CS4221/CS5421

Tutorial 5: Dependencies, entity-relationship modelling and the Chase

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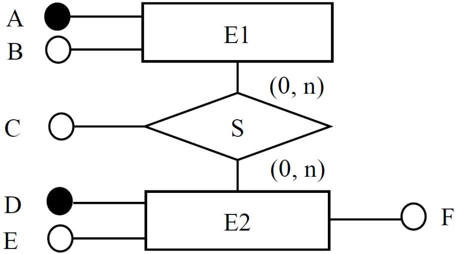
Question 1

Consider the entity-relationship diagram of Figure 1.



(a) Without other knowledge than that captured by the entity-relationship diagram, what are the **functional** and **multi-valued** dependencies?

Question 1 (Cont.)

Solution: The entity-relationship diagram tells us the following functional dependencies.

$\Sigma = \{\{A\} \rightarrow \{B\}, \{D\} \rightarrow \{E, F\}, \{A, D\} \rightarrow \{C\}\}$

Multi-valued dependencies depend on the translation of this design into tables. The entity- relationship diagram and its translation will only result in "not interesting" multi-valued dependencies, those are trivial or correspond to the table or the functional dependencies.

If we canonically translate this design into three tables, we have $\{A\} \twoheadrightarrow \{B\}$ and $\{C\} \twoheadrightarrow \{A, D\}$ and many more (none of them being useful for normalisation), for instance.

Question 1 (Cont.)

Note that additional knowledge of the application could tell us additional functional and multivalued dependencies not captured by the design.

For instance, we could know that $\{E\} \rightarrow \{F\}$ (which would suggest that the entity-relationship design is probably missing entities and relationships that have been merged too early), which we could use to produce a design in the Boyce-Codd normal by splitting the table $R_3(D, E, F)$ into two tables $R_{3.1} = (\underline{D}, E)$ and $R_{3.2} = (\underline{E}, F)$.

For instance, we could know that $\{E\} \twoheadrightarrow \{F\}$, which we could use to produce a design in the fifth normal by splitting the table $R_3(D, E, F)$ into two tables $R_{3.1} = (\underline{D}, E)$ and $R_{3.2} = (\underline{E, F})$.

Question 2

Consider the relational schema $R = \{A, B, C, D, E\}$ with the following set of functional and multi-valued dependencies.

$\Sigma = \{\{C\} \to \{A\}, \{D\} \to \{D, B\}, \{B\} \to \{E\}, \{E\} \twoheadrightarrow \{A, D\}, \{A, B, D\} \to \{A, B, C, D\}, \{B\} \to \{D\}\}$

(a) Prove that $\{E\} \to \{D\}$ using the Armstrong and multi-valued dependencies axioms.

Solution:

1. We know that $\{E\} \twoheadrightarrow \{A, D\}$.
2. We know that $\{B\} \to \{D\}$.
3. We see that $\{D\} \subset \{A, D\}$.
4. We see that $\{B\} \cap \{A, D\} = \emptyset$
5. Therefore $\{E\} \to \{D\}$ by Coalescence of (1), (2), (3) and (4).

Q.E.D.

Question 3

Consider the relation $R(A, B, C, D, E, G)$ with the following set, F, of functional and multi-valued dependencies.

$$F = \{\{A, B\} \rightarrow \{C\}, \{A, B\} \twoheadrightarrow \{E\}, \{C, D\} \twoheadrightarrow \{A, B\}\}$$

Prove that the decomposition of R into $R_1(A, B, C, D, G)$ and $R_2(C, D, E)$ is lossless using the Chase algorithm (as shown in the lecture).

Solution:

1. Initial table.

A	B	C	D	E	G
a1	b1	c1	d1	e1	g1
a2	b2	c2	d2	e2	g2

2. We want to chase $\{C, D\} \twoheadrightarrow \{E\}$, make C and D values the same.

A	B	C	D	E	G
a1	b1	**c1**	**d1**	e1	g1
a2	b2	**c1**	**d1**	e2	g2

Question 3 (Cont.)

3. Apply $\{C, D\} \twoheadrightarrow \{A, B\}$ by copying two tuples that have the same C and D values but swapping their A and B values.

A	B	C	D	E	G
a1	b1	c1	d1	e1	g1
a2	b2	c1	d1	e2	g2
a2	**b2**	c1	d1	e1	g1
a1	**b1**	c1	d1	e2	g2

4. Apply $\{A, B\} \twoheadrightarrow \{E\}$ by copying two tuples that have the same A and B values but swapping their A and E values.

A	B	C	D	E	G
a1	b1	c1	d1	e1	g1
a2	b2	c1	d1	e2	g2
a2	b2	c1	d1	e1	g1
a1	b1	c1	d1	e2	g2
a1	b1	c1	d1	**e2**	g1
a2	b2	c1	d1	**e1**	g2
a2	b2	c1	d1	**e2**	g1
a1	b1	c1	d1	**e1**	g2

Question 3 (Cont.)

5. Applying $\{A, B\} \to \{C\}$ do not change the table.
6. Sort the table and proved that $\{C, D\} \twoheadrightarrow \{E\}$

A	B	C	D	E	G
a1	b1	c1	d1	e1	g1
a1	b1	c1	d1	e2	g1
a2	b2	c1	d1	e1	g2
a2	b2	c1	d1	e2	g2
a1	b2	c1	d1	e1	g1
a1	b2	c1	d1	e2	g1
a2	b1	c1	d1	e2	g2
a2	b1	c1	d1	e1	g2

Q.E.D

Question 4

Consider the relation $R(A, B, C, D, E, F, G)$ with the following set, Σ, of functional dependencies.

$$\Sigma = \{\{A, B\} \rightarrow \{C\}, \{C\} \rightarrow \{D, E\}, \{E\} \rightarrow \{D\}, \{F\} \rightarrow \{G\}\}$$

Prove that the decomposition of R into $R_1(A, B, C, D, E)$ and $R_2(A, B, F, G)$ is lossless using the Chase algorithm (as shown in the lecture).

Solution:

1. Initial table.

A	B	C	D	E	F	G
a1	b1	c1	d1	e1	f1	g1
a2	b2	c2	d2	e2	f2	g2

2. We want to chase $\{A, B\} \twoheadrightarrow \{C, D, E\}$, make A and B values the same.

A	B	C	D	E	F	G
a1	**b1**	c1	d1	e1	f1	g1
a1	**b1**	c2	d2	e2	f2	g2

Question 4 (Cont.)

3. Apply $\{A, B\} \rightarrow \{C\}$, make C with the same A and B values the same.

A	B	C	D	E	F	G
a1	b1	**c1**	d1	e1	f1	g1
a1	b1	**c1**	d2	e2	f2	g2

4. Apply $\{C\} \rightarrow \{D, E\}$, make D and E with the same C values the same.

A	B	C	D	E	F	G
a1	b1	c1	**d1**	**e1**	f1	g1
a1	b1	c1	**d1**	**e1**	f2	g2

5. Applying $\{E\} \rightarrow \{D\}$ does not change the table.
6. Applying $\{F\} \rightarrow \{G\}$ does not change the table.
7. Proved that $\{A, B\} \twoheadrightarrow \{C, D, E\}$.

Q.E.D

Question 5

Consider the relation $R(A, B, C, D, E)$ with the following set, F, of functional dependencies.

$$\Sigma = \{\{A\} \to \{B, C\}, \{B\} \to \{A\}, \{C\} \to \{D\}\}$$

Check whether the decomposition of R into $R_1(A, E)$, $R_2(C, D)$ and $R_3(A, B, C)$ is lossless using the Chase algorithm with the distinguished attributes.

Solution:

1. Initial table.

	A	B	C	D	E
R_1	a				a
R_2			a	a	
R_3	a	a	a		

2. Apply $\{A\} \to \{B, C\}$.

	A	B	C	D	E
R_1	a	**a**	**a**		a
R_2			a	a	
R_3	a	a	a		

Question 5 (Cont.)

3. Applying $\{B\} \to \{A\}$ does not change the table.
4. Apply $\{C\} \to \{D\}$.

	A	**B**	**C**	**D**	**E**
R_1	a	a	a	**a**	a
R_2			a	a	
R_3	a	a	a		

5. R_1 has distinguished variables in all of the columns, therefore the decomposition is lossless.

Q.E.D

For any further question, please feel free to email me:

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Cases in the extra practice are contributed by our students.