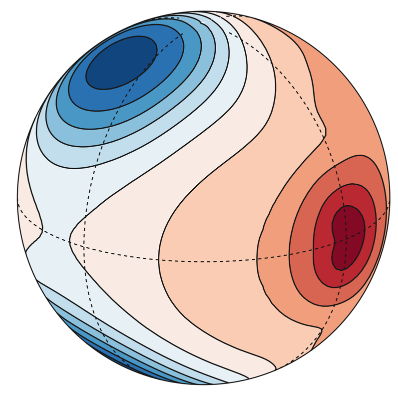
Wave-Mean Flow Interaction on Tidally Locked Planets

UNIVERSITY OF OXFORD

Mark Hammond and Raymond T. Pierrehumbert

Tidally locked planets with atmospheres have a global circulation with a superrotating eastward equatorial jet, and a hot-spot shifted eastwards from their substellar point. We used the shallow-water equations linearised about this eastward flow to show that the effect of the flow explains the form of the global circulation, particularly the hot-spot shift and the positions of the cold spots on the night-side.



1 - Tidally Locked Atmospheres

Tidally locked planets always present the same face to the star they orbit.

Observations of such planets show that their hottest part is shifted up to tens of degrees eastward of the substellar point, and that their night-sides are warmer than expected from radiative equilibrium.

This implies a global circulation from day-side to night-side. GCM simulations show the atmospheres on such planets have an eastward superrotating jet and a hot-spot shifted east of the substellar point.

Figure 1 shows the stationary response to a forcing which is sinusoidal in longitude, for the one-layer shallow-water equations linearised about zero background flow (Matsuno 1966).

This system predicts an eastward equatorial acceleration, so predicts the equatorial jet on these planets (Showman & Polvani 2011). However, the hot and cold spots are in the wrong place.

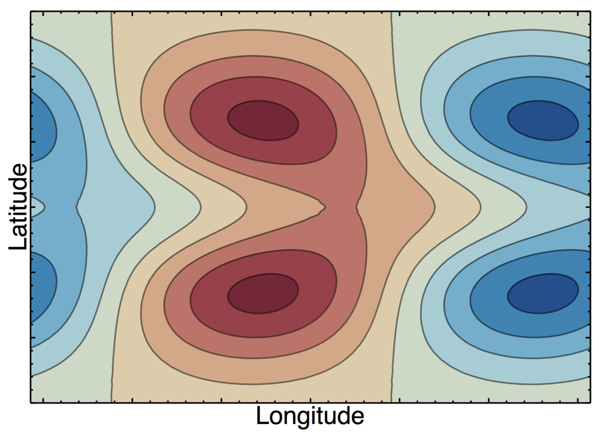


Figure 1: Stationary solution to the linear shallow-water equations with constant sinusoidal forcing.

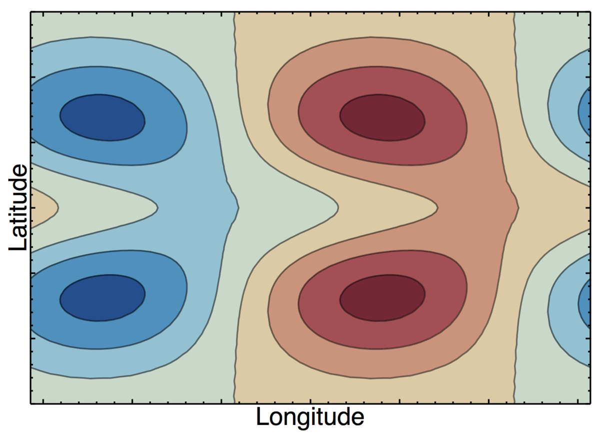


Figure 2: Stationary response in Figure 1, Doppler-shifted by uniform eastward flow

The linear shallow-water equations are:

$$\frac{\partial u}{\partial t} - \beta y v + \frac{\partial \phi}{\partial x} = 0$$
$$\frac{\partial v}{\partial t} + \beta y u + \frac{\partial \phi}{\partial y} = 0$$
$$\frac{\partial \phi}{\partial t} + c^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

With damping, forcing, and sinusoidal variation in x, they become:

$$\begin{pmatrix} \alpha & -y & ik \\ y & \alpha & \frac{\partial}{\partial y} \\ ik & \frac{\partial}{\partial y} & \alpha \end{pmatrix} \begin{pmatrix} u \\ v \\ h \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ Q(y) \end{pmatrix}$$

Matsuno (1966) describes the free and forced solutions. We added the effect of the equatorial jet to the model. Figure 2 shows how a uniform flow Doppler-shifts the forced response eastwards (Tsai et al. 2014).

Figure 3 shows the height perturbation from the jet, if it is geostrophically balanced. Figure 4 shows the sum of the Doppler-shifted solution, and the jet height perturbation. Its form matches our solutions below, and the hot and cold spots are in the right place.

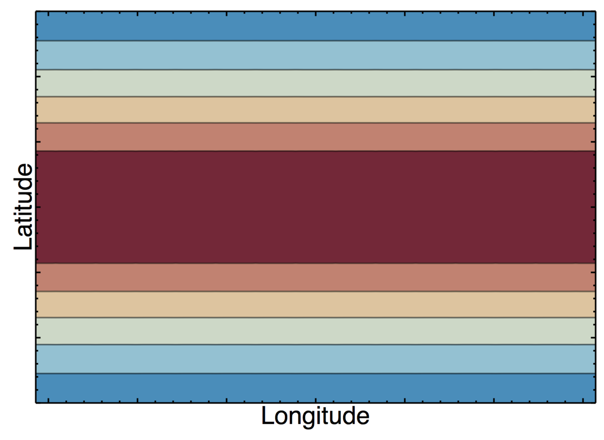


Figure 3: Height perturbation due to geostrophically balanced uniform flow

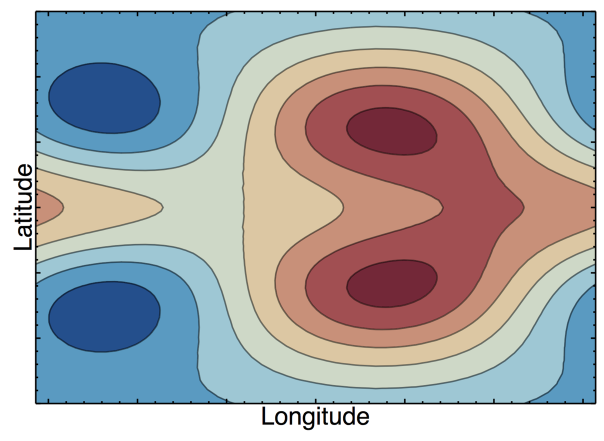


Figure 4: Sum of the fields in Figures 2 and 3, showing how the jet height creates the single hot-spot and two cold spots

2 - Effect of Shear Flow

We linearised the one-layer shallow-water equations on an equatorial beta-plane about a zonally uniform shear flow U(y) and a height perturbation H(y) due to geostrophic balance. This is more accurate than our approximation above.

Our aim was to build on previous work by including the effect of the eastward equatorial jet on tidally locked planetary atmospheric dynamics.

The previous systems in Showman and Polvani (2011) (linearised about zero flow) and Tsai et al. (2014) (linearised about uniform flow and height) have the same analytic solutions as in Matsuno (1966).

Our new system has no analytic solution, so we solve it with a pseudo-spectral method. We solve the equations:

$$\begin{pmatrix} \alpha + ik_x\bar{U}(y) & \frac{\partial \bar{U}(y)}{\partial y} - y & ik \\ y & \alpha + ik_x\bar{U}(y) & \frac{\partial}{\partial y} \\ ik\bar{H}' & -y\bar{U}(y) + \bar{H}'\frac{\partial}{\partial y} & \alpha + k_x\bar{U}(y) \end{pmatrix} \begin{pmatrix} u \\ v \\ h \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ Q(y) \end{pmatrix}$$

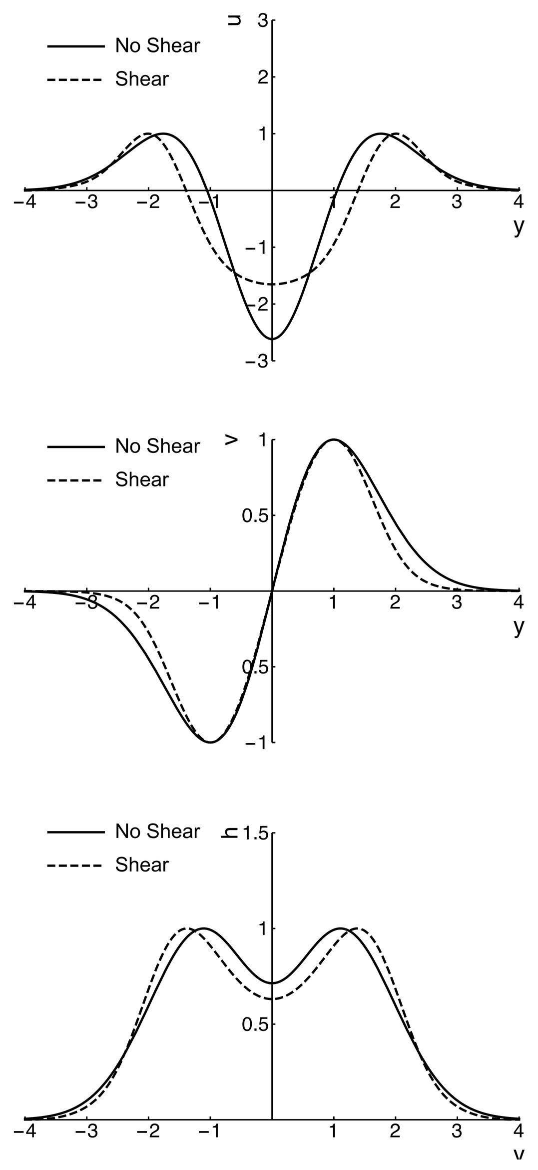


Figure 5: From top, effect of shear flow U(y) on i) zonal flow u, ii) meridional flow v, and iii) height h of forced wave response.

We express the solution as a series of Hermite functions which correspond to the free modes of the system. A non-zero flow U(y) and height H(y) excite more of these free modes in the stationary response.

Figure 5 shows the effect of the shear flow on the meridional (real) parts of the forced solution, and Figure 6 shows its effect on the total forced solution on the beta-plane.

The beta-plane does not represent the latitude of the planet realistically, so we also solved the following equations for the same system on a sphere:

$$\begin{pmatrix} \alpha_{dyn} + im\bar{U}/\cos\phi & \frac{\partial \bar{U}\cos\phi}{\partial\phi} - \sin\phi & \frac{imG}{\cos\phi} \\ 2\bar{U}\tan\phi + \sin\phi & \alpha_{dyn} + \frac{im\bar{U}}{\cos\phi} & G\frac{\partial}{\partial\phi} \\ \frac{im(1+\bar{H})}{\cos\phi} & \frac{1}{\cos\phi}\frac{\partial(1+\bar{H})\cos\phi)}{\partial\phi} & \alpha_{rad} + \frac{im\bar{U}}{\cos\phi} \end{pmatrix} \begin{pmatrix} u \\ v \\ h \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ Q(\phi) \end{pmatrix}$$

We expressed this solution as a series of Legendre polynomials. Figure 7 shows the resulting solution, which now corresponds to a latitude-longitude field.

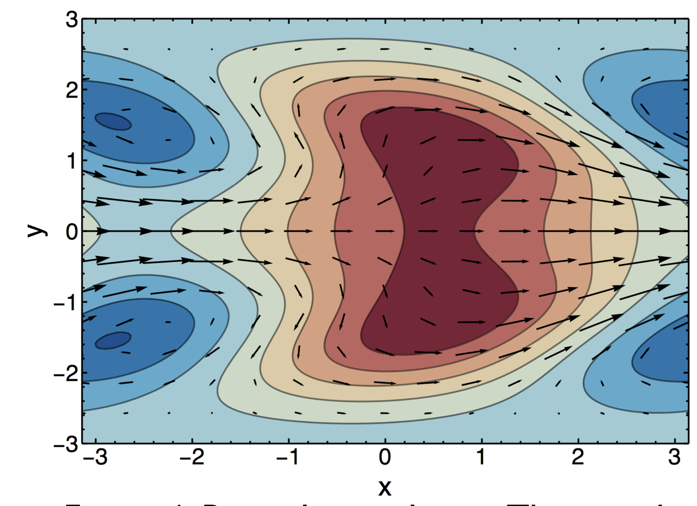


Figure 6: Beta-plane solution. This matches Figures 4 and 8, showing that the shear flow is vital to the global circulation.

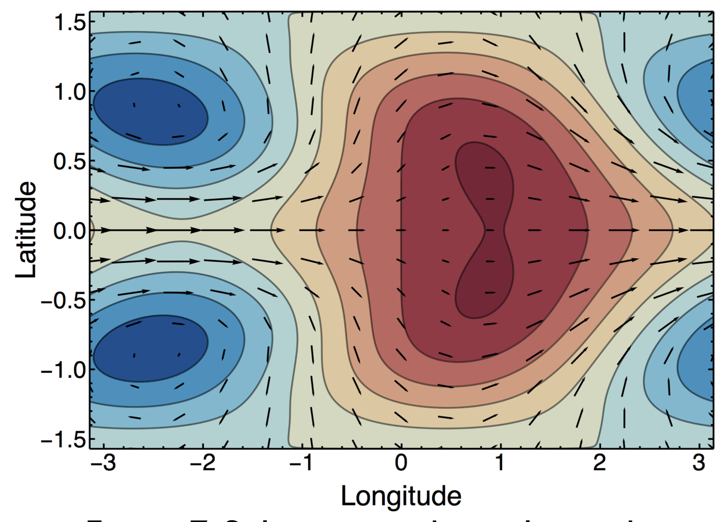


Figure 7: Solution in spherical coordinates. The form is the same as Figure 6, but is now on a latitude-longitude field.

3 - Comparing to Simulations

Figure 8 shows the mid-atmosphere temperature and wind fields for a tidally locked planet with a 10 day period. It also shows the streamfunction and winds of the same simulation minus their zonal mean, showing the eastward Doppler-shift discussed above.

The shallow-water model predicts a largely zonally symmetric circulation for weakly forced or rapidly rotating planets. Figure 9 shows how our GCM results agree with this prediction.

Figure 10 shows the wavenumber-1 component of the mid-atmosphere pressure field, as the model spins up. The Rossby wave shifts from west to east as predicted.

Figure 11 shows the longitude of the maximum of this wavenumber-1 response, and the mean eastward flow speed. As the flow increases, the wave moves east.

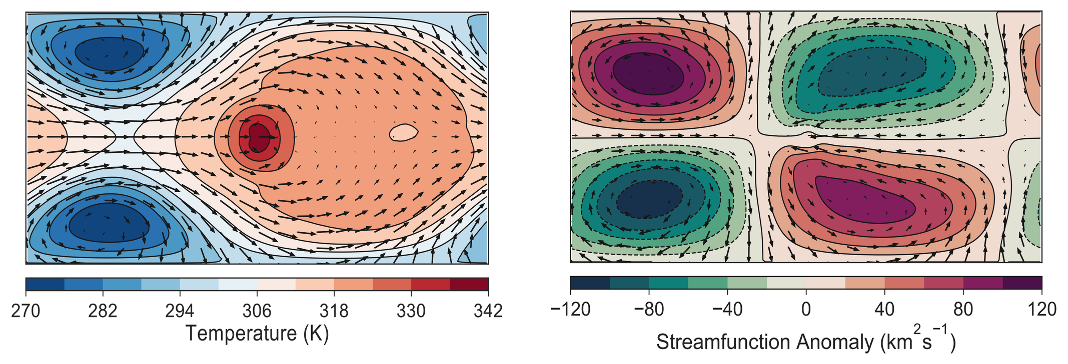


Figure 8: i) Simulation of Earth-sized tidally locked planet ii) The same simulation minus its zonal mean.

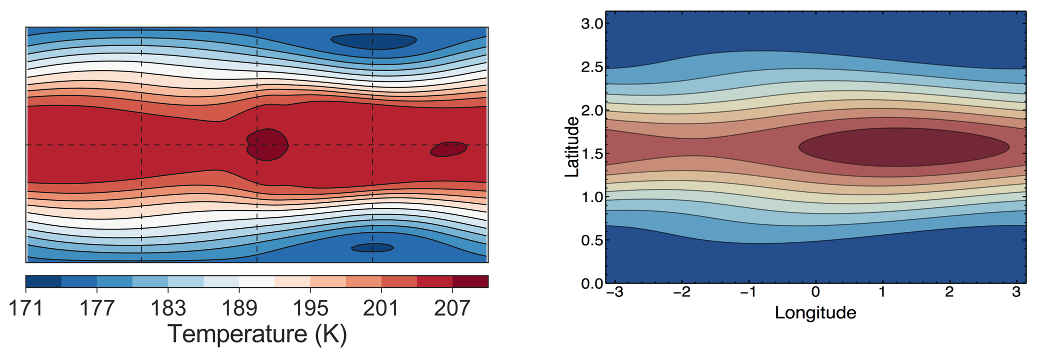


Figure 9: i) Simulation of cool tidally locked planet with 5 day period ii) Solution to shallow-water shear system with high rotation rate.

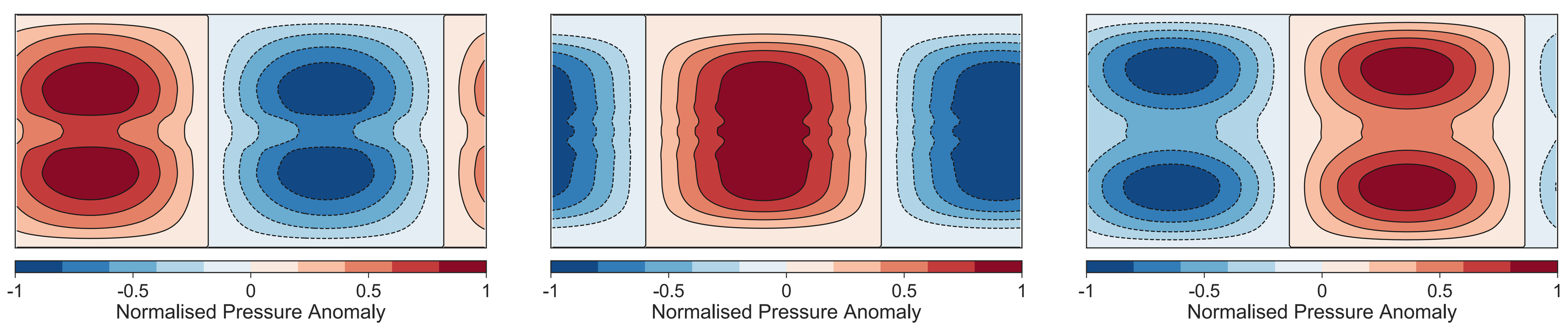


Figure 10: Mid-atmosphere pressure anomaly during spin-up of simulations, at 6 days, 8 days, and 22 days.

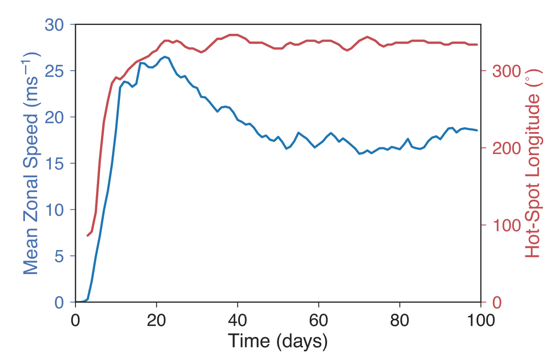


Figure 11: Mean zonal flow speed timeseries, and position of maximum of wavenumber-1 pressure field.

4 - Conclusions

We linearised the shallow-water equations about an equatorial jet and its associated height perturbation, in a single-layer model of a tidally locked planet.

The solutions successfully predicted the behaviour of simulations of such atmospheres, particularly the shift of the wavenumber-1 standing wave as the equatorial jet forms.

Figure 12 summarises our work, showing how linearising the shallow-water model about the eastward jet explains the form of the circulation.

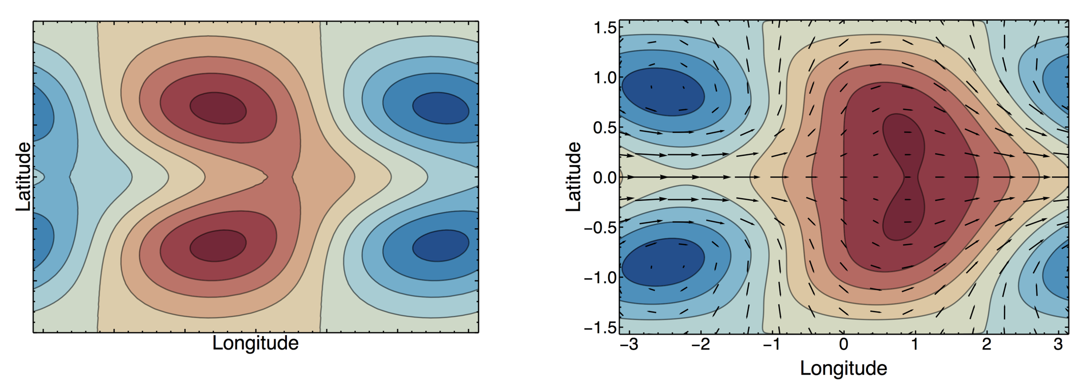


Figure 12: Response to tidally locked instellation in the shallow-water equations, linearised about i) zero flow and ii) shear flow

References

Matsuno, T. 1966, Journal of the Meteorological Society of Japan Ser II, 44, 25
Showman, A. P., & Polvani, L. M. 2011, The Astrophysical Journal, 738, 71
Tsai, S. M., Dobbs-Dixon, I., & Gu, P. G. 2014, Astrophysical Journal, 793, 141