Q3 Understanding MVD

A	B	C	D	E	G	
a1	b1	c1	d1	e1	g1	t1
a2	b2	c1	d1	e2	g2	t2
a2	b2	c1	d1	e1	g1	
a1	b1	c1	d1	e2	g2	
a1	b1	c1	d1	e2	g1	t3
a2	b2	c1	d1	e1	g2	
a2	b2	c1	d1	e2	g1	
a1	b1	c1	d1	e1	g2	

CD ->> ABG

$$(\forall t_1 \in r\ \forall t_2 \in r\ (t_1.X = t_2.X \rightarrow$$

$$\exists t_3 \in r\ (t_3.X = t_1.X \land t_3.Y = t_1.Y \land t_3.Z = t_2.Z)))$$

← formal definition (Credit to Zemmy)

X = CD
Y = ABG
Z = R - {A,B,C,D,G} = E

You can assume for $R = X \cup Y \cup Z$ and to prove $X \twoheadrightarrow Y$

There must be a Cartesian Product of all possible values' combination observed in the table.

	A	B	C	D	E	G	
row 1	a1	b1	c1	d1	e1	g1	t1
... 2	a2	b2	c1	d1	e2	g2	t2
... 3	a2	b2	c1	d1	e1	g1	
... 4	a1	b1	c1	d1	e2	g2	
... 5	a1	b1	c1	d1	e2	g1	t3
... 6	a2	b2	c1	d1	e1	g2	
... 7	a2	b2	c1	d1	e2	g1	
... 8	a1	b1	c1	d1	e1	g2	

CD ->> ABG

$$(\forall t_1 \in r\ \forall t_2 \in r\ (t_1.X = t_2.X \rightarrow$$

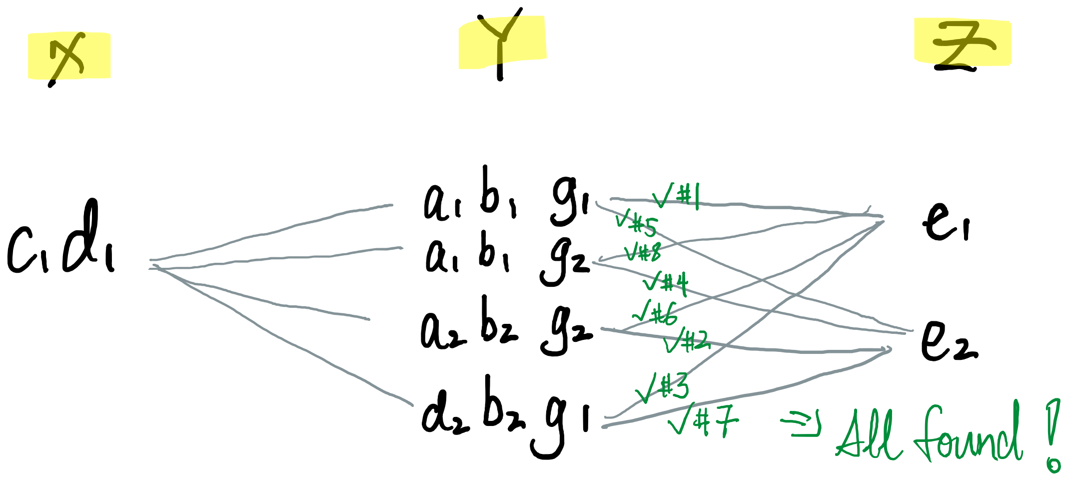
$$\exists t_3 \in r\ (t_3.X = t_1.X \wedge t_3.Y = t_1.Y \wedge t_3.Z = t_2.Z)))$$

X = CD

Y = ABG

Z = R - {A,B,C,D,G} = E

A Quick Trick — let's build a map of all possible values of X, Y, Z and check if they are whether FULLY-CONNECTED.



There is another case showing a MVD violation

(Thanks Prof. for providing this case)

$R(A,B,C,D,E)$

$\Sigma = \{\{C\} \rightarrow \{B\}, \{E\} \twoheadrightarrow \{C,D\}\}$

Prove $\{E\} \twoheadrightarrow \{D\}$ by Chase:

Step 1 Initiate Table

a_1	b_1	c_1	d_1	e_1
a_2	b_2	c_2	d_2	e_2

→ Step 2: Unify LHS value of the Chase

a_1	b_1	c_1	d_1	e
a_2	b_2	c_2	d_2	e

Step 3 Apply $\{E\} \twoheadrightarrow \{CD\}$

	a_1	b_1	c_1	d_1	e
	a_2	b_2	c_2	d_2	e
+	a_1	b_1	c_2	d_2	e
+	a_2	b_2	c_1	d_1	e

↓

See next page for further steps ...

Table after Step 3:

a_1	b_1	c_1	d_1	e
a_2	b_2	c_2	d_2	e
a_1	b_1	c_2	d_2	e
a_2	b_2	c_1	d_1	e

Step 4 Apply $\{C\} \to \{B\}$

We have $b_1 = b_2 = b$

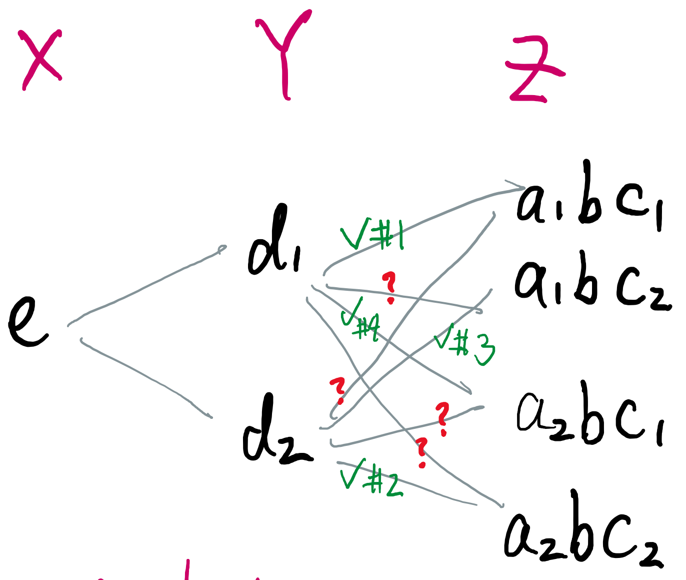
a_1	b	c_1	d_1	e	← row 1
a_2	b	c_2	d_2	e	←… 2
a_1	b	c_2	d_2	e	←… 3
a_2	b	c_1	d_1	e	←… 4

Step 5 Verify the Chase $\{E\} \twoheadrightarrow \{D\}$

According to the definition, we have:

$X: E$, $Y: D$, $Z: ABC$

The we analyse via the mapping trick:



Therefore we fail to prove!?

We found 4 combinations missing from the table

For any Question

Please feel free to

email us