CS4221/CS5421

Tutorial 3: Dependency

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Updated on 12 Feb (Saturday):

- Added some notes regarding finding all minimal covers on page 10 and 12.

Question 1

Consider the relational schema $R = \{A, B, C, D, E\}$ with the following set of functional dependencies.

$$\Sigma = \{\{A, B\} \rightarrow \{C\}, \{D\} \rightarrow \{D, B\}, \{B\} \rightarrow \{E\}, \{E\} \rightarrow \{D\}, \{A, B, D\} \rightarrow \{A, B, C, D\}\}.$$

Question 1 (a-b) Attribute Closure & Candidate Keys

(a) Compute all the closures of the the sets of attributes that are not equal to themselves, are not super-keys or are candidate keys. What information is not essential and could be removed.

Solution:

(Let's start with single attribute first)

$\{A\}^+ = \{A\}$ *(omitted)*

$\{B\}^+ = \{B, D, E\}$

$\{C\}^+ = \{C\}$ *(omitted)*

$\{D\}^+ = \{B, D, E\}$

$\{E\}^+ = \{B, D, E\}$

(Two attributes' combination)

$\underline{\{A, B\}^+} = \{A, B, C, D, E\}$

$\{A, C\}^+ = \{A, C\}$ *(omitted)*

$\underline{\{A, D\}^+} = \{A, B, C, D, E\}$

$\underline{\{A, E\}^+} = \{A, B, C, D, E\}$

$\{B, C\}^+ = \{B, C, D, E\}$

$\{B, D\}^+ = \{B, D, E\}$

$\{B, E\}^+ = \{B, D, E\}$

$\{C, D\}^+ = \{B, C, D, E\}$

$\{C, E\}^+ = \{B, C, D, E\}$

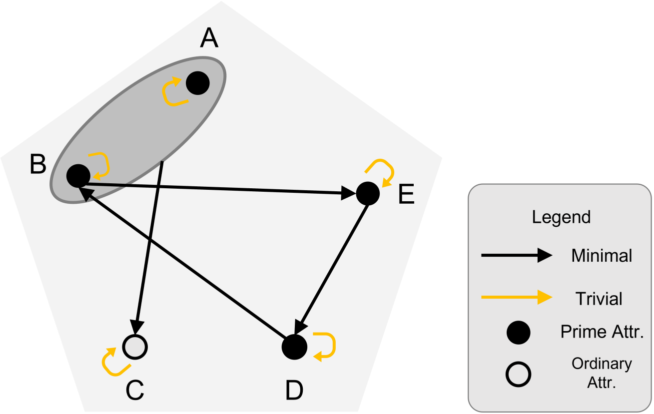
$\{D, E\}^+ = \{B, D, E\}$

Other attribute closures need not be computed.

Question 1 (a-b) Cont.

We see the candidate keys are $\{A, B\}$, $\{A, D\}$ and $\{A, E\}$.
This is the answer for 1(b)

We can also visualize the FD as a figure below:



Question 1 (a-b) Cont.

Now let's remove those information that is not essential from this closure:

We can remove from the RHS the members of the set on the LHS, as it also keeps all the essential information:

$\{A, B\}^+ = \{C, D, E\}$
$\{A, D\}^+ = \{B, C, E\}$
$\{A, E\}^+ = \{B, C, D\}$
$\{B\}^+ = \{D, E\}$
$\{D\}^+ = \{B, E\}$
$\{E\}^+ = \{B, D\}$
$\{B, C\}^+ = \{D, E\}$
$\{B, D\}^+ = \{E\}$

$\{B, E\}^+ = \{D\}$
$\{C, D\}^+ = \{B, E\}$
$\{C, E\}^+ = \{B, D\}$
$\{D, E\}^+ = \{B\}$
$\{B, C, D\}^+ = \{E\}$
$\{B, C, E\}^+ = \{D\}$
$\{C, D, E\}^+ = \{B\}$

Question 1 (a-b) Cont.

Remark: We can even remove an equality if the LHS is a superset of another equality LHS and its RHS is a subset of the RHS (e.g., give $\{E\}^+ = \{B, D\}$ and $\{C, D, E\}^+ = \{B\}$, we can omit the second one). It also keeps all the essential information:

$\{A, B\}^+ = \{C, D, E\}$
$\{A, D\}^+ = \{B, C, E\}$
$\{A, E\}^+ = \{B, C, D\}$
$\{B\}^+ = \{D, E\}$
$\{D\}^+ = \{B, E\}$
$\{E\}^+ = \{B, D\}$

(b) What are the candidate keys of R with Σ?

The candidate keys are $\{A, B\}$, $\{A, D\}$ and $\{A, E\}$.

Question 1 (c) Minimal Cover

(c) Find a minimal cover of R with Σ that can be reached from Σ using the algorithm from the lecture.

Solution: Use the three step approach then you should be able to find one.

We start from Σ:

$\{A, B\} \rightarrow \{C\}$
$\{D\} \rightarrow \{D, B\}$
$\{B\} \rightarrow \{E\}$
$\{E\} \rightarrow \{D\}$
$\{A, B, D\} \rightarrow \{A, B, C, D\}$

Step 1, we simplify the right-hand sides to obtain Σ':

$\{A, B\} \rightarrow \{C\}$
$\{D\} \rightarrow \{D\}$
$\{D\} \rightarrow \{B\}$
$\{B\} \rightarrow \{E\}$
$\{E\} \rightarrow \{D\}$

$\{A, B, D\} \rightarrow \{A\}$
$\{A, B, D\} \rightarrow \{B\}$
$\{A, B, D\} \rightarrow \{C\}$
$\{A, B, D\} \rightarrow \{D\}$

Question 1 (c) Cont.

Step 2, we construct Σ'', which is equivalent to Σ' (and to Σ), by minimizing the LHS of every functional dependency in Σ'. We may have the choice between several possible simplifications. We only need to choose one:

$\{A, B\} \rightarrow \{C\}$
$\{D\} \rightarrow \{D\}$
$\{D\} \rightarrow \{B\}$
$\{B\} \rightarrow \{E\}$
$\{E\} \rightarrow \{D\}$

$\{A, \cancel{B, D}\} \rightarrow \{A\}$ because $\{A\} \rightarrow \{A\}$
$\{\cancel{A}, B, \cancel{D}\} \rightarrow \{B\}$ because $\{B\} \rightarrow \{B\}$
$\cancel{\{A, B, D\} \rightarrow \{C\}}$ because $\{A, B\} \rightarrow \{C\}$
$\{\cancel{A, B}, D\} \rightarrow \{D\}$because $\{D\} \rightarrow \{D\}$

Question 1 (c) Cont.

Step 3, we construct Σ''', which is equivalent to Σ'' (and to Σ), by minimizing the whole set Σ'' (e.g., removing trivial ones). We may have the choice between several possible simplifications. The choice depends on the order in which we consider the functional dependency:

$\{A, B\} \rightarrow \{C\}$
$\cancel{\{D\} \rightarrow \{D\}}$ because it is trivial.
$\{D\} \rightarrow \{B\}$
$\{B\} \rightarrow \{E\}$
$\{E\} \rightarrow \{D\}$

The (not unique) result is:

$$\Sigma''' = \{\{A, B\} \rightarrow \{C\}, \{D\} \rightarrow \{B\}, \{B\} \rightarrow \{E\}, \{E\} \rightarrow \{D\}\}$$

NOTES FROM TA: There is another minimal cover can be derived by this 3-step method, with the order of dependency reshuffled. You may download and refer to the Luminus files (Tutorials/Dependency/Tutorial Dependency (some answers).pdf) for more details.
Both minimal cover are so called "reachable with the algorithm".

Question 1 (d) Minimal Cover

(d) Find all the minimal covers of R with Σ

Solution: There are **15** different minimal covers.

(1) $\Sigma''' = \{\{A, B\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$

(2) $\Sigma''' = \{\{A, B\} \to \{C\}, \{B\} \to \{D\}, \{E\} \to \{B\}, \{D\} \to \{E\}\}$

(3) $\Sigma''' = \{\{A, B\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{B\}\}$

(4) $\Sigma''' = \{\{A, B\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$

(5) $\Sigma''' = \{\{A, B\} \to \{C\}, \{E\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$

(6) $\Sigma''' = \{\{A, D\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$

(7) $\Sigma''' = \{\{A, D\} \to \{C\}, \{B\} \to \{D\}, \{E\} \to \{B\}, \{D\} \to \{E\}\}$

(8) $\Sigma''' = \{\{A, D\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{B\}\}$

(9) $\Sigma''' = \{\{A, D\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$

(10) $\Sigma''' = \{\{A, D\} \to \{C\}, \{E\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$

(11) $\Sigma''' = \{\{A, E\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$

(12) $\Sigma''' = \{\{A, E\} \to \{C\}, \{B\} \to \{D\}, \{E\} \to \{B\}, \{D\} \to \{E\}\}$

(13) $\Sigma''' = \{\{A, E\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{B\}\}$

(14) $\Sigma''' = \{\{A, E\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$

(15) $\Sigma''' = \{\{A, E\} \to \{C\}, \{E\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$

Question 1 (d) Minimal Cover

NOTES FROM TA: These 15 minimal covers contains those cannot be obtained (aka. not reachable) through the 3-step method. You may design your algorithm to look into the **closure** (Σ^+) rather than the original Σ to find all of them.

You may confuse at the first glance. But don't worry, it's easy :D

Let's look at the first 5 minimal covers, with the visualization of Σ as the reference:

$\Sigma''' = \{\{A, B\} \rightarrow \{C\}, \{D\} \rightarrow \{B\}, \{B\} \rightarrow \{E\}, \{E\} \rightarrow \{D\}\}$

$\Sigma''' = \{\{A, B\} \rightarrow \{C\}, \{B\} \rightarrow \{D\}, \{E\} \rightarrow \{B\}, \{D\} \rightarrow \{E\}\}$

$\Sigma''' = \{\{A, B\} \rightarrow \{C\}, \{B\} \rightarrow \{D\}, \{D\} \rightarrow \{B\}, \{B\} \rightarrow \{E\}, \{E\} \rightarrow \{B\}\}$

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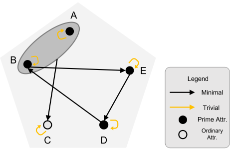

We find for each $\{??\} \rightarrow \{C\}$, there are **five** legal minimal covers.

Then we know there are **three** $\{??\} \rightarrow \{C\}$ possible $\{??\} \rightarrow \{C\}$.

Thus there exists $3 \times 5 = 15$ different minimal covers.

Question 1 (e) Armstrong Axioms

(e) Prove, using the three Armstrong axioms, that the following set of functional dependencies is equivalent to Σ.

$\Sigma'''' = \{\{A, B\} \rightarrow \{C, D, E\}, \{A, D\} \rightarrow \{B, C, E\}, \{A, E\} \rightarrow \{B, C, D\}, \{B\} \rightarrow \{D, E\}, \{D\} \rightarrow \{B, E\}, \{E\} \rightarrow \{B, D\}\}$.

Solution: We prove that every functional dependency in one of Σ and Σ'''' can obtained from those in the other set.

For example $\{B\} \rightarrow \{D, E\}$ can be obtained from Σ.

1. We know that $\{B\} \rightarrow \{E\}$.
2. We know that $\{E\} \rightarrow \{D\}$.
3. Therefore we have $\{E\} \rightarrow \{D, E\}$ by Augmentation of (2) with $\{E\}$.
4. Therefore we have $\{B\} \rightarrow \{D, E\}$ by Transitivity of (1) and (3).

Q.E.D.

In fact you can prove by change (3) to:
Therefore we have $\{B\} \rightarrow \{D\}$ by by Transitivity of (1) and (2).
Then we have $\{B\} \rightarrow \{D, E\}$ by Augmentation of (3) with (1).

For any further question, please feel free to email me:

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