

# BT5110 Data Management and Warehousing

## Tutorial 5: Normalisation (Extra Practice)

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4 - 8 Oct, 2021



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Updated on 8 Oct (Friday):

- Fixed some errors that have been raised in-class for the Case 4 and In-class Case 2. Refer to the red color notice text in relevant pages, and pay attention while viewing the Zoom recording.

Updated on 10 Oct (Sunday):

- Fixed the underline notation to indicate keys of  $R_1$  in page 11.  
i.e.,  $\underline{S}, \underline{M} \rightarrow \underline{S}, \underline{M}$
- Fixed the decomposition mistake of Case 3 in page 16.  
i.e.,  $R_1$  in the first option is not in BCNF, therefore a further decomposition is needed.

Updated on 12 Oct (Tuesday):

- Fixed the mistake in computing the minimal cover set and consequential sub-questions of In-class Case 2 in page 28-30.

# Agenda

- Extra Case No.1 - Warehouse management system
- Extra Case No.2 - University transcript issuing system
- Extra Case No.3 (Abstract) - *Non dependency preserving decomposition*
- Extra Case No.4 (Abstract) ‡ - *Candidate keys with different sizes*
- In-class Case No.1 (Abstract)
- In-class Case No.2 (Abstract) ‡ - *Trick of finding candidate key(s)*

‡ Updated in the latest version as some errors raised in-class have been fixed

## Boss Level Extra - Case 1

We are designing a warehouse management system for a lot of warehouses. Each warehouse ( $W$ ) has one manager ( $M$ ), and each manager only manage one warehouse. There could be many products ( $P$ ) in per warehouse. For each product we also record its stock number ( $S$ ).

### Questions:

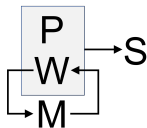
- (1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures  $\Sigma^+$ .
- (2) Compute the **compact minimal cover** of  $R$  with all FDs  $\Sigma$ .
- (3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
- (4) If it is not 3NF, **synthesis** the relations to make it 3NF.
- (5) If it is not BCNF, **decompose** the relations to make it BCNF and verify the **dependency preservation**.

## 👹 Boss Level Extra - Case 1 (Solution)

**W**: warehouse; **M**: manager; **P**: product; **S**: stock.

$$R = \{W, M, P, S\}$$

$$\Sigma = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}, \{W, P\} \rightarrow \{S\}\}.$$



## Boss Level Extra - Case 1 (Solution)

$$R = \{W, M, P, S\}$$

$$\Sigma = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}, \{W, P\} \rightarrow \{S\}\}.$$

### Solution:

(1) The attribute closure of  $R$  is:

$$\begin{aligned}\Sigma^+ = & \{\{W\}^+ \rightarrow \{W, M\}, \\ & \{M\}^+ \rightarrow \{W, M\}, \\ & \{P\}^+ \rightarrow \{P\}, \\ & \{S\}^+ \rightarrow \{S\}, \\ & \{M, W\}^+ \rightarrow \{W, M\}, \\ & \{M, P\}^+ \rightarrow \{W, M, P, S\}, \\ & \{M, S\}^+ \rightarrow \{W, M, S\}, \\ & \{W, P\}^+ \rightarrow \{W, M, P, S\}, \\ & \{W, S\}^+ \rightarrow \{W, M, S\}, \dots\}.\end{aligned}$$

Now we find candidate keys:  $\{W, P\}$  and  $\{M, P\}$ .

Prime attributes:  $W, M, P$

## Boss Level Extra - Case 1 (Solution)

(2) The compact minimal cover is:

$$\{W\} \rightarrow \{M\},$$

$$\{M\} \rightarrow \{W\},$$

$$\{W, P\} \rightarrow \{S\}.$$

(3) Yes it is 2NF, 3NF, but not BCNF (e.g.,  $\{W\} \rightarrow \{M\}$ , where  $\{W\}$  is not a superkey).

(4) Omitted as it is 3NF.

(5) Decomposition at  $\{W\} \rightarrow \{M\}$ :

$$R_1 = (\underline{W}, \underline{M}),$$

$$R_2 = (\underline{W}, \underline{P}, S).$$

It is (luckily) dependency preserving.

## Boss Level Extra - Case 2

We are designing a transcript issuing system for our university. Each student is identified by its matric number/student ID, written as  $S$ . We are going to record a grade ( $G$ ) for each student ( $S$ ) and each module ( $M$ ). We also record students' names ( $N$ ) and their faculty ( $F$ ). In case any verification is needed, we also save the dean's name for each department ( $D$ ) so that people can contact him/her.

### Questions:

- (1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures  $\Sigma^+$ .
- (2) Compute the **compact minimal cover** of  $R$  with all FDs  $\Sigma$ .
- (3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
- (4) If it is not 3NF, **synthesis** the relations to make it 3NF.
- (5) If it is not BCNF, **decompose** the relations to make it BCNF and verify the **dependency preservation**.

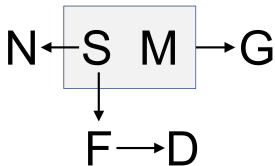


## 👹 Boss Level Extra - Case 2 (Solution)

**S**: student ID; **M**: module; **N**: name; **F**: faculty; **G**: grade; **D**: dean

$$R = \{S, M, G, N, F, D\}$$

$$\Sigma = \{\{S, M\} \rightarrow \{G\}, \{S\} \rightarrow \{N, F\}, \{F\} \rightarrow \{D\}\}.$$



## Boss Level Extra - Case 2 (Solution)

$$R = \{S, M, G, N, F, D\}$$

$$\Sigma = \{\{S, M\} \rightarrow \{G\}, \{S\} \rightarrow \{N, F\}, \{F\} \rightarrow \{D\}\}.$$

**Solution:**

(1) The attribute closure of  $R$  is:

$$\begin{aligned}\Sigma^+ = & \{\{S\}^+ \rightarrow \{S, N, F, D\}, \\ & \{M\}^+ \rightarrow \{M\}, \\ & \{G\}^+ \rightarrow \{G\}, \\ & \{N\}^+ \rightarrow \{N\}, \\ & \{F\}^+ \rightarrow \{F, D\}, \\ & \{D\}^+ \rightarrow \{D\}, \\ & \{S, M\}^+ \rightarrow \{S, M, N, F, D, G\}, \\ & \{S, G\}^+ \rightarrow \{S, G, N, F, D\}, \dots\end{aligned}$$

Now we find candidate keys:  $\{S, M\}$ .

Prime attributes:  $S, M$

## Boss Level Extra - Case 2 (Solution)

(2) The compact minimal cover is:

$$\{S, M\} \rightarrow \{G\},$$

$$\{S\} \rightarrow \{N, F\},$$

$$\{F\} \rightarrow \{D\}.$$

(3) No it is not 2NF (e.g.,  $\{S\} \rightarrow \{N, F\}$ ,  $\{N, F\}$  are not prime attributes and  $S$  is a subset of candidate key). Therefore it is not 3NF, and not BCNF, too.

(4) Synthesis result has 3 relations and is (luckily) BCNF:

$$R_1 = (\underline{S}, M, G),$$

$$R_2 = (\underline{S}, N, F),$$

$$R_3 = (\underline{F}, D).$$

## Boss Level Extra - Case 2 (Solution)

(5) Decomposition at  $\{F\} \rightarrow \{D\}$ :

$$R_1 = (\underline{F}, D),$$

$$R_2 = (\underline{S}, \underline{M}, G, N, F).$$

However,  $R_2$  needs to be further decomposed, at  $\{S\} \rightarrow \{N\}$ :

$$R_{2.1} = (\underline{S}, N, F).$$

$$R_{2.2} = (\underline{S}, \underline{M}, G).$$

As the result, the BCNF decomposition is (luckily) dependency preserving and is given below:

$$R_1 = (\underline{F}, D),$$

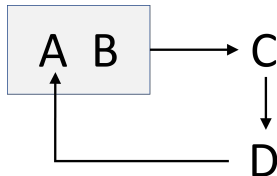
$$R_{2.1} = (\underline{S}, N, F).$$

$$R_{2.2} = (\underline{S}, \underline{M}, G).$$

*(In fact this decomposition result is same with the synthesis one)*

## Boss Level Extra - Case 3

This time we deal with abstract relations with functional dependencies shown as in the figure below:



### Questions:

- (1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures  $\Sigma^+$ .
- (2) Compute the **compact minimal cover** of  $R$  with all FDs  $\Sigma$ .
- (3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
- (4) If it is not 3NF, **synthesis** the relations to make it 3NF.
- (5) If it is not BCNF, **decompose** the relations to make it BCNF and verify the **dependency preservation**.

## Boss Level Extra - Case 3 (Solution)

### Solution:

(1) The attribute closure of  $R = \{A, B, C, D\}$  is:

$$\begin{aligned}\Sigma^+ = & \{\{A\}^+ \rightarrow \{A\}, \\ & \{B\}^+ \rightarrow \{B\}, \\ & \{C\}^+ \rightarrow \{A, C, D\}, \\ & \{D\}^+ \rightarrow \{A, D\}, \\ & \{A, B\}^+ \rightarrow \{A, B, C, D\}, \\ & \{A, C\}^+ \rightarrow \{A, C, D\}, \\ & \{A, D\}^+ \rightarrow \{A, D\}, \\ & \{B, C\}^+ \rightarrow \{A, B, C, D\} \\ & \{B, D\}^+ \rightarrow \{A, B, C, D\}, \dots\end{aligned}$$

Now we find candidate keys:  $\{A, B\}$  or  $\{B, C\}$  or  $\{B, D\}$ .

Prime attributes:  $A, B, C, D$  (There is no non-prime attribute for this case)

## Boss Level Extra - Case 3 (Solution)

(2) The compact minimal cover is:

$$\{A, B\} \rightarrow \{C\}$$

$$\{C\} \rightarrow \{D\}$$

$$\{D\} \rightarrow \{A\}.$$

(3) Yes it is 2NF and 3NF (because all attributes are prime attributes).  
However, it is not BCNF (e.g.,  $\{C\} \rightarrow \{D\}$  and  $\{D\} \rightarrow \{A\}$ ).

(4) Omitted as it is 3NF.

## Boss Level Extra - Case 3 (Solution)

(5) Decomposition at  $\{C\} \rightarrow \{D\}$ :

$$R_1 = (A, \underline{C}, D),$$

$$R_2 = (\underline{B}, \underline{C}).$$

The  $R_1$  is not in BCNF (e.g.,  $\{D\} \rightarrow \{A\}$ ), let's further decompose it:

$$R_{1.1} = (A, \underline{D}),$$

$$R_{1.2} = (\underline{C}, D),$$

In this way all 3 relations ( $R_{1.1}$ ,  $R_{1.2}$  and  $R_2$ ) are BCNF, but we lose a dependency  $\{A, B\} \rightarrow \{C\}$ .

How about we change the entry point of decomposition?

Decomposition at  $\{D\} \rightarrow \{A\}$ :

$$R_1 = (A, \underline{D}),$$

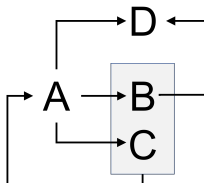
$$R_2 = (B, \underline{C}, D).$$

In this way both relations are BCNF, but we (still) lose that dependency  $\{A, B\} \rightarrow \{C\}$ .



## 👹 Boss Level Extra - Case 4

Another abstract relations with functional dependencies shown as in the figure below:



### Questions:

- (1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures  $\Sigma^+$ .
- (2) Compute the **compact minimal cover** of  $R$  with all FDs  $\Sigma$ .
- (3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
- (4) If it is not 3NF, **synthesis** the relations to make it 3NF.
- (5) If it is not BCNF, **decompose** the relations to make it BCNF and verify the **dependency preservation**.

## Boss Level Extra - Case 4 (Solution)

### Solution:

(1) The attribute closure of  $R = \{A, B, C, D\}$  is:

$$\begin{aligned}\Sigma^+ = & \{ \{A\}^+ \rightarrow \{A, B, C, D\}, \\ & \{B\}^+ \rightarrow \{B, D\}, \\ & \{C\}^+ \rightarrow \{C\}, \\ & \{D\}^+ \rightarrow \{D\}, \\ & \{A, B\}^+ \rightarrow \{A, B, C, D\}, \\ & \{A, C\}^+ \rightarrow \{A, B, C, D\}(\text{trivial}), \\ & \{A, D\}^+ \rightarrow \{A, B, C, D\}(\text{trivial}), \\ & \{B, C\}^+ \rightarrow \{A, B, C, D\} \\ & \{B, D\}^+ \rightarrow \{B, D\}, \dots\end{aligned}$$

Now we find candidate keys:  $\{A\}$  or  $\{B, C\}$ .

Although  $\{A\} \rightarrow \{B, C\}$ , both of them are candidate keys because (1) their closures functionally determine all attributes of  $R$ ; (2) both of them are minimal superkeys (cannot be further simplified).

Prime attributes:  $A, B, C$ .

## Boss Level Extra - Case 4 (Solution)

### Updated version (an error raised in class corrected)

(2) First let's list all FDs given in the question:

$\{A\} \rightarrow \{B, C, D\}$ ,  $\{B\} \rightarrow \{D\}$ ,  $\{B, C\} \rightarrow \{A\}$ .

Step 1: Simplify the RHS:

$\{A\} \rightarrow \{B\}$

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{D\}$

$\{B\} \rightarrow \{D\}$

$\{B, C\} \rightarrow \{A\}$ .

Step 2: Simplify the LHS

(There is only 1 FD with multiple attributes on the LHS and that FD cannot be simplified as there does not exist any FD implies A)

## Boss Level Extra - Case 4 (Solution)

Step 3: Simplify the set:

$$\{A\} \rightarrow \{B\}$$

$$\{A\} \rightarrow \{C\}$$

$$\cancel{\{A\} \rightarrow \{D\}} \text{ (because } \{A\} \rightarrow \{B\} \text{ and } \{B\} \rightarrow \{D\})$$

$$\{B\} \rightarrow \{D\}$$

$$\{B, C\} \rightarrow \{A\}.$$

Finally, the compact minimal cover is:

$$\{A\} \rightarrow \{B, C\}$$

$$\{B\} \rightarrow \{D\}$$

$$\{B, C\} \rightarrow \{A\}.$$

## Boss Level Extra - Case 4 (Solution)

(3) No it is not 2NF (e.g.,  $\{B\} \rightarrow \{D\}$ , where  $D$  is a non-prime attribute but  $B$  is a subset of candidate key). Therefore, it is not 3NF and not BCNF, too. (all because of  $\{B\} \rightarrow \{D\}$ )

**Updated version (an error raised in class corrected)**

(4) Synthesis result has 2 relations and each one is (luckily) BCNF:

$$R_1 = (\underline{A}, \underline{B}, \underline{C}),$$

$$R_2 = (\underline{B}, \underline{D}),$$

$$\underline{R_3} = (\underline{B}, \underline{C}, \underline{A}). \text{ (duplicate with } R_1)$$

## Boss Level Extra - Case 4 (Solution)

### Updated version (an error raised in class corrected)

(5) Decomposition at  $\{B\} \rightarrow \{D\}$ :

$$R_1 = (\underline{B}, D),$$

$$R_2 = (\underline{A}, \underline{B}, C).$$

In this way both relations are BCNF, and luckily we obtain the exact same result as the 3NF synthesis, therefore it is dependency preserving.

## Boss Level Extra - In-class Case 1

Given a relation  $R = \{A, B, C\}$  with functional dependency set:  
 $\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}.$

### Questions:

- (1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures  $\Sigma^+$ .
- (2) Compute the **compact minimal cover** of  $R$  with all FDs  $\Sigma$ .
- (3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
- (4) If it is not 3NF, **synthesis** the relations to make it 3NF.
- (5) If it is not BCNF, **decompose** the relations to make it BCNF and verify the **dependency preservation**.

## Boss Level Extra - In-class Case 1 (Solution)

$$R = \{A, B, C\}$$

$$\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}.$$

**Solution:**

(1) The attribute closure of  $R$  is:

$$\begin{aligned}\Sigma^+ = & \{\{A\}^+ \rightarrow \{A, B, C\}, \\ & \{B\}^+ \rightarrow \{A, B, C\}, \\ & \{C\}^+ \rightarrow \{C\}, \\ & \{A, B\}^+ \rightarrow \{A, B, C\}, \\ & \{A, C\}^+ \rightarrow \{A, B, C\}, \\ & \{B, C\}^+ \rightarrow \{A, B, C\}, \\ & \{A, B, C\}^+ \rightarrow \{A, B, C\}\}.\end{aligned}$$

Now we find candidate keys:  $\{A\}$  and  $\{B\}$ .

Prime attributes:  $A, B$



## Boss Level Extra - In-class Case 1 (Solution)

$$\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}.$$

(2) The minimal cover is:

$$\begin{aligned} &\{A\} \rightarrow \{B\}, \\ &\{B\} \rightarrow \{C\}, \\ &\{B, C\} \rightarrow \{A\}. \end{aligned}$$

This is the compact minimal cover, too.

(3) Yes it is 2NF, 3NF, and BCNF, too.

(4) Omitted as it is 3NF.

(5) Omitted as it is BCNF.

## Boss Level Extra - In-class Case 2

Given a relation  $R = \{A, B, C, D, E\}$  with functional dependency set:  
 $\Sigma = \{\{C, D\} \rightarrow \{E\}, \{A, B\} \rightarrow \{B\}, \{A, C, D\} \rightarrow \{E\}, \{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}, \{A\} \rightarrow \{A\}\}.$

### Questions:

- (1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures  $\Sigma^+$ .
- (2) Compute the **compact minimal cover** of  $R$  with all FDs  $\Sigma$ .
- (3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
- (4) If it is not 3NF, **synthesis** the relations to make it 3NF.
- (5) If it is not BCNF, **decompose** the relations to make it BCNF and verify the **dependency preservation**.

## Boss Level Extra - In-class Case 2 (Solution)

$$\Sigma = \{\{C, D\} \rightarrow \{E\}, \{A, B\} \rightarrow \{B\}, \{A, C, D\} \rightarrow \{E\}, \{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}, \{A\} \rightarrow \{A\}\}.$$

**Solution:**

**Updated version (an error raised in class corrected)**

(1) The attribute closure of  $R$  is:

$$\Sigma^+ = \{\{A\}^+ \rightarrow \{A, E\},$$

$$\{B\}^+ \rightarrow \{B\},$$

$$\{C\}^+ \rightarrow \{C\},$$

$$\{D\}^+ \rightarrow \{D\},$$

$$\{E\}^+ \rightarrow \{E\},$$

$$\{A, B\}^+ \rightarrow \{A, B, E\},$$

$$\{A, C\}^+ \rightarrow \{A, C, E\},$$

$$\{A, D\}^+ \rightarrow \{A, B, C, D, E\},$$

$$\{A, E\}^+ \rightarrow \{A, B, E\}$$

...

$$\{A, C, D, E\}^+ \rightarrow \{A, B, C, D, E\}$$

$$\{A, B, C, D, E\}^+ \rightarrow \{A, B, C, D, E\}.$$

**Hint:** After removing all trivial FDs, we find  $A$  and  $D$  have never appeared in the right-hand side, which means the candidate key must contain  $A$  and  $D$ . We just need to find the minimal set that implies  $\{B, C, E\}$ . (Luckily both  $A$  and  $D$  together can determine  $\{B, C, E\}$ )

Then we can find candidate key:  $\{A, D\}$ . And the prime attributes:  $A, D$

## Boss Level Extra - In-class Case 2 (Solution)

$\Sigma = \{\{C, D\} \rightarrow \{E\}, \{A, B\} \rightarrow \{B\}, \{A, C, D\} \rightarrow \{E\}, \{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}, \{A\} \rightarrow \{A\}\}.$

(2) The compact minimal cover is:

### STEP 1

~~$\{C, D\} \rightarrow \{E\},$~~   
 ~~$\{A, B\} \rightarrow \{B\},$~~   
 ~~$\{A, C, D\} \rightarrow \{E\},$~~   
 $\{A\} \rightarrow \{E\},$   
 $\{D, E\} \rightarrow \{B\}.$   
 $\{D, E\} \rightarrow \{C\}.$   
 ~~$\{A\} \rightarrow \{A\}$~~

### STEP 3

~~$\{C, D\} \rightarrow \{E\},$~~   
(Can be derived by  $\{D, E\}$ )  
 $\{A\} \rightarrow \{E\},$   
 $\{D, E\} \rightarrow \{B\}.$   
 $\{D, E\} \rightarrow \{C\}.$

### STEP 3

The finalized set is:  
 $\{A\} \rightarrow \{E\},$   
 $\{D, E\} \rightarrow \{B, C\}.$

### Updated version (an error raised in class corrected)

(3) Yes it is in 2NF. But not in 3NF, and not in BCNF, too.

E.g.,  $\{C, D\} \rightarrow \{E\}$  is not trivial, the LHS is not a superkey and the RHS is not a prime attribute, therefore it violates 3NF (in fact all four FDs in the compact minimal cover violate 3NF and BCNF).

## Boss Level Extra - In-class Case 2 (Solution)

### Updated version (an error raised in class corrected)

(4) Recall the compact minimal cover:

$$\{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}.$$

Let's do 3NF synthesis:

$$R_1 = (\underline{A}, E),$$

$$R_2 = (B, C, \underline{D}, \underline{E}).$$

(The candidate key is not included in any relation fragment above, so we add a new fragment manually)

$$R_3 = (\underline{A}, \underline{D}).$$

All 3 fragments are in BCNF.

## Boss Level Extra - In-class Case 2 (Solution)

### Updated version (an error raised in class corrected)

(5) Recall the compact minimal cover:

$$\{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}.$$

Decomposition at  $\{D, E\} \rightarrow \{B, C\}$ :

$$R_1 = (B, C, \underline{D}, \underline{E}),$$

$$R_2 = (\underline{A}, \underline{D}, \underline{E}).$$

We can find that:

$$\Sigma_1 = \{\{D, E\} \rightarrow \{B, C\}\}$$

$$\Sigma_2 = \{\{A\} \rightarrow \{E\}\}$$

We find only  $R_1$  is BCNF,  $\{\{A\} \rightarrow \{E\}\}$  violates BCNF in  $R_2$  as the candidate key of it should be  $\{A, D\}$ . So we further decompose  $R_2$ .

$$R_{2.1} = (\underline{A}, \underline{E}),$$

$$R_{2.2} = (\underline{A}, \underline{D}).$$

Thus all three fragments are in BCNF. Luckily we do not loss any functional dependency through this decomposition.

This decomposition result is same as the one we obtained in 3NF synthesis.

For any further question, please feel free to email me:

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