BT5110 Data Management and Warehousing
Tutorial 5: Normalisation
(Extra Practice)

Mark Meng Huasong

School of Computing
National University of Singapore

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Updated on 8 Oct (Friday):

- Fixed some errors that have been raised in-class for the Case 4 and In-class Case 2. Refer to the red color notice text in relevant pages, and pay attention while viewing the Zoom recording.

Updated on 10 Oct (Sunday):

- Fixed the underline notation to indicate keys of R_1 in page 11.
  i.e., $\underline{S}, \underline{M} \to \underline{S}, M$
- Fixed the decomposition mistake of Case 3 in page 16.
  i.e., R_1 in the first option is not in BCNF, therefore a further decomposition is needed.

Updated on 17 Oct (Sunday):

- Updated the BCNF decomposition and highlight the process of finding "projected functional dependency" in the In-class Case 2 in page 30-32.

Updated on 9 Nov (Tuesday):

- Fixed the underline notation to indicate keys of R_2 in page 7.
  i.e., $\underline{W}, \underline{P} \to \underline{W}, P$
- Fixed the underline notation to indicate keys of R_2 and further decomposition in page 12.
  i.e., $\underline{S}, \underline{M} \to \underline{S}, M$

Agenda

- Extra Case No.1 - Warehouse management system
- Extra Case No.2 - University transcript issuing system
- Extra Case No.3 (Abstract) - *Non dependency preserving decomposition*
- Extra Case No.4 (Abstract) ‡ - *Candidate keys with different sizes*
- In-class Case No.1 (Abstract)
- In-class Case No.2 (Abstract) ‡ - *Trick of finding candidate key(s)*

‡ Updated in the latest version as some errors raised in-class have been fixed

♛ Boss Level Extra - Case 1

We are designing a warehouse management system for a lot of warehouses. Each warehouse (W) has one manager (M), and each manager only manage one warehouse. There could be many products (P) in per warehouse. For each product we also record its stock number (S).

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

☗ Boss Level Extra - Case 1 (Solution)

W: warehouse; **M**: manager; **P**: product; **S**: stock.

$R = \{W, M, P, S\}$
$\Sigma = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}, \{W, P\} \rightarrow \{S\}\}$.
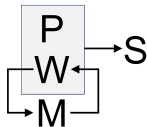
♛ Boss Level Extra - Case 1 (Solution)

$R = \{W, M, P, S\}$

$\Sigma = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}, \{W, P\} \rightarrow \{S\}\}$.

Solution:

(1) The attribute closure of R is:

$$\Sigma^+ = \{\{W\}^+ \rightarrow \{W, M\},$$
$$\{M\}^+ \rightarrow \{W, M\},$$
$$\{P\}^+ \rightarrow \{P\},$$
$$\{S\}^+ \rightarrow \{S\},$$
$$\{M, W\}^+ \rightarrow \{W, M\},$$
$$\{M, P\}^+ \rightarrow \{W, M, P, S\},$$
$$\{M, S\}^+ \rightarrow \{W, M, S\},$$
$$\{W, P\}^+ \rightarrow \{W, M, P, S\},$$
$$\{W, S\}^+ \rightarrow \{W, M, S\}, ...\}.$$

Now we find candidate keys: $\{W, P\}$ and $\{M, P\}$.

Prime attributes: W, M, P

♚ Boss Level Extra - Case 1 (Solution)

(2) The compact minimal cover is:

$\{W\} \rightarrow \{M\}$,
$\{M\} \rightarrow \{W\}$,
$\{W, P\} \rightarrow \{S\}$.

(3) Yes it is 2NF, 3NF, but not BCNF (e.g., $\{W\} \rightarrow \{M\}$, where $\{W\}$ is not a superkey).

(4) Omitted as it is 3NF.

(5) Decomposition at $\{W\} \rightarrow \{M\}$:

$R_1 = (\underline{W}, \underline{M})$,
$R_2 = (\underline{W}, \underline{P}, S)$.

It is (luckily) dependency preserving.

♜ Boss Level Extra - Case 2

We are designing a transcript issuing system for our university. Each student is identified by its matric number/student ID, written as S. We are going to record a grade (G) for each student (S) and each module (M). We also record students' names (N) and their faculty (F). In case any verification is needed, we also save the dean's name for each department (D) so that people can contact him/her.

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

S: student ID; **M**: module; **N**: name; **F**: faculty; **G**: grade; **D**: dean

$R = \{S, M, G, N, F, D\}$
$\Sigma = \{\{S, M\} \rightarrow \{G\}, \{S\} \rightarrow \{N, F\}, \{F\} \rightarrow \{D\}\}.$
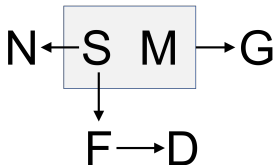
☗ Boss Level Extra - Case 2 (Solution)

$R = \{S, M, G, N, F, D\}$

$\Sigma = \{\{S, M\} \rightarrow \{G\}, \{S\} \rightarrow \{N, F\}, \{F\} \rightarrow \{D\}\}.$

Solution:

(1) The attribute closure of R is:

$$\Sigma^+ = \{\{S\}^+ \rightarrow \{S, N, F, D\},$$
$$\{M\}^+ \rightarrow \{M\},$$
$$\{G\}^+ \rightarrow \{G\},$$
$$\{N\}^+ \rightarrow \{N\},$$
$$\{F\}^+ \rightarrow \{F, D\},$$
$$\{D\}^+ \rightarrow \{D\},$$
$$\{S, M\}^+ \rightarrow \{S, M, N, F, D, G\},$$
$$\{S, G\}^+ \rightarrow \{S, G, N, F, D\}, ...$$

Now we find candidate keys: $\{S, M\}$.

Prime attributes: S, M

♛ Boss Level Extra - Case 2 (Solution)

(2) The compact minimal cover is:

$\{S, M\} \rightarrow \{G\}$,
$\{S\} \rightarrow \{N, F\}$,
$\{F\} \rightarrow \{D\}$.

(3) No it is not 2NF (e.g., $\{S\} \rightarrow \{N, F\}$, {N, F} are not prime attributes and S is a subset of candidate key). Therefore it is not 3NF, and not BCNF, too.

(4) Synthesis result has 3 relations and is (luckily) BCNF:

$R_1 = (\underline{S, M}, G)$,
$R_2 = (\underline{S}, N, F)$,
$R_3 = (\underline{F}, D)$.

♛ Boss Level Extra - Case 2 (Solution)

(5) Decomposition at $\{F\} \to \{D\}$:

$R_1 = (\underline{F}, D)$,
$R_2 = (\underline{S, M}, G, N, F)$.

However, R_2 needs to be further decomposed, at $\{S\} \to \{N\}$:

$R_{2.1} = (\underline{S}, N, F)$.
$R_{2.2} = (\underline{S, M}, G)$.

As the result, the BCNF decomposition is (luckily) dependency preserving and is given below:
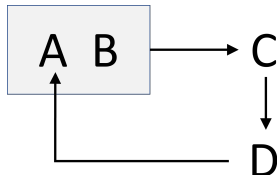
$R_1 = (\underline{F}, D)$,
$R_{2.1} = (\underline{S}, N, F)$.
$R_{2.2} = (\underline{S, M}, G)$.

(In fact this decomposition result is same with the synthesis one)

☻ Boss Level Extra - Case 3

This time we deal with abstract relations with functional dependencies shown as in the figure below:

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

♜ Boss Level Extra - Case 3 (Solution)

Solution:
(1) The attribute closure of $R = \{A, B, C, D\}$ is:

$$\Sigma^+ = \{\{A\}^+ \to \{A\},$$
$$\{B\}^+ \to \{B\},$$
$$\{C\}^+ \to \{A, C, D\},$$
$$\{D\}^+ \to \{A, D\},$$
$$\{A, B\}^+ \to \{A, B, C, D\},$$
$$\{A, C\}^+ \to \{A, C, D\},$$
$$\{A, D\}^+ \to \{A, D\},$$
$$\{B, C\}^+ \to \{A, B, C, D\}$$
$$\{B, D\}^+ \to \{A, B, C, D\}, \ldots$$

Now we find candidate keys: $\{A, B\}$ or $\{B, C\}$ or $\{B, D\}$.
Prime attributes: A, B, C, D (There is no non-prime attribute for this case)

☻ Boss Level Extra - Case 3 (Solution)

(2) The compact minimal cover is:

$\{A, B\} \rightarrow \{C\}$
$\{C\} \rightarrow \{D\}$
$\{D\} \rightarrow \{A\}$.

(3) Yes it is 2NF and 3NF (because all attributes are prime attributes). However, it is not BCNF (e.g., $\{C\} \rightarrow \{D\}$ and $\{D\} \rightarrow \{A\}$).

(4) Omitted as it is 3NF.

☸ Boss Level Extra - Case 3 (Solution)

(5) Decomposition at $\{C\} \rightarrow \{D\}$:

$R_1 = (A, \underline{C}, D)$,
$R_2 = (\underline{B}, \underline{C})$.

The R_1 is not in BCNF (e.g., $\{D\} \rightarrow \{A\}$), let's further decompose it:

$R_{1.1} = (A, \underline{D})$,
$R_{1.2} = (\underline{C}, D)$,

In this way all 3 relations ($R_{1.1}$, $R_{1.2}$ and R_2) are BCNF, but we lose a dependency $\{A, B\} \rightarrow \{C\}$.

How about we change the entry point of decomposition?

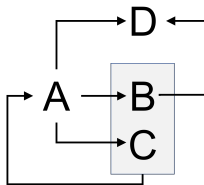Decomposition at $\{D\} \rightarrow \{A\}$:

$R_1 = (A, \underline{D})$,
$R_2 = (B, \underline{C}, D)$.

In this way both relations are BCNF, but we (still) lose that dependency $\{A, B\} \rightarrow \{C\}$.

♛ Boss Level Extra - Case 4

Another abstract relations with functional dependencies shown as in the figure below:

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

♛ Boss Level Extra - Case 4 (Solution)

Solution:

(1) The attribute closure of $R = \{A, B, C, D\}$ is:

$$\Sigma^+ = \{\{A\}^+ \rightarrow \{A, B, C, D\},$$
$$\{B\}^+ \rightarrow \{B, D\},$$
$$\{C\}^+ \rightarrow \{C\},$$
$$\{D\}^+ \rightarrow \{D\},$$
$$\{A, B\}^+ \rightarrow \{A, B, C, D\},$$
$$\{A, C\}^+ \rightarrow \{A, B, C, D\}(trivial),$$
$$\{A, D\}^+ \rightarrow \{A, B, C, D\}(trivial),$$
$$\{B, C\}^+ \rightarrow \{A, B, C, D\}$$
$$\{B, D\}^+ \rightarrow \{B, D\}, ...$$

Now we find candidate keys: $\{A\}$ or $\{B, C\}$.

Although $\{A\} \rightarrow \{B, C\}$, both of them are candidate keys because (1) their closures functionally determine all attributes of R; (2) both of them are minimal superkeys (cannot be further simplified).

Prime attributes: A, B, C.

☗ Boss Level Extra - Case 4 (Solution)

(2) First let's list all FDs given in the question:

$\{A\} \rightarrow \{B, C, D\}$, $\{B\} \rightarrow \{D\}$, $\{B, C\} \rightarrow \{A\}$.

Step 1: Simplify the RHS:

$\{A\} \rightarrow \{B\}$
$\{A\} \rightarrow \{C\}$
$\{A\} \rightarrow \{D\}$
$\{B\} \rightarrow \{D\}$
$\{B, C\} \rightarrow \{A\}$.

Step 2: Simplify the LHS
(There is only 1 FD with multiple attributes on the LHS and that FD cannot be simplified as there does not exist any FD implies A)

Step 3: Simplify the set:

$\{A\} \to \{B\}$
$\{A\} \to \{C\}$
$\cancel{\{A\} \to \{D\}}$ (because $\{A\} \to \{B\}$ and $\{B\} \to \{D\}$)
$\{B\} \to \{D\}$
$\{B, C\} \to \{A\}$.

Finally, the compact minimal cover is:

$\{A\} \to \{B, C\}$
$\{B\} \to \{D\}$
$\{B, C\} \to \{A\}$.

(3) No it is not 2NF (e.g., $\{B\} \rightarrow \{D\}$, where D is a non-prime attribute but B is a subset of candidate key). Therefore, it is not 3NF and not BCNF, too. (all because of $\{B\} \rightarrow \{D\}$)

(4) Synthesis result has 2 relations and each one is (luckily) BCNF:

$R_1 = (\underline{A}, \underline{B}, C)$,
$R_2 = (\underline{B}, D)$,
$\cancel{R_3 = (\underline{B}, \underline{C}, A)}$. (duplicate with R_1)

(5) Decomposition at $\{B\} \rightarrow \{D\}$:

$R_1 = (\underline{B}, D)$,
$R_2 = (\underline{A}, \underline{B}, C)$.

In this way both relations are BCNF, and luckily we obtain the exact same result as the 3NF synthesis, therefore it is dependency preserving.

♛ Boss Level Extra - In-class Case 1

Given a relation $R = \{A, B, C\}$ with functional dependency set:
$\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}$.

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

♛ Boss Level Extra - In-class Case 1 (Solution)

$R = \{A, B, C\}$
$\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}$.

Solution:
(1) The attribute closure of R is:

$$\Sigma^+ = \{\{A\}^+ \rightarrow \{A, B, C\},$$
$$\{B\}^+ \rightarrow \{A, B, C\},$$
$$\{C\}^+ \rightarrow \{C\},$$
$$\{A, B\}^+ \rightarrow \{A, B, C\},$$
$$\{A, C\}^+ \rightarrow \{A, B, C\},$$
$$\{B, C\}^+ \rightarrow \{A, B, C\},$$
$$\{A, B, C\}^+ \rightarrow \{A, B, C\}\}.$$

Now we find candidate keys: $\{A\}$ and $\{B\}$.
Prime attributes: A, B

☠ Boss Level Extra - In-class Case 1 (Solution)

$\Sigma = \{\{A\} \to \{B\}, \{B\} \to \{C\}, \{A, B\} \to \{C\}, \{B, C\} \to \{A\}\}$.

(2) The minimal cover is:

$\{A\} \to \{B\}$,
$\{B\} \to \{C\}$,
$\{B, C\} \to \{A\}$.

This is the compact minimal cover, too.

(3) Yes it is 2NF, 3NF, and BCNF, too.

(4) Omitted as it is 3NF.

(5) Omitted as it is BCNF.

Given a relation $R = \{A, B, C, D, E\}$ with functional dependency set:
$\Sigma = \{\{C, D\} \to \{E\}, \{A, B\} \to \{B\}, \{A, C, D\} \to \{E\}, \{A\} \to \{E\}, \{D, E\} \to \{B, C\}, \{A\} \to \{A\}\}$.

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

$\Sigma = \{\{C, D\} \to \{E\}, \{A, B\} \to \{B\}, \{A, C, D\} \to \{E\}, \{A\} \to \{E\}, \{D, E\} \to \{B, C\}, \{A\} \to \{A\}\}.$

Solution:

(1) The attribute closure of R is:

$\Sigma^+ = \{\{A\}^+ \to \{A, E\},$

$\{B\}^+ \to \{B\},$

$\{C\}^+ \to \{C\},$

$\{D\}^+ \to \{D\},$

$\{E\}^+ \to \{E\},$

$\{A, B\}^+ \to \{A, B, E\},$

$\{A, C\}^+ \to \{A, C, E\},$

$\{A, D\}^+ \to \{A, B, C, D, E\},$

$\{A, E\}^+ \to \{A, B, E\}\}$

...

$\{A, C, D, E\}^+ \to \{A, B, C, D, E\}\}$

$\{A, B, C, D, E\}^+ \to \{A, B, C, D, E\}\}.$

Hint: After removing all trivial FDs, we find A and D have never appeared in the right-hand side, which means the candidate key must contains A and D. We just need to find the minimal set that implies $\{B, C, E\}$. (Luckily both A and D together can determine $\{B, C, E\}$)

Then we can find candidate key: $\{A, D\}$. And the prime attributes: A, D

$\Sigma = \{\{C, D\} \rightarrow \{E\}, \{A, B\} \rightarrow \{B\}, \{A, C, D\} \rightarrow \{E\}, \{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}, \{A\} \rightarrow \{A\}\}$.

(2) The compact minimal cover is:

STEP 1

Simplify the RHS by the Union Rule, i.e., if $\{X\} \rightarrow \{YZ\}$, then split into $\{X\} \rightarrow \{Y\}$ and $\{X\} \rightarrow \{Z\}$

$\{C, D\} \rightarrow \{E\}$,
$\{A, B\} \rightarrow \{B\}$,
$\{A\} \rightarrow \{E\}$,
$\{A, C, D\} \rightarrow \{E\}$,
$\{D, E\} \rightarrow \{B\}$.
$\{D, E\} \rightarrow \{C\}$.
$\{A\} \rightarrow \{A\}$

STEP 2

Simplify the LHS, i.e., if given $\{X\} \rightarrow \{Y\}$ and $\{XB\} \rightarrow \{Y\}$, then we omit $\{XB\} \rightarrow \{Y\}$

$\{C, D\} \rightarrow \{E\}$,
$\{A, B\} \rightarrow \{B\}$,
$\{A, C, D\} \rightarrow \{E\}$,
$\{A\} \rightarrow \{E\}$,
$\{D, E\} \rightarrow \{B\}$.
$\{D, E\} \rightarrow \{C\}$.
$\{A\} \rightarrow \{A\}$

STEP 3 (compact)

Check if every FD is non-trivial and minimal, i.e., if given $\{X\} \rightarrow \{Y\}$, we need to ensure there does not exist an A s.t. $\{X\} \rightarrow \{A\}$ and $\{A\} \rightarrow \{Y\}$

The finalized set is:
$\{C, D\} \rightarrow \{E\}$,
$\{A\} \rightarrow \{E\}$,
$\{D, E\} \rightarrow \{B, C\}$
$\{A\} \rightarrow \{A\}$.

🐲 Boss Level Extra - In-class Case 2 (Solution)

(3) It is not in 2NF, therefore it is not in 3NF, and not in BCNF, too.

E.g., $\{A\} \to \{E\}$ is not trivial, the LHS is a subset of the candidate key $\{A, D\}$ and the RHS is not a prime attribute, therefore it violates 2NF.

For 3NF and beyond, we find all FDs in the compact minimal cover are violations (all RHS are not prime attributes and all LHS are not superkeys).

(4) Recall the compact minimal cover:

$\{C, D\} \to \{E\}$, $\{A\} \to \{E\}$, $\{D, E\} \to \{B, C\}$.

Let's do 3NF synthesis:

$R_1 = (\underline{C, D}, E)$, $R_2 = (\underline{A}, E)$, $R_3 = (B, C, \underline{D, E})$.

(The candidate key is not included in any relation fragment above, so we add a new fragment manually)

$R_4 = (\underline{A, D})$.

All fragments are in BCNF.

♚ Boss Level Extra - In-class Case 2 (Solution)

(5) Recall the compact minimal cover:
$\{C, D\} \rightarrow \{E\}$, $\{A\} \rightarrow \{E\}$, $\{D, E\} \rightarrow \{B, C\}$.
Let's start decomposition at $\{A\} \rightarrow \{E\}$:
$R_1 = (\underline{A}, E)$, We can find that $\Sigma_1 = \{\{A\} \rightarrow \{E\}\}$, and
$R_2 = (\underline{A, D}, B, C)$.
Now it's the most challenging part: **find all projected FDs in the fragment**. Any mistake in this step would affect your determining if the fragment is in BCNF (according to the new candidate key(s)), and consequentially mislead your further decomposition.

Find the projected FDs in a fragment

Here we find none of FDs in the minimal cover could be preserved in R_2. But does it means we lost all FDs? How to find all remaining FDs in fragments?

♛ Boss Level Extra - In-class Case 2 (Solution)

Although E is not presented in R_2, we still have $\{A, D\} \rightarrow \{B, C\}$ because of $\{A\} \rightarrow \{E\}$ and $\{D, E\} \rightarrow \{B, C\}$.

One way to find those FDs is to go back and look at the attribute closure, by removing those attributes do not exist in the current fragment, as follows:

$\Sigma^+ = \{\{A\}^+ \rightarrow \{A, \cancel{E}\},$

$\{B\}^+ \rightarrow \{B\},$

$\{C\}^+ \rightarrow \{C\},$

$\{D\}^+ \rightarrow \{D\},$

$\cancel{\{E\}^+ \rightarrow \{E\}},$

$\{A, B\}^+ \rightarrow \{A, B, \cancel{E}\},$

$\{A, C\}^+ \rightarrow \{A, C, \cancel{E}\},$

$\{A, D\}^+ \rightarrow \{A, B, C, D, \cancel{E}\},$

$\cancel{\{A, E\}^+ \rightarrow \{A, B, E\}}\}$

...

$\{A, C, D, \cancel{E}\}^+ \rightarrow \{A, B, C, D, \cancel{E}\}\}$

$\{A, B, C, D, \cancel{E}\}^+ \rightarrow \{A, B, C, D, \cancel{E}\}\}.$

Then we know $\{A, D\}$ is the candidate key for R_2.

So the first round of BCNF decomposition gives us 2 fragments:

$R_1 = \{\underline{A}, E\}$ with $\Sigma_1 = \{\{A\} \rightarrow \{E\}\}$;
and $R_2 = \{A, D, B, C\}$ with $\Sigma_2 = \{\{A, \underline{D}\} \rightarrow \{B, C\}\}$.

We find both fragment are in BCNF, however we loss all FDs except $\{A\} \rightarrow \{E\}$ from the minimal cover set.

♛ Boss Level Extra - In-class Case 2 (Solution)

If you try to use a FD other than $\{A\} \to \{E\}$ in BCNF decomposition, for example $\{C, D\} \to \{E\}$, you may have two fragments $R_1 = \{B, C, D, E\}$ (candidate keys to be unveiled later) and $R_2 = \{\underline{A, C, D}\}$.

Finding candidate key(s) and minimal cover of R_1 is the next challenge:

$\Sigma_1^+ = \{\ldots$

$\quad \{B, E\}^+ \to \{B, E\},$

$\quad \{C, D\}^+ \to \{B, C, D, E\},$

$\quad \{C, E\}^+ \to \{C, E\},$

$\quad \{D, E\}^+ \to \{B, C, D, E\},$

$\quad \ldots$

$\quad \{C, D, E\}^+ \to \{B, C, D, E\},$

$\quad \{B, C, D, E\}^+ \to \{B, C, D, E\}\}.$

With the LHS, we identify R_1's candidate key(s) as $\{D, E\}$ and $\{C, D\}$, and the FDs in the minimal cover of that fragment is $\{\{C, D\} \to \{E\}, \{D, E\} \to \{B, C\}\}$.

Both of these two FDs are in BCNF.

According to the textbook referred in this module (Database Management Systems, Ramakrishnan & Gehrke), there is another (more) ineffective algorithm to decompose $\{A, B\}$ into $\{C, D, E\}$. You can borrow the book from NUS library if you have interest to read.

For any further question, please feel free to email me:

huasong.meng@u.nus.edu