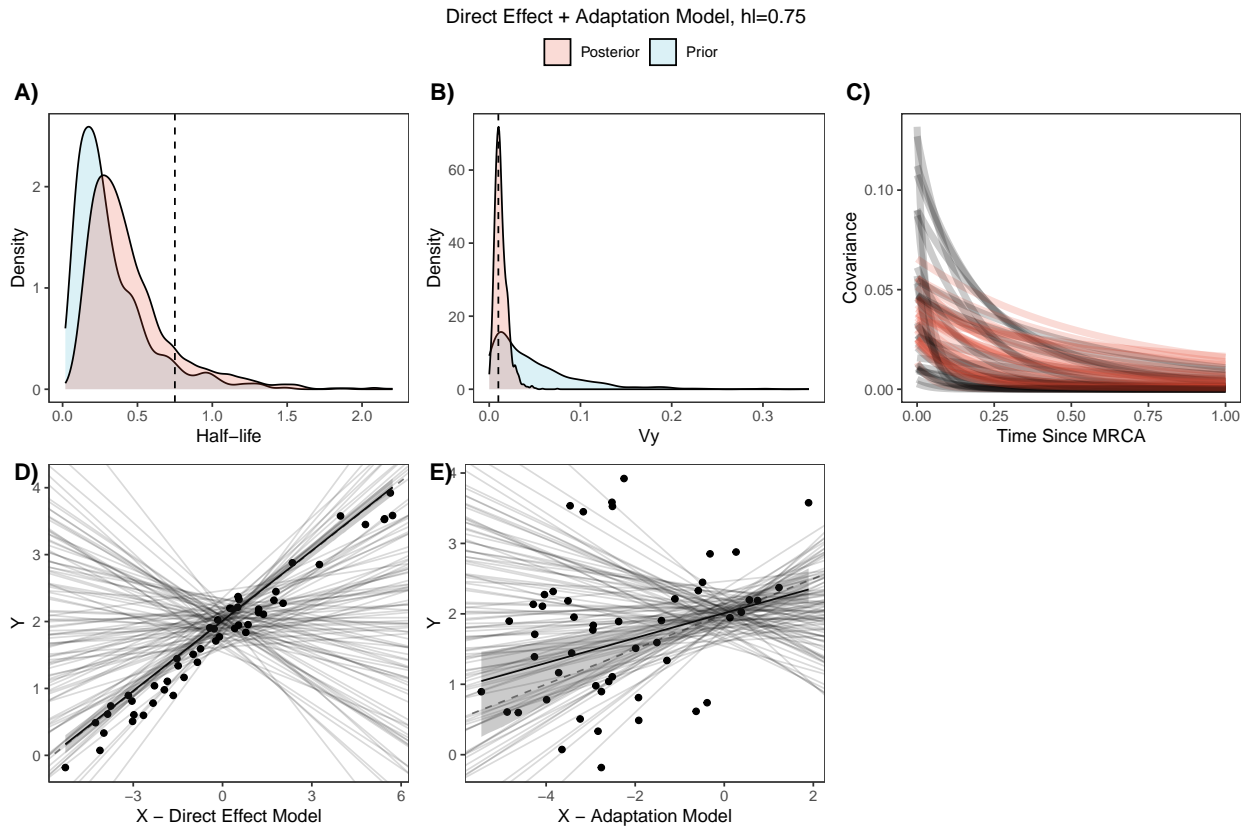


Figure 10: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

6 Multi-Optima models

6.1 Multi-Optima Model

6.1.1 $hl = 0.1$

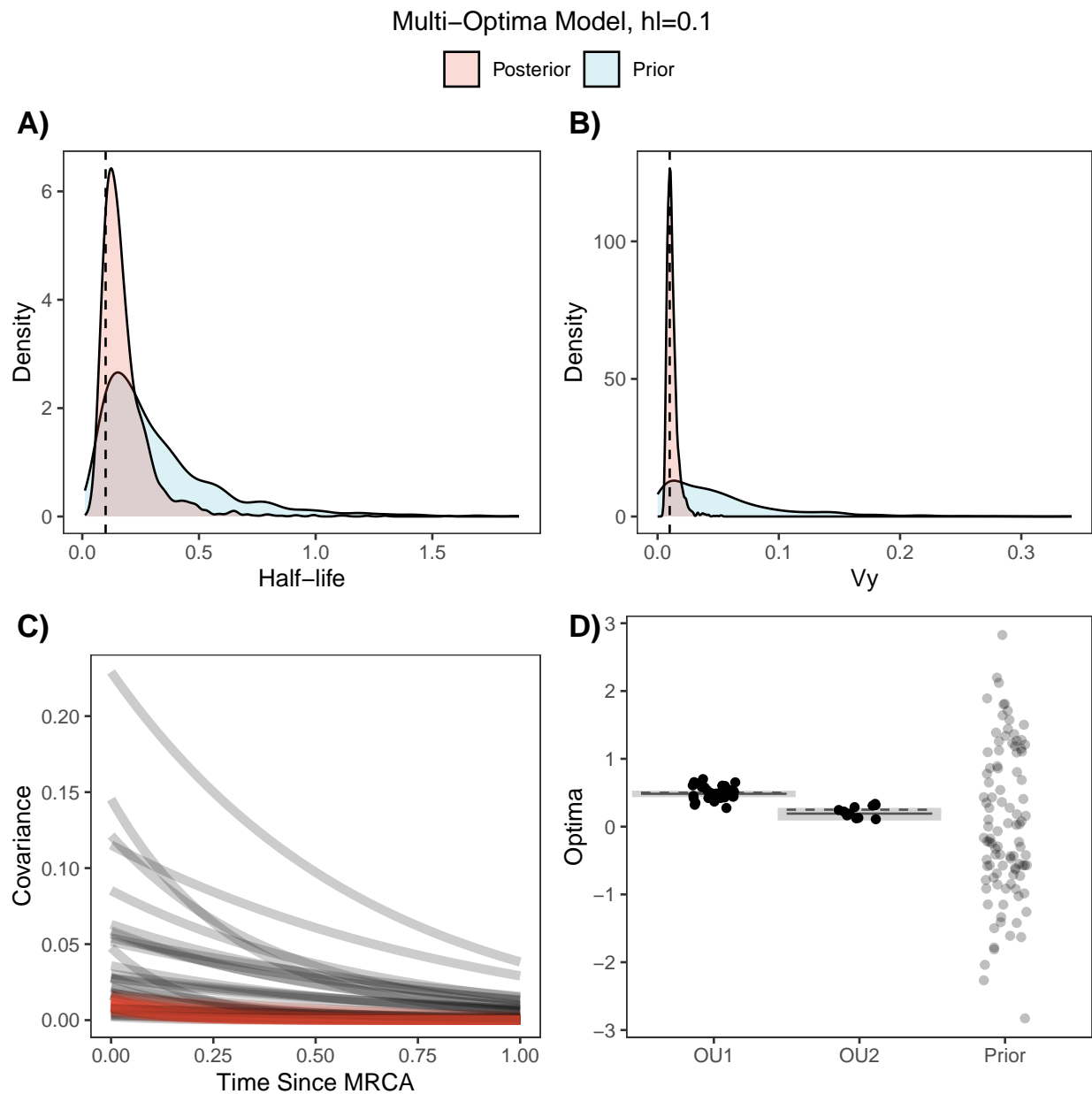


Figure 11: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.1.2 $hl = 0.25$

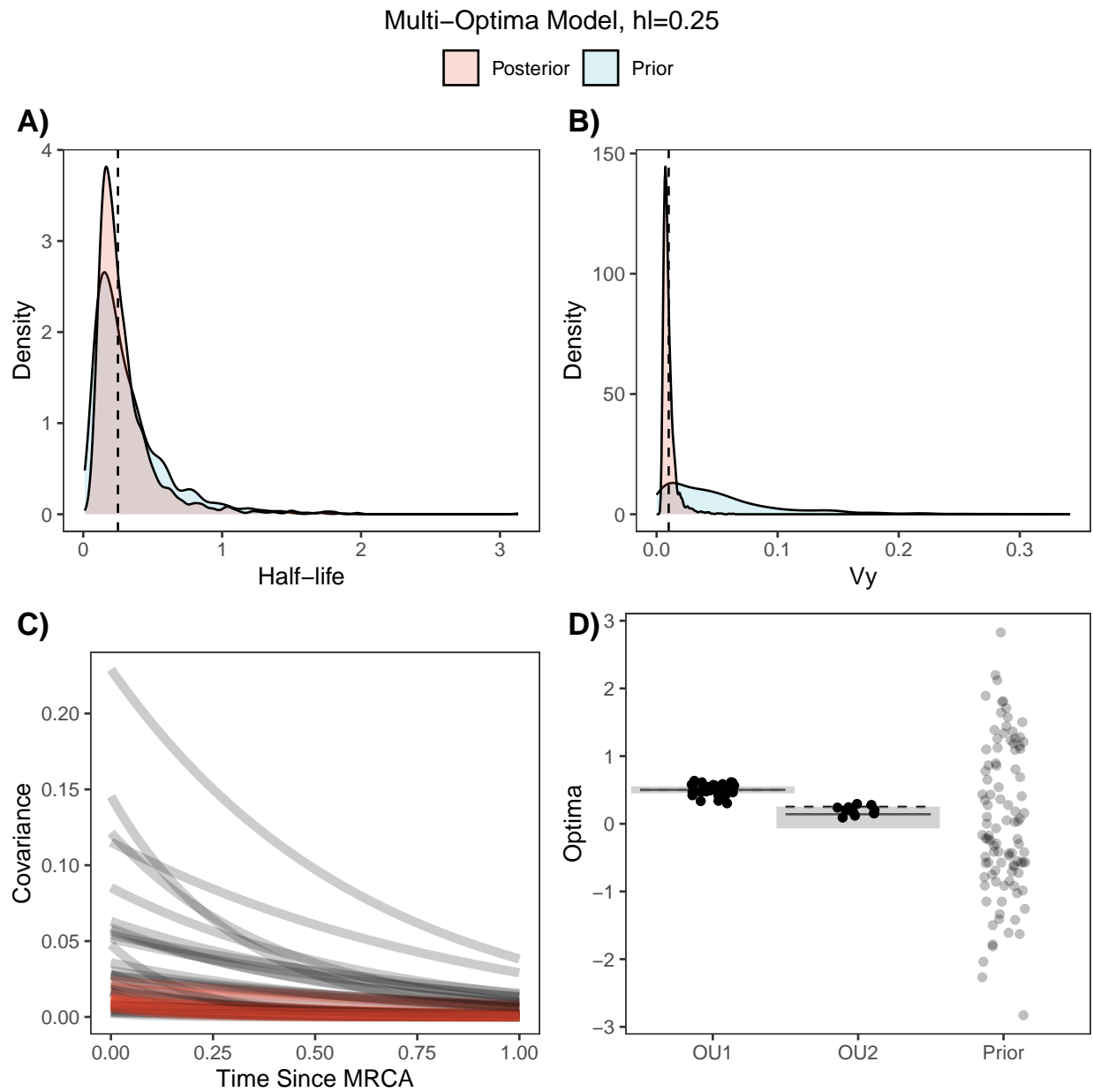


Figure 12: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.1.3 $hl = 0.75$

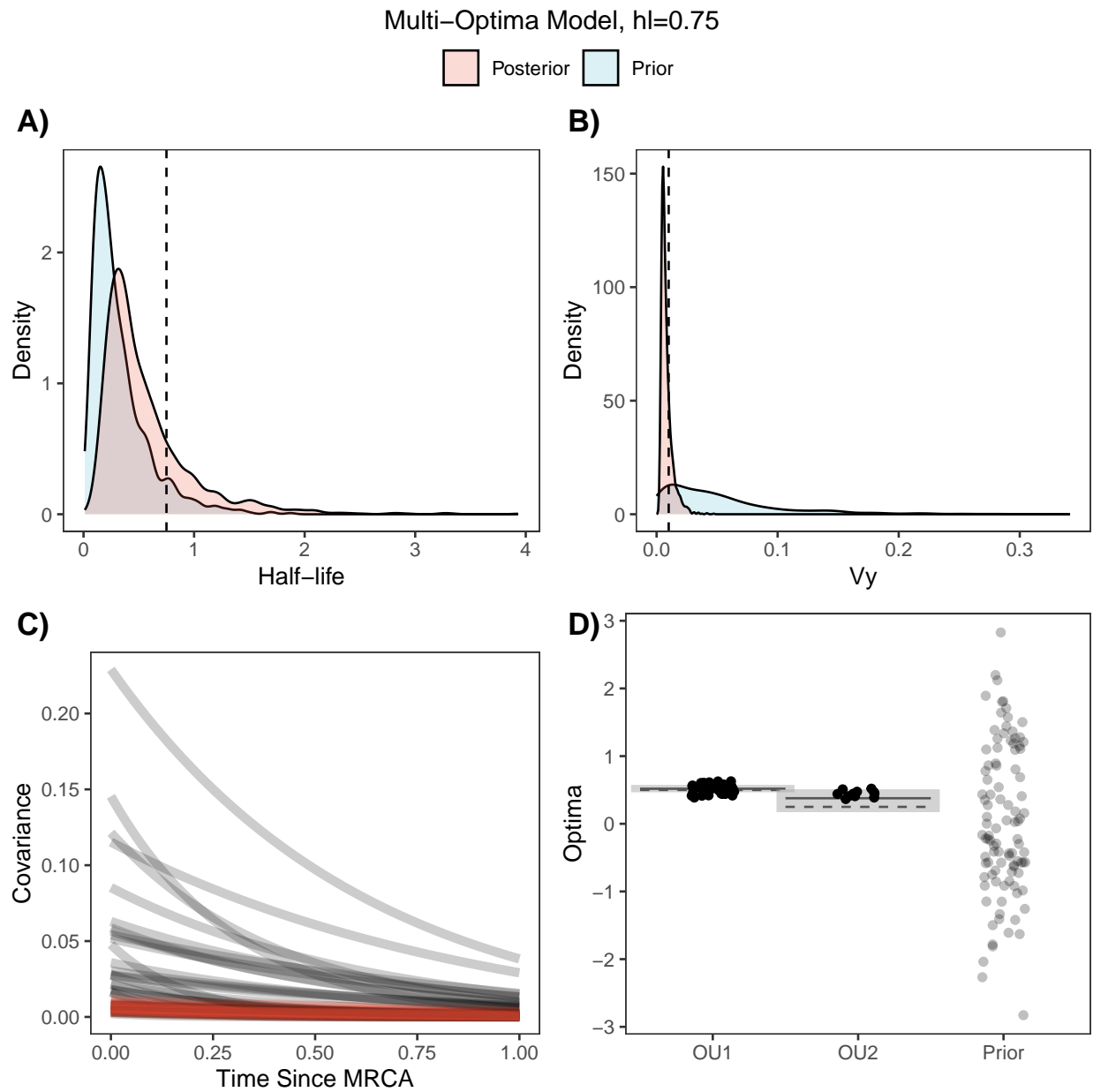


Figure 13: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.2 Multilevel Multi-Optima Model - Varying Intercepts

6.2.1 $hl = 0.1$

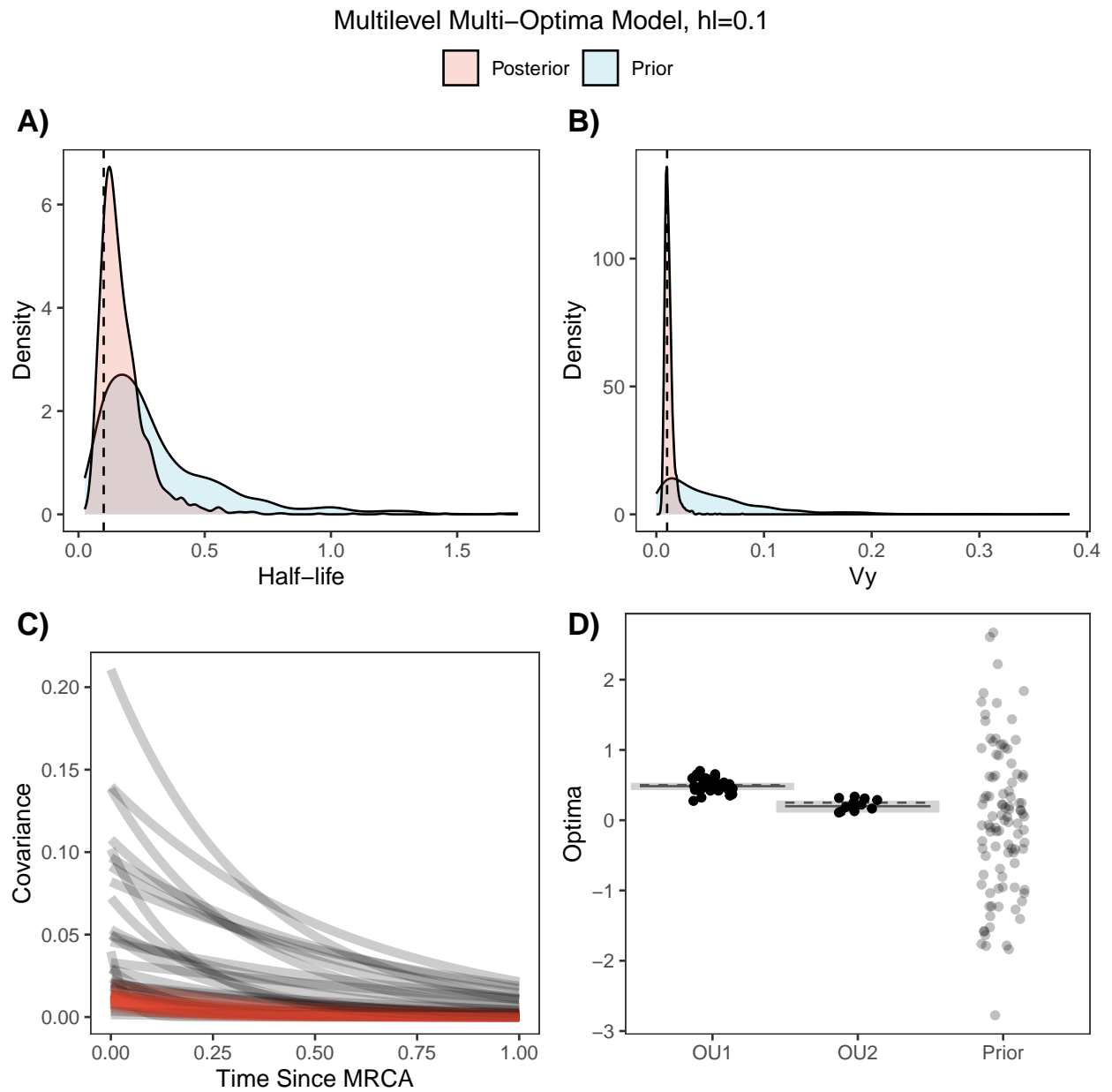


Figure 14: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$ Dotted lines in A,B,D) are true values of the parameter.

6.2.2 $hl = 0.25$

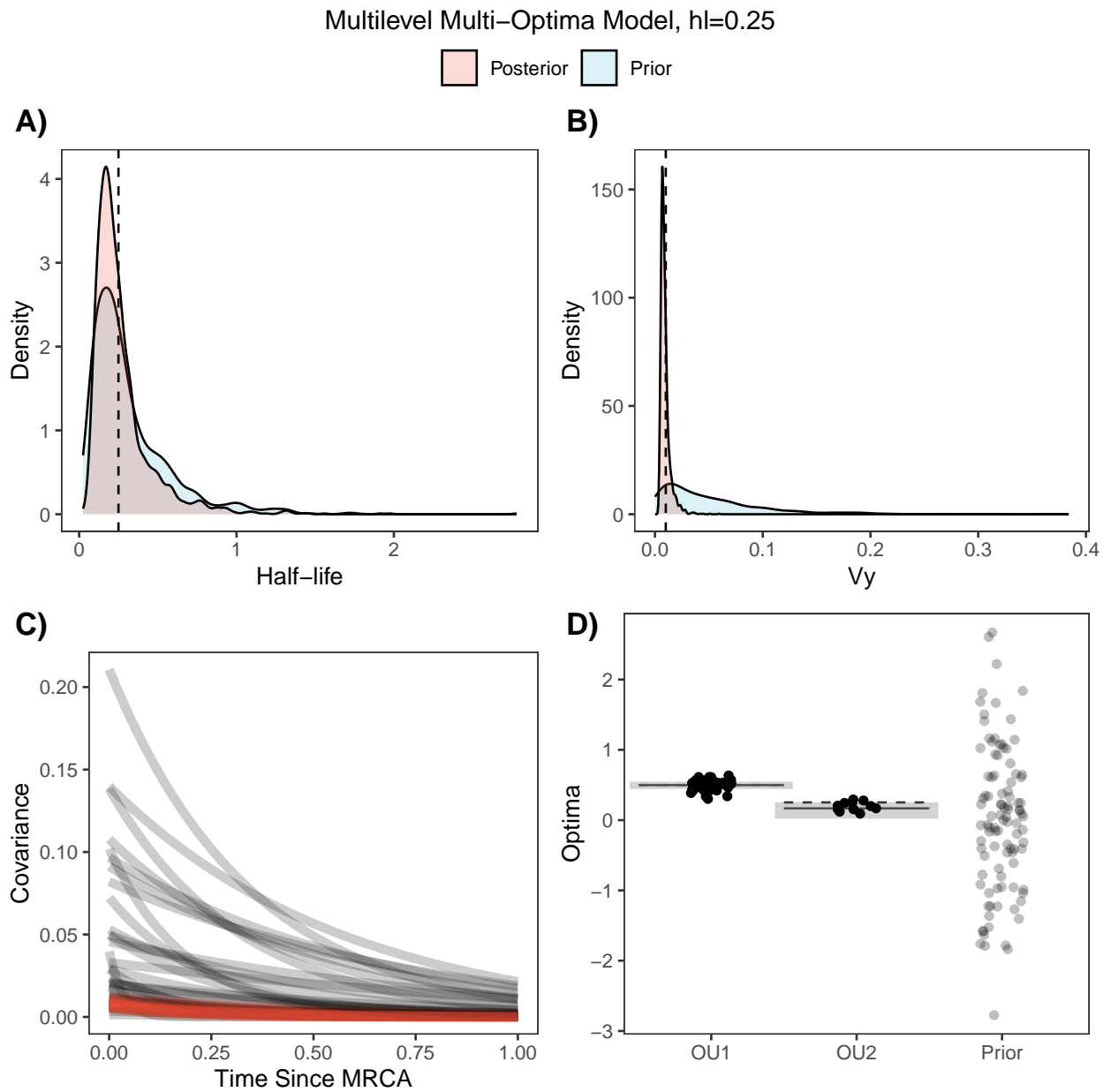


Figure 15: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.2.3 $hl = 0.75$

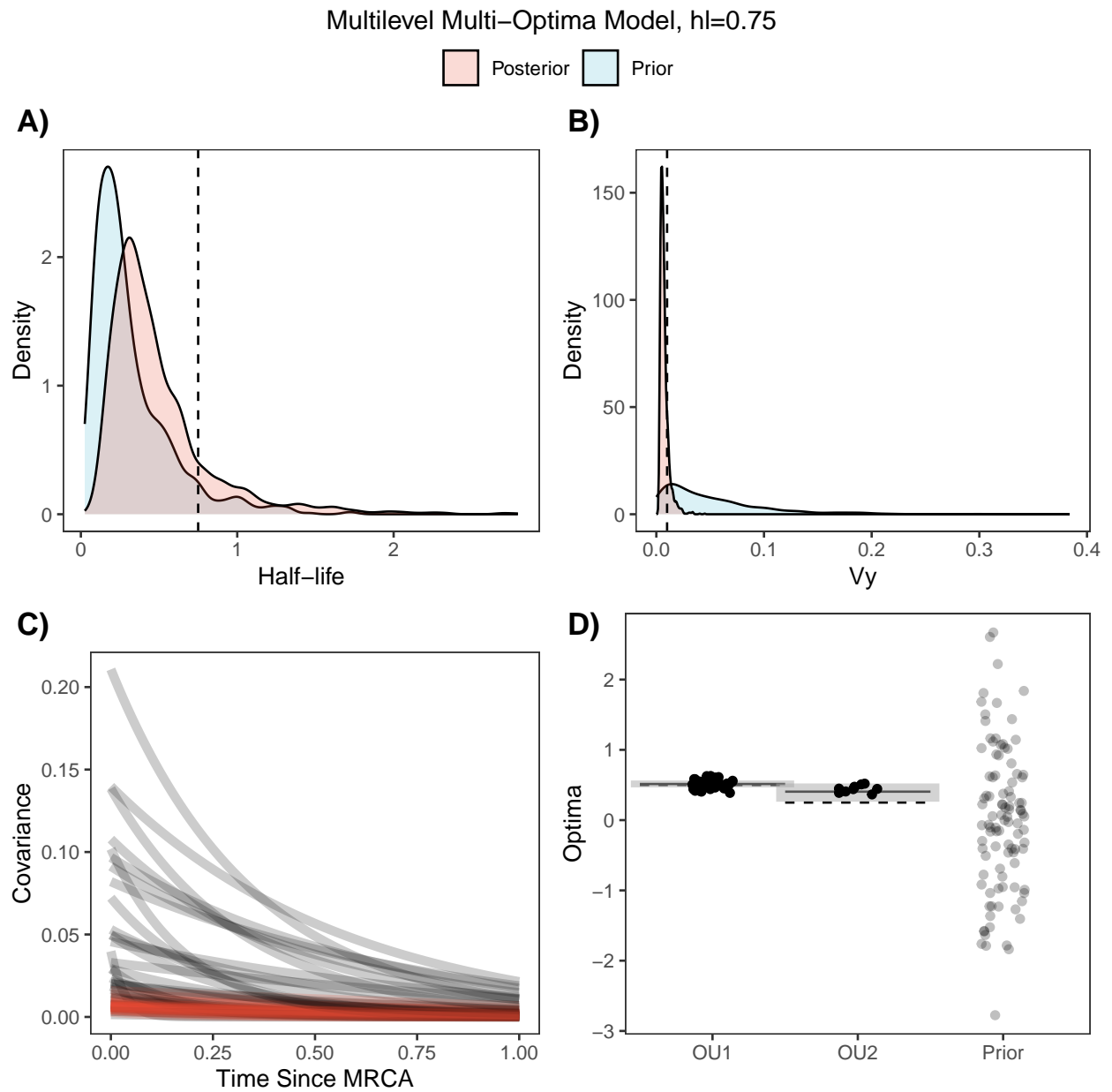


Figure 16: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.3 Multilevel Multi-Optima Model - Varying Intercepts - Non-centered

6.3.1 $hl = 0.1$

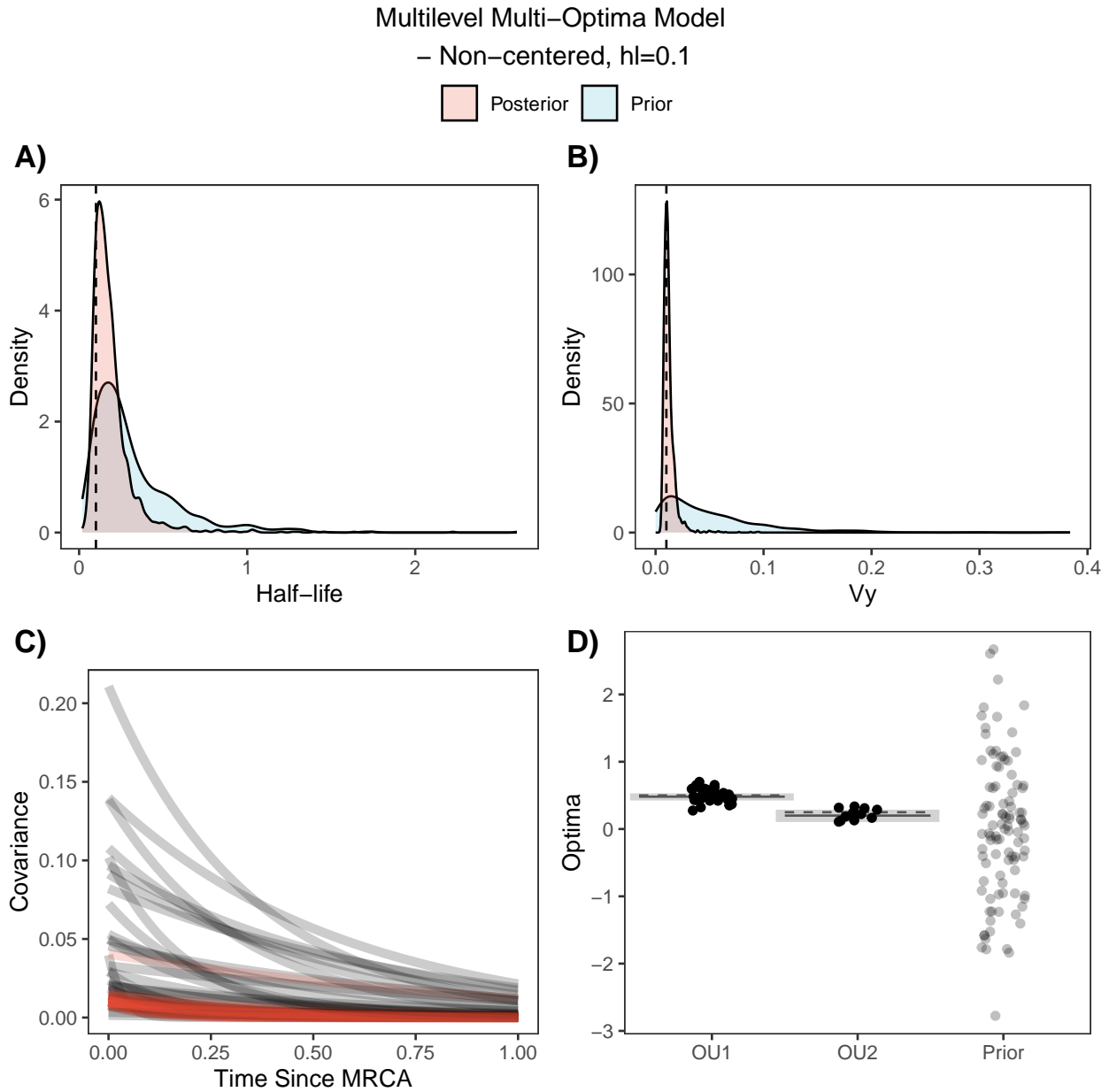


Figure 17: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.3.2 $hl = 0.25$

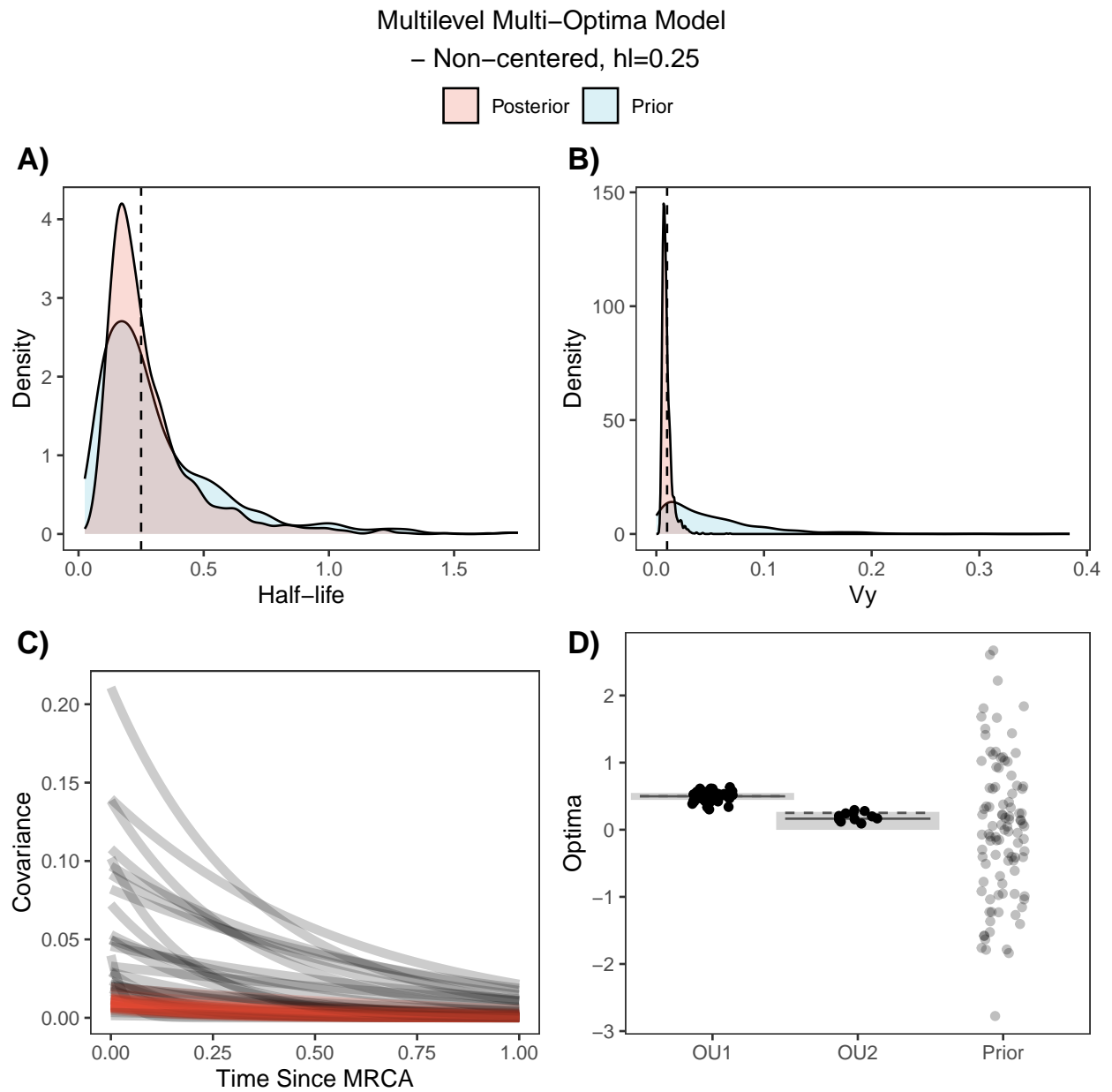


Figure 18: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

6.3.3 $hl = 0.75$

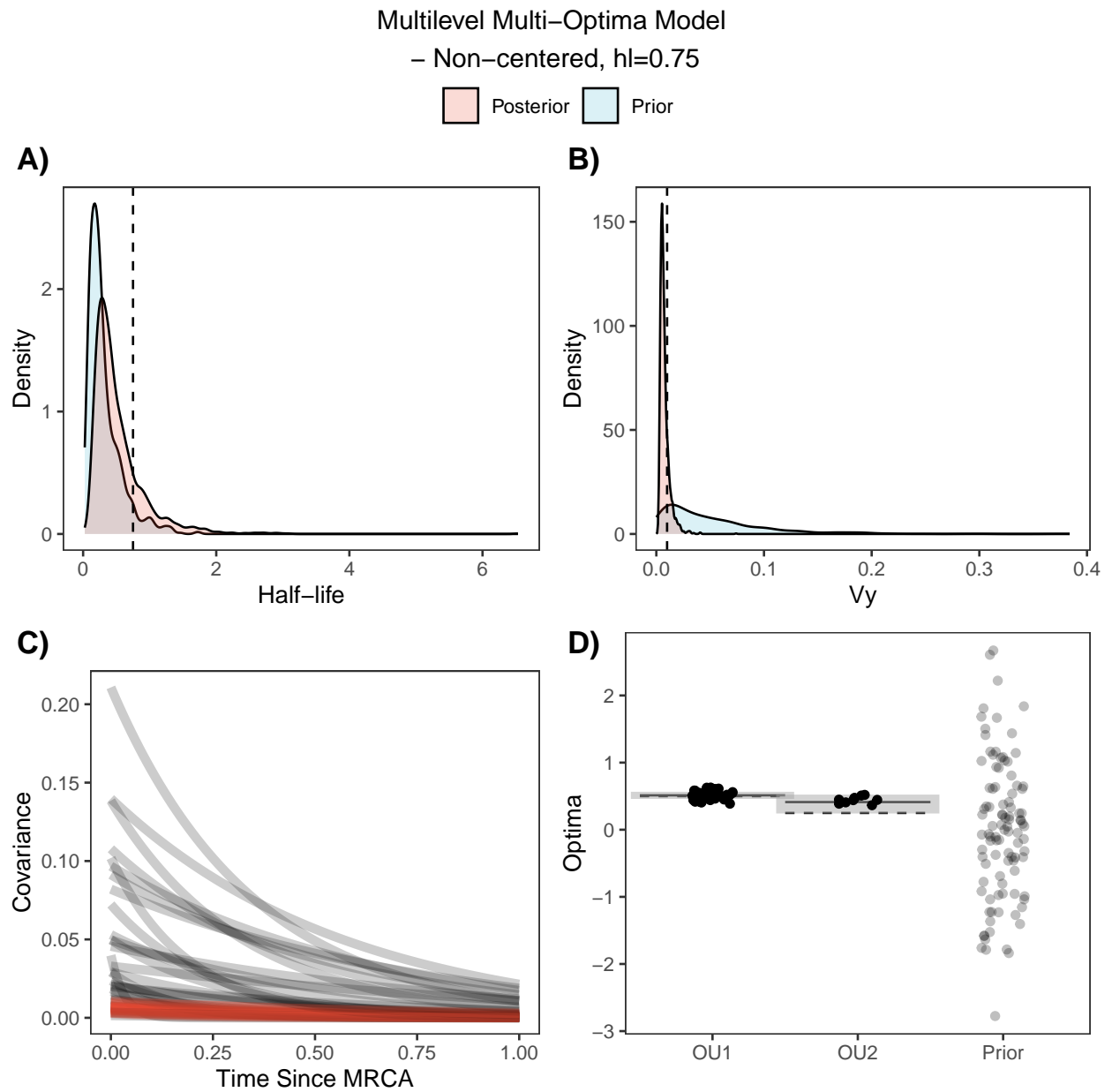


Figure 19: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7 Multi-Optima Direct Effect Models

7.1 Multi-Optima Direct Effect Model

7.1.1 $hl = 0.1$

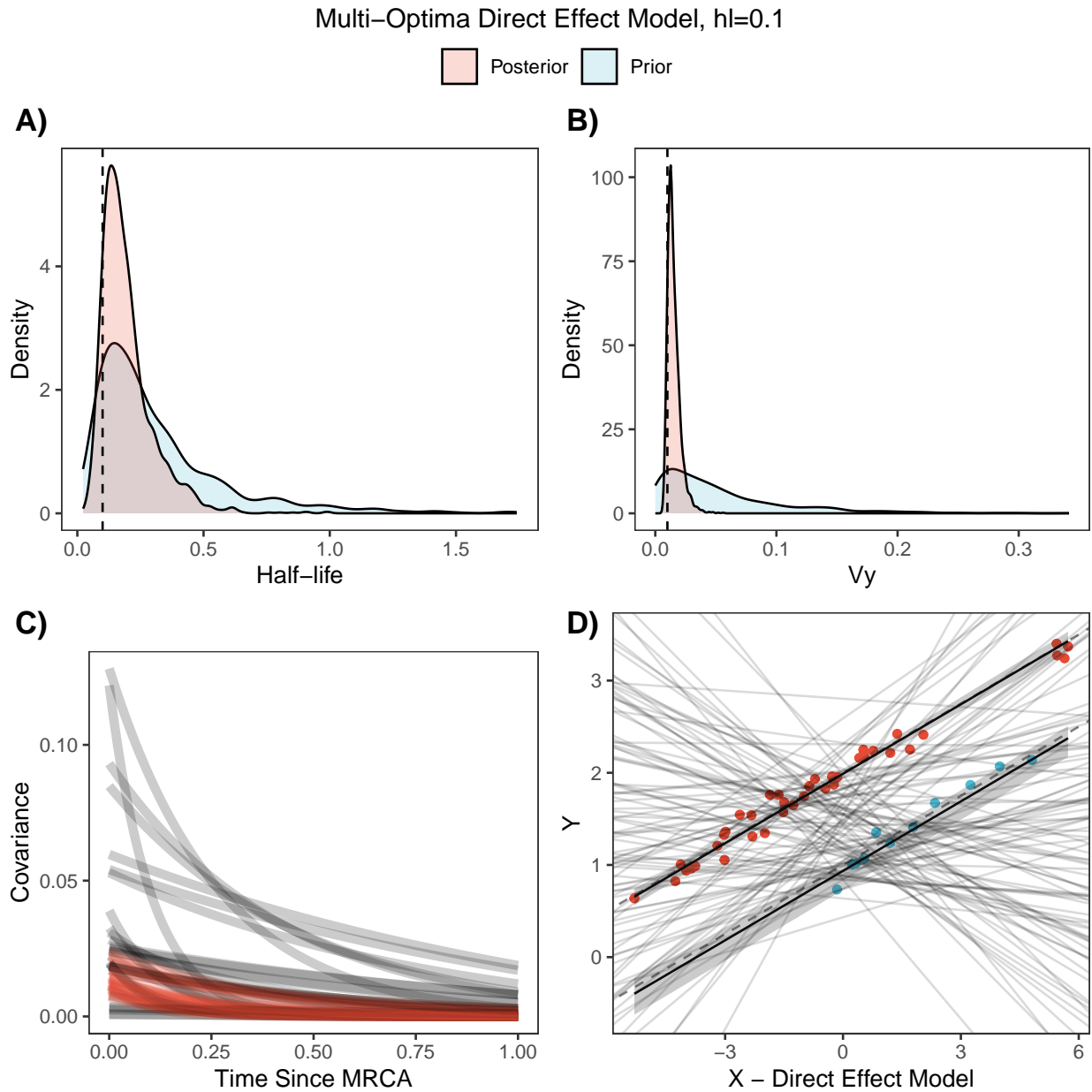


Figure 20: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.1.2 $hl = 0.25$

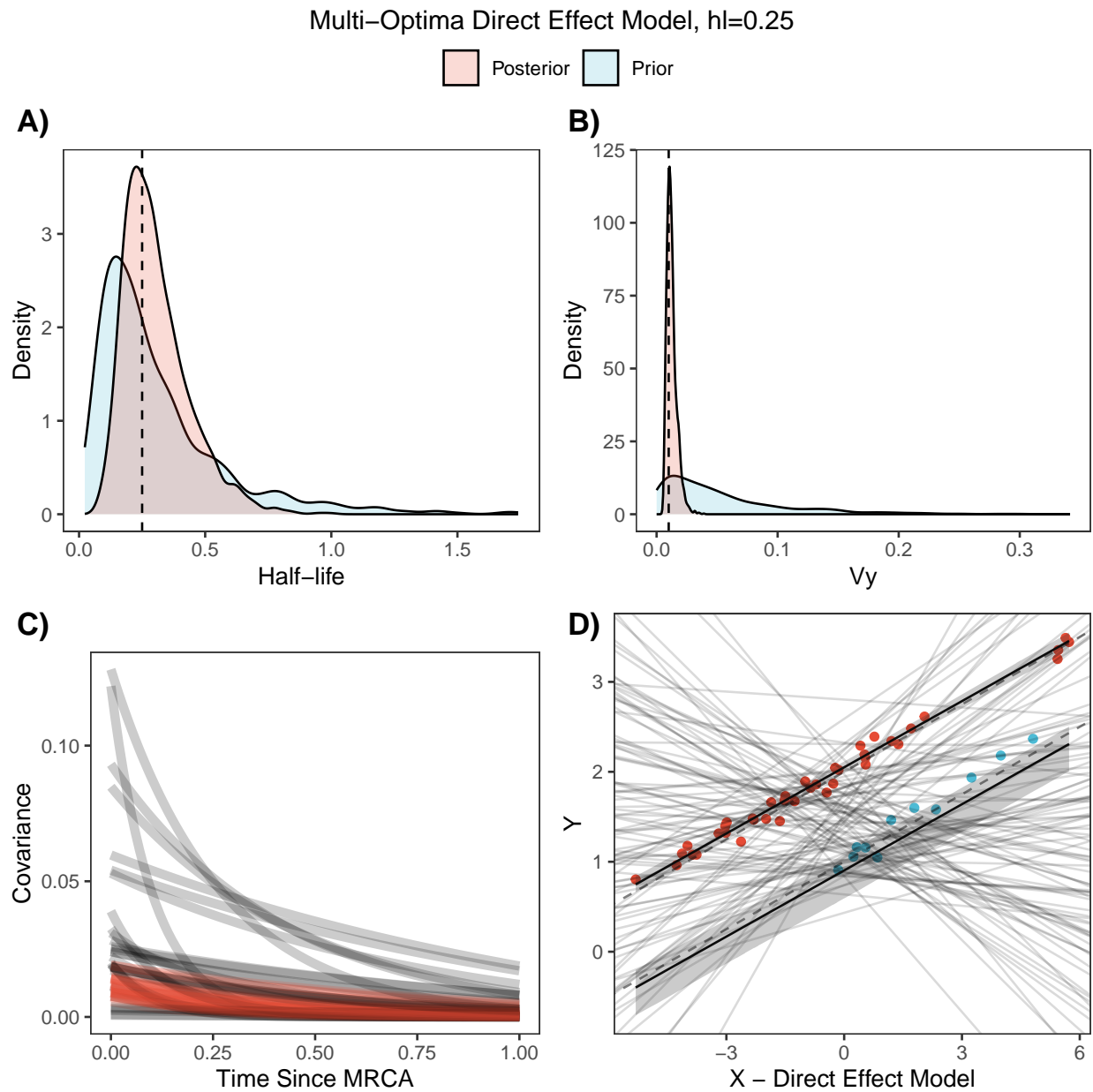


Figure 21: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.1.3 $hl = 0.75$

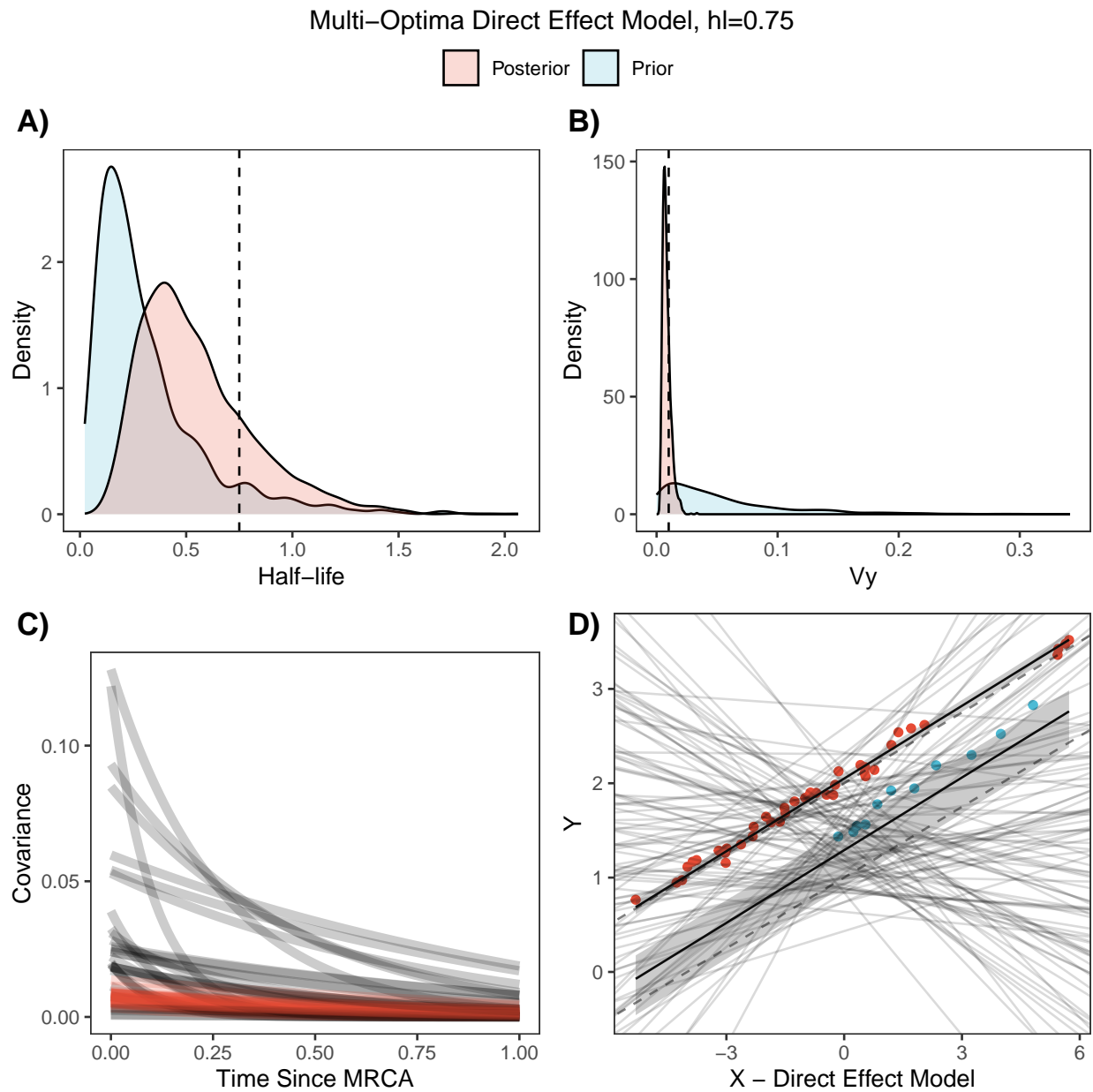


Figure 22: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.2 Multilevel Multi-Optima Direct Effect Model - Varying Intercepts

7.2.1 $hl = 0.1$

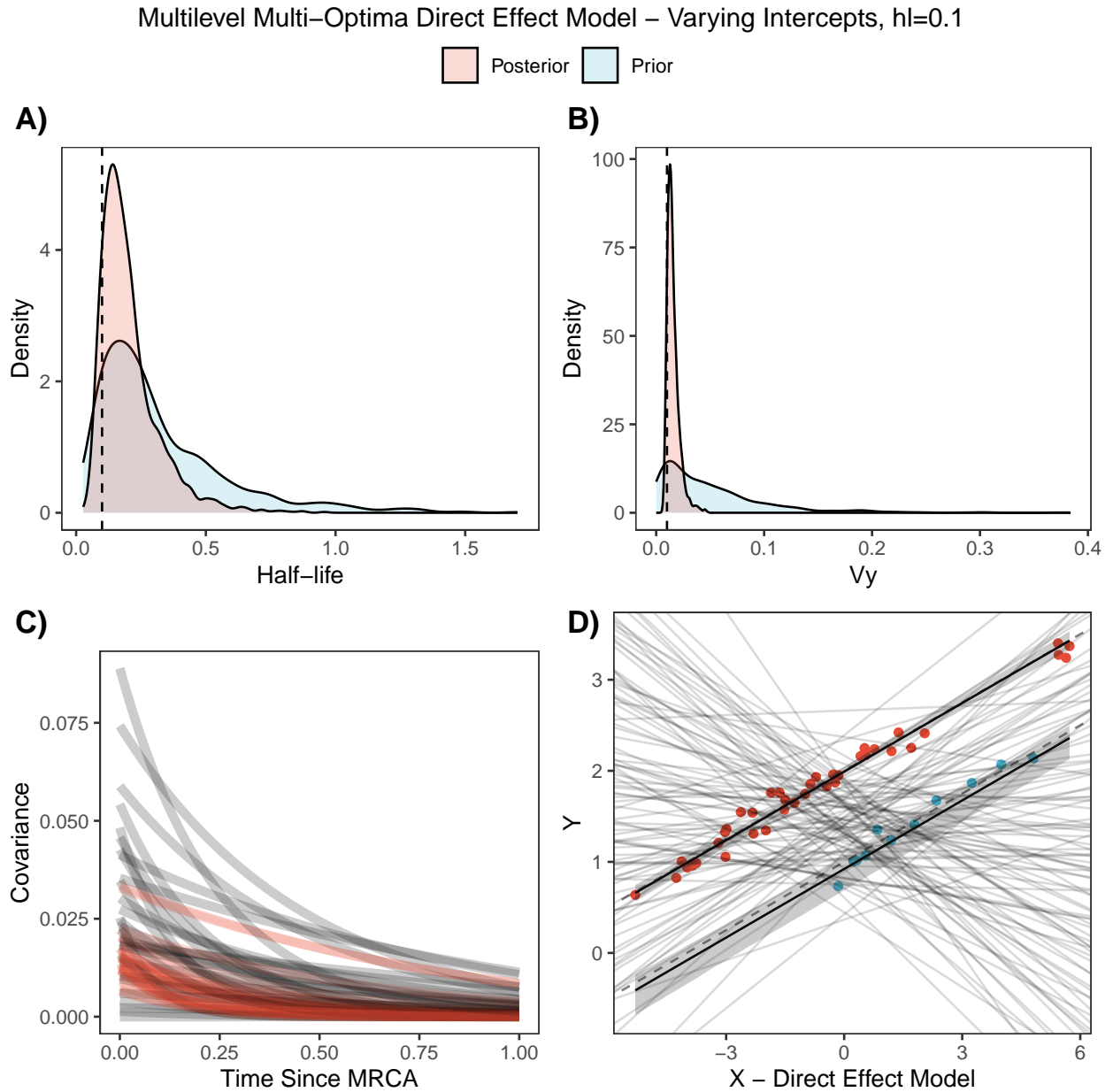


Figure 23: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.2.2 $hl = 0.25$

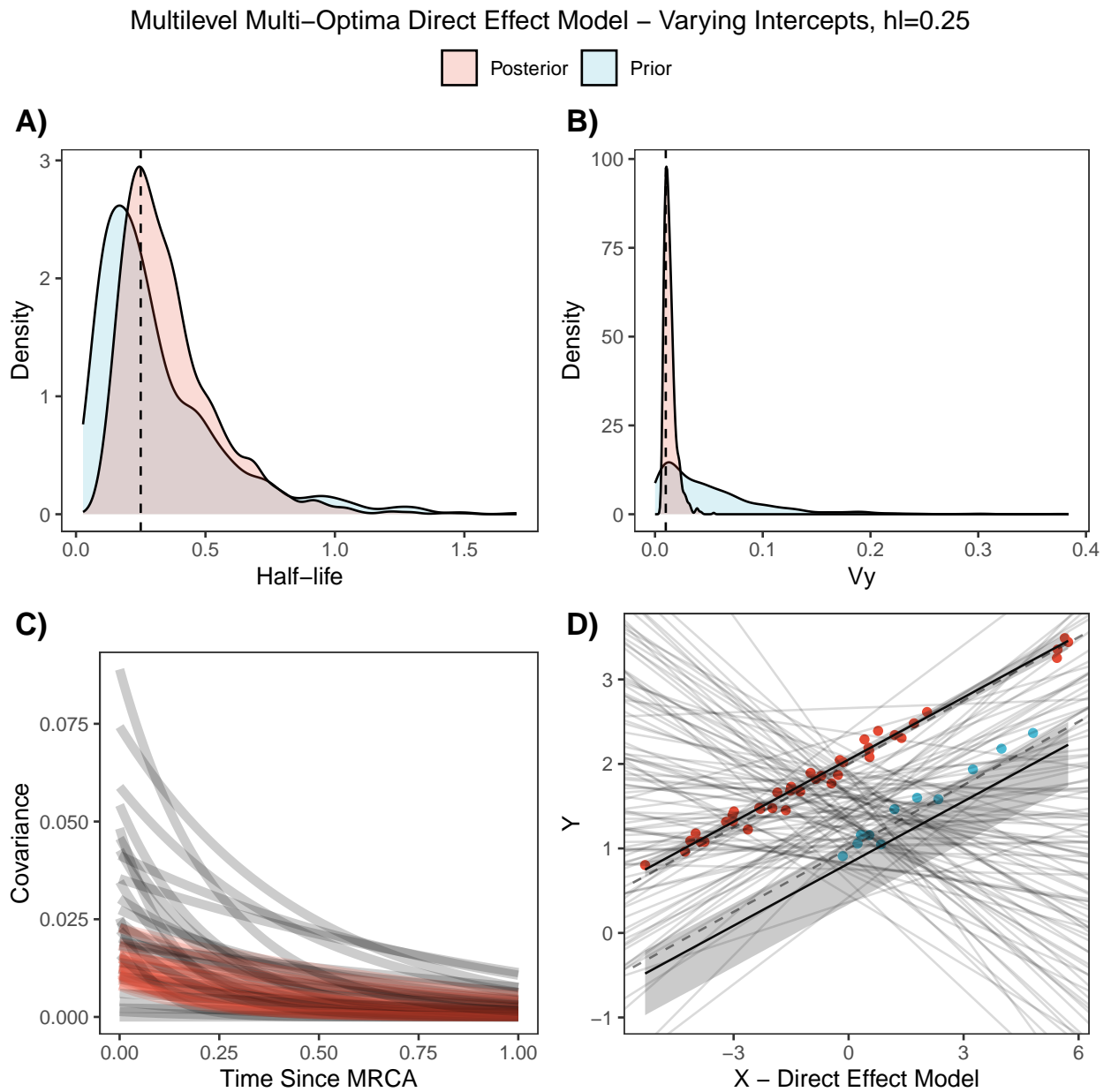


Figure 24: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.2.3 $hl = 0.75$

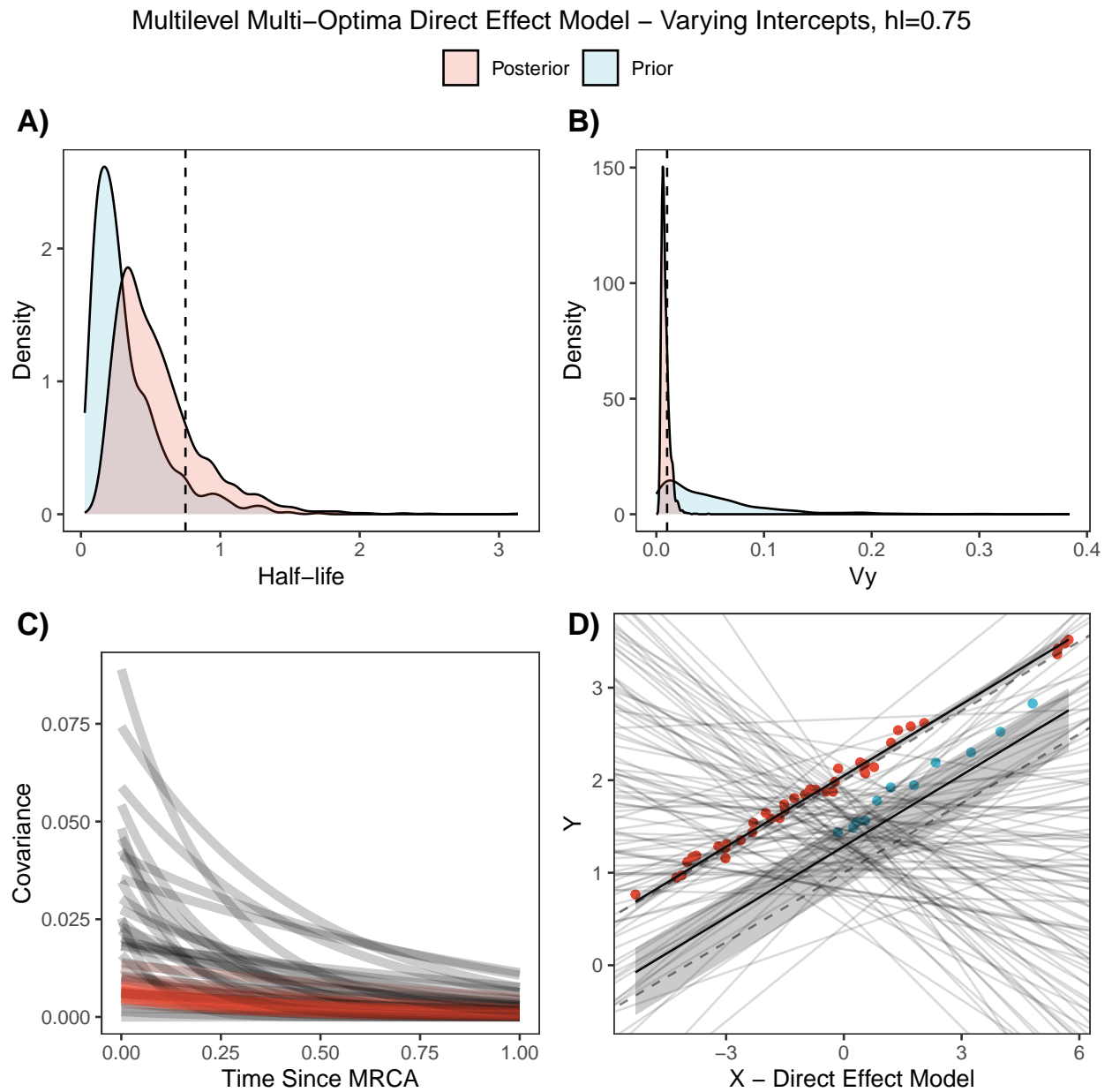


Figure 25: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.3 Multilevel Multi-Optima Direct Effect Model - Varying Intercepts - Non-centered

7.3.1 $hl = 0.1$

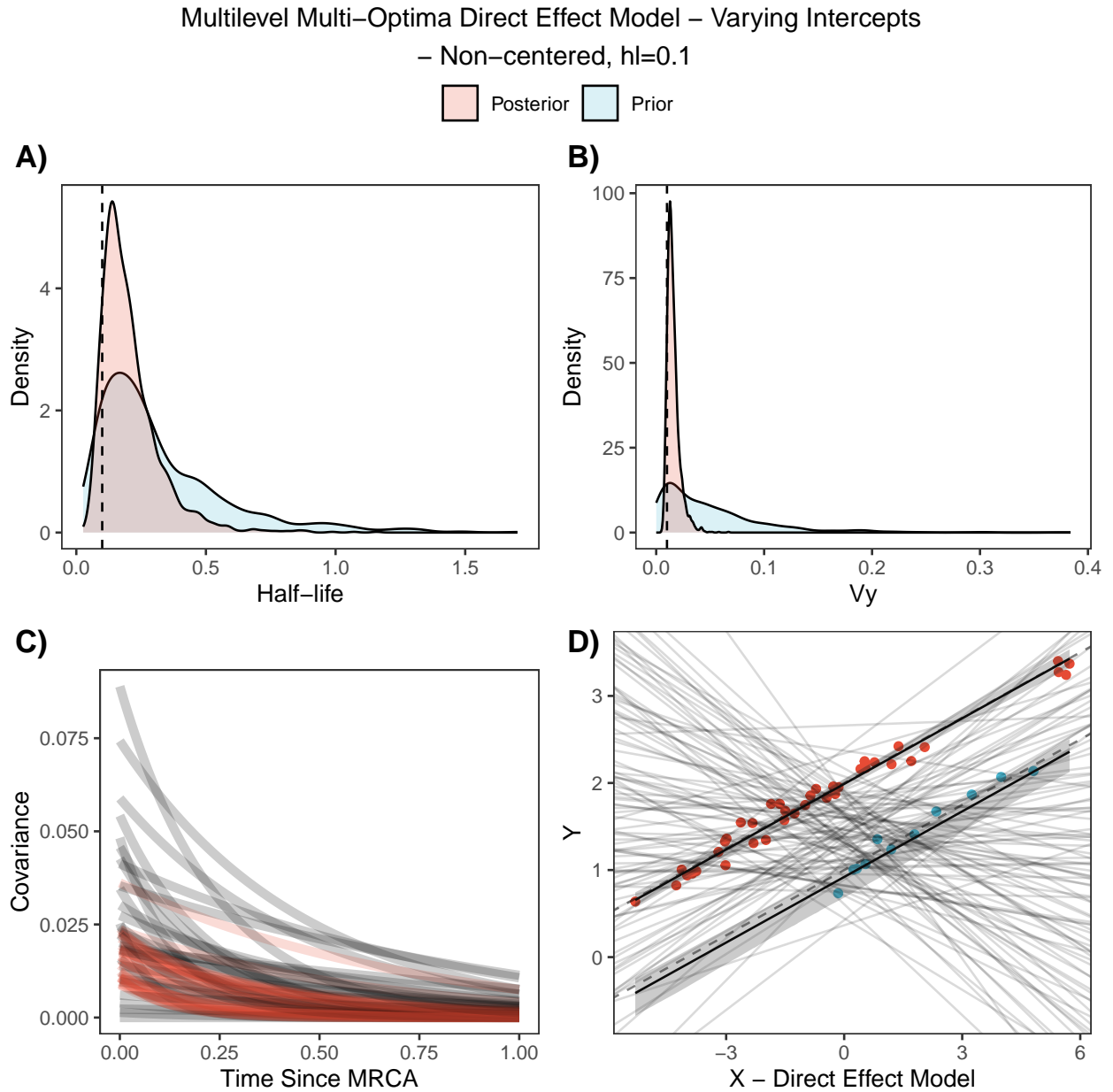


Figure 26: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.3.2 $hl = 0.25$

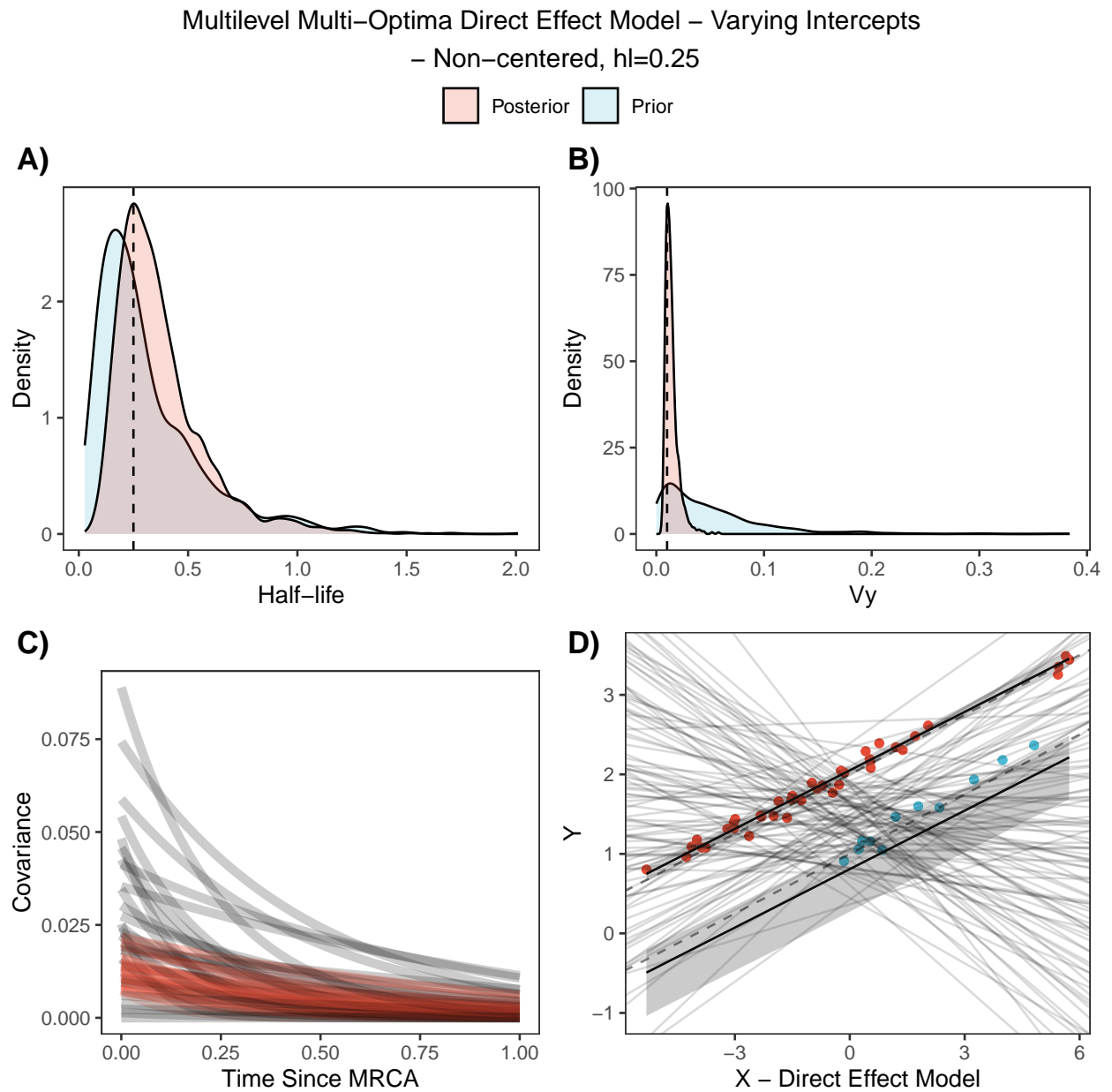


Figure 27: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.3.3 $hl = 0.75$

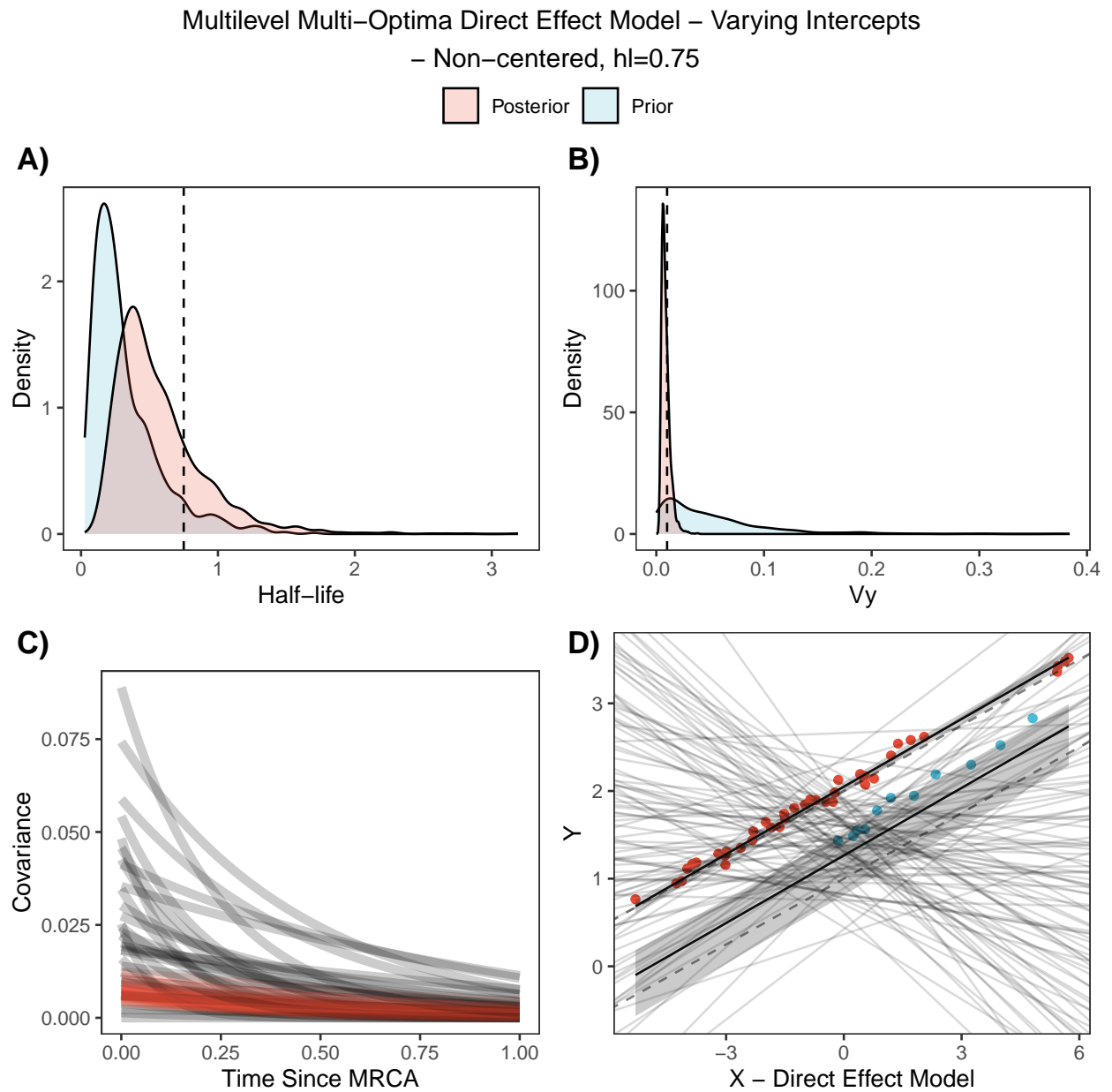


Figure 28: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A, B, D) are true values of the parameter.

7.4 Multi-Optima Direct Effect Model - Varying Effects

7.4.1 $hl = 0.1$

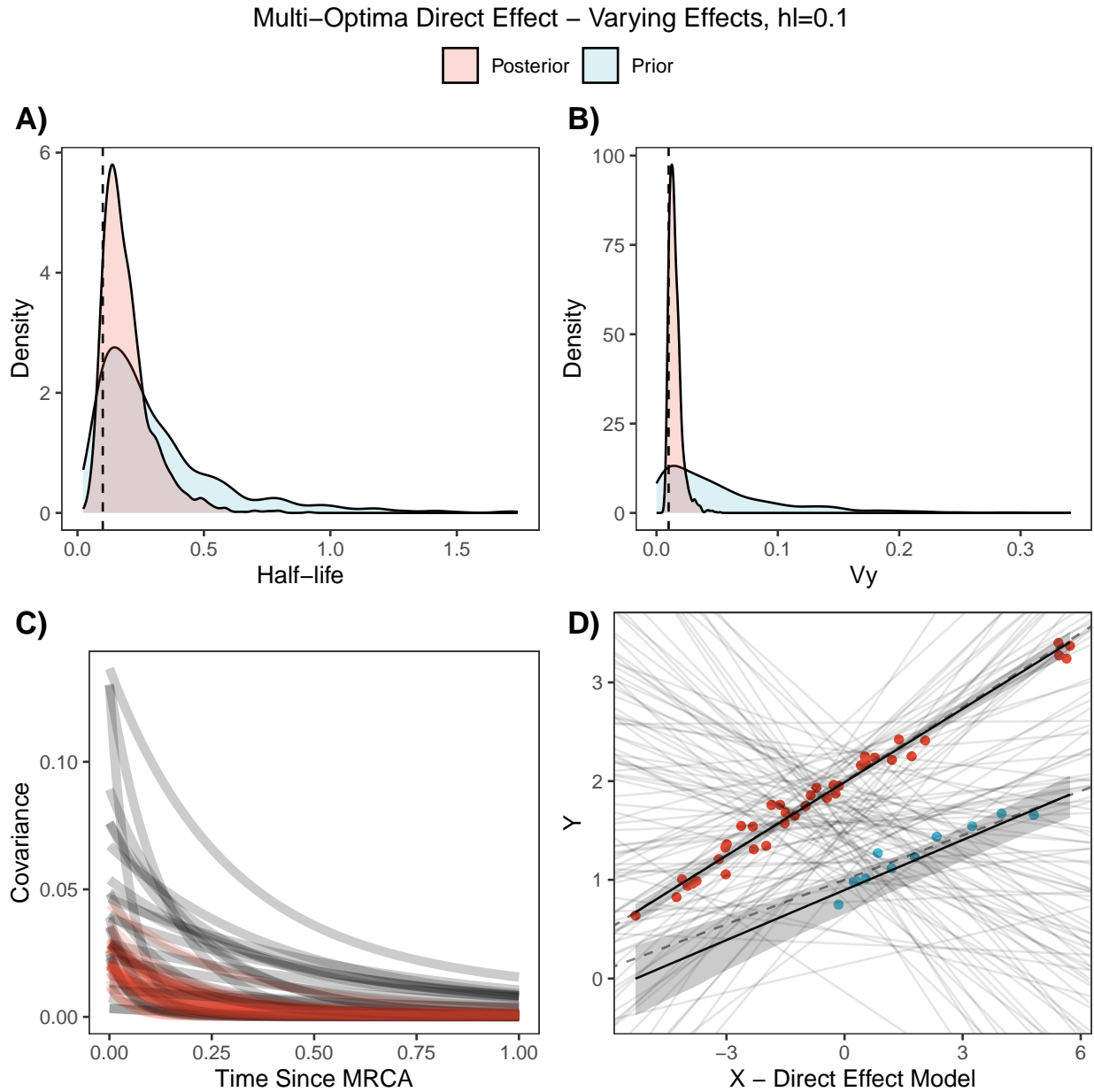


Figure 29: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.4.2 $hl = 0.25$

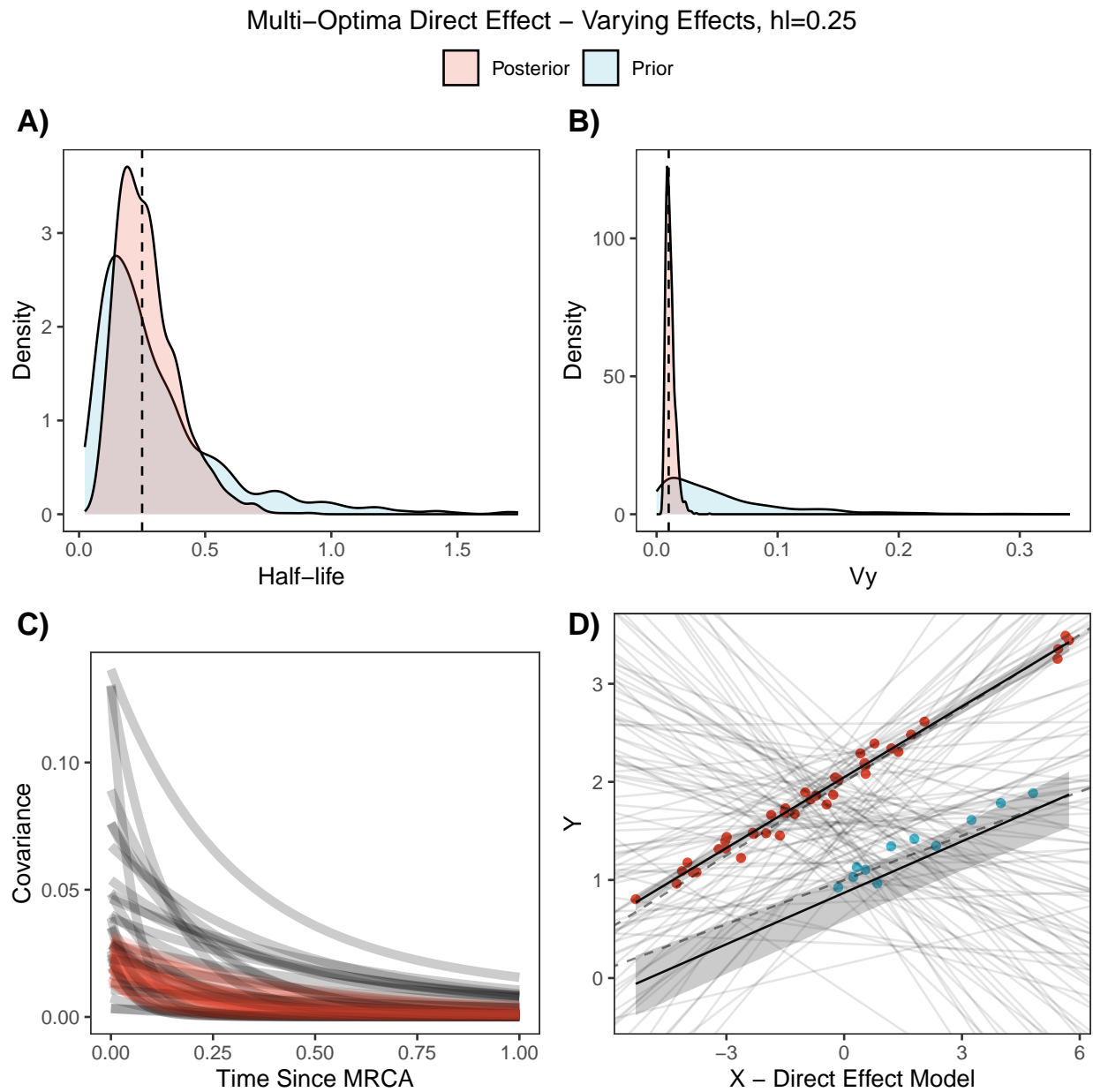


Figure 30: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.4.3 $hl = 0.75$

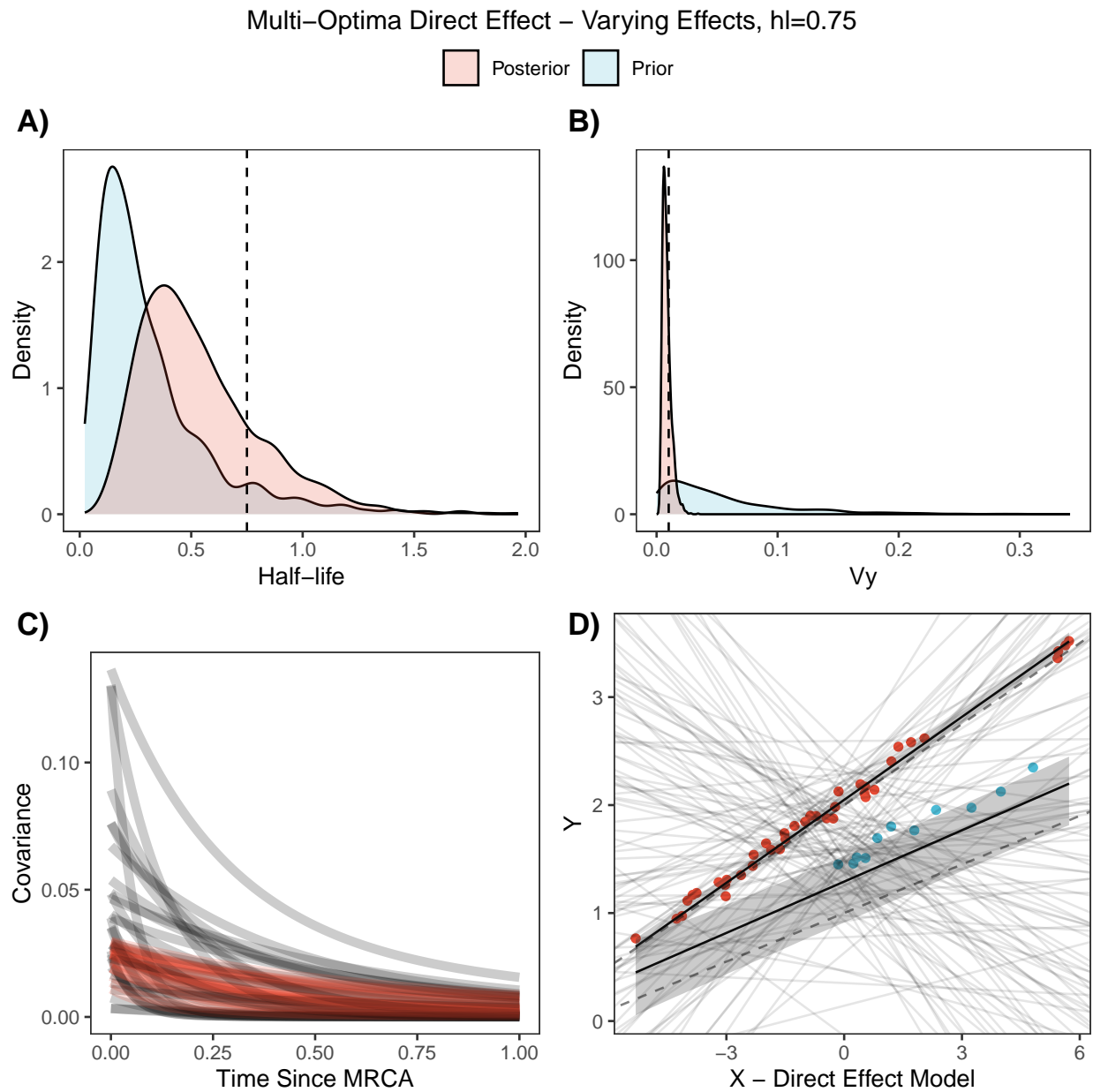


Figure 31: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.5 Multilevel Multi-Optima Direct Effect Model - Varying Effects

7.5.1 $hl = 0.1$

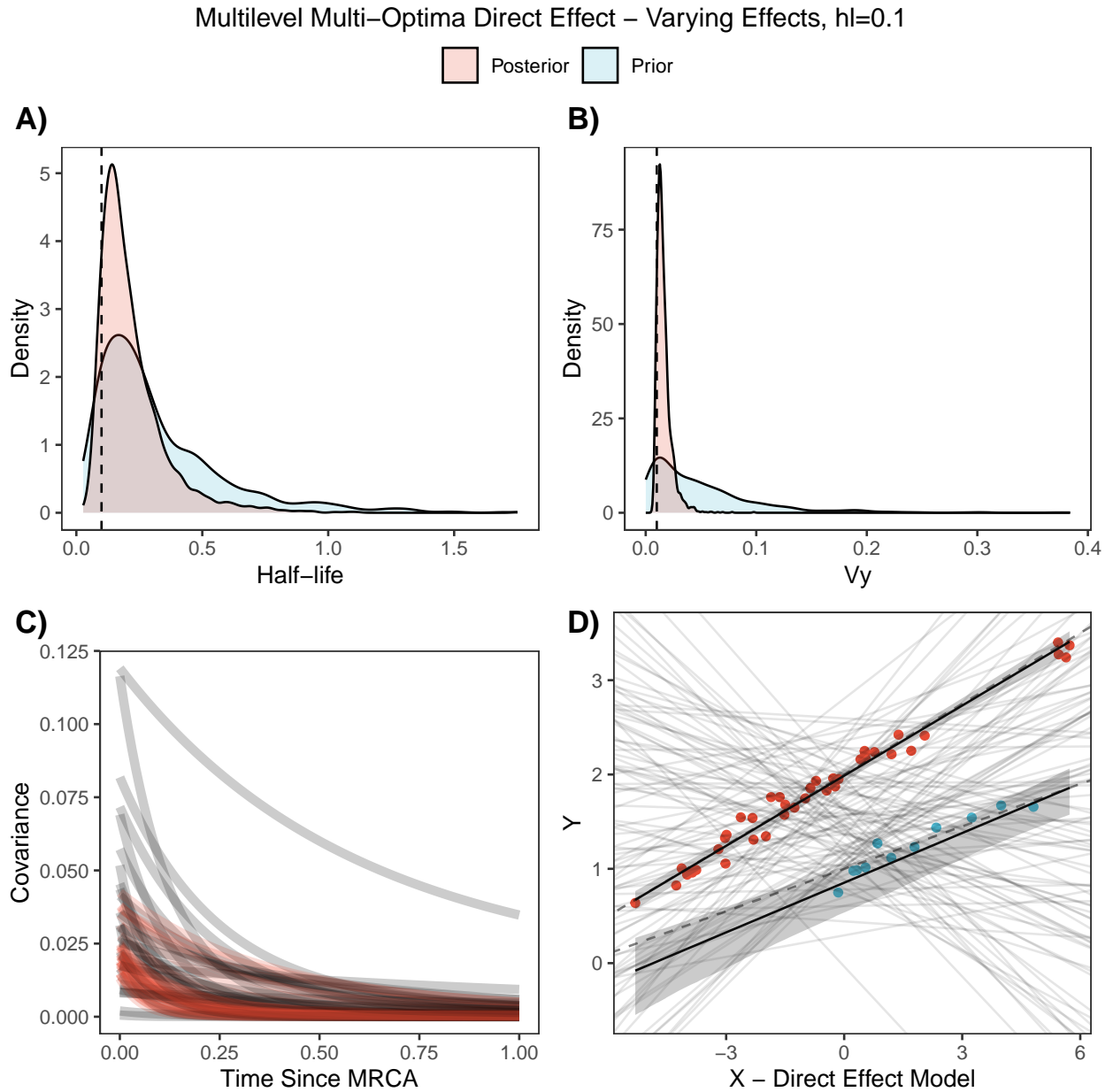


Figure 32: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.5.2 $hl = 0.25$

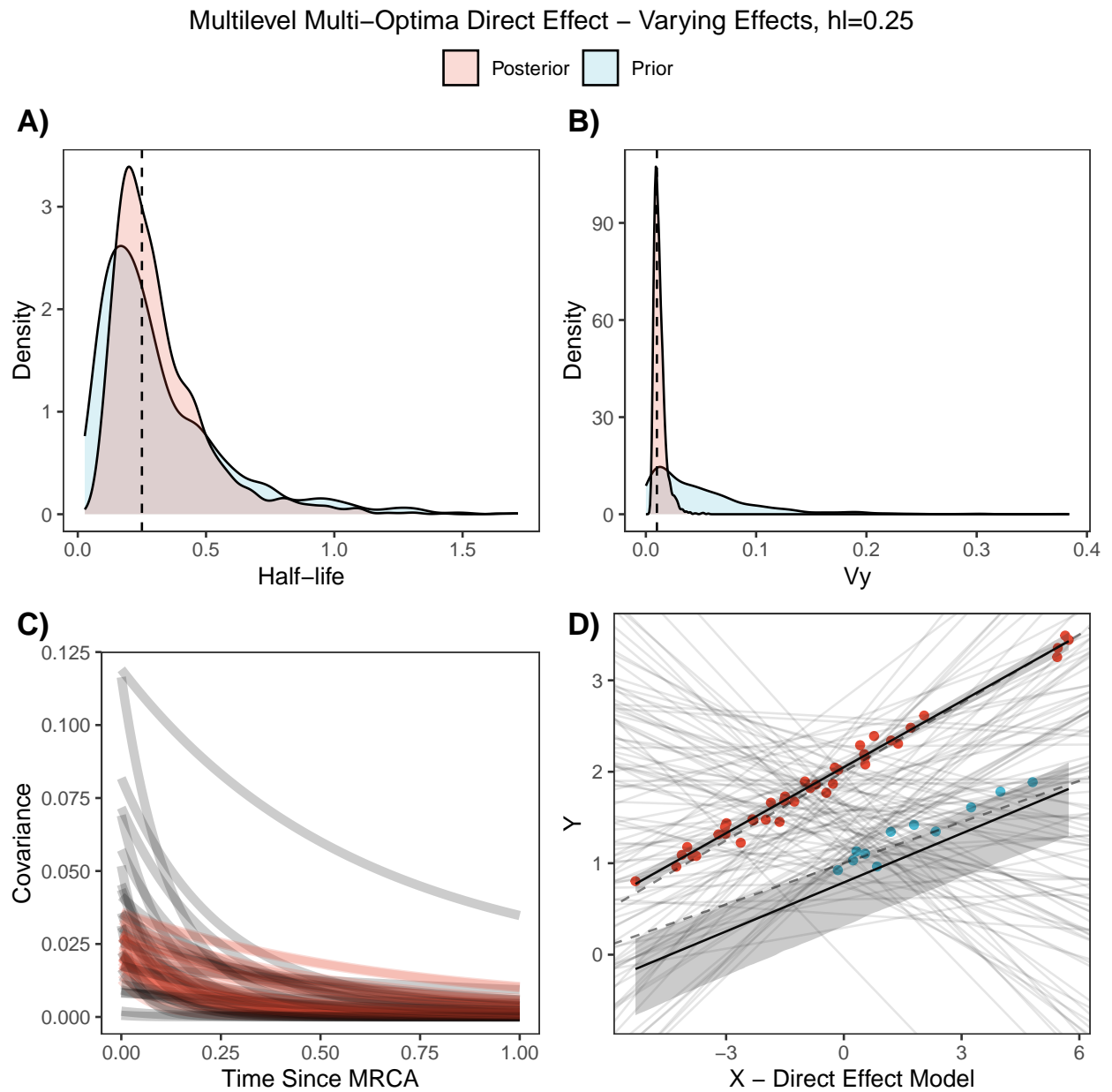


Figure 33: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.5.3 $hl = 0.75$

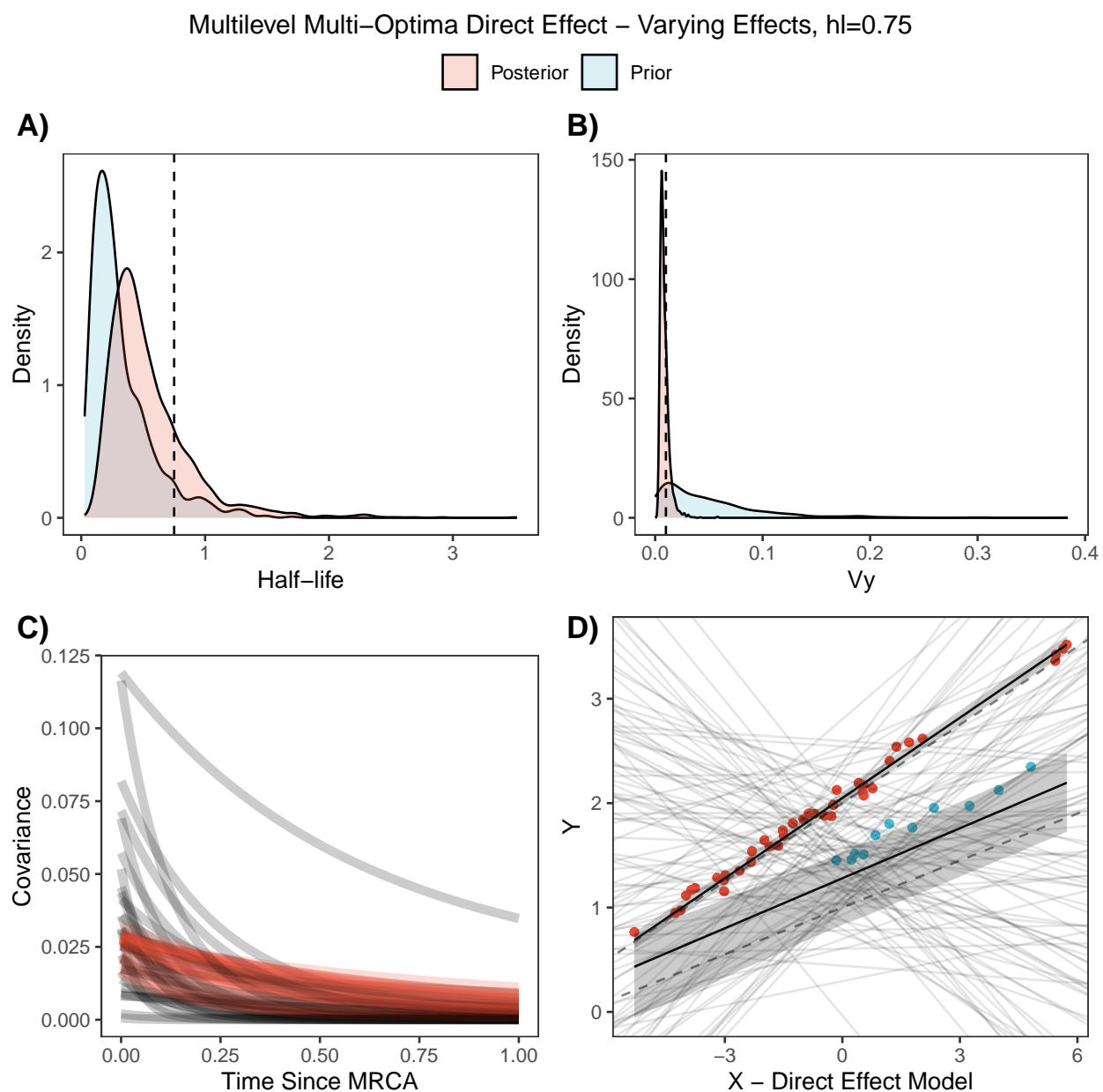


Figure 34: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.6 Multilevel Multi-Optima Direct Effect Model - Varying Effects - Non-centered

7.6.1 $hl = 0.1$

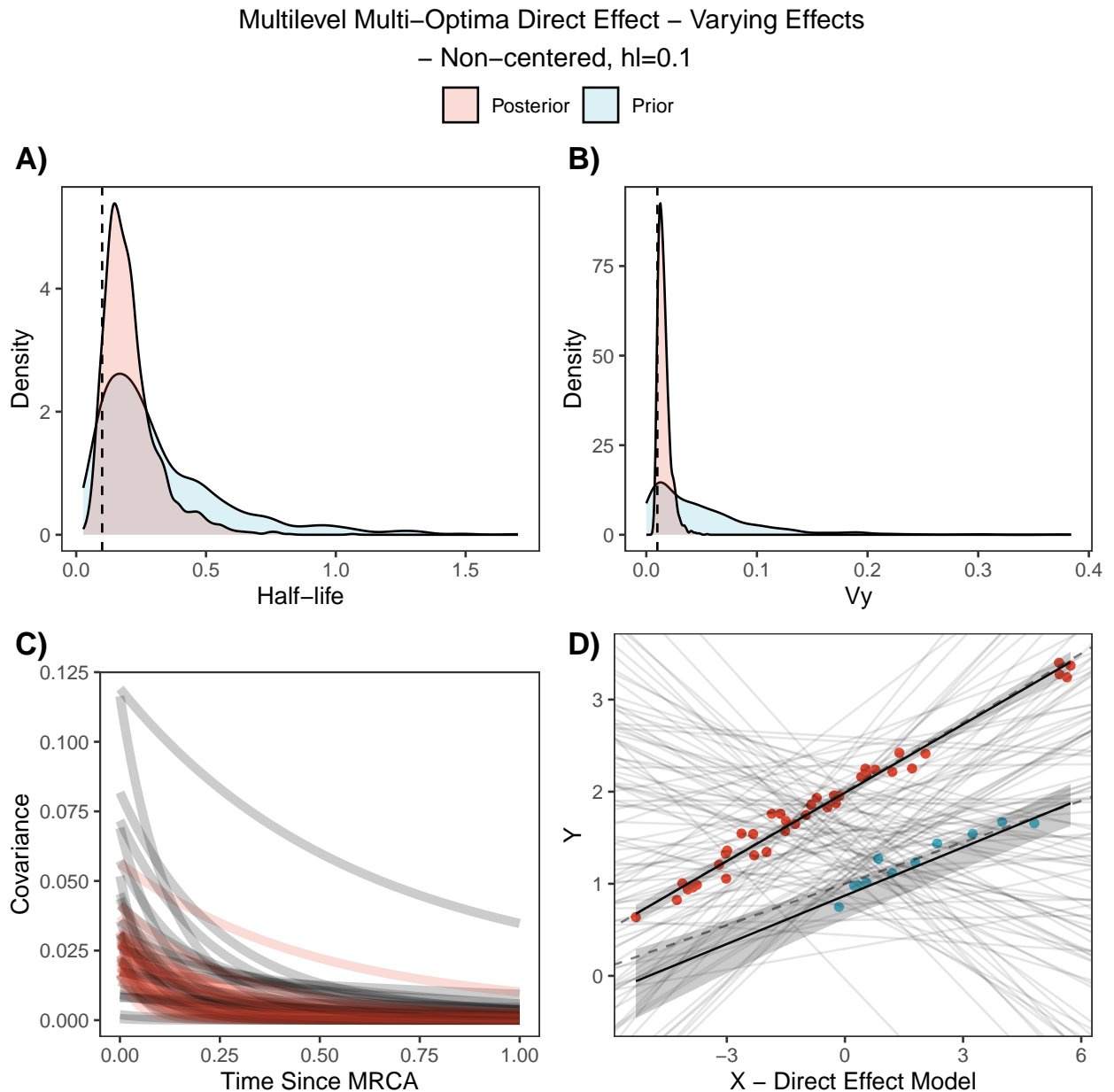


Figure 35: Pri or vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

7.6.2 $hl = 0.25$

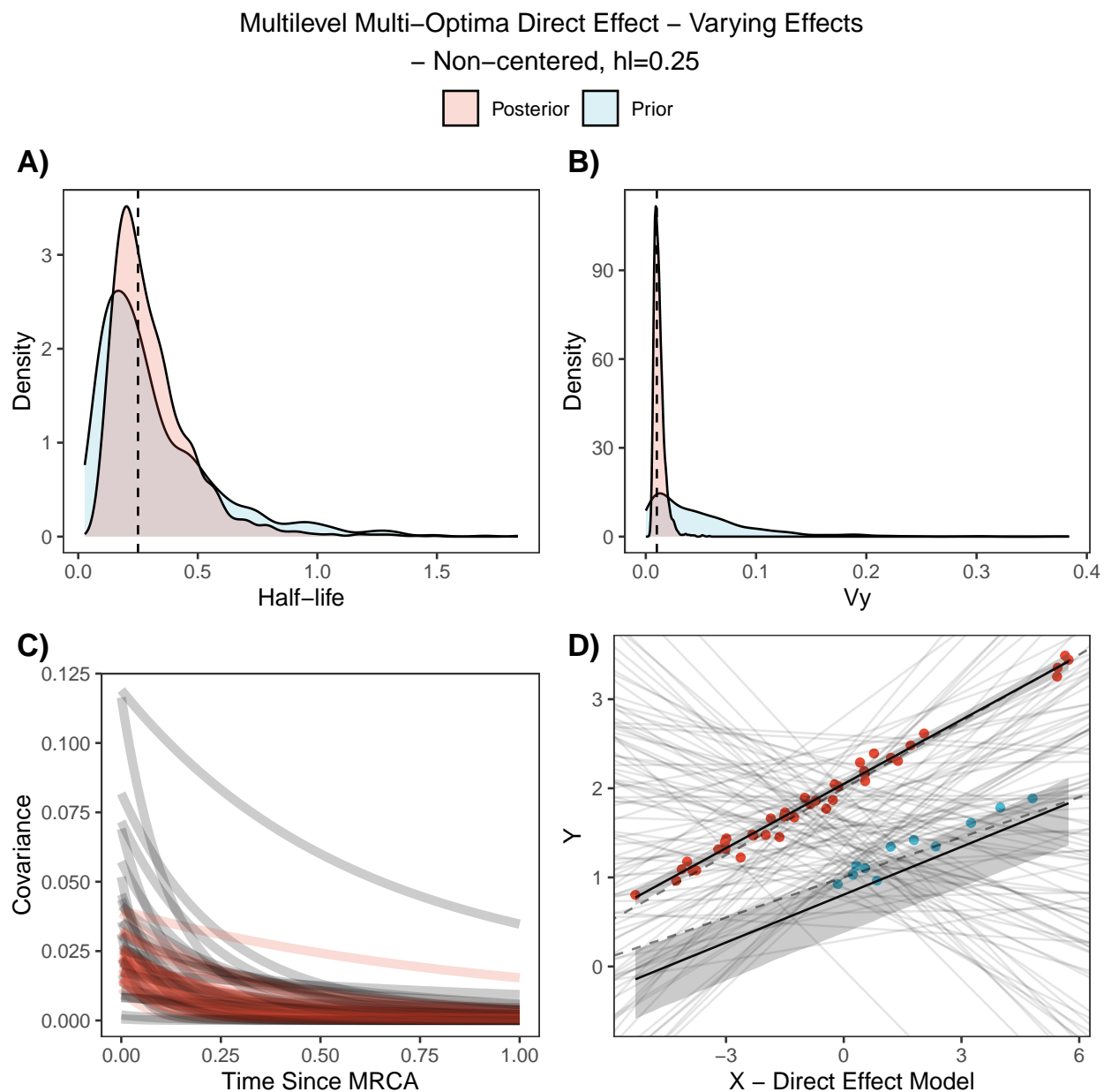


Figure 36: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A, B, D) are true values of the parameter.

7.6.3 $hl = 0.75$

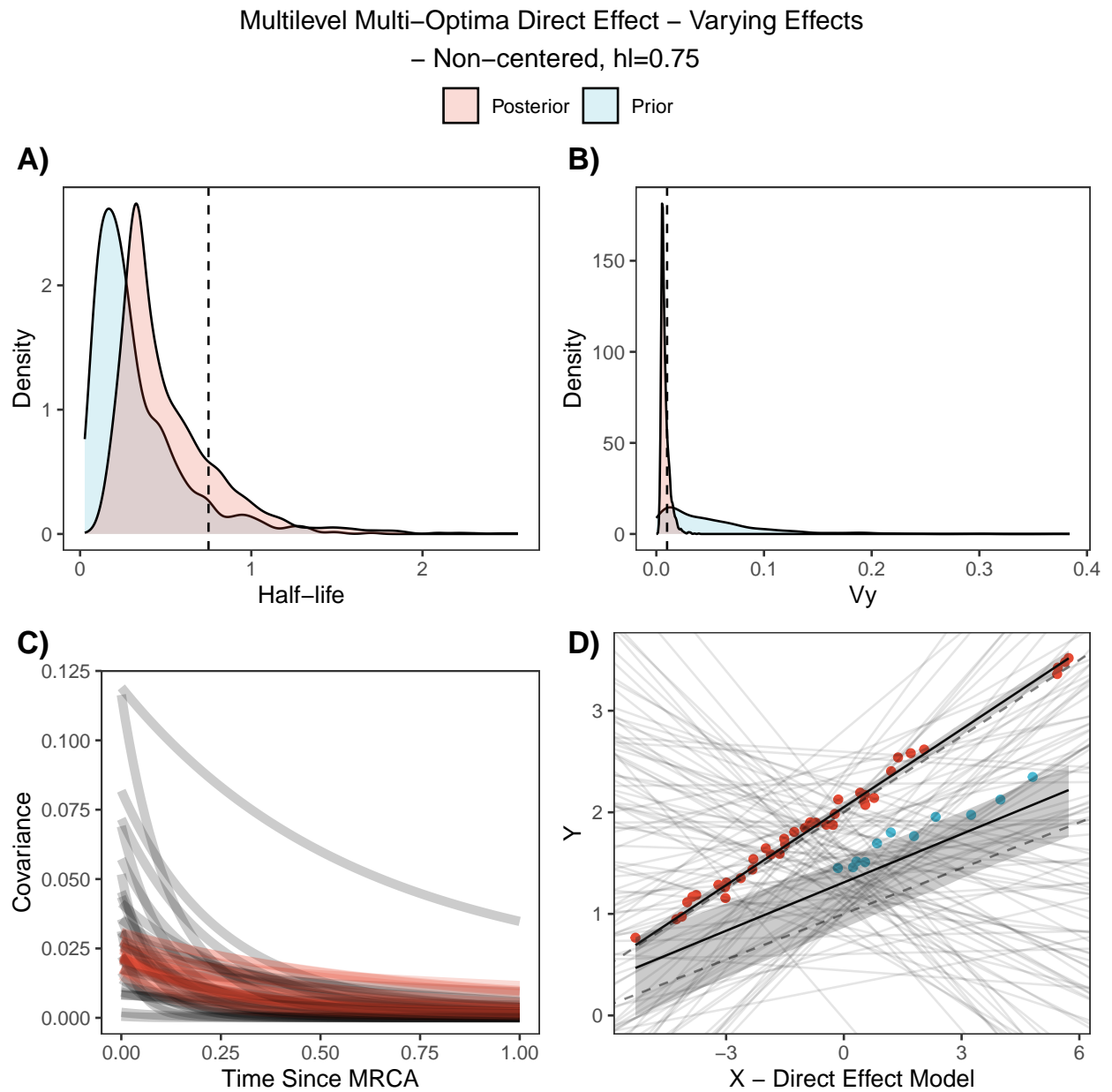


Figure 37: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8 Multi-Optima Adaptation Models

8.1 Multi-Optima Adaptation Model

8.1.1 $hl = 0.1$

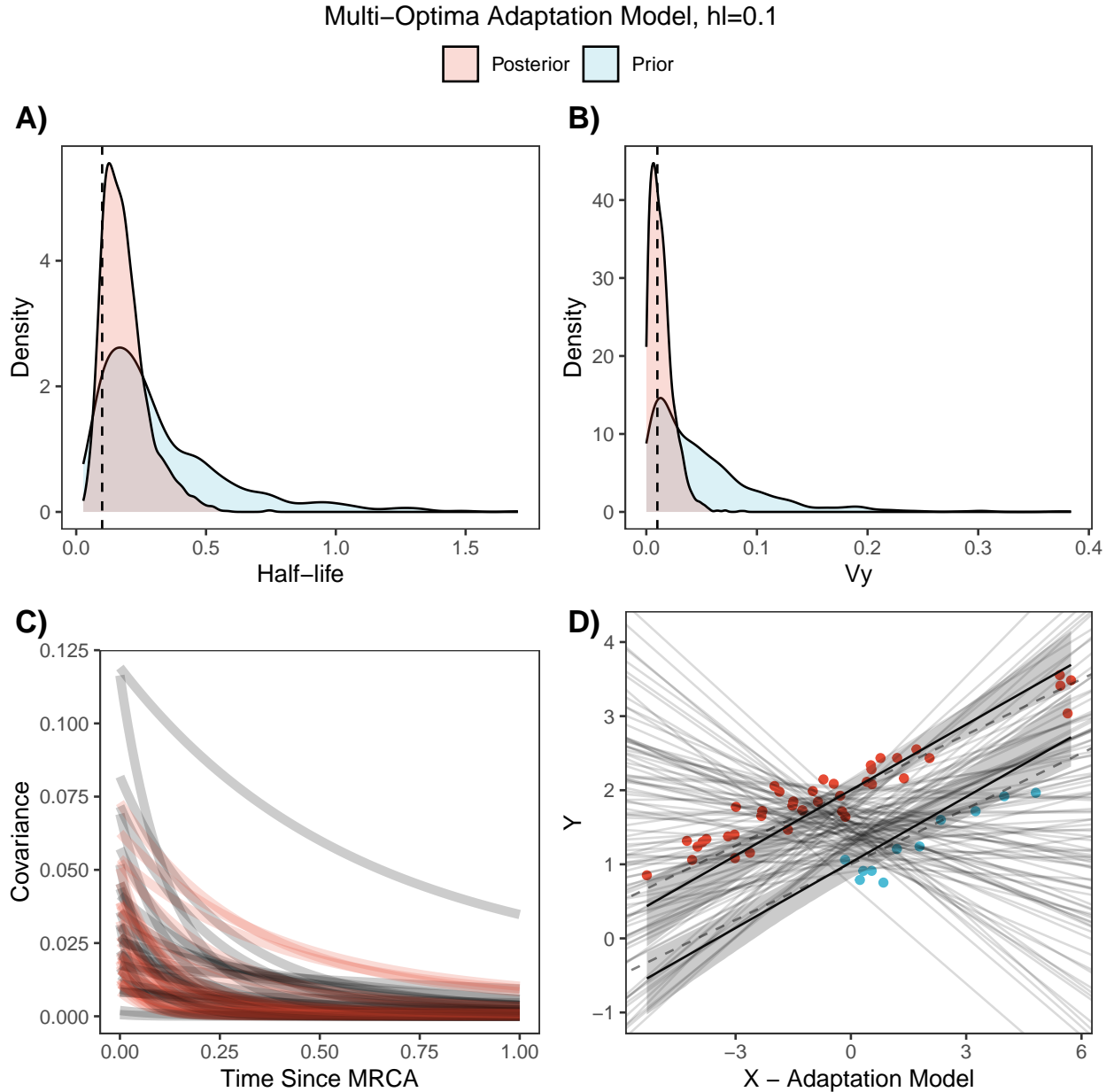


Figure 38: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.1.2 $hl = 0.25$

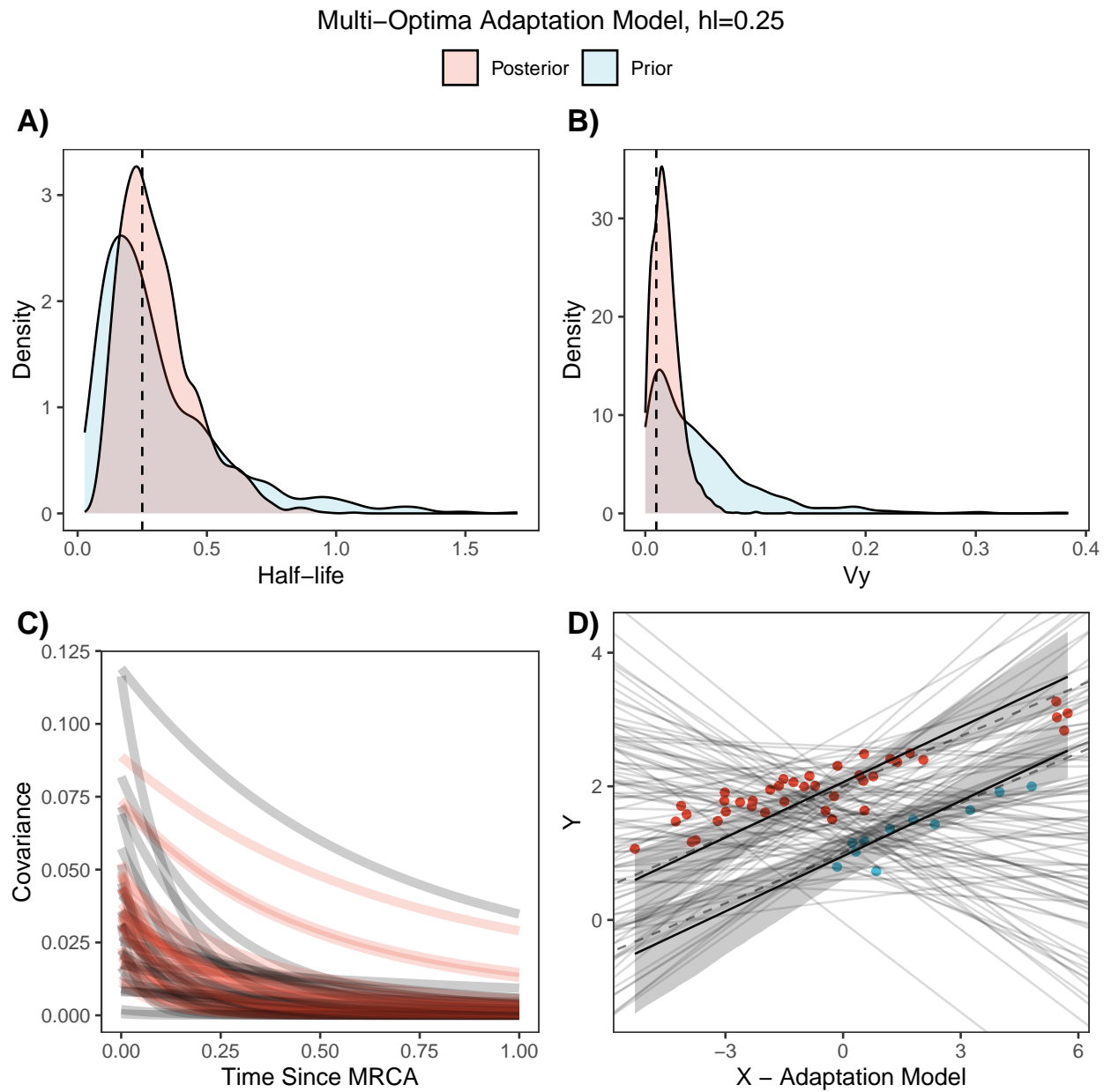


Figure 39: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.1.3 $hl = 0.75$

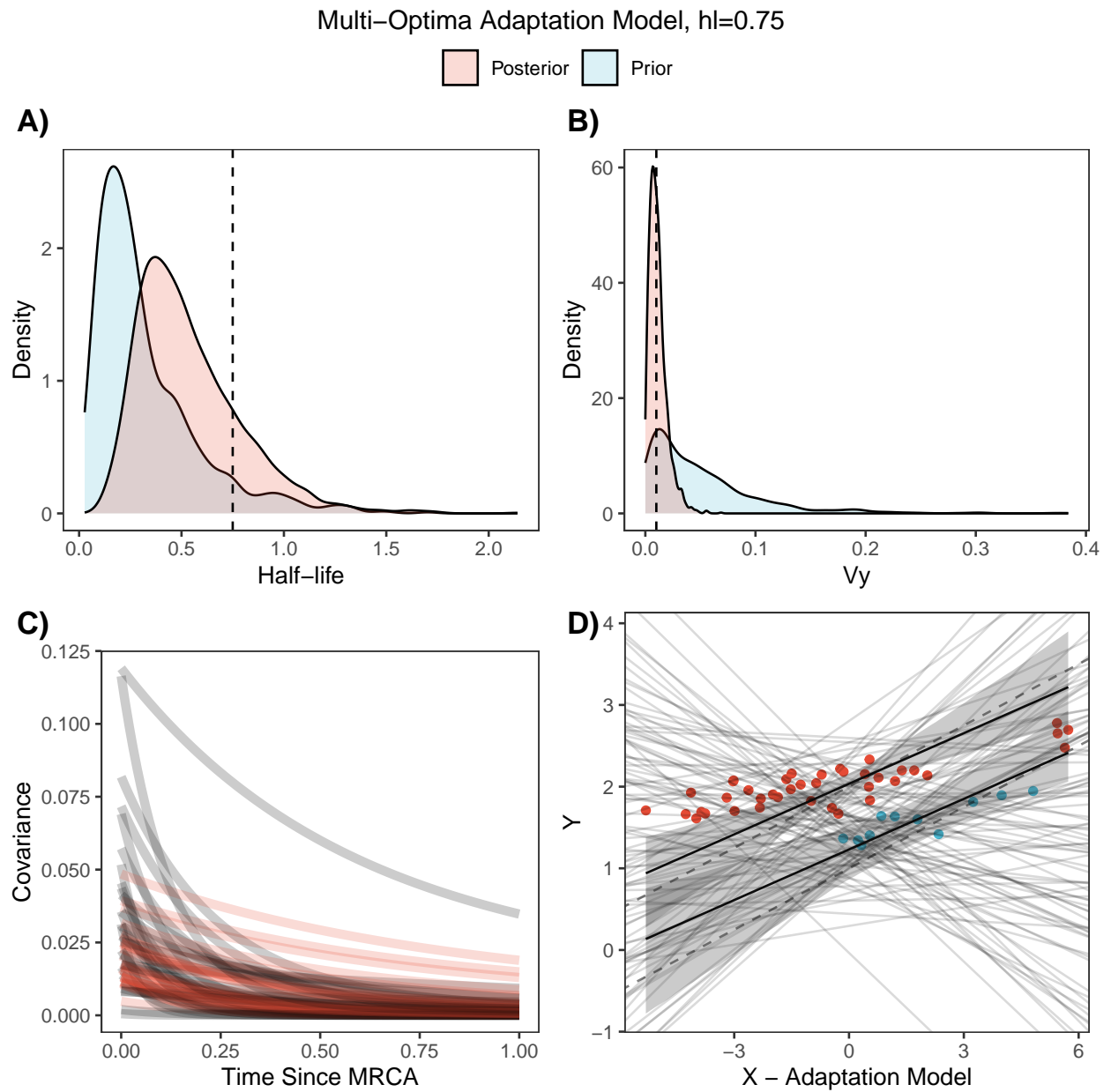


Figure 40: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.2 Multilevel Multi-Optima Adaptation Model - Varying Intercepts

8.2.1 $hl = 0.1$

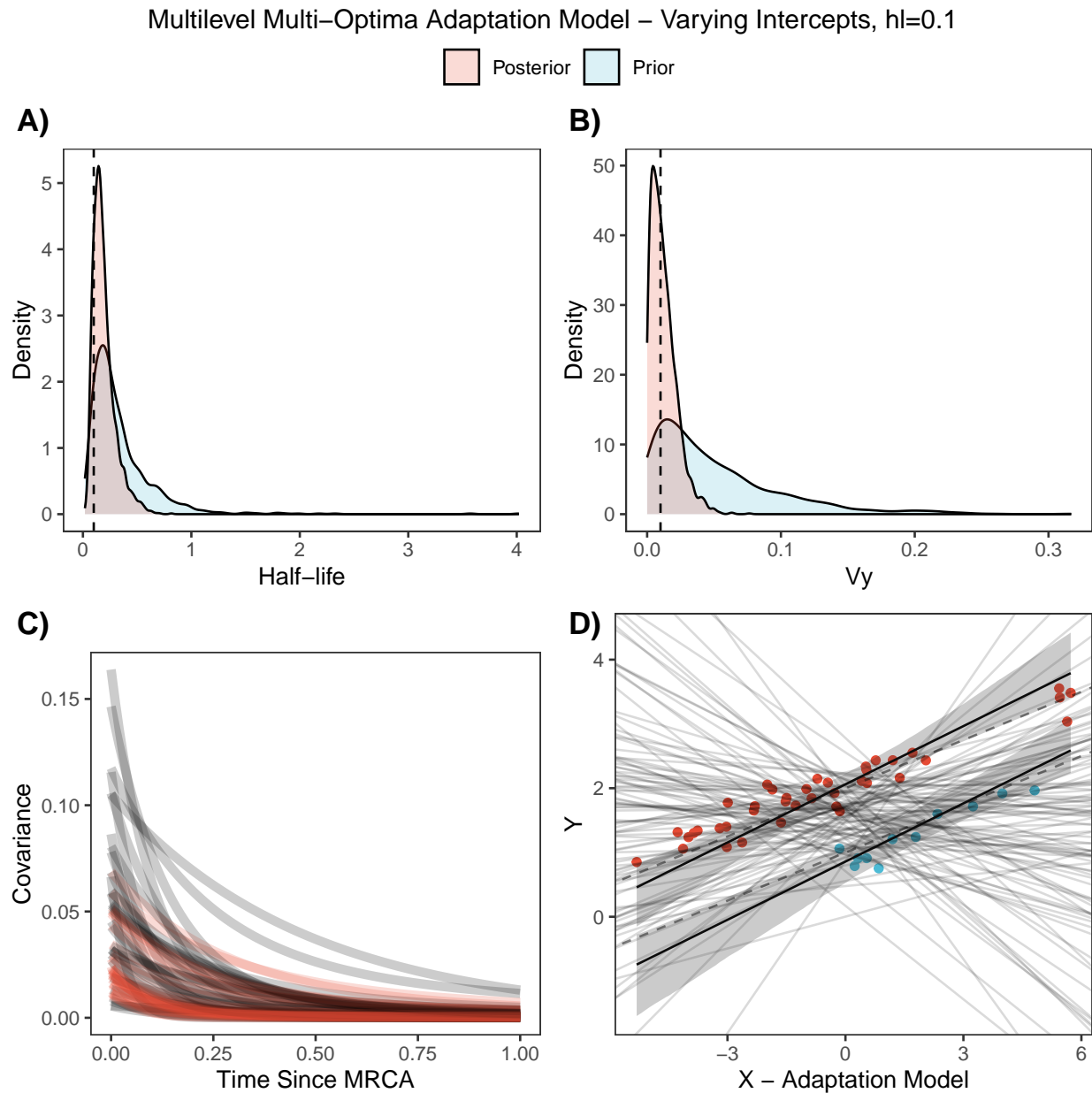


Figure 41: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.2.2 $hl = 0.25$

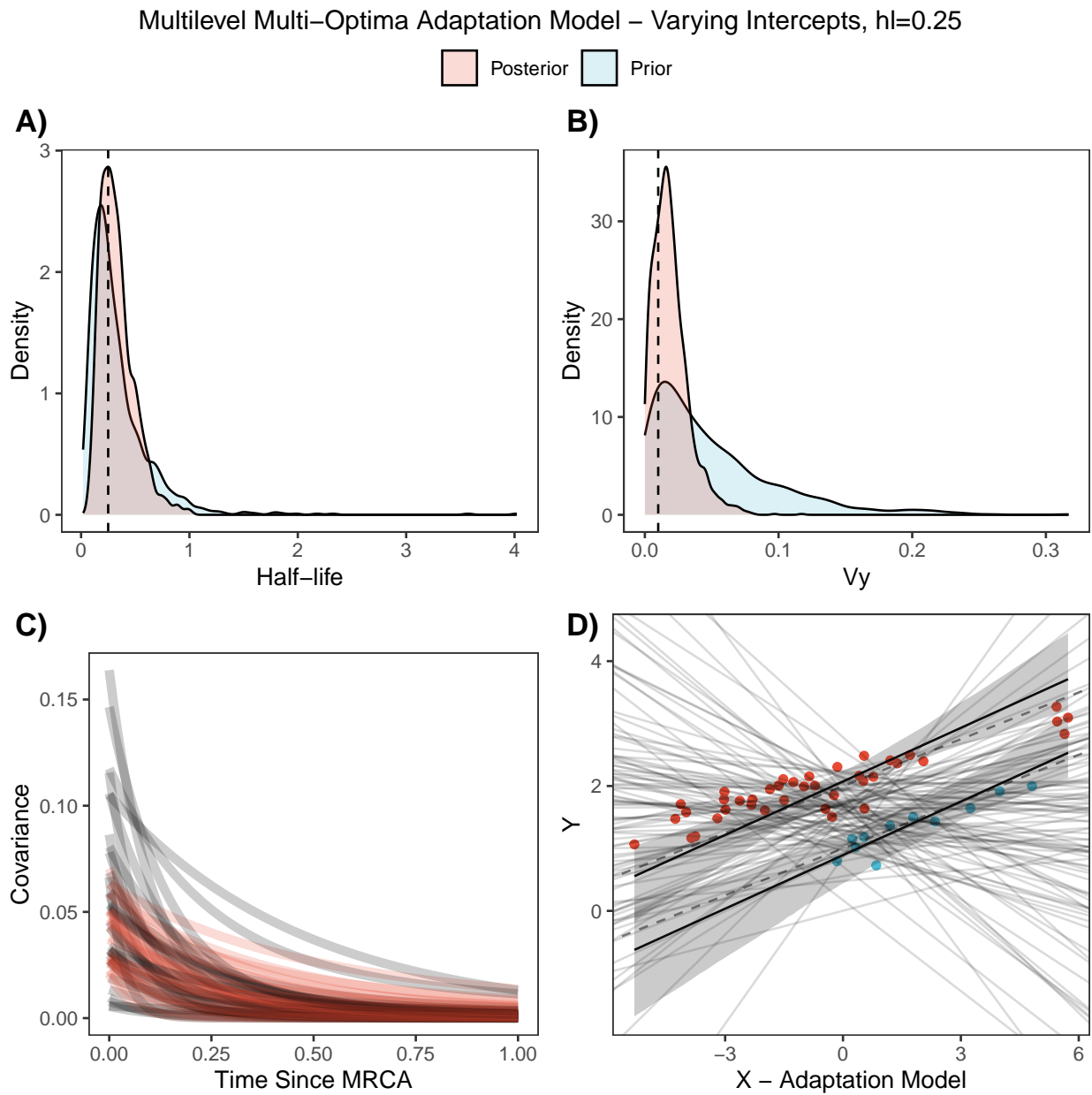


Figure 42: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.2.3 $hl = 0.75$

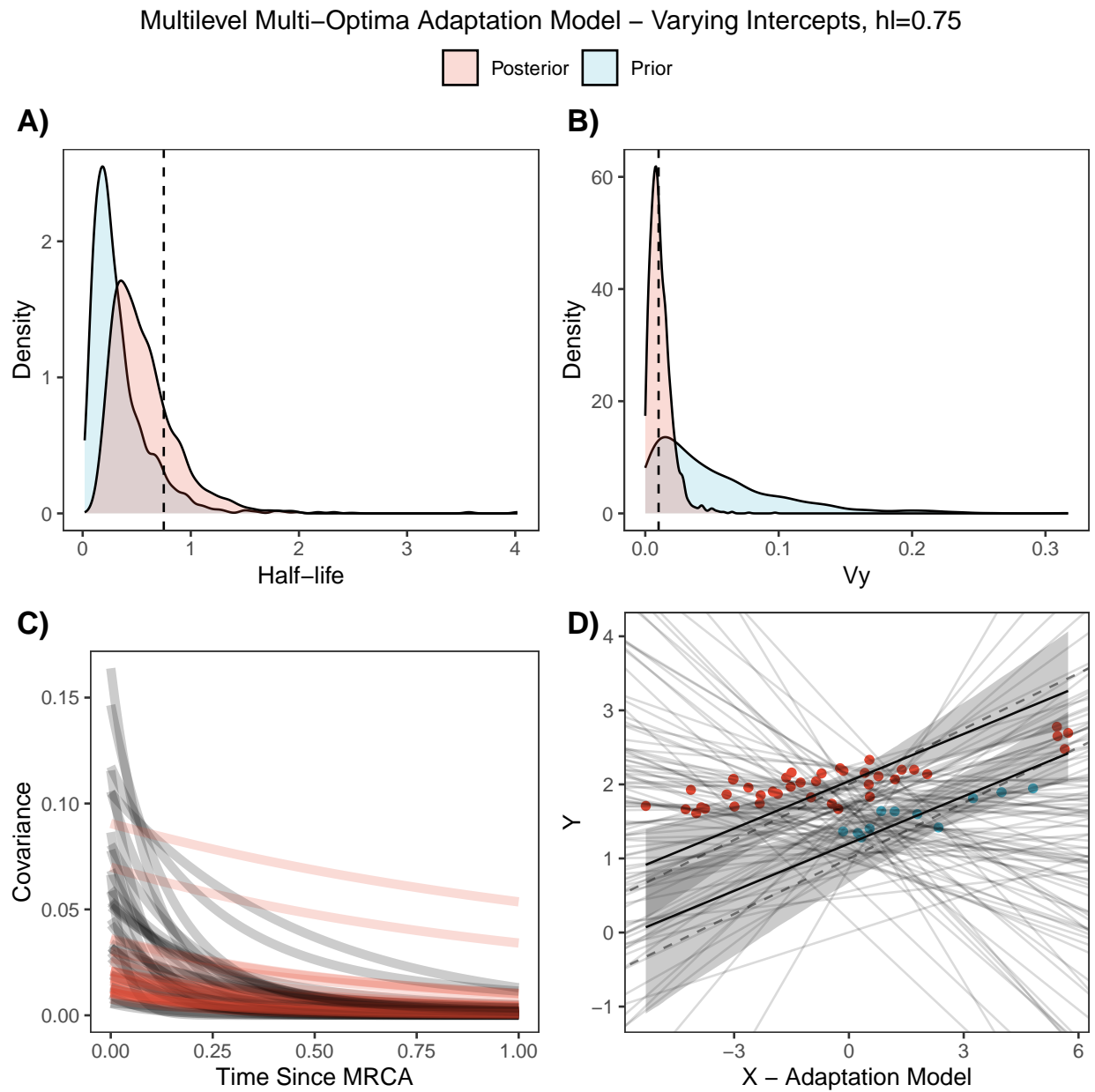


Figure 43: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.3 Multilevel Multi-Optima Adaptation Model - Varying Intercepts - Non-centered

8.3.1 $hl = 0.1$

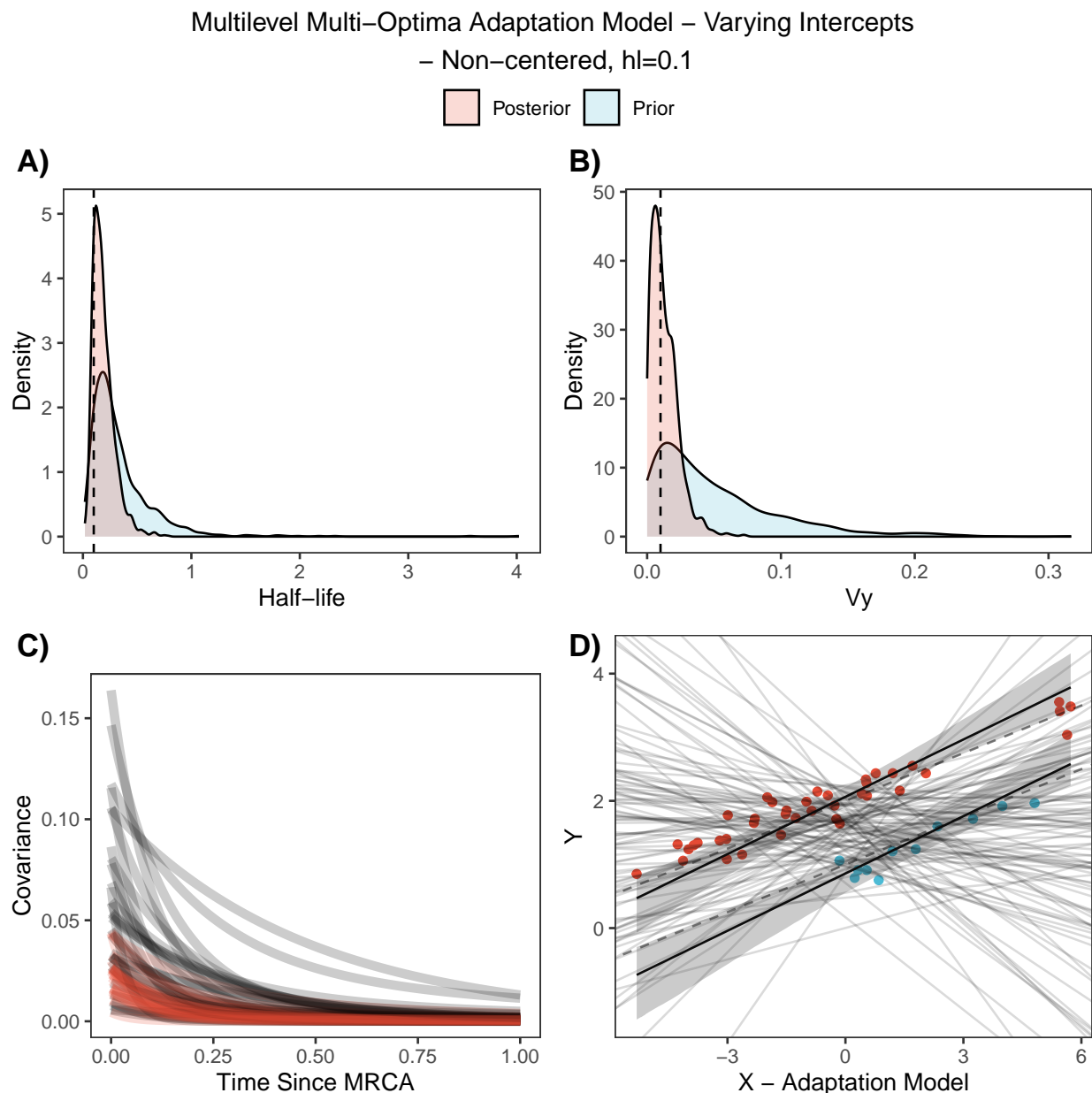


Figure 44: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.3.2 $hl = 0.25$

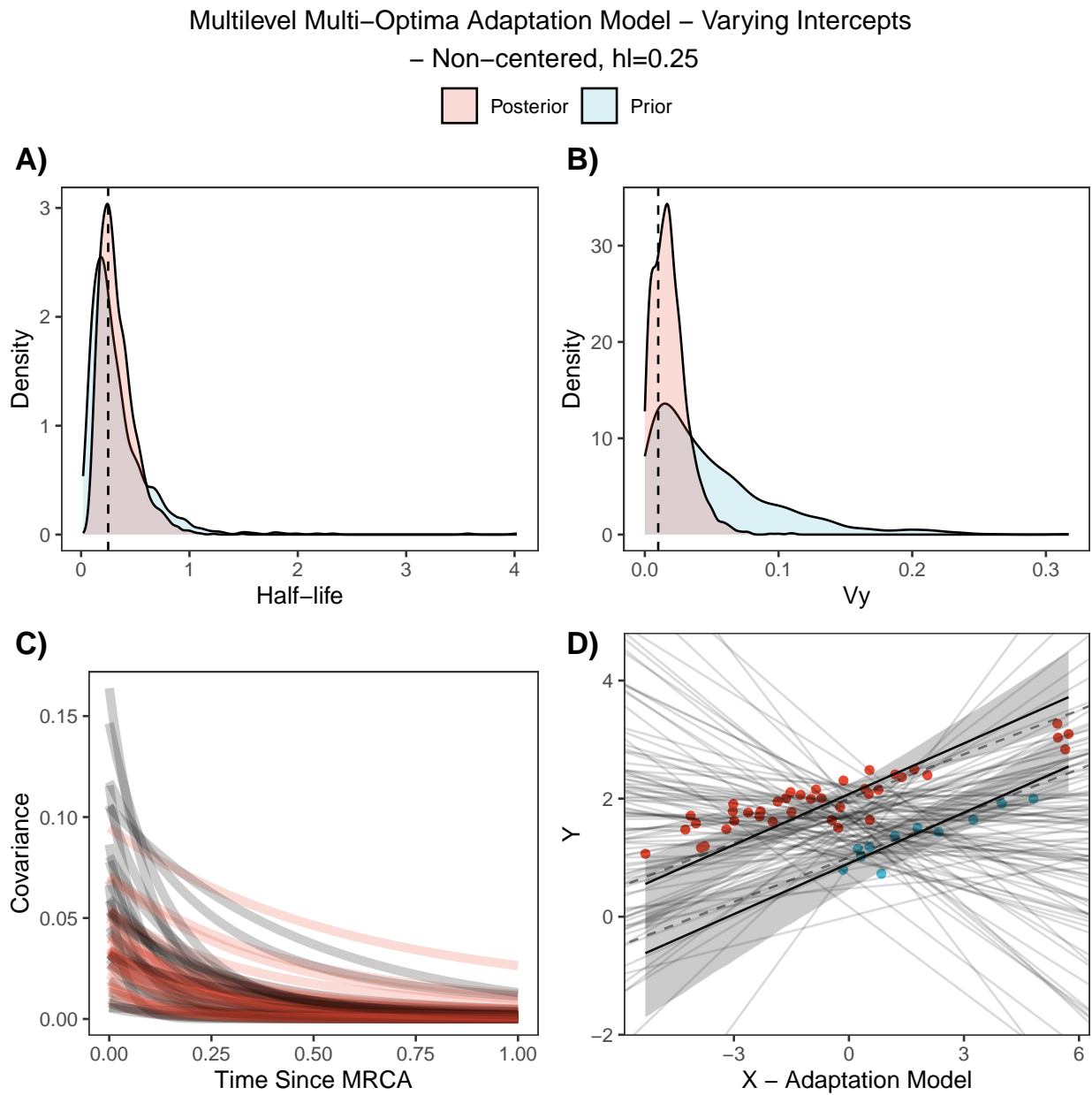


Figure 45: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.3.3 $hl = 0.75$

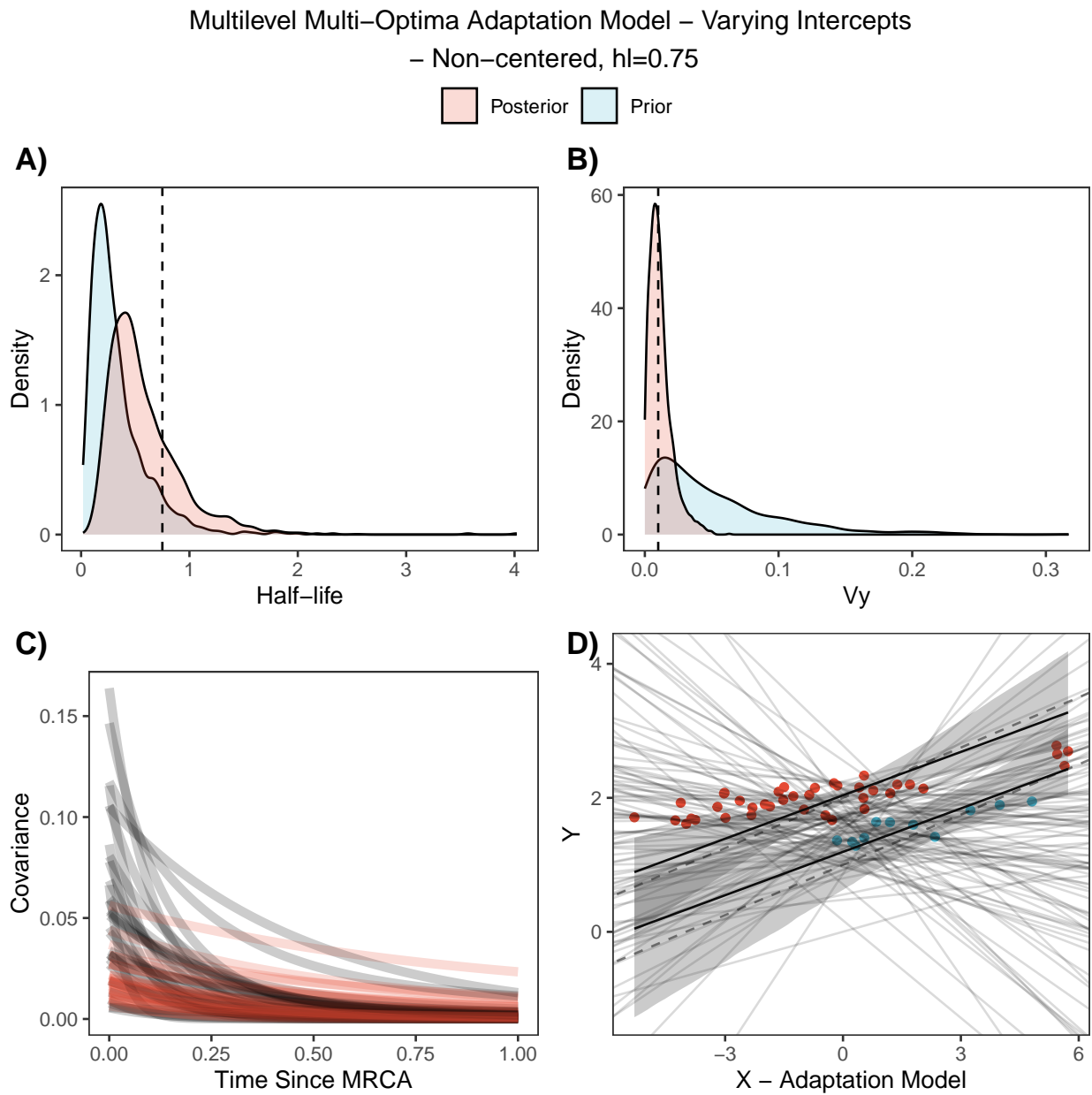


Figure 46: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.4 Multi-Optima Adaptation Model - Varying Effects

8.4.1 $hl = 0.1$

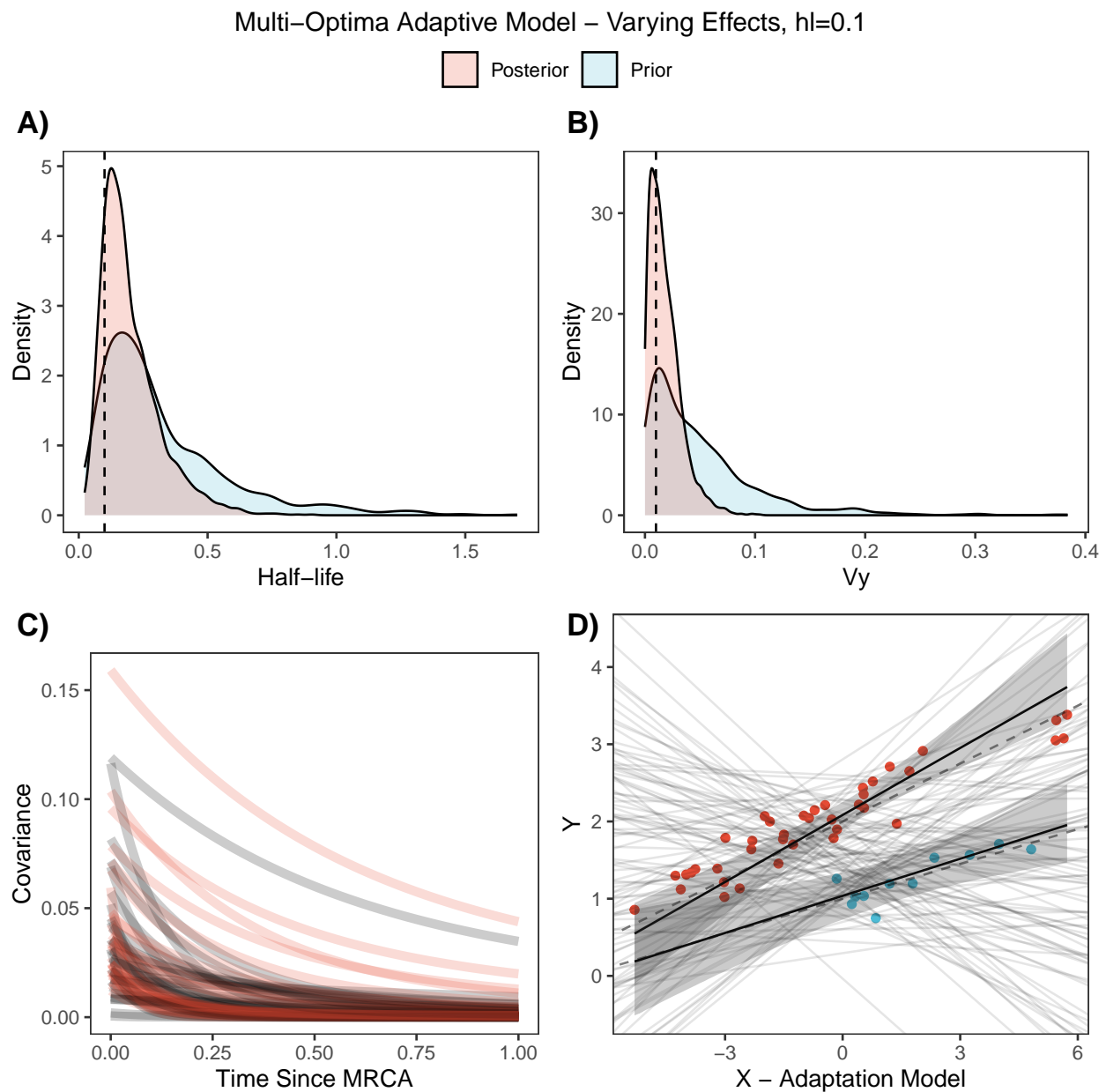


Figure 47: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.4.2 $hl = 0.25$

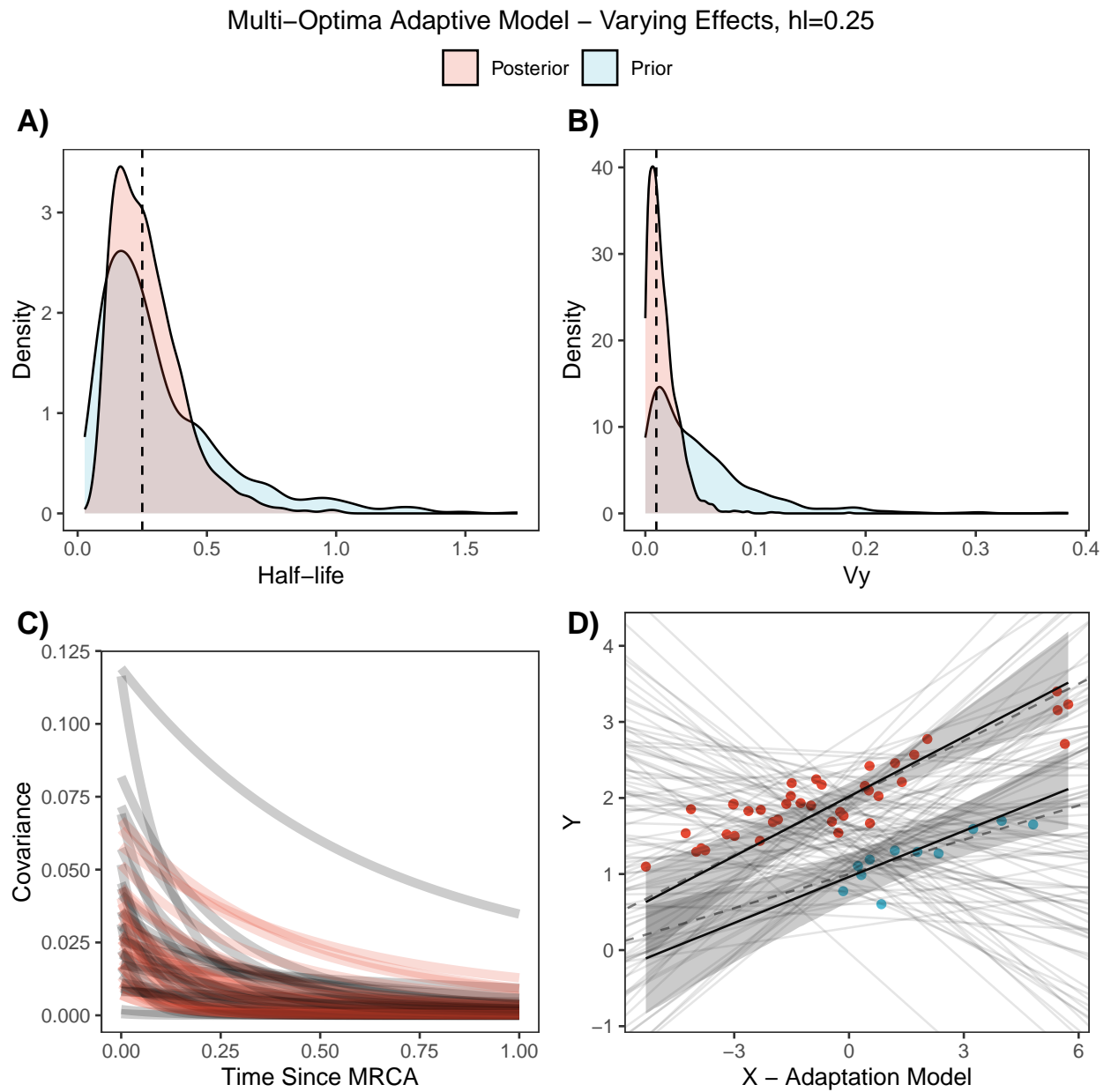


Figure 48: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.4.3 $hl = 0.75$

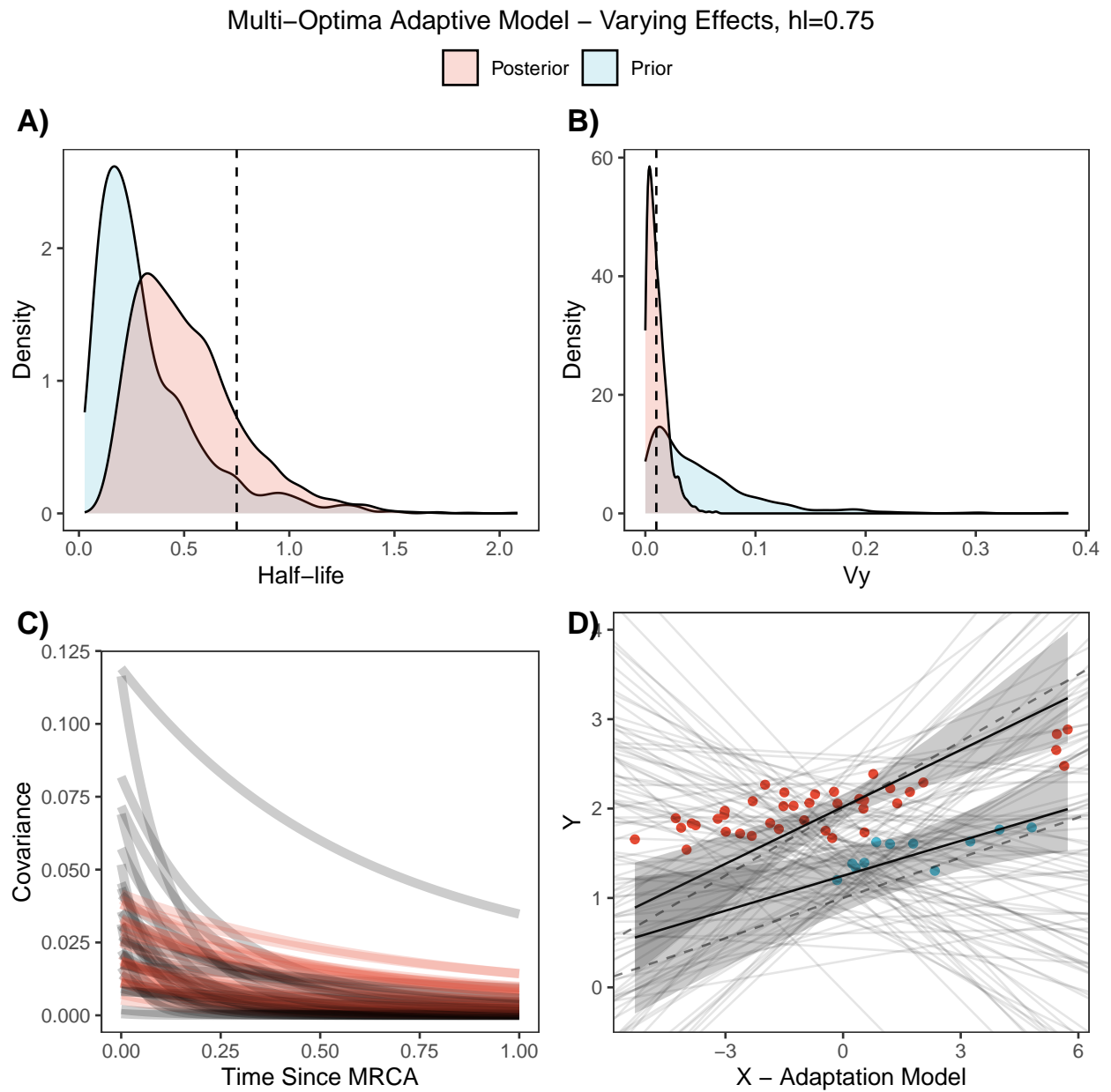


Figure 49: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.5 Multilevel Multi-Optima Adaptation Model - Varying Effects

8.5.1 $hl = 0.1$

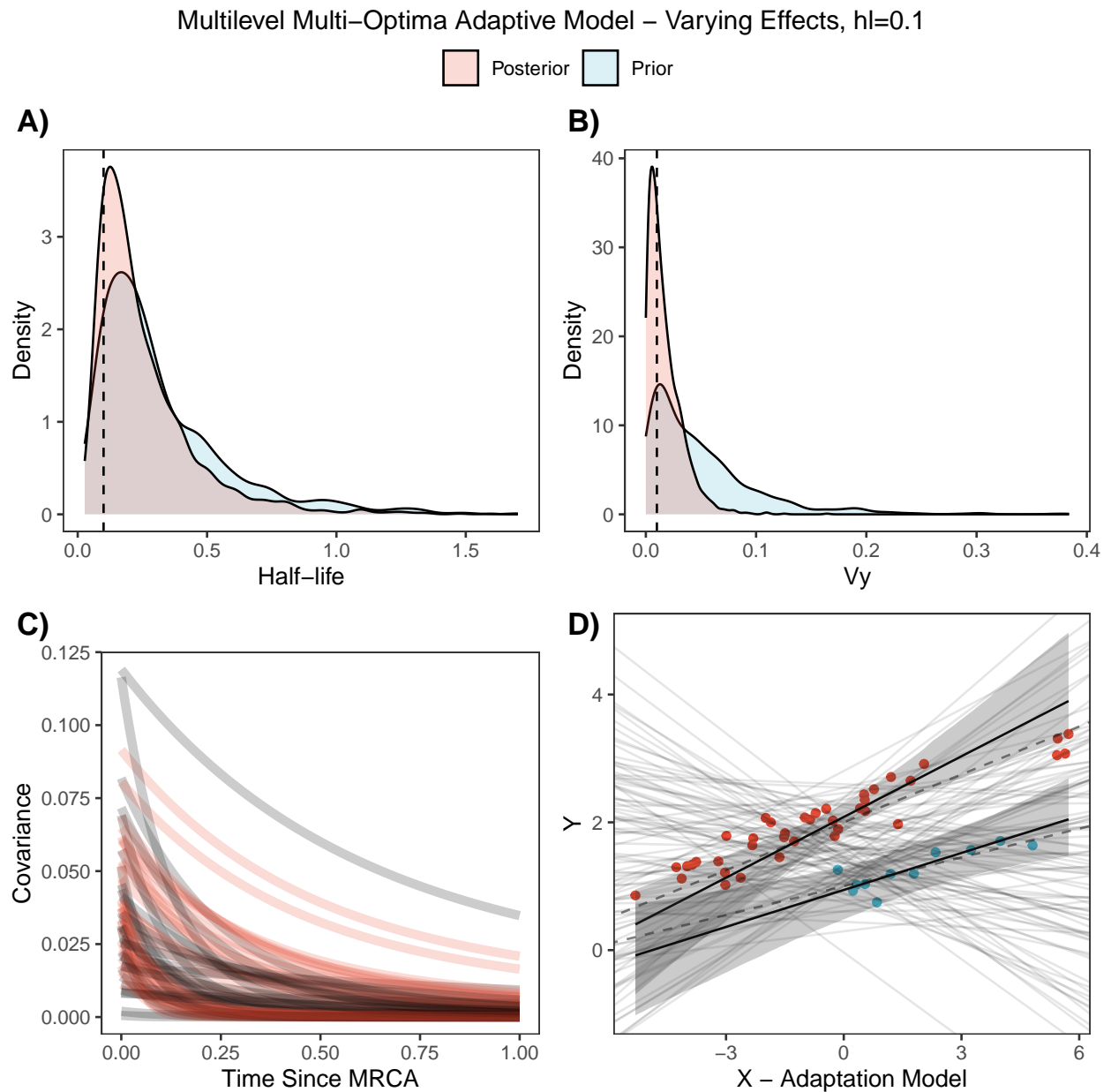


Figure 50: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.5.2 $hl = 0.25$

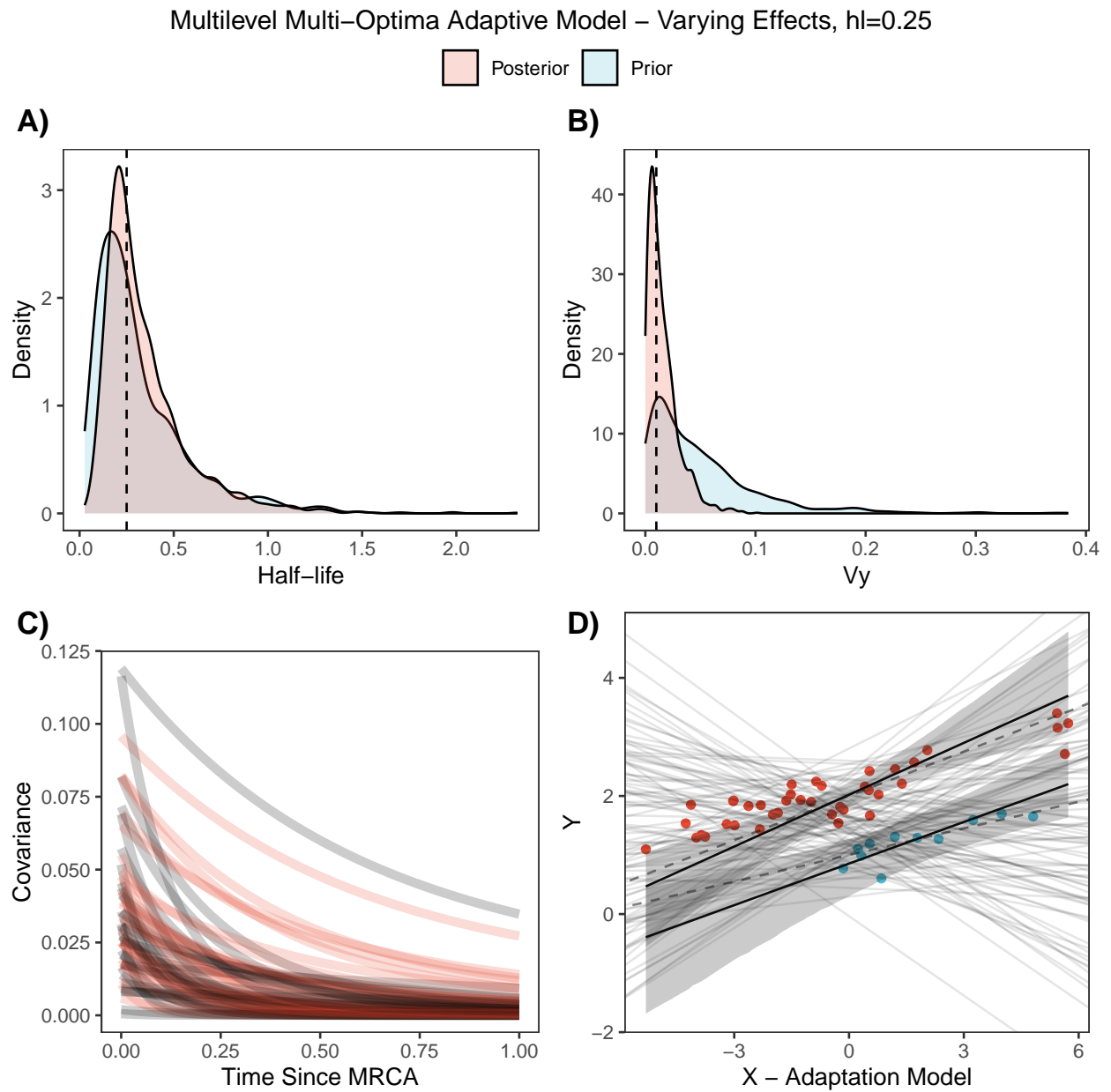


Figure 51: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.5.3 $hl = 0.75$

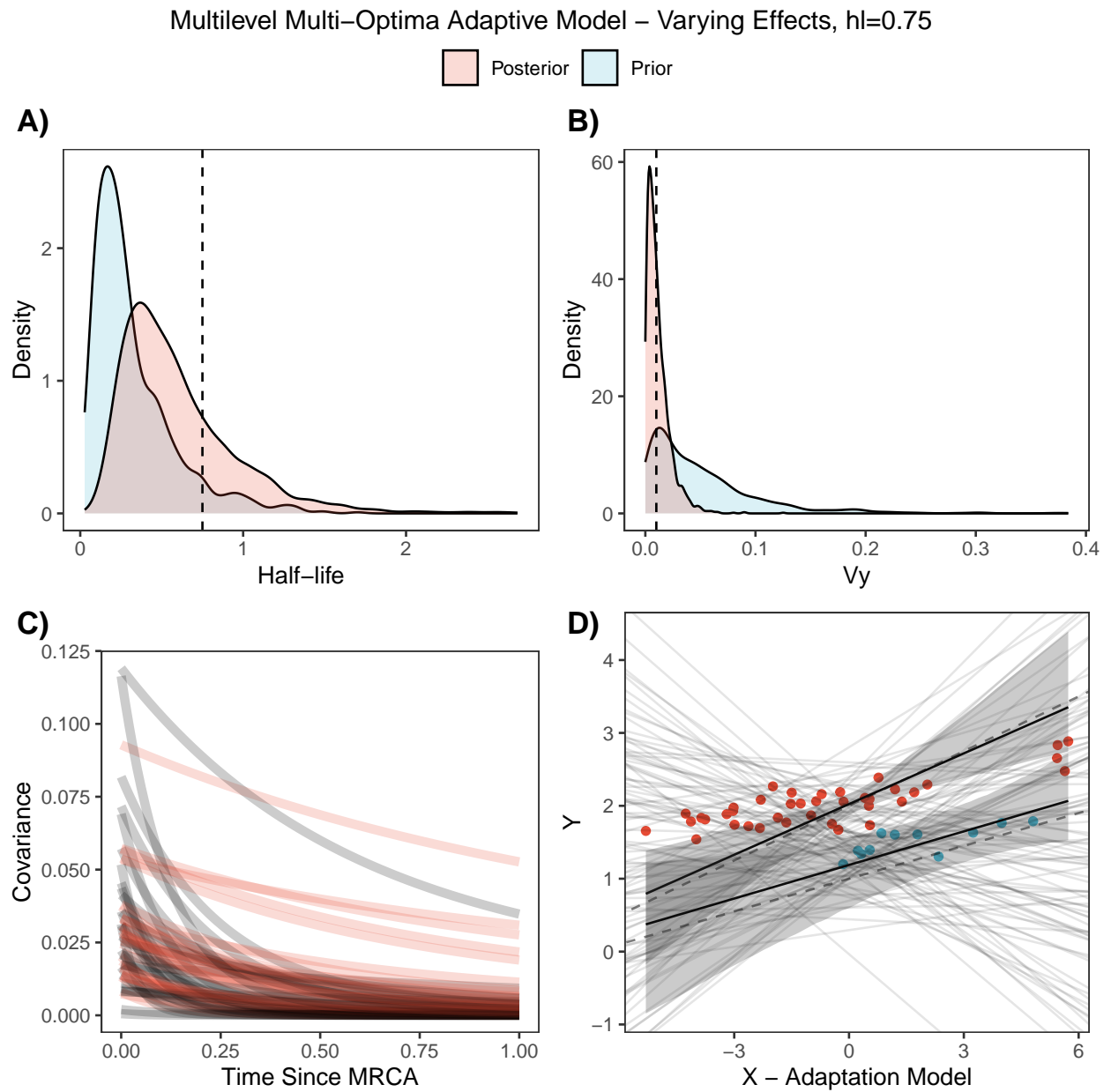


Figure 52: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.6 Multilevel Multi-Optima Adaptation Model - Varying Effects - Non-centered

8.6.1 $hl = 0.1$

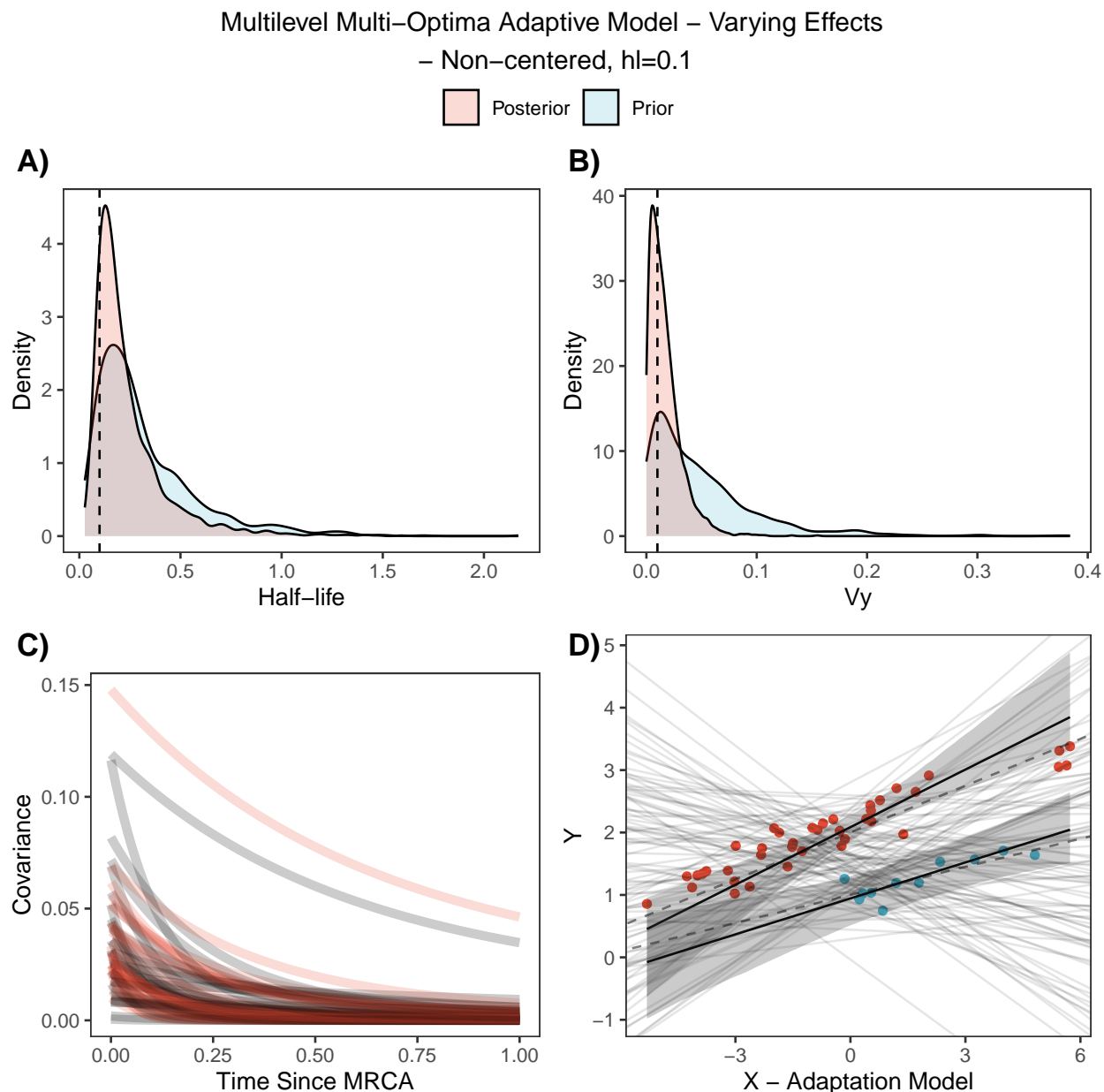


Figure 53: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

8.6.2 $hl = 0.25$

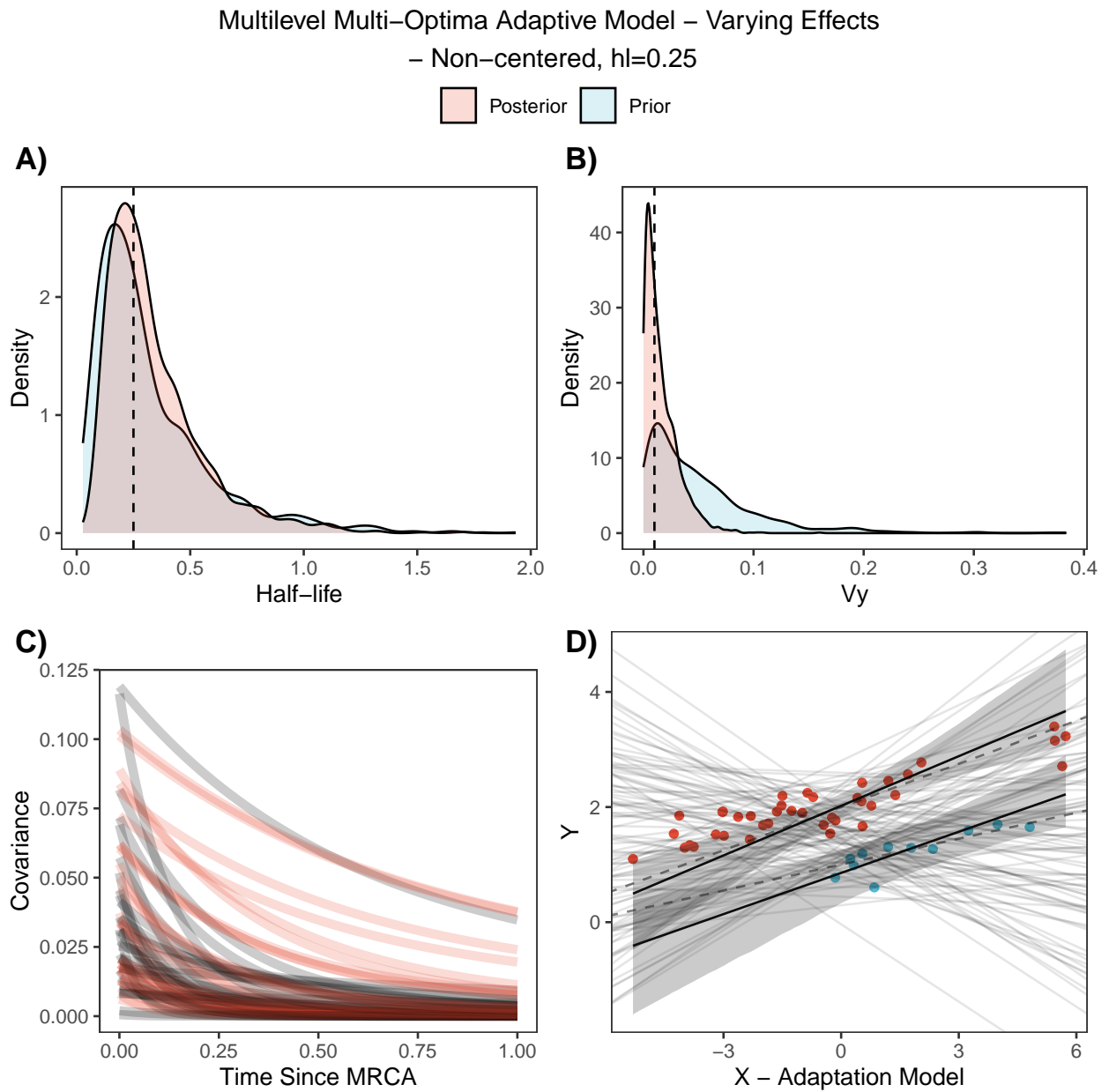


Figure 54: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A, B, D) are true values of the parameter.

8.6.3 $hl = 0.75$

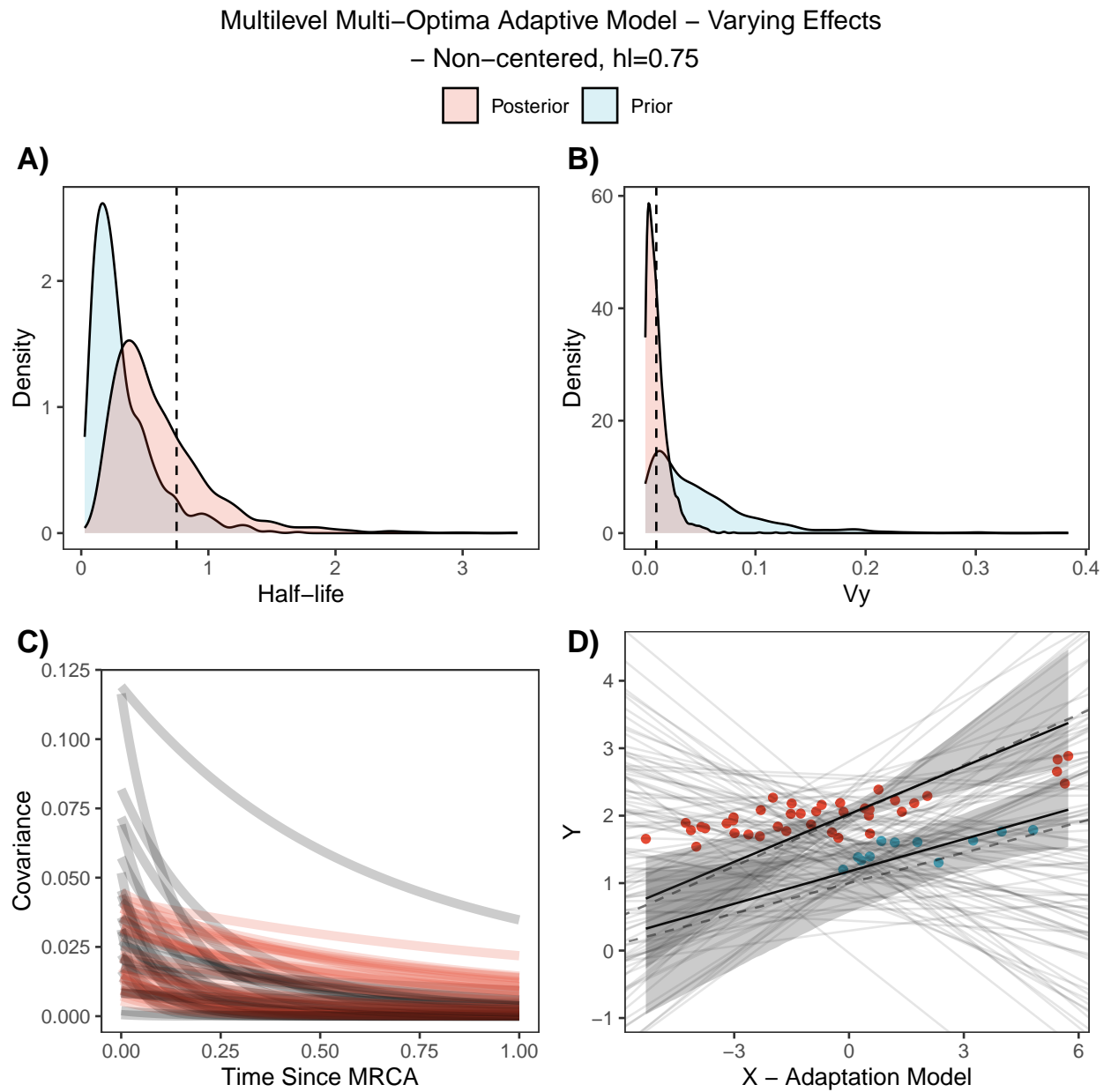


Figure 55: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with species values in black and priors in light grey. Dotted lines overlaying estimated optima are true values of the parameter. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(0, 1)$. Dotted lines in A,B,D) are true values of the parameter.

9 Multi-Optima Direct Effect Adaptation Models

9.1 Multi-Optima Direct Effect Adaptation Model

9.1.1 $hl = 0.1$

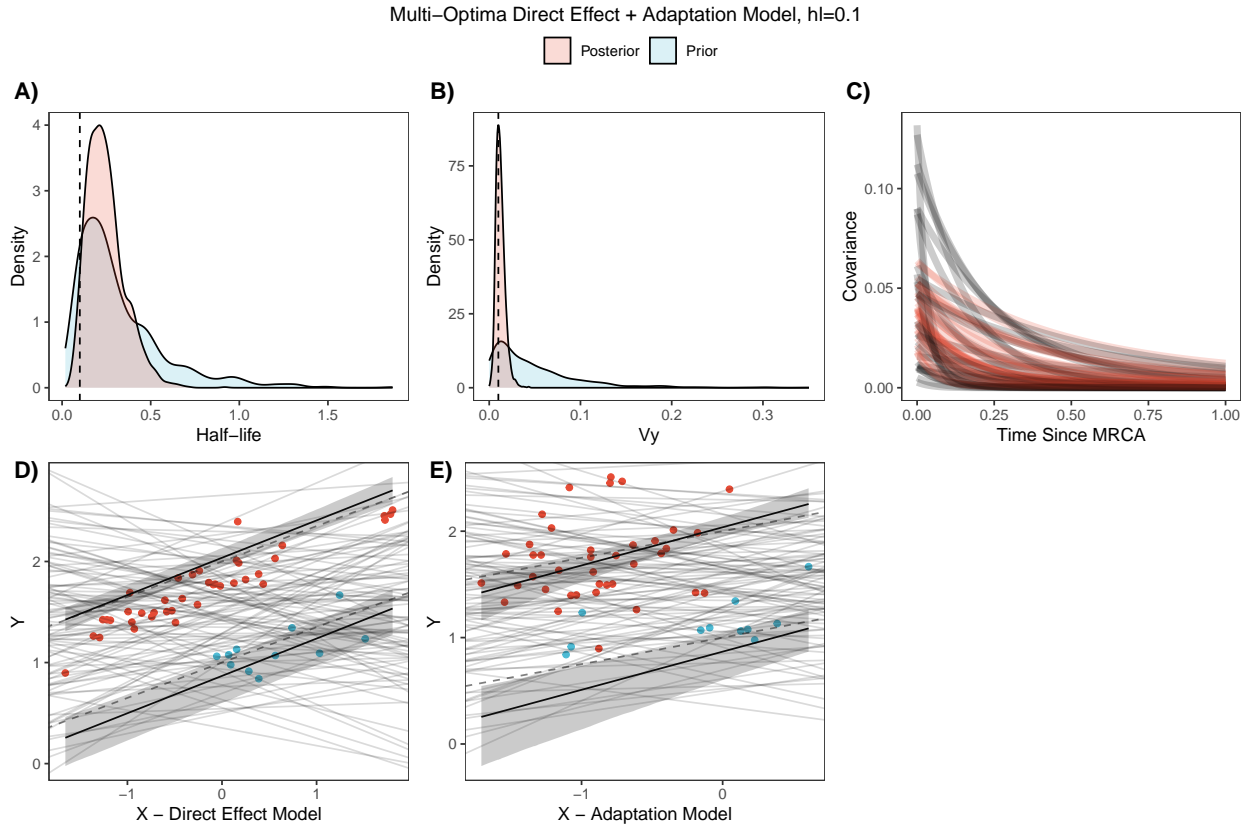


Figure 56: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.1.2 $hl = 0.25$

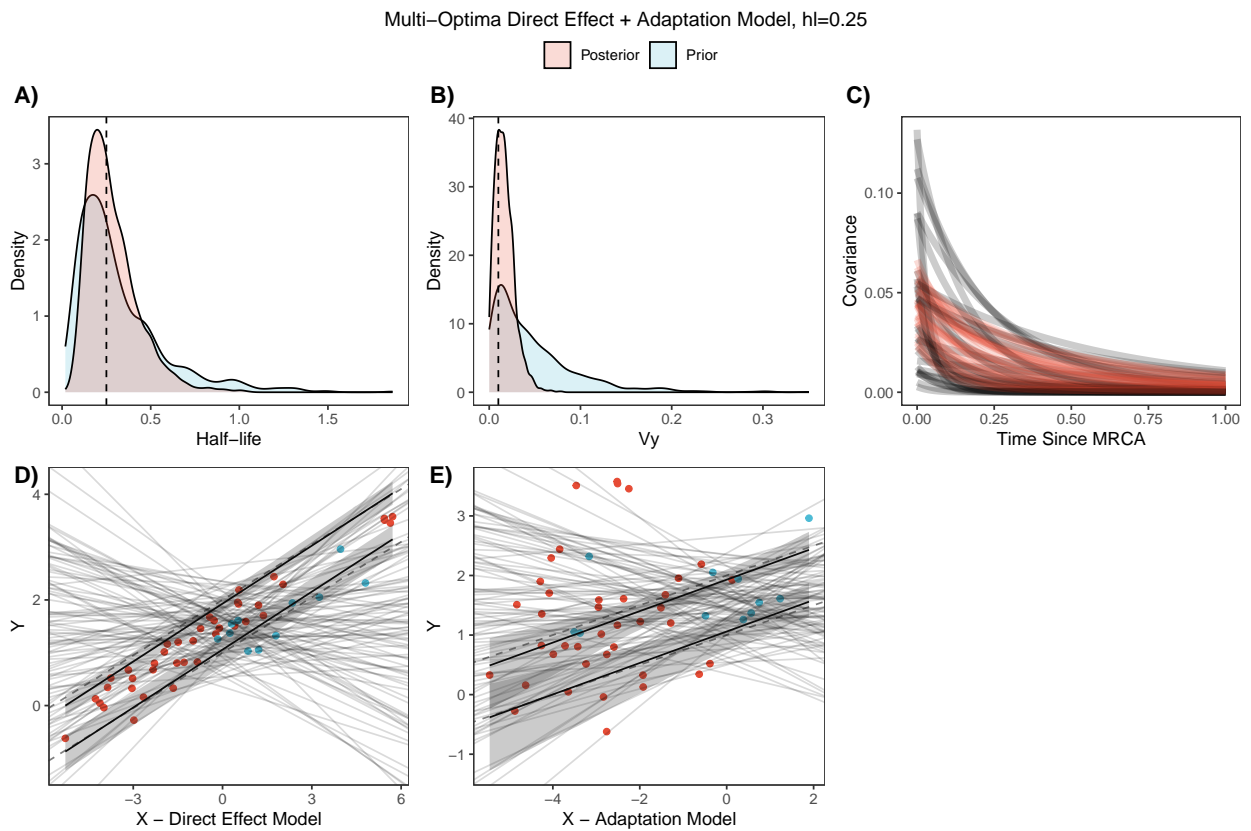


Figure 57: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.1.3 $hl = 0.75$

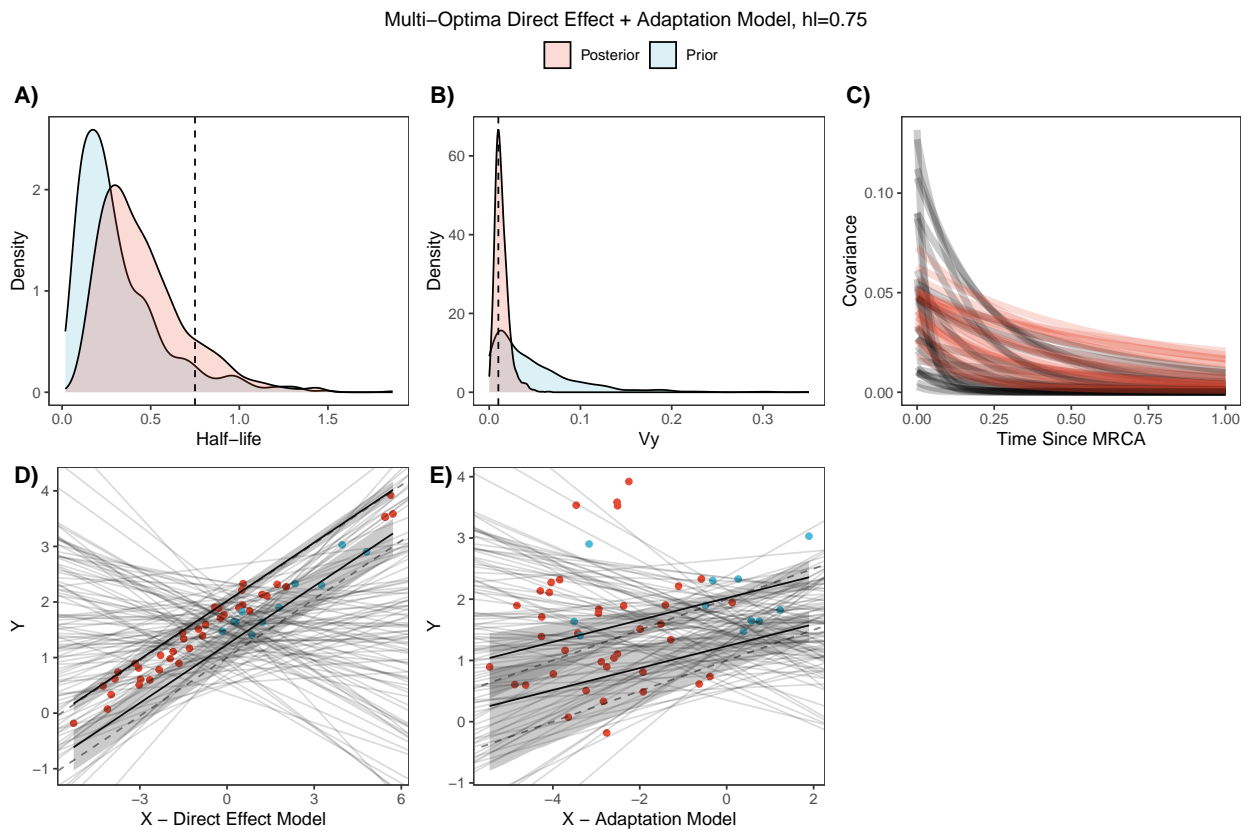


Figure 58: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.2 Multilevel Multi-Optima Direct Effect Adaptation Model - Varying Intercepts

9.2.1 $hl = 0.1$

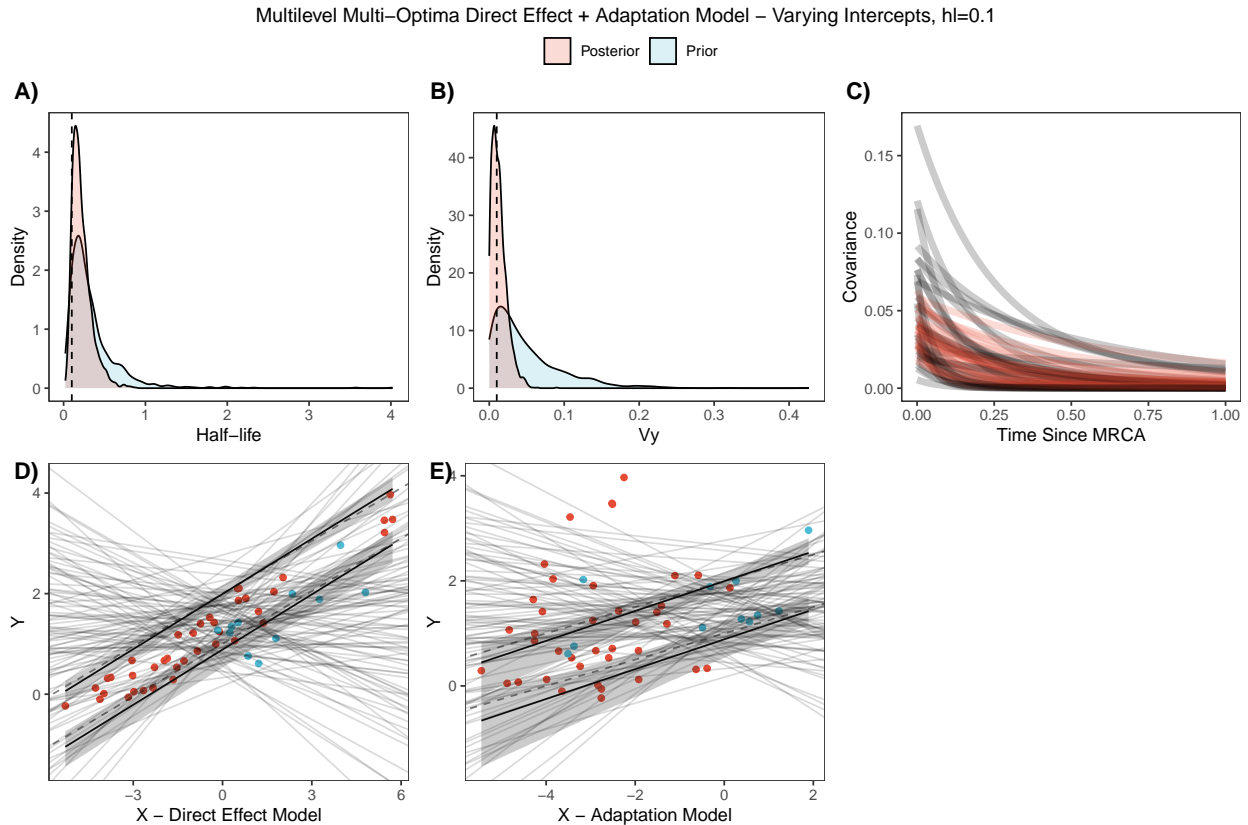


Figure 59: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.2.2 $hl = 0.25$

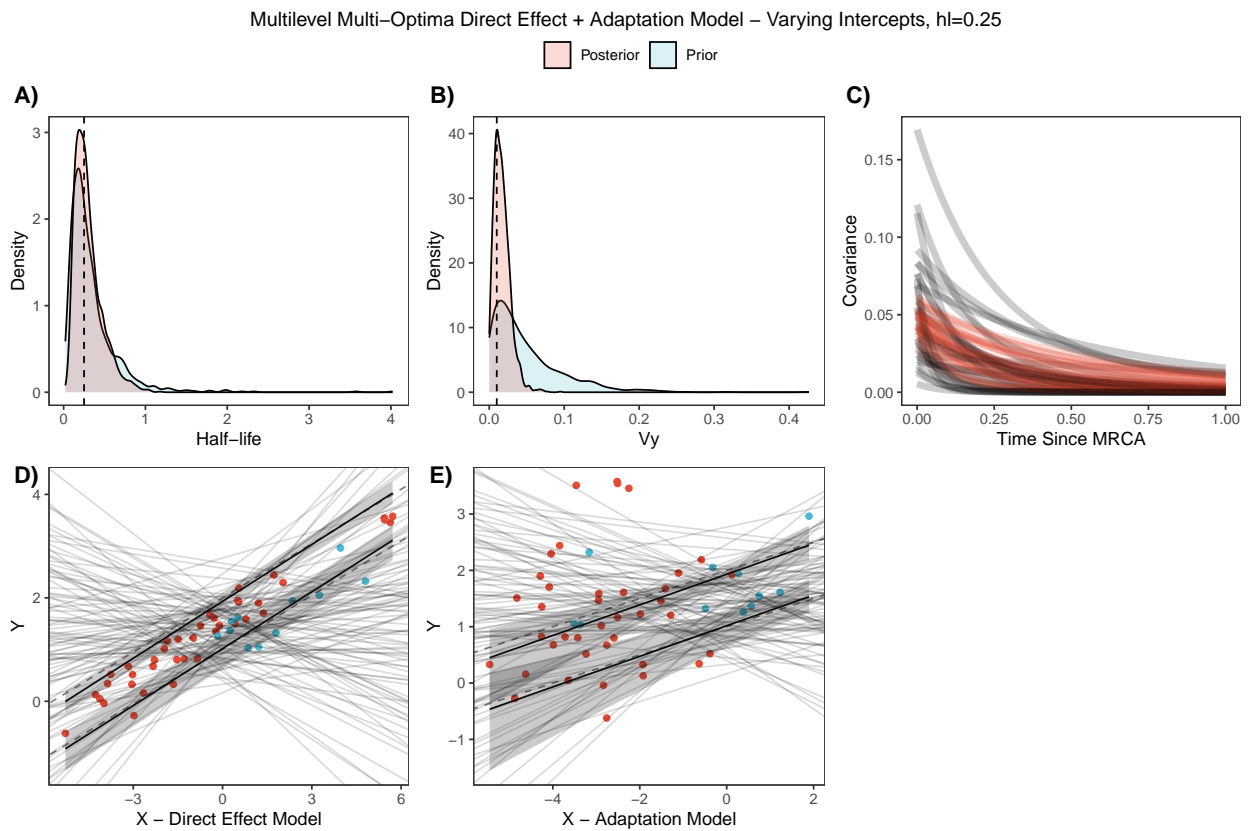


Figure 60: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.2.3 $hl = 0.75$

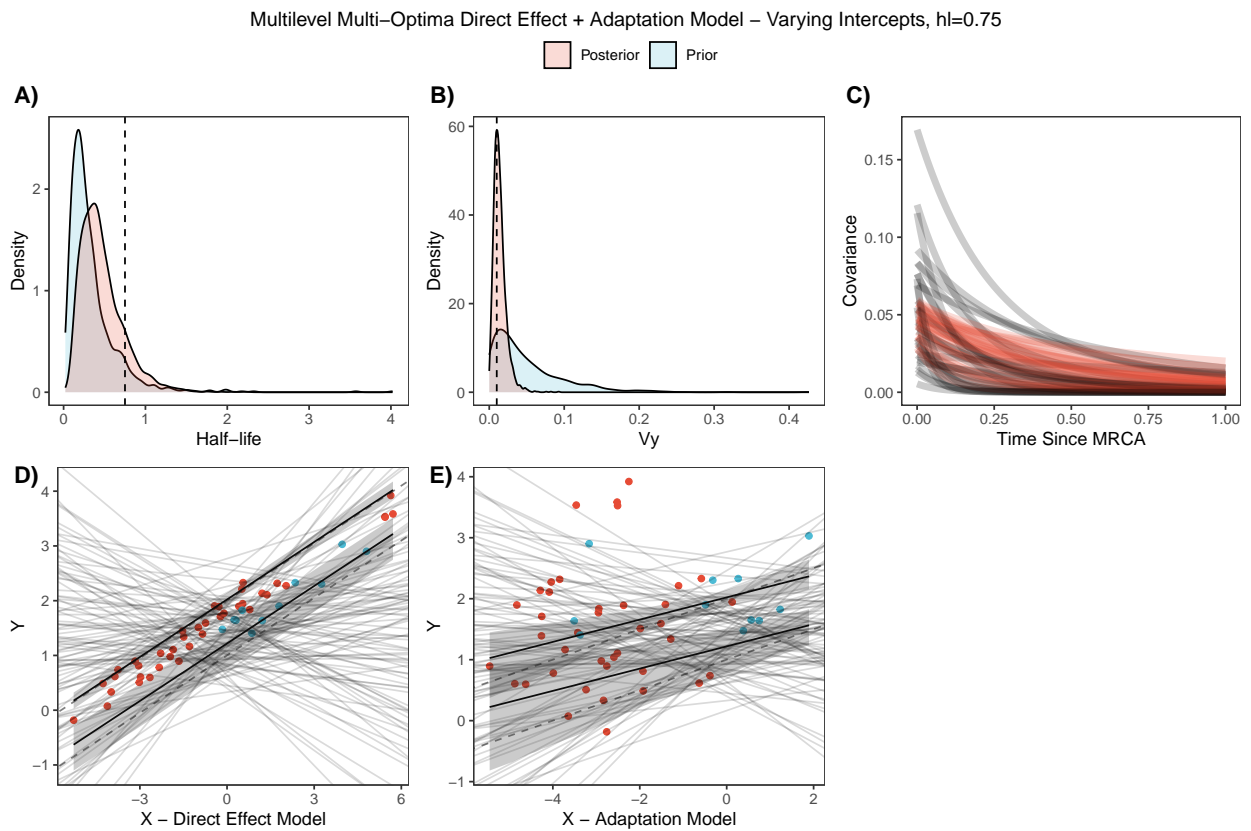


Figure 61: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.3 Multilevel Multi-Optima Direct Effect Adaptation Model - Varying Intercepts - Non-centered

9.3.1 $hl = 0.1$

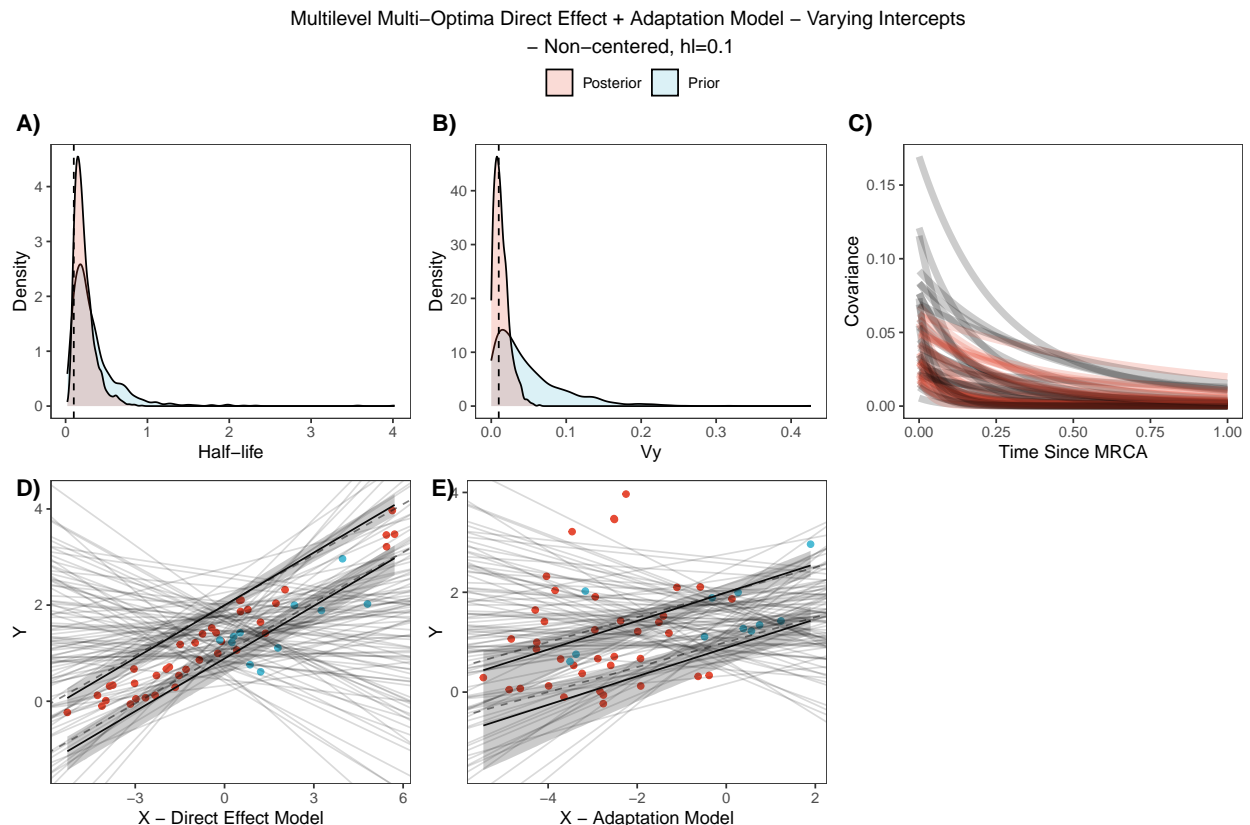


Figure 62: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.3.2 $hl = 0.25$

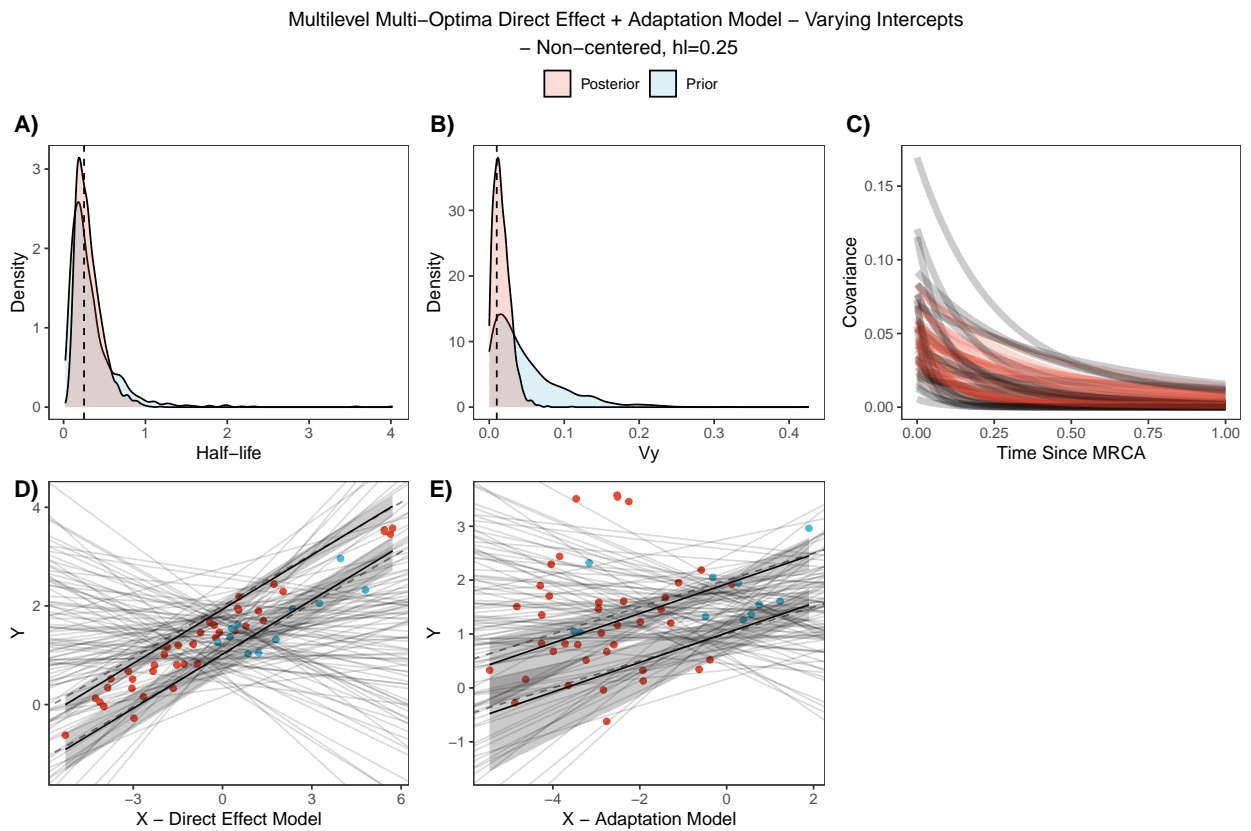
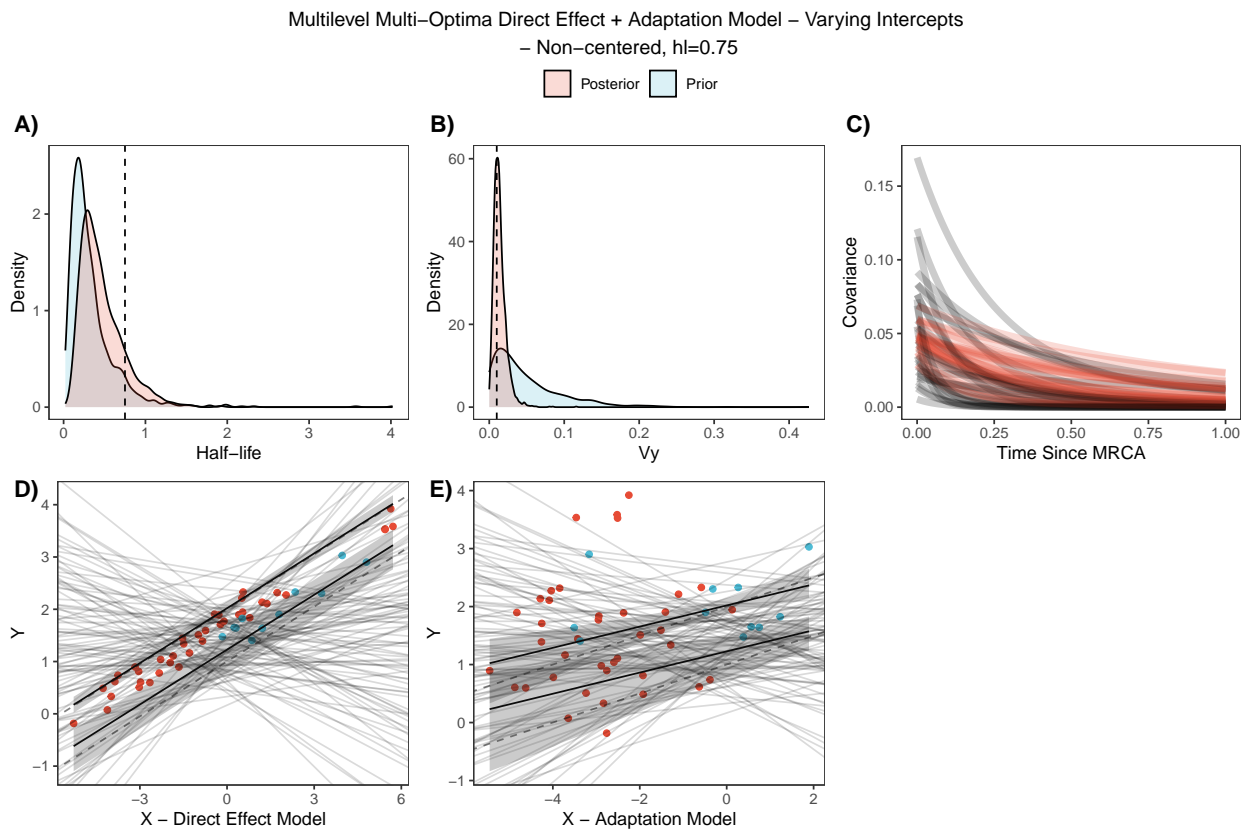


Figure 63: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.3.3 $hl = 0.75$



9.4 Multi-Optima Direct Effect Adaptation Model - Varying Effects

9.4.1 $hl = 0.1$

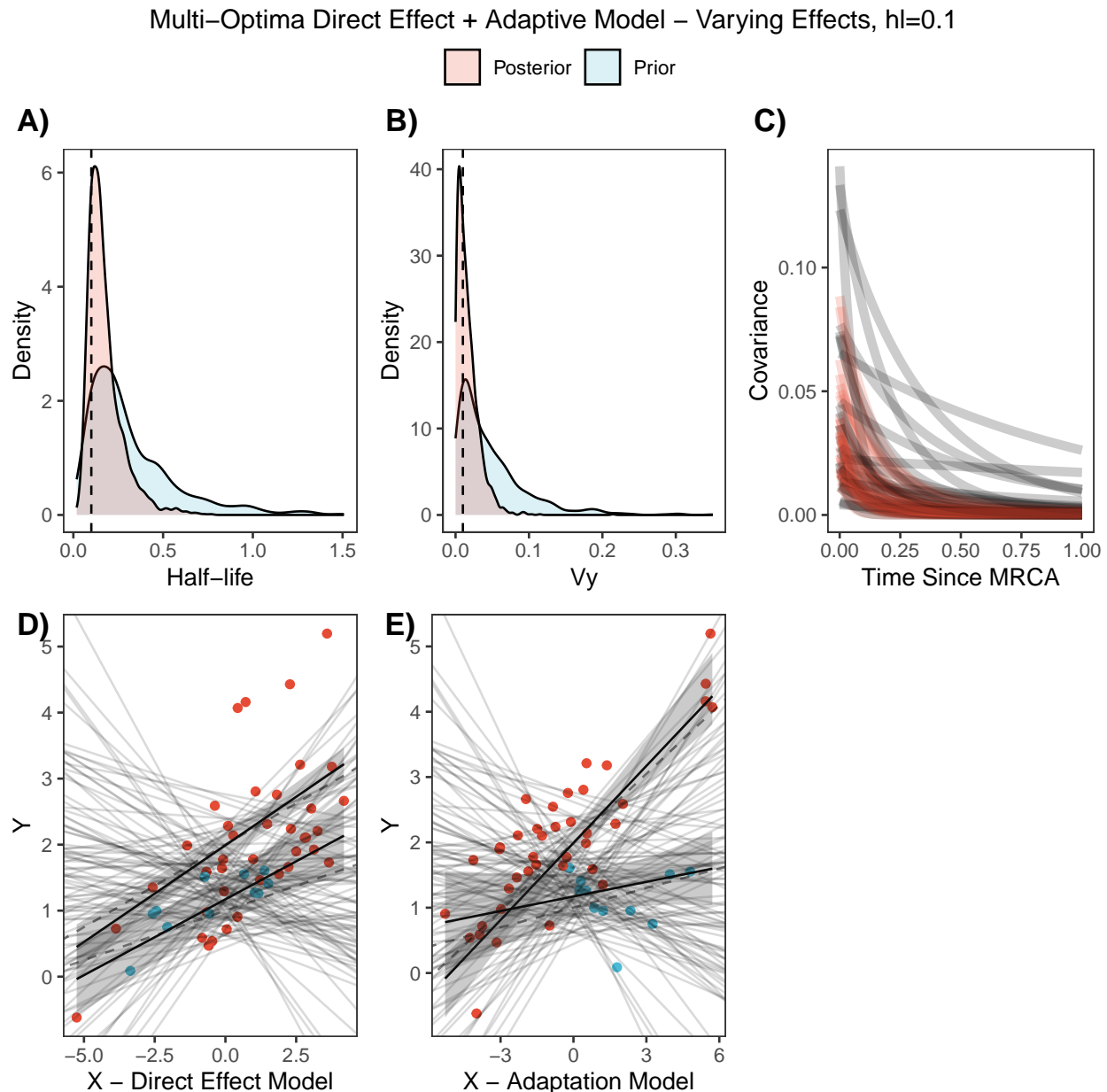


Figure 65: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.4.2 $hl = 0.25$

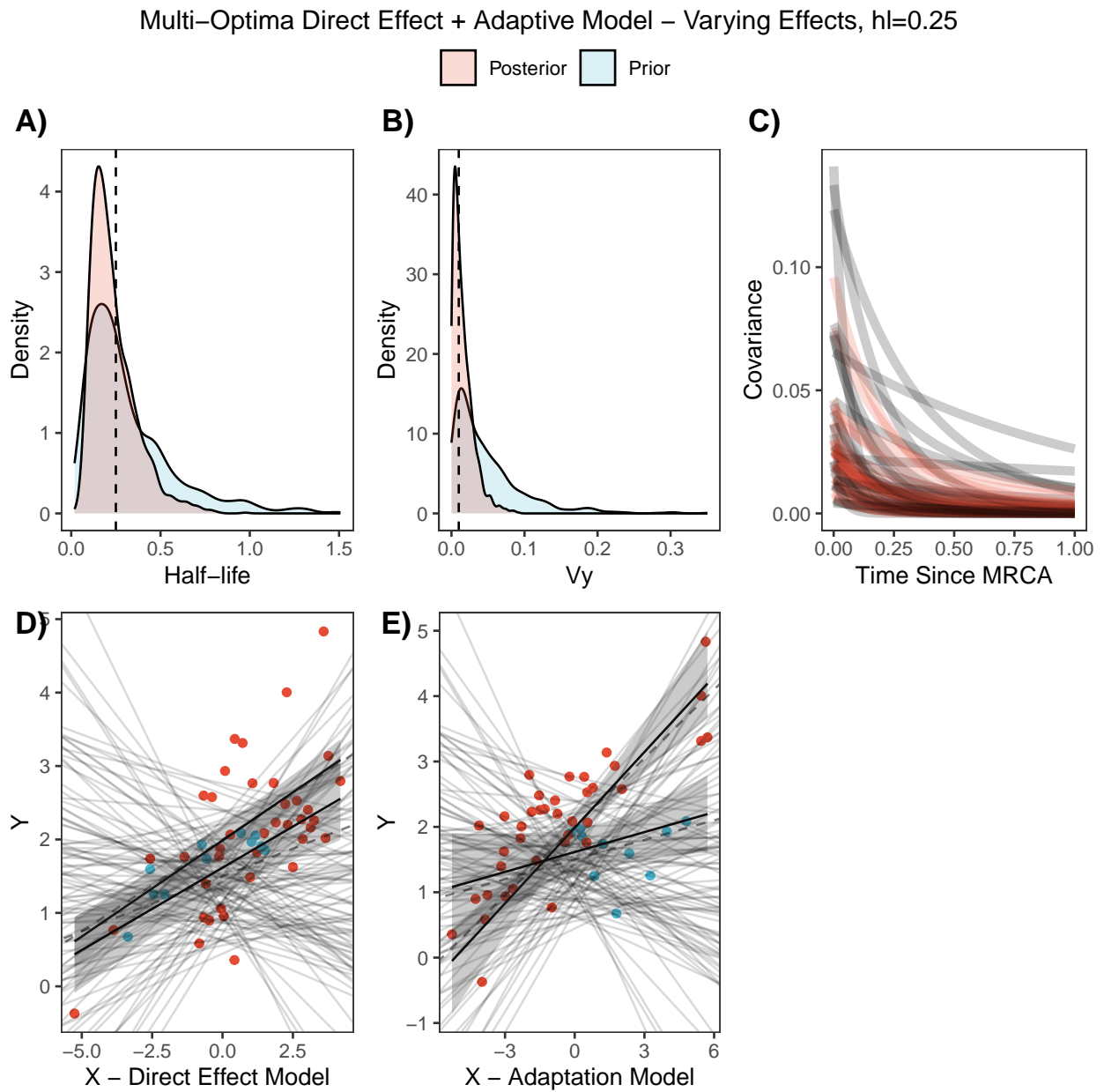


Figure 66: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.4.3 $hl = 0.75$

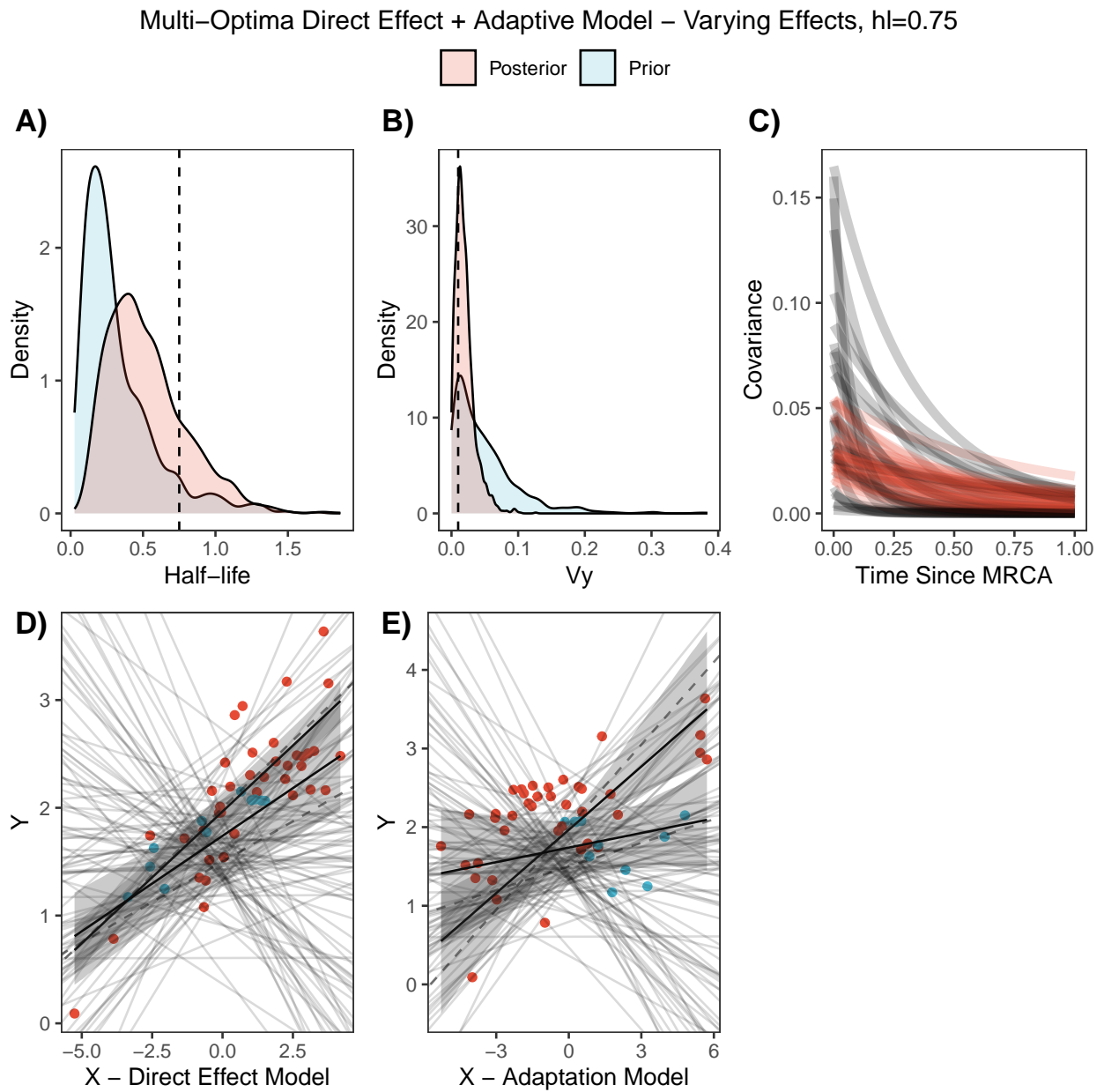


Figure 67: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.5 Multilevel Multi-Optima Direct Effect Adaptation Model - Varying Effects

9.5.1 $hl = 0.1$

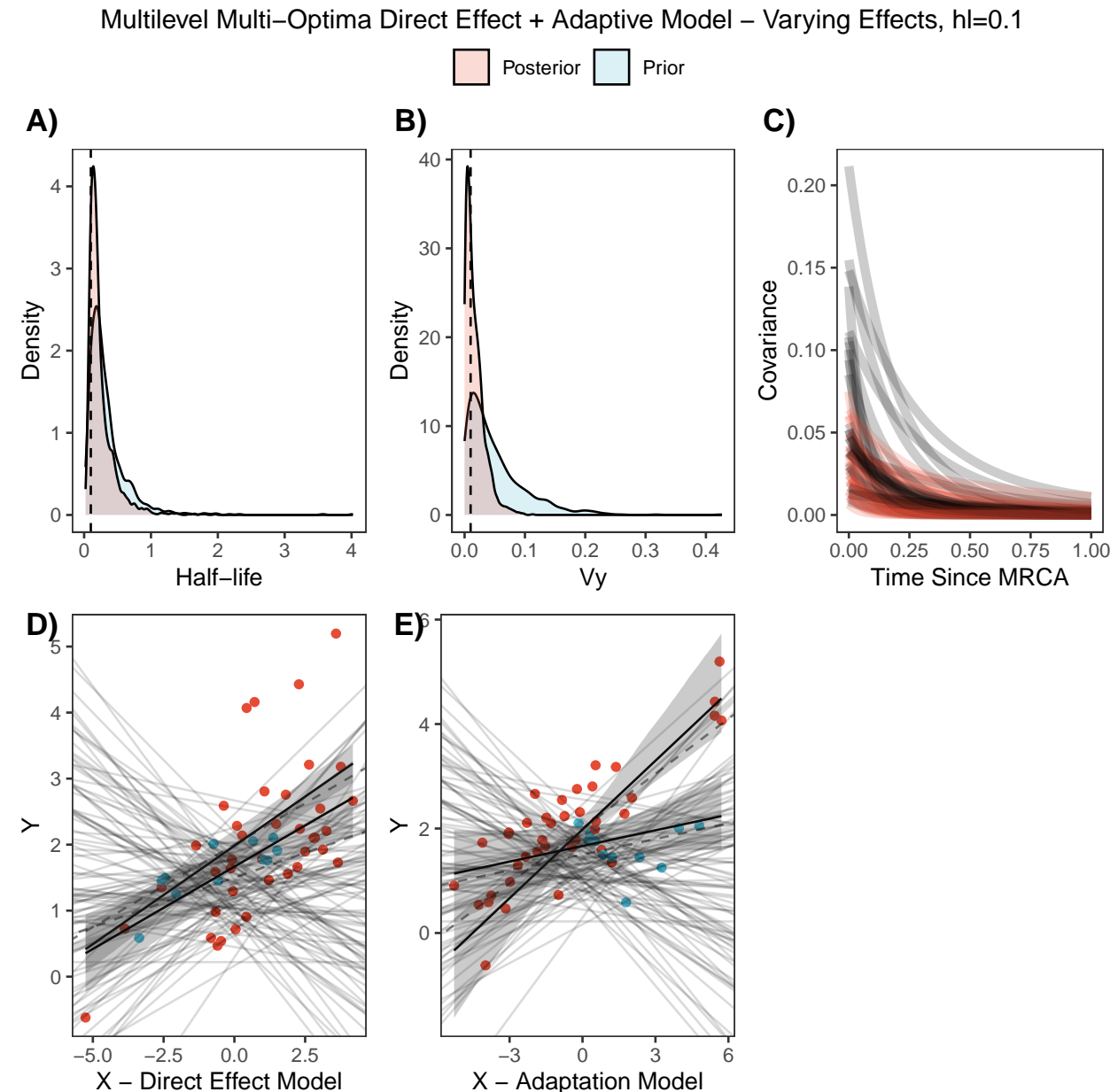


Figure 68: Prior vs. posterior for A) Half-life (hl); B) Vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.5.2 $hl = 0.25$

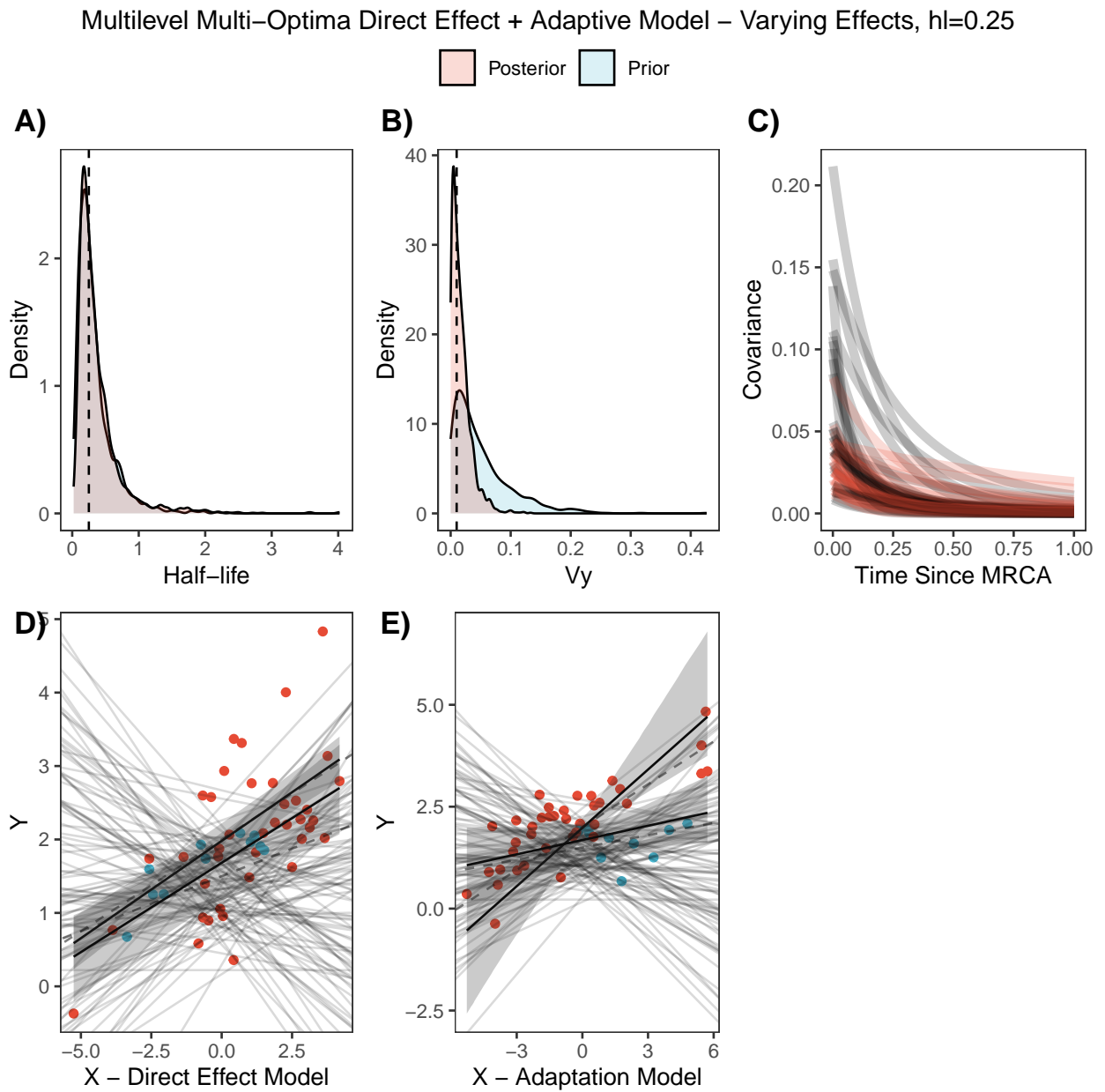


Figure 69: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.5.3 $hl = 0.75$

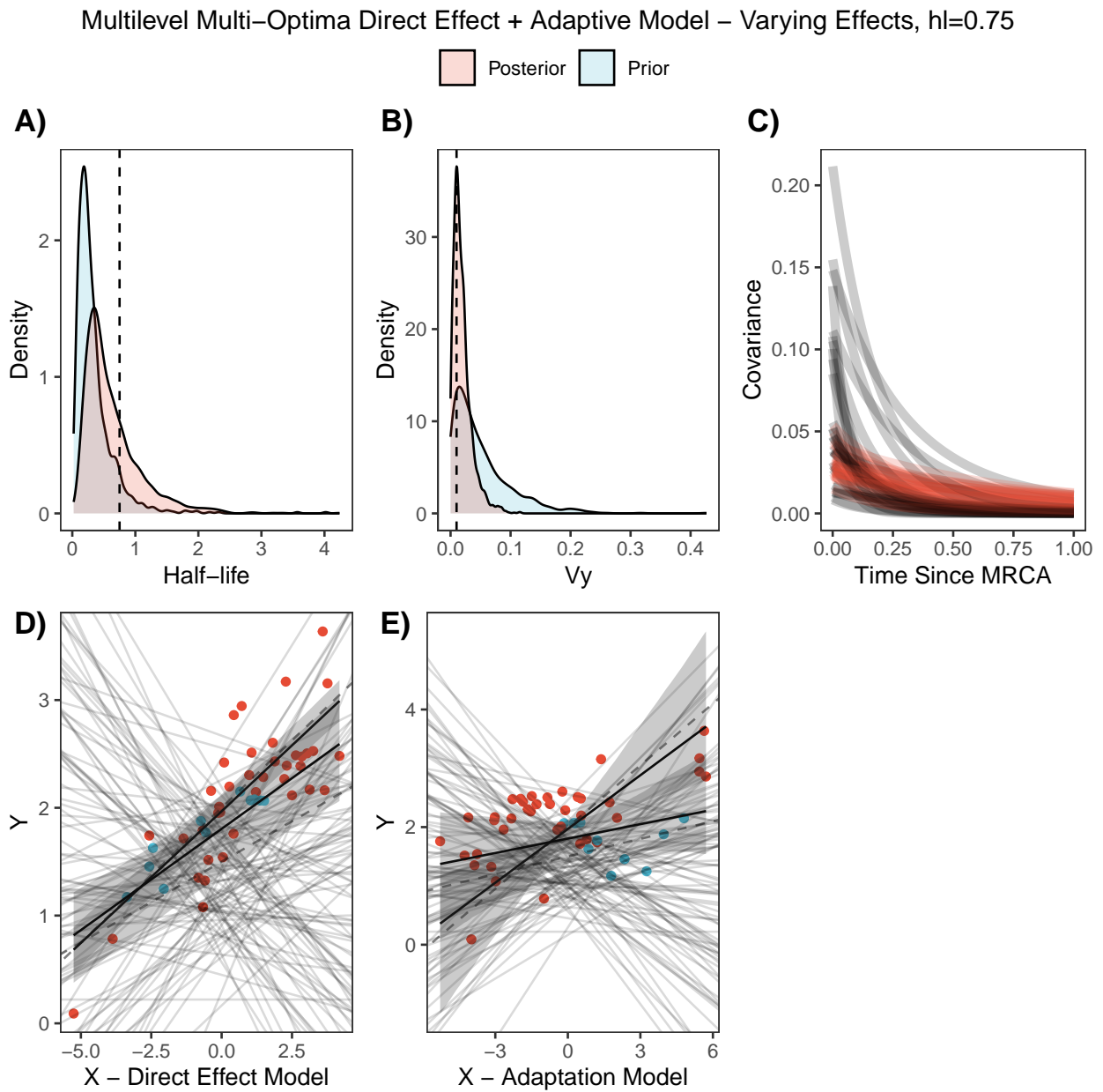


Figure 70: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.6 Multilevel Multi-Optima Direct Effect Adaptation Model - Varying Effects - Non-centered

9.6.1 $hl = 0.1$

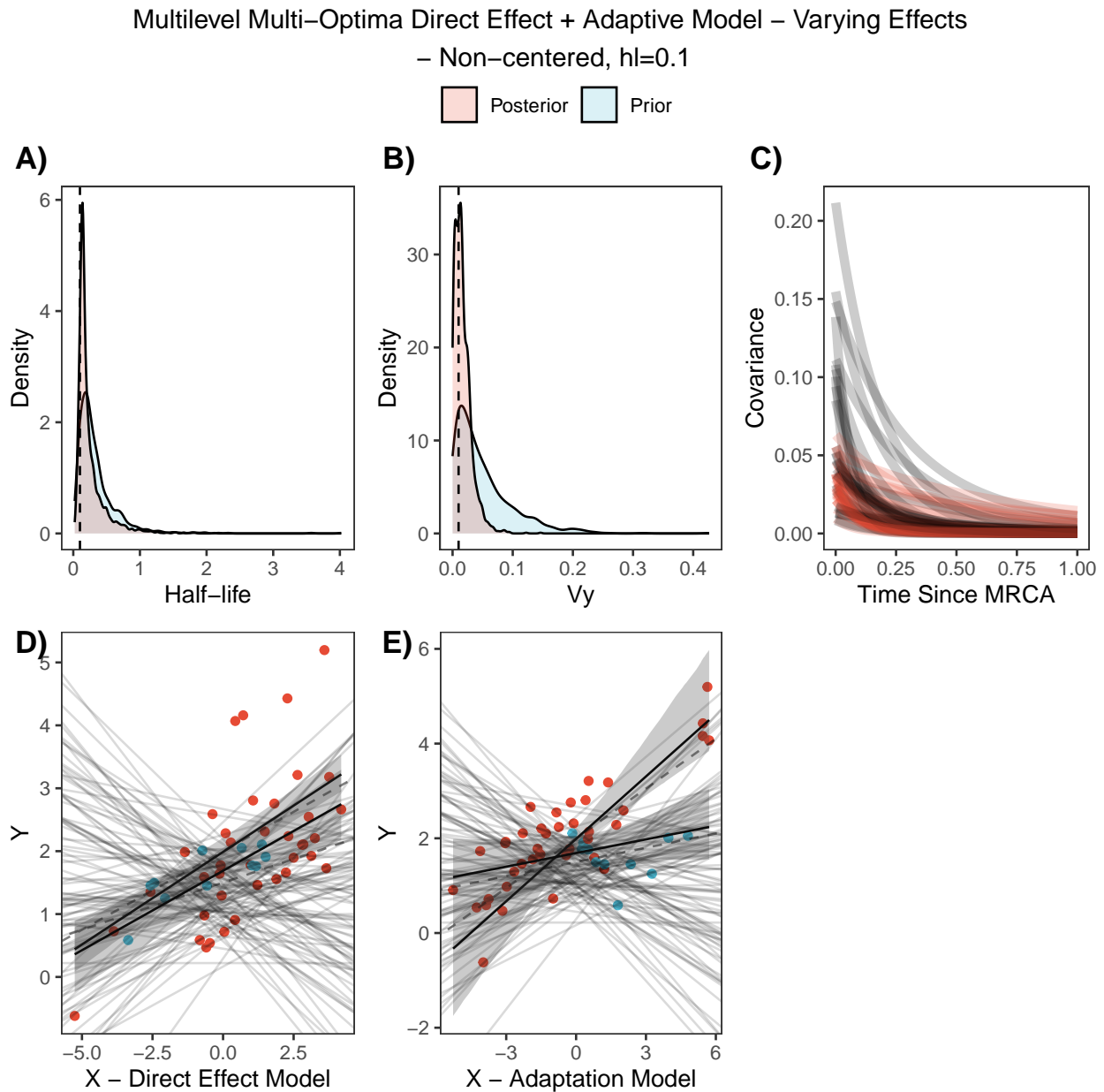


Figure 71: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.1$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.6.2 $hl = 0.25$

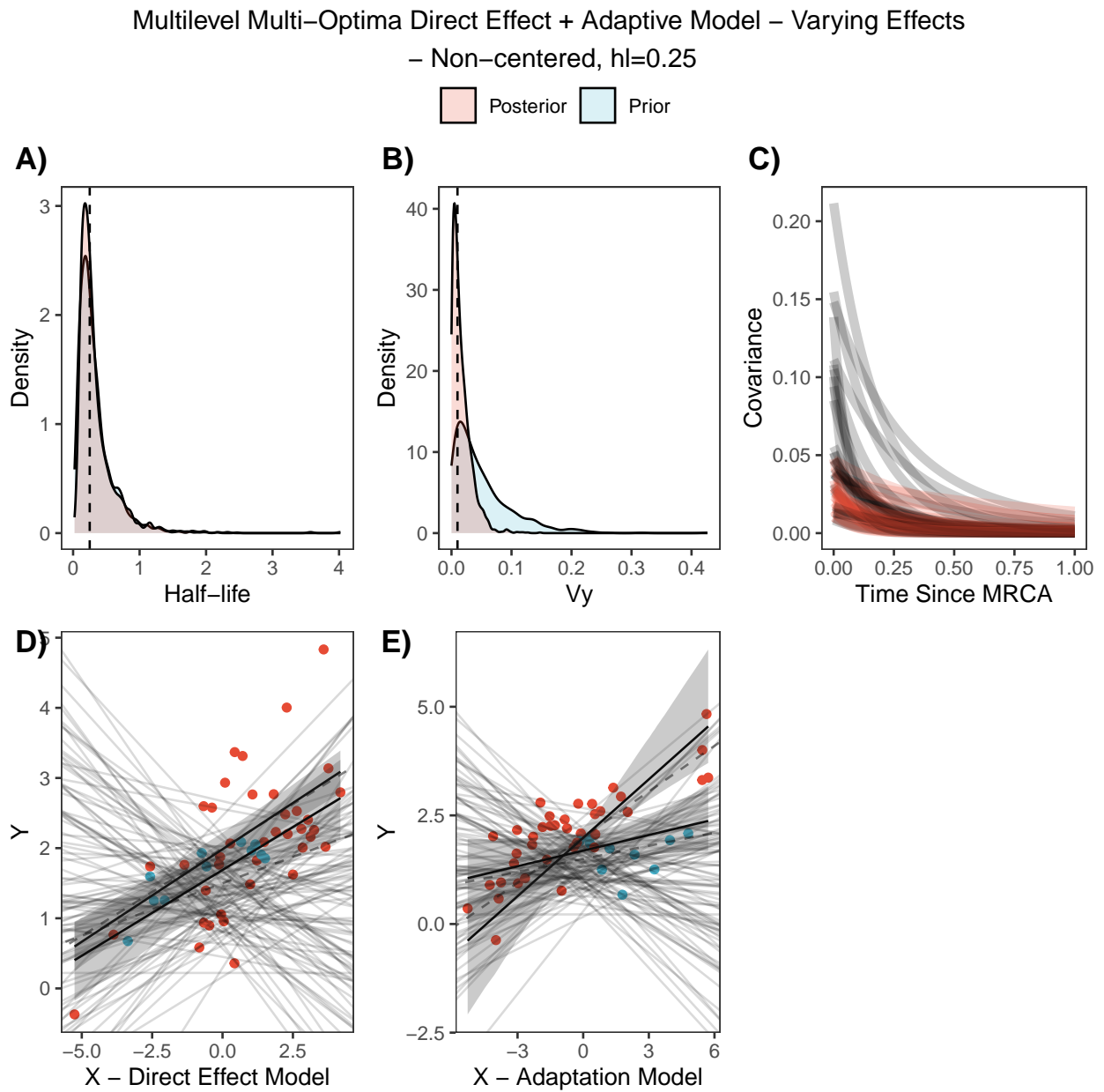


Figure 72: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.25$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.

9.6.3 $hl = 0.75$

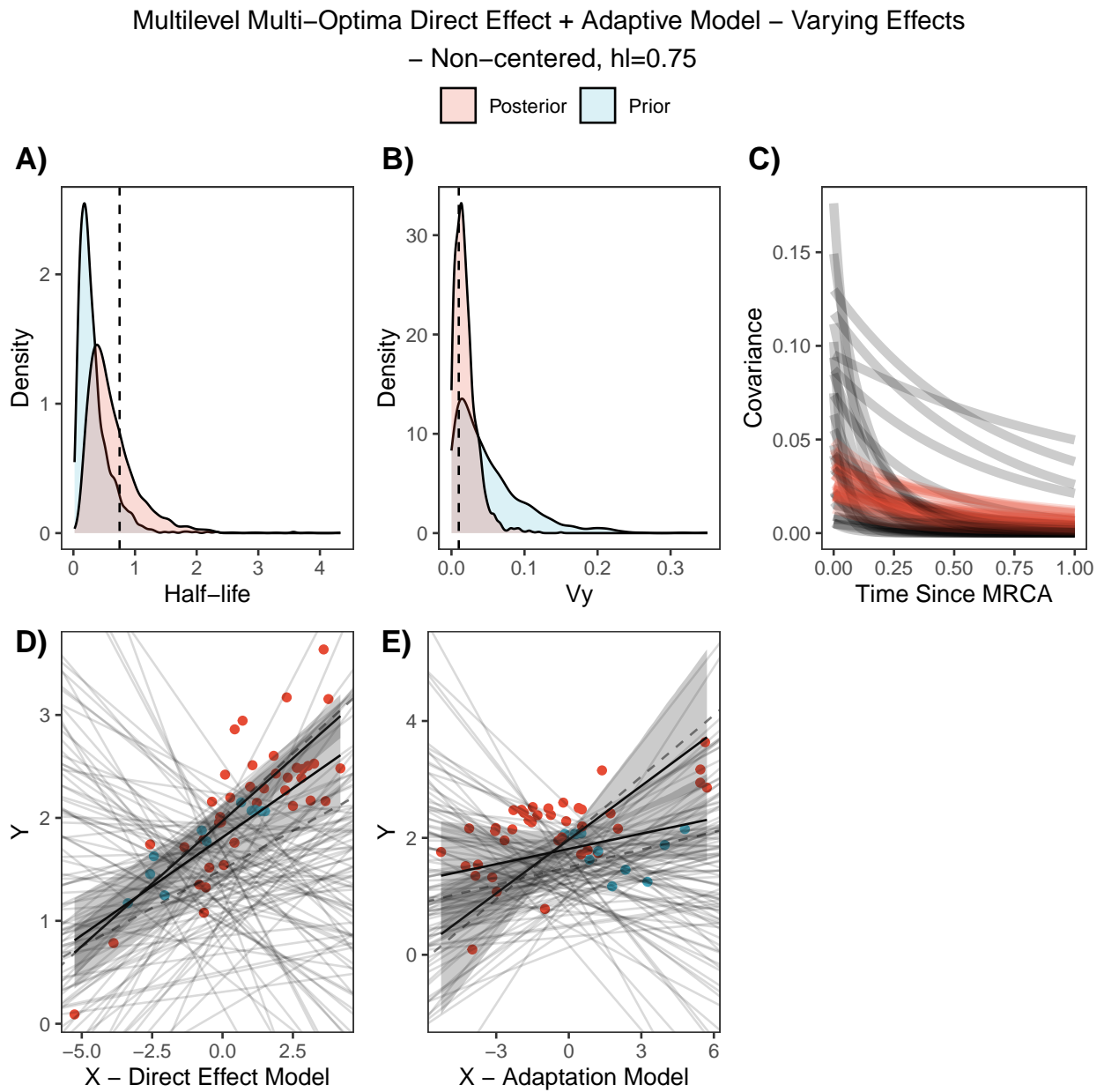


Figure 73: Prior vs. posterior for A) Half-life (hl); B) vy ; C) Covariance between residuals from the regression as a reflection of time since MRCA with priors (grey lines) and posterior (light red line); and D,E) Posterior prediction means (black line) and 89% compatibility intervals (light grey region) for posterior predictions, with priors in light grey. Species values are shown in the dark circles, with dotted line showing true parameter values of intercept and slope. For this simulation parameter values were set to: $hl = 0.75$; $Vy = 20$, $\theta = 2$, $\beta = 0.25$. Priors were set to: $hl \sim \text{lognormal}(\log(0.25), 0.75)$; $Vy \sim \text{exponential}(5)$; $\theta \sim \text{normal}(2, 0.2)$; $\beta \sim \text{normal}(0, 0.25)$. Dotted lines in A,B,D,E) are true values of the parameter.