CS4221/CS5421

Normal Forms and Normalisation
(Extra Practice)

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Week 7, 2022 Spring

Agenda

- Extra Case No.1 - Warehouse management system
- Extra Case No.2 - University transcript issuing system
- Extra Case No.3 (Abstract) - *Non dependency preserving decomposition*
- Extra Case No.4 (Abstract) - *Candidate keys with different sizes*
- Extra Case No.5 (Abstract)
- Extra Case No.6 (Abstract) - *Trick in finding candidate key(s)*

Extra - Case 1

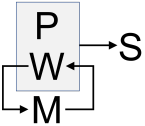
We are designing a warehouse management system for a lot of warehouses. Each warehouse (W) has one manager (M), and each manager only manage one warehouse. There could be many products (P) in per warehouse. For each product we also record its stock number (S).

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

Extra - Case 1 (Solution)

W: warehouse; **M**: manager; **P**: product; **S**: stock.

$R = \{W, M, P, S\}$
$\Sigma = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}, \{W, P\} \rightarrow \{S\}\}$.



Extra - Case 1 (Solution)

$R = \{W, M, P, S\}$
$\Sigma = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}, \{W, P\} \rightarrow \{S\}\}$.

Solution:
(1) The attribute closure of R is:

$$\begin{aligned}
\Sigma^+ = \{&\{W\}^+ \rightarrow \{W, M\},\\
&\{M\}^+ \rightarrow \{W, M\},\\
&\{P\}^+ \rightarrow \{P\},\\
&\{S\}^+ \rightarrow \{S\},\\
&\{M, W\}^+ \rightarrow \{W, M\},\\
&\{M, P\}^+ \rightarrow \{W, M, P, S\},\\
&\{M, S\}^+ \rightarrow \{W, M, S\},\\
&\{W, P\}^+ \rightarrow \{W, M, P, S\},\\
&\{W, S\}^+ \rightarrow \{W, M, S\}, ...\}.
\end{aligned}$$

Now we find candidate keys: $\{W, P\}$ and $\{M, P\}$.
Prime attributes: W, M, P

Extra - Case 1 (Solution)

(2) The compact minimal cover is:

$\{W\} \rightarrow \{M\}$,
$\{M\} \rightarrow \{W\}$,
$\{W, P\} \rightarrow \{S\}$.

(3) Yes it is 2NF, 3NF, but not BCNF (e.g., $\{W\} \rightarrow \{M\}$, where $\{W\}$ is not a superkey).

(4) Omitted as it is 3NF.

(5) Decomposition at $\{W\} \rightarrow \{M\}$:

$R_1 = (\underline{W}, \underline{M})$, with $\Sigma_1 = \{\{W\} \rightarrow \{M\}, \{M\} \rightarrow \{W\}\}$
$R_2 = (\underline{W, P}, S)$, with $\Sigma_2 = \{\{W, P\} \rightarrow \{S\}\}$

It is (luckily) dependency preserving.

Extra - Case 2

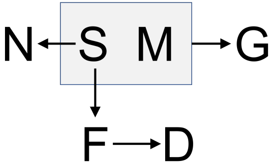
We are designing a transcript issuing system for our university. Each student is identified by its matric number/student ID, written as S. We are going to record a grade (G) for each student (S) and each module (M). We also record students' names (N) and their faculty (F). In case any verification is needed, we also save the dean's name for each department (D) so that people can contact him/her.

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

Extra - Case 2 (Solution)

S: student ID; **M**: module; **N**: name; **F**: faculty; **G**: grade; **D**: dean

$R = \{S, M, G, N, F, D\}$
$\Sigma = \{\{S, M\} \rightarrow \{G\}, \{S\} \rightarrow \{N, F\}, \{F\} \rightarrow \{D\}\}.$



Extra - Case 2 (Solution)

$R = \{S, M, G, N, F, D\}$
$\Sigma = \{\{S, M\} \to \{G\}, \{S\} \to \{N, F\}, \{F\} \to \{D\}\}.$

Solution:
(1) The attribute closure of R is:

$$\begin{aligned}
\Sigma^+ = \{&\{S\}^+ \to \{S, N, F, D\},\\
&\{M\}^+ \to \{M\},\\
&\{G\}^+ \to \{G\},\\
&\{N\}^+ \to \{N\},\\
&\{F\}^+ \to \{F, D\},\\
&\{D\}^+ \to \{D\},\\
&\{S, M\}^+ \to \{S, M, N, F, D, G\},\\
&\{S, G\}^+ \to \{S, G, N, F, D\}, \ldots
\end{aligned}$$

Now we find candidate keys: $\{S, M\}$.
Prime attributes: S, M

Extra - Case 2 (Solution)

(2) The compact minimal cover is:

$\{S, M\} \rightarrow \{G\}$,
$\{S\} \rightarrow \{N, F\}$,
$\{F\} \rightarrow \{D\}$.

(3) No it is not 2NF (e.g., $\{S\} \rightarrow \{N, F\}$, {N, F} are **not** prime attributes and S **is** a subset of candidate key). Therefore it is not 3NF, and not BCNF, too.
(Besides $\{S\} \rightarrow \{N, F\}$, $\{F\} \rightarrow \{D\}$ violates 3NF and BCNF, too)

(4) Synthesis result has 3 relations and is (luckily) BCNF:

$R_1 = (\underline{S, M}, G)$, with $\Sigma_1 = \{\{S, M\} \rightarrow \{G\}\}$
$R_2 = (\underline{S}, N, F)$, with $\Sigma_2 = \{\{S\} \rightarrow \{N, F\}\}$
$R_3 = (\underline{F}, D)$, with $\Sigma_3 = \{\{F\} \rightarrow \{D\}\}$

Extra - Case 2 (Solution)

(5) Decomposition at $\{F\} \rightarrow \{D\}$:

$R_1 = (\underline{F}, D)$, with $\Sigma_1 = \{\{F\} \rightarrow \{D\}\}$, which is in BCNF
$R_2 = (\underline{S, M}, G, N, F)$, with $\Sigma_2 = \{\{S, M\} \rightarrow \{G\}, \{S\} \rightarrow \{N, F\}\}$

The $\{S\} \rightarrow \{N, F\}$ violates BCNF because the LHS is not a superkey, therefore R_2 needs to be further decomposed, at $\{S\} \rightarrow \{N, F\}$:

$R_{2.1} = (\underline{S}, N, F)$, with $\Sigma_{2.1} = \{\{S\} \rightarrow \{N, F\}\}$
$R_{2.2} = (\underline{S, M}, G)$, with $\Sigma_{2.2} = \{\{S, M\} \rightarrow \{G\}\}$

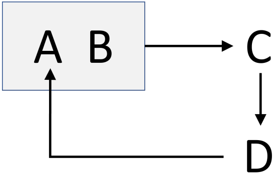
As the result, the BCNF decomposition is (luckily) dependency preserving and is given below:

$R_1 = (\underline{F}, D)$,
$R_{2.1} = (\underline{S}, N, F)$.
$R_{2.2} = (\underline{S, M}, G)$.

(In fact this decomposition result is same with the synthesis one)

Extra - Case 3

This time we deal with abstract relations with functional dependencies shown as in the figure below:



Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

Extra - Case 3 (Solution)

Solution:
(1) The attribute closure of $R = \{A, B, C, D\}$ is:

$$
\begin{aligned}
\Sigma^+ = \{ & \{A\}^+ \rightarrow \{A\}, \\
& \{B\}^+ \rightarrow \{B\}, \\
& \{C\}^+ \rightarrow \{A, C, D\}, \\
& \{D\}^+ \rightarrow \{A, D\}, \\
& \{A, B\}^+ \rightarrow \{A, B, C, D\}, \\
& \{A, C\}^+ \rightarrow \{A, C, D\}, \\
& \{A, D\}^+ \rightarrow \{A, D\}, \\
& \{B, C\}^+ \rightarrow \{A, B, C, D\} \\
& \{B, D\}^+ \rightarrow \{A, B, C, D\}, \ldots
\end{aligned}
$$

Now we find candidate keys: $\{A, B\}$ or $\{B, C\}$ or $\{B, D\}$.
Prime attributes: A, B, C, D (There is no non-prime attribute for this case)

Extra - Case 3 (Solution)

(2) The compact minimal cover is:

$\{A, B\} \rightarrow \{C\}$
$\{C\} \rightarrow \{D\}$
$\{D\} \rightarrow \{A\}$.

(3) Yes it is 2NF and 3NF (because all attributes are prime attributes). However, it is not BCNF (e.g., $\{C\} \rightarrow \{D\}$ and $\{D\} \rightarrow \{A\}$).

(4) Omitted as it is 3NF.

Extra - Case 3 (Solution)

(5) Decomposition at $\{C\} \rightarrow \{D\}$:

$R_1 = (A, \underline{C}, D)$, with $\Sigma_1 = \{\{C\} \rightarrow \{D\}, \{D\} \rightarrow \{A\}\}$
$R_2 = (\underline{B, C})$, with $\Sigma_2 = \emptyset$

The R_1 is not in BCNF (e.g., $\{D\} \rightarrow \{A\}$, the LHS is not superkey), let's further decompose it:

$R_{1.1} = (A, \underline{D})$, with $\Sigma_{1.1} = \{\{D\} \rightarrow \{A\}\}$
$R_{1.2} = (\underline{C}, D)$, with $\Sigma_{1.2} = \{\{C\} \rightarrow \{D\}\}$

In this way all 3 relations ($R_{1.1}$, $R_{1.2}$ and R_2) are BCNF, but we lose a dependency $\{A, B\} \rightarrow \{C\}$.

How about we change the entry point of decomposition?

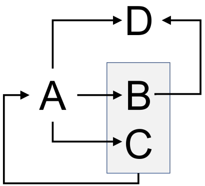
Decomposition at $\{D\} \rightarrow \{A\}$:

$R_1 = (A, \underline{D})$,
$R_2 = (\underline{B, C}, D)$.

Then R_2 needs to be further decomposed to
$R_2.1 = (\underline{C}, D)$, $R_{2.2} = (\underline{B, C})$. to ensure all of them are BCNF. However, we (still) lose that dependency $\{A, B\} \rightarrow \{C\}$.

Extra - Case 4

Another abstract relations with functional dependencies shown as in the figure below:



Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

Extra - Case 4 (Solution)

Solution:
(1) The attribute closure of $R = \{A, B, C, D\}$ is:

$$\begin{aligned}
\Sigma^+ = \{&\{A\}^+ \rightarrow \{A, B, C, D\},\\
&\{B\}^+ \rightarrow \{B, D\},\\
&\{C\}^+ \rightarrow \{C\},\\
&\{D\}^+ \rightarrow \{D\},\\
&\{A, B\}^+ \rightarrow \{A, B, C, D\},\\
&\{A, C\}^+ \rightarrow \{A, B, C, D\}(\text{not minimal}),\\
&\{A, D\}^+ \rightarrow \{A, B, C, D\}(\text{not minimal}),\\
&\{B, C\}^+ \rightarrow \{A, B, C, D\}\\
&\{B, D\}^+ \rightarrow \{B, D\}, \ldots
\end{aligned}$$

Now we find candidate keys: $\{A\}$ or $\{B, C\}$.
Although $\{A\} \rightarrow \{B, C\}$, both of them are candidate keys because (1) their closures functionally determine all attributes of R; (2) both of them are minimal superkeys (cannot be further simplified).
Prime attributes: A, B, C.

Extra - Case 4 (Solution)

(2) First let's list all FDs given in the question:

$\{A\} \rightarrow \{B, C, D\}$, $\{B\} \rightarrow \{D\}$, $\{B, C\} \rightarrow \{A\}$.

Step 1: Simplify the RHS:

$\{A\} \rightarrow \{B\}$
$\{A\} \rightarrow \{C\}$
$\{A\} \rightarrow \{D\}$
$\{B\} \rightarrow \{D\}$
$\{B, C\} \rightarrow \{A\}$.

Step 2: Simplify the LHS
(There is only 1 FD with multiple attributes on the LHS and that FD cannot be simplified as there does not exist any FD implies A)

Extra - Case 4 (Solution)

Step 3: Simplify the set:

$\{A\} \rightarrow \{B\}$
$\{A\} \rightarrow \{C\}$
$\cancel{\{A\} \rightarrow \{D\}}$ (because $\{A\} \rightarrow \{B\}$ and $\{B\} \rightarrow \{D\}$)
$\{B\} \rightarrow \{D\}$
$\{B, C\} \rightarrow \{A\}$.

Finally, the compact minimal cover is:

$\{A\} \rightarrow \{B, C\}$
$\{B\} \rightarrow \{D\}$
$\{B, C\} \rightarrow \{A\}$.

Extra - Case 4 (Solution)

(3) No it is not 2NF (e.g., $\{B\} \rightarrow \{D\}$, where D is a non-prime attribute but B is a subset of candidate key). Therefore, it is not 3NF and not BCNF, too. (all because of $\{B\} \rightarrow \{D\}$)

(4) Synthesis result has 2 relations and each one is (luckily) BCNF:

$R_1 = (\underline{A}, \underline{B, C})$,
$R_2 = (\underline{B}, D)$,
$\cancel{R_3 = (\underline{B, C}, \underline{A})}$. (duplicate with R_1)

Extra - Case 4 (Solution)

(5) Decomposition at $\{B\} \rightarrow \{D\}$:

$R_1 = (\underline{B}, D)$,
$R_2 = (\underline{A}, \underline{B, C})$.

In this way both relations are BCNF, and luckily we obtain the exact same result as the 3NF synthesis, therefore it is dependency preserving.

Extra - Case 5

Given a relation $R = \{A, B, C\}$ with functional dependency set: $\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}$.

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.

Extra - Case 5 (Solution)

$R = \{A, B, C\}$
$\Sigma = \{\{A\} \to \{B\}, \{B\} \to \{C\}, \{A, B\} \to \{C\}, \{B, C\} \to \{A\}\}.$

Solution:
(1) The attribute closure of R is:

$$\begin{aligned}\Sigma^+ = \{&\{A\}^+ \to \{A, B, C\},\\ &\{B\}^+ \to \{A, B, C\},\\ &\{C\}^+ \to \{C\},\\ &\{A, B\}^+ \to \{A, B, C\},\\ &\{A, C\}^+ \to \{A, B, C\},\\ &\{B, C\}^+ \to \{A, B, C\},\\ &\{A, B, C\}^+ \to \{A, B, C\}\}.\end{aligned}$$

Now we find candidate keys: $\{A\}$ and $\{B\}$.
Prime attributes: A, B

Extra - Case 5 (Solution)

$$\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}, \{A, B\} \rightarrow \{C\}, \{B, C\} \rightarrow \{A\}\}.$$

(2) The minimal cover is:

$\{A\} \rightarrow \{B\}$,
$\{B\} \rightarrow \{C\}$,
$\{B\} \rightarrow \{A\}$.

The compact minimal cover is $\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{A, C\}$.

(3) Yes it is 2NF, 3NF, and BCNF, too.

(Hint: All LHS are superkeys)

(4) Omitted as it is 3NF.

(5) Omitted as it is BCNF.

Extra - Case 6

Given a relation $R = \{A, B, C, D, E\}$ with functional dependency set: $\Sigma = \{\{C, D\} \rightarrow \{E\}, \{A, B\} \rightarrow \{B\}, \{A, C, D\} \rightarrow \{E\}, \{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}, \{A\} \rightarrow \{A\}\}$.

Questions:
(1) Find **candidate key(s)** and **prime attribute(s)** from attribute closures Σ^+.
(2) Compute the **compact minimal cover** of R with all FDs Σ.
(3) Determine if it is **2NF**? If yes, is it **3NF**? If yes, is it **BCNF**?
(4) If it is not 3NF, **synthesis** the relations to make it 3NF.
(5) If it is not BCNF, **decomposite** the relations to make it BCNF and verify the **dependency preservation**.
(This question's BCNF decomposition requests some tricks. It is out of the scope so don't worry if you feel hard to understand)

Extra - Case 6 (Solution)

$\Sigma = \{\{C, D\} \to \{E\}, \{A, B\} \to \{B\}, \{A, C, D\} \to \{E\}, \{A\} \to \{E\}, \{D, E\} \to \{B, C\}, \{A\} \to \{A\}\}.$

Solution:

(1) The attribute closure of R is:

$$
\begin{aligned}
\Sigma^+ = \{&\{A\}^+ \to \{A, E\},\\
&\{B\}^+ \to \{B\},\\
&\{C\}^+ \to \{C\},\\
&\{D\}^+ \to \{D\},\\
&\{E\}^+ \to \{E\},\\
&\{A, B\}^+ \to \{A, B, E\},\\
&\{A, C\}^+ \to \{A, C, E\},\\
&\{A, D\}^+ \to \{A, B, C, D, E\},\\
&\{A, E\}^+ \to \{A, E\}\}\\
&\ldots\\
&\{A, C, D, E\}^+ \to \{A, B, C, D, E\}\}\\
&\{A, B, C, D, E\}^+ \to \{A, B, C, D, E\}\}.
\end{aligned}
$$

Hint: After removing all trivial FDs, we find A and D have never appeared in the right-hand side, which means the candidate key must contains A and D. We just need to find the minimal set that implies $\{B, C, E\}$. (Luckily both A and D together can determine $\{B, C, E\}$)

Then we can find candidate key: $\{A, D\}$. And the prime attributes: A, D

Extra - Case 6 (Solution)

$\Sigma = \{\{C, D\} \rightarrow \{E\}, \{A, B\} \rightarrow \{B\}, \{A, C, D\} \rightarrow \{E\}, \{A\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}, \{A\} \rightarrow \{A\}\}$.

(2) The compact minimal cover is:

STEP 1

Simplify the RHS by the Union Rule, i.e., if $\{X\} \rightarrow \{YZ\}$, then split into $\{X\} \rightarrow \{Y\}$ and $\{X\} \rightarrow \{Z\}$

$\{C, D\} \rightarrow \{E\}$,
$\{A, B\} \rightarrow \{B\}$,
$\{A\} \rightarrow \{E\}$,
$\{A, C, D\} \rightarrow \{E\}$,
$\{D, E\} \rightarrow \{B\}$.
$\{D, E\} \rightarrow \{C\}$.
$\{A\} \rightarrow \{A\}$

STEP 2

Simplify the LHS, i.e., if given $\{X\} \rightarrow \{Y\}$ and $\{XB\} \rightarrow \{Y\}$, then we omit $\{XB\} \rightarrow \{Y\}$

$\{C, D\} \rightarrow \{E\}$,
$\cancel{\{A, B\} \rightarrow \{B\}}$,
$\cancel{\{A, C, D\} \rightarrow \{E\}}$,
$\{A\} \rightarrow \{E\}$,
$\{D, E\} \rightarrow \{B\}$.
$\{D, E\} \rightarrow \{C\}$.
$\{A\} \rightarrow \{A\}$

STEP 3 (compact)

Check if every FD is non-trivial and minimal, i.e., if given $\{X\} \rightarrow \{Y\}$, we need to ensure there does not exist an A s.t. $\{X\} \rightarrow \{A\}$ and $\{A\} \rightarrow \{Y\}$

The finalized set is:
$\{C, D\} \rightarrow \{E\}$,
$\{A\} \rightarrow \{E\}$,
$\{D, E\} \rightarrow \{B, C\}$
$\cancel{\{A\} \rightarrow \{A\}}$.

Extra - Case 6 (Solution)

(3) It is not in 2NF, therefore it is not in 3NF, and not in BCNF, too.

E.g., $\{A\} \to \{E\}$ is not trivial, the LHS is a subset of the candidate key $\{A, D\}$ and the RHS is not a prime attribute, therefore it violates 2NF.

For 3NF and beyond, we find all FDs in the compact minimal cover are violations (all RHS are not prime attributes and all LHS are not superkeys).

(4) Recall the compact minimal cover:

$\{C, D\} \to \{E\}$, $\{A\} \to \{E\}$, $\{D, E\} \to \{B, C\}$.

Let's do 3NF synthesis:

$R_1 = (\underline{C, D}, E)$ (subsumed by R_3), $R_2 = (\underline{A}, E)$, $R_3 = (B, C, \underline{D, E})$.

(The candidate key is not included in any relation fragment above, so we add a new fragment manually)

$R_4 = (\underline{A, D})$.

The results are R_2, R_3 and R_4.

Extra - Case 6 (Solution)

(5) Recall the compact minimal cover:
$\{C, D\} \rightarrow \{E\}$, $\{A\} \rightarrow \{E\}$, $\{D, E\} \rightarrow \{B, C\}$.
Let's start decomposition at $\{A\} \rightarrow \{E\}$:
$R_1 = (\underline{A}, E)$, We can find that $\Sigma_1 = \{\{A\} \rightarrow \{E\}\}$, and
$R_2 = (\underline{A, D}, B, C)$.

Now it's the most challenging part: **find all projected FDs in the fragment**. Any mistake in this step would affect your determining if the fragment is in BCNF (according to the new candidate key(s)), and consequentially mislead your further decomposition.

Find the projected FDs in a fragment

Here we find none of FDs in the minimal cover could be preserved in R_2. But does it means we lost all FDs? How to find all remaining FDs in fragments?

Extra - Case 6 (Solution)

Although E is not presented in R_2, we still have $\{A, D\} \rightarrow \{B, C\}$ because of $\{A\} \rightarrow \{E\}$ and $\{D, E\} \rightarrow \{B, C\}$.
One way to find those FDs is to go back and look at the attribute closure, by removing those attributes do not exist in the current fragment, as follows:

$$
\begin{aligned}
\Sigma^+ = \{&\{A\}^+ \rightarrow \{A, \cancel{E}\},\\
&\{B\}^+ \rightarrow \{B\},\\
&\{C\}^+ \rightarrow \{C\},\\
&\{D\}^+ \rightarrow \{D\},\\
&\cancel{\{E\}^+ \rightarrow \{E\}},\\
&\{A, B\}^+ \rightarrow \{A, B, \cancel{E}\},\\
&\{A, C\}^+ \rightarrow \{A, C, \cancel{E}\},\\
&\{A, D\}^+ \rightarrow \{A, B, C, D, \cancel{E}\},\\
&\cancel{\{A, E\}^+ \rightarrow \{A, B, E\}\}}\\
&\ldots\\
&\{A, C, D, \cancel{E}\}^+ \rightarrow \{A, B, C, D, \cancel{E}\}\}\\
&\{A, B, C, D, \cancel{E}\}^+ \rightarrow \{A, B, C, D, \cancel{E}\}\}.
\end{aligned}
$$

Then we know $\{A, D\}$ is the candidate key for R_2.

So the first round of BCNF decomposition gives us 2 fragments:

$R_1 = \{\underline{A}, E\}$ with
$\Sigma_1 = \{\{A\} \rightarrow \{E\}\}$;
and $R_2 = \{\underline{A, D}, B, C\}$ with
$\Sigma_2 = \{\{A, D\} \rightarrow \{B, C\}\}$.

We find both fragment are in BCNF, however we loss all FDs except $\{A\} \rightarrow \{E\}$ from the minimal cover set.

Extra - Case 6 (Solution)

If you try to use a FD other than $\{A\} \rightarrow \{E\}$ in BCNF decomposition, for example $\{C, D\} \rightarrow \{E\}$, you may have two fragments $R_1 = \{B, C, D, E\}$ (candidate keys to be unveiled later) and $R_2 = \{\underline{A, D}, C\}$.
Finding candidate key(s) and minimal cover of R_1 is the next challenge:

$$\Sigma_1^+ = \{\ldots$$
$$\{B, E\}^+ \rightarrow \{B, E\},$$
$$\{C, D\}^+ \rightarrow \{B, C, D, E\},$$
$$\{C, E\}^+ \rightarrow \{C, E\},$$
$$\{D, E\}^+ \rightarrow \{B, C, D, E\},$$
$$\ldots$$
$$\{C, D, E\}^+ \rightarrow \{B, C, D, E\},$$
$$\{B, C, D, E\}^+ \rightarrow \{B, C, D, E\}\}.$$

With the LHS, we identify R_1's candidate key(s) as $\{D, E\}$ and $\{C, D\}$, and the FDs in the minimal cover of that fragment is $\{\{C, D\} \rightarrow \{E\}, \{D, E\} \rightarrow \{B, C\}\}$.
Both of these two FDs are in BCNF.

According to the textbook referred in this module (Database Management Systems, Ramakrishnan & Gehrke), there is another (more) ineffective algorithm to decompose $\{A, B\}$ into $\{C, D, E\}$. You can borrow the book from NUS library if you have interest to read.

For any further question, please feel free to email me:

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