Tutorial: Dependencies

1. Consider the relational schema $R = \{A, B, C, D, E\}$ with the following set of functional dependencies.

$$\Sigma = \{\{A, B\} \to \{C\}, \{D\} \to \{D, B\}, \{B\} \to \{E\}, \{E\} \to \{D\}, \{A, B, D\} \to \{A, B, C, D\}\}$$

(a) Compute all the closures of the the sets of attributes that are not equal to themselves, are not super-keys or are candidate keys. What information is not essential and could be removed.

Solution:

$$\{A, B\}^+ = \{A, B, C, D, E\}$$
$$\{A, D\}^+ = \{A, B, C, D, E\}$$
$$\{A, E\}^+ = \{A, B, C, D, E\}$$
$$\{B\}^+ = \{B, D, E\}$$
$$\{D\}^+ = \{B, D, E\}$$
$$\{E\}^+ = \{B, D, E\}$$
$$\{B, D\}^+ = \{B, D, E\}$$
$$\{B, E\}^+ = \{B, D, E\}$$
$$\{D, E\}^+ = \{B, D, E\}$$
$$\{B, C\}^+ = \{B, C, D, E\}$$
$$\{D, C\}^+ = \{B, C, D, E\}$$
$$\{E, C\}^+ = \{B, C, D, E\}$$
$$\{B, D, C\}^+ = \{B, C, D, E\}$$
$$\{B, E, C\}^+ = \{B, C, D, E\}$$
$$\{D, E, C\}^+ = \{B, C, D, E\}$$

We see that the candidate keys are $\{A, B\}$, $\{A, D\}$ and $\{A, E\}$.
Remark: You can even remove from the right-hand side the members of the set on the left-hand side. It aslo keeps all the essential information (how to prove it?).

$$\{A, B\}^+ = \{C, D, E\}$$
$$\{A, D\}^+ = \{B, C, E\}$$
$$\{A, E\}^+ = \{B, C, D\}$$
$$\{B\}^+ = \{D, E\}$$
$$\{D\}^+ = \{B, E\}$$
$$\{E\}^+ = \{B, D\}$$
$$\{B, D\}^+ = \{E\}$$
$$\{B, E\}^+ = \{D\}$$
$$\{D, E\}^+ = \{B\}$$
$$\{B, C\}^+ = \{D, E\}$$
$$\{D, C\}^+ = \{B, E\}$$
$$\{E, C\}^+ = \{B, D\}$$
$$\{B, D, C\}^+ = \{E\}$$
$$\{B, E, C\}^+ = \{D\}$$

$$\{D, E, C\}^+ = \{B\}$$

Remark: You can even remove an equality if the left-hand side is a superset of another equality left-hand-side and its right-hand side is a subset of the right-hand side. It also keeps all the essential information (how to prove it?).

$$\{A, B\}^+ = \{C, D, E\}$$

$$\{A, D\}^+ = \{B, C, E\}$$

$$\{A, E\}^+ = \{B, C, D\}$$

$$\{B\}^+ = \{D, E\}$$

$$\{D\}^+ = \{B, E\}$$

$$\{E\}^+ = \{B, D\}$$

(b) What are the candidate keys of R with Σ?

Solution: The candidate keys are $\{A, B\}$, $\{A, D\}$ and $\{A, E\}$ (see question 1).

(c) Find a minimal cover of R with Σ that can be reached from Σ using the algorithm from the lecture.

Solution:

Step 1: We construct Σ', which is equivalent to Σ, by minimizing the right-hand side of every functional dependency in Σ.

$$\Sigma' = \{\{A, B\} \to \{C\}, \underline{\{D\} \to \{D\}}, \underline{\{D\} \to \{B\}}, \{B\} \to \{E\}, \{E\} \to \{D\},$$
$$\underline{\{A, B, D\} \to \{A\}}, \underline{\{A, B, D\} \to \{B\}}, \underline{\{A, B, D\} \to \{C\}}, \underline{\{A, B, D\} \to \{D\}}\}$$

Step 2: We construct Σ'', which is equivalent to Σ' (and to Σ), by minimizing the left-hand side of every functional dependency in Σ'. We may have the choice between several possible simplifications. We only need to choose one.

$$\Sigma'' = \{\{A, B\} \to \{C\}, \{D\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\},$$
$$\underline{\{A\} \to \{A\}}, \underline{\{B\} \to \{B\}}, \underline{\cancel{\{A, B\} \to \{C\}}}, \underline{\cancel{\{D\} \to \{D\}}}\}$$

or

$$\Sigma'' = \{\{A, B\} \to \{C\}, \{D\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\},$$
$$\underline{\{A\} \to \{A\}}, \underline{\{B\} \to \{B\}}, \underline{\{A, D\} \to \{C\}}, \underline{\cancel{\{D\} \to \{D\}}}\}$$

Notice that we can keep all the candidates with a minimal left-hand side. Namely, it suffices, in the example to keep:

$$\Sigma'' = \{\underline{\{A, B\} \to \{C\}}, \{D\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\},$$
$$\{A\} \to \{A\}, \{B\} \to \{B\}, \underline{\{A, D\} \to \{C\}}, \{D\} \to \{D\}\}$$

Step 3: We construct Σ''', which is equivalent to Σ'' (and to Σ), by minimizing the whole set Σ''. We may have the choice between several possible simplifications. The choice depends on the order in which we consider the functional dependencies.

$$\Sigma''' = \{\{A, B\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$$

or

$$\Sigma''' = \{\{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}, \{A, D\} \to \{C\}\}$$

(d) Find all the minimal covers of R with Σ.

Solution:

$$\Sigma''' = \{\{A, B\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$$

$$\{\{A, D\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$$

$$\{\{A, E\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{D\}\}$$

$$\{\{A, B\} \to \{C\}, \{D\} \to \{E\}, \{E\} \to \{B\}, \{B\} \to \{D\}\}$$

$$\{\{A, D\} \to \{C\}, \{D\} \to \{E\}, \{E\} \to \{B\}, \{B\} \to \{D\}\}$$

$$\{\{A, E\} \to \{C\}, \{D\} \to \{E\}, \{E\} \to \{B\}, \{B\} \to \{D\}\}$$

$$\{\{A, D\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{B\}\}$$

$$\{\{A, D\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{D\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$$

$$\{\{A, D\} \to \{C\}, \{B\} \to \{E\}, \{E\} \to \{B\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$$

$$\{\{A, E\} \to \{C\}, \{B\} \to \{D\}, \{D\} \to \{B\}, \{B\} \to \{E\}, \{E\} \to \{B\}\}$$

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$$\{\{A, B\} \to \{C\}, \{D\} \to \{B\}, \{B\} \to \{D\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$$

$$\{\{A, B\} \to \{C\}, \{B\} \to \{E\}, \{E\} \to \{B\}, \{E\} \to \{D\}, \{D\} \to \{E\}\}$$

(e) Prove, using the three Armstrong axioms, that the following set of functional dependencies is equivalent to Σ.

$$\Sigma'''' = \{\{A, B\} \rightarrow \{C, D, E\}, \{A, D\} \rightarrow \{B, C, E\}, \{A, E\} \rightarrow \{B, C, D\},$$
$$\{B\} \rightarrow \{D, E\}, \{D\} \rightarrow \{B, E\}, \{E\} \rightarrow \{B, D\}\}$$

Solution: We prove that every functional dependency in one of Σ and Σ'''' can obtained from those in the other set.
For example $\{B\} \rightarrow \{D, E\}$ can be obtained from Σ.

1. We know that $\{B\} \rightarrow \{E\}$.
2. We know that $\{E\} \rightarrow \{D\}$.
3. Therefore $\{E\} \rightarrow \{D, E\}$ by Augmentation of (2) with $\{E\}$.
4. Therefore $\{B\} \rightarrow \{D, E\}$, by Transitivity of (1) and (3).

   Q.E.D.

Post the other answers on the Luminus Forum (type in Latex and post the PDF).