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Two-Factor Mixed and Within-Participants ANOVA

PSYC214: Statistics For Group Comparisons

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Week 8

Learning Objectives

- Two-factor mixed and within-participants designs
- Focus on procedures rather than how the analysis is calculated
- How to interpret ANOVA tables and graphs
- Handling significant main effects, and simple main effects, of factors with three or more levels

Introduction

- We have now covered the three most mathematically straightforward designs:
  1. splitting total variability into between-group variability and within-group variability — one factor between-participants designs
  2. splitting within-group variability into between-participant variability and residual variability — one factor within-participants designs
  3. splitting between-group variability into main effect and interaction variability — 2×2 between-participants design
- There is little new to learn from analysing more complicated designs by hand

A Two-Factor Mixed Design

- Mixed design ANOVAs are particularly versatile and are often used in psychology
- These designs have at least one between-participants factor and at least one within-participants factor
- Allows the researcher to capitalise on the benefits of between- and within-participants designs within the same design
- Let's consider an example based on the Stroop task

The Stroop Task

- In the Stroop task, participants must name the ink colour of a colour word as quickly as possible:
  - on *congruent trials*, the ink colour and colour name are consistent
  - on *incongruent trials*, the ink colour and colour name are inconsistent
- Stroop effect = longer RTs for incongruent, compared to congruent, trials
- A measure of **response inhibition**

Congruent Trials

RED

GREEN

Incongruent Trials

RED

GREEN

Example of A Mixed Design

- A researcher wants to know if response inhibition is impaired in patients with Schizophrenia using the Stroop task
- She employs a 2×2 mixed design:
  - patient group: healthy *vs.* schizophrenia
  - trial type: congruent *vs.* incongruent
- *patient group* is necessarily a between-participants factor
- *trial type* is a within-participants factor
- There are $2 \times 2 = 4$ conditions; two groups of participants (healthy *vs.* schizophrenia) each complete two conditions of the experiment (congruent *vs.* incongruent trials)

Hypothetical Data For Mixed-Design Stroop Experiment

			Factor B: Trial type (within participants)	
			Level A_1 *congruent*	*Level* A_2 *incongruent*
Factor *A*:	Level B_1 healthy	P_1	680	790
Group (between participants)		P_2	616	746
		P_3	530	670
		P_4	630	830
		P_5	694	794
	Level B_2 schizophrenia	P_6	630	852
		P_7	610	875
		P_8	602	863
		P_9	660	912
		P_{10}	673	928

Hypothetical Data For Mixed Design-Stroop Experiment

		Factor B: Trial type		
		Level A_1 *congruent*	*Level A_2* *incongruent*	*Overall*
Factor *A*:	Level B_1 healthy	630	776	703
Group	Level B_2 schizophrenia	635	885	760
	Overall	632.5	830.5	731.5

Error Terms In A Mixed-Design ANOA

- Recall that a between-participants design uses the within-group variance as its error term
- By contrast, a within-participants design uses the residual variance as its error term
- A mixed-design ANOVA produces two error terms:
  - one for the between-participants main effect
  - one for the within-participants main effect and the interaction
- You must be careful to ensure when reporting the ANOVA that the correct degrees of freedom are read from the table

ANOVA Table For Mixed-Design Stroop Experiment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (group)	19531.250	1	19531.250	4.307	0.072
Error S/A (Bet-ss)	36281.000	8	4535.125		
B (trial type)	187211.250	1	187211.250	411.793	< .001
$A \times B$	16531.250	1	16531.250	36.362	< .001
Error B $\times$ *S/A*	3637.000	8	454.625		

- One error term is labelled *Error S/A (Bet-ss)* and has been used to calculate the *F* ratio for the between-participants factor
- *Error B*$\times$*S/A* has been used to calculate the F ratio for every component linked to factor B—the within-participants factor and interaction

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Simple Main Effects Table For Mixed-Design Stroop Experiment

- The interaction is significant, so we need to test the simple main effects
- There are different approaches to testing simple main effects in mixed designs
- The simplest approach uses **pooled error terms**
- We begin by calculating the between-group variance for each simple main effect, as we did in last week's lecture
- The calculations are identical to those used for the between-participants design (see the Week 6 lecture slides)
- Each pair of simple main effects is tested for significance using the same error term (hence pooled error term approach)

Simple Main Effects Table For Mixed-Design Stroop Experiment

- The error term for testing the significance of the between-participant effects is the pooled within-group variance for the four cells
- This is calculated identically to a fully between-participants design $\{SS_{S/AB} = [Y] - [AB]; df_{S/AB} = ab(s-1)\}$
- This is used to test the significance of the two simple main effects of the between-participants factor:
  - group at congruent
  - group at incongruent

Simple Main Effects Table For Mixed-Design Stroop Experiment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
Group at					
congruent	62.500	1	62.500	0.024	0.880
incongruent	36000.00	1	36000.000	14.908	0.005
Error term	38636.96	16	2414.81		
Trial type at					
healthy	46240.000	1	46240.000	101.710	< .001
schizophrenia	157502.500	1	157502.500	346.445	< .001
Error term	3637.000	8	454.625		

Simple Main Effects Table For Mixed-Design Stroop Experiment

- The other error term is the within-participants factor error term from the initial ANOVA (*Error B*$\times$*S/A*)
- This is used to test the two within-participants simple main effects:
  - trial type at healthy
  - trial type at schizophrenia

Simple Main Effects Table For Mixed-Design Stroop Experiment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
Group at					
congruent	62.500	1	62.500	0.024	0.880
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Trial type at					
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Simple Main Effects Table For Mixed-Design Stroop Experiment

- An alternative to this approach would be to calculate a separate *t*-test for each pair of means being compared
- We would use independent-samples *t*-tests to test the simple main effects of the between-participants factor
- We would use repeated-measures *t*-tests to test the simple main effects of the within-participants factor

Simple Main Effects Table For Mixed-Design Stroop Experiment

- Once you have calculated the simple main effects, generate an interaction plot
- Locate the simple main effects in the graph to facilitate interpretation of the interaction

Interaction Plot

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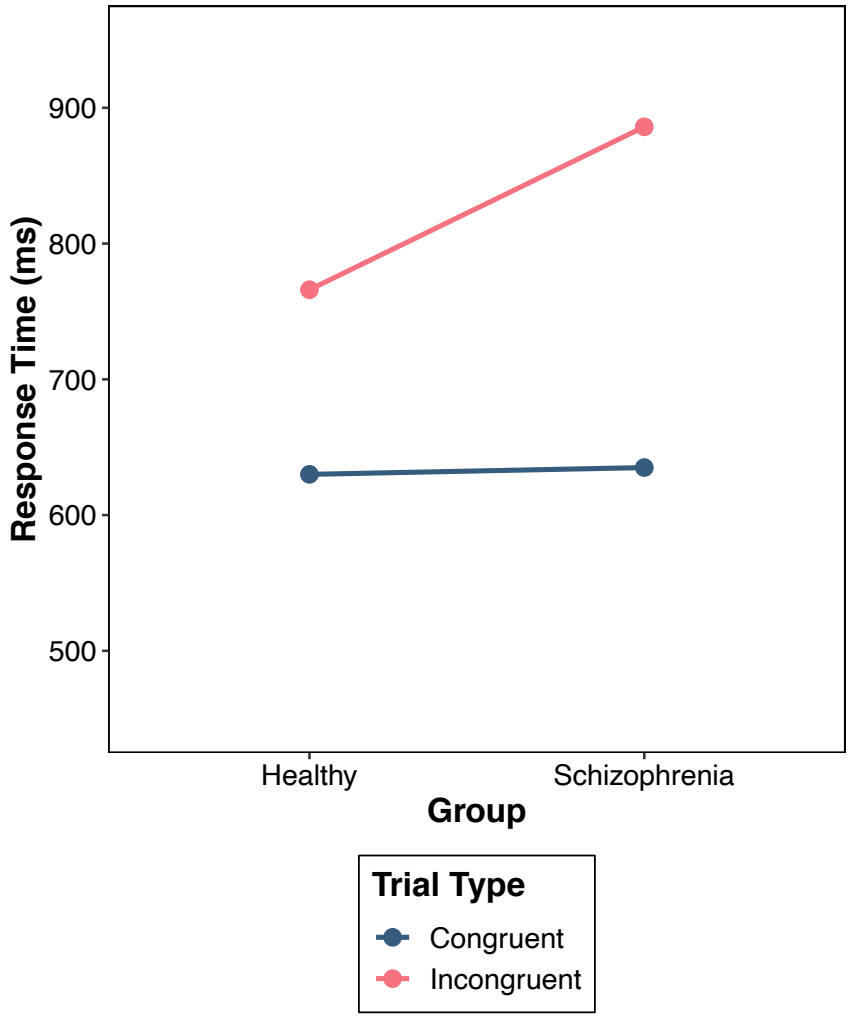
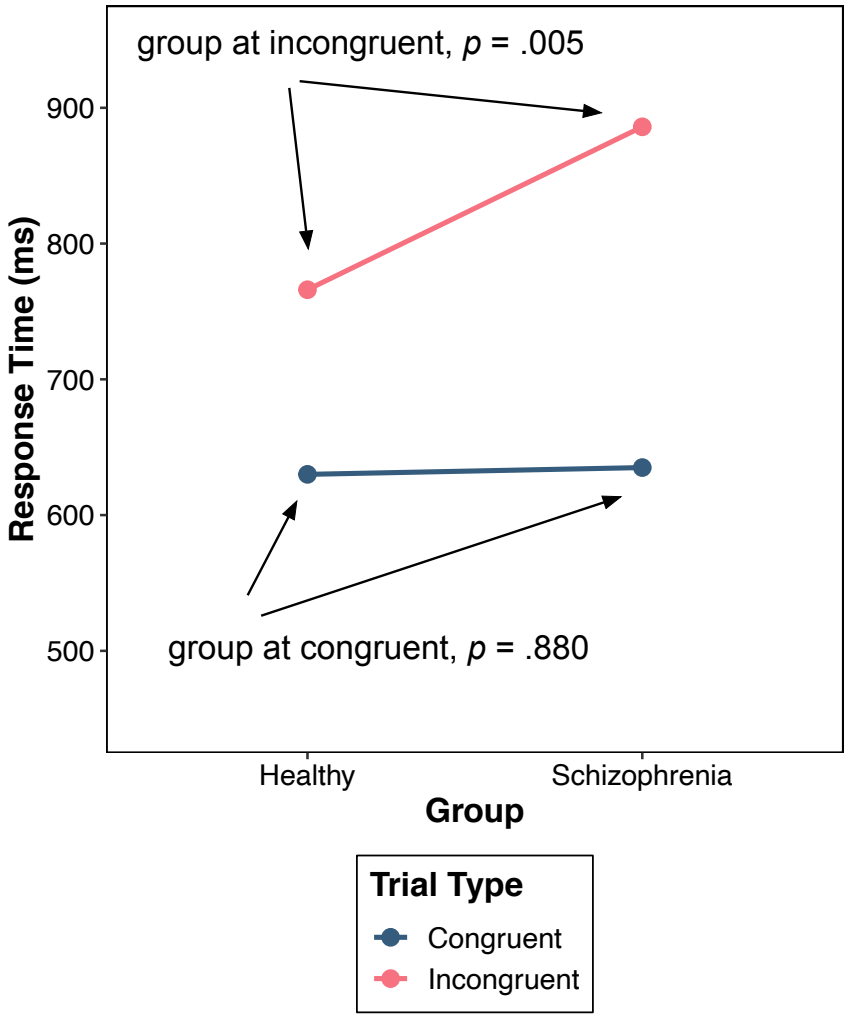


Interaction Plot



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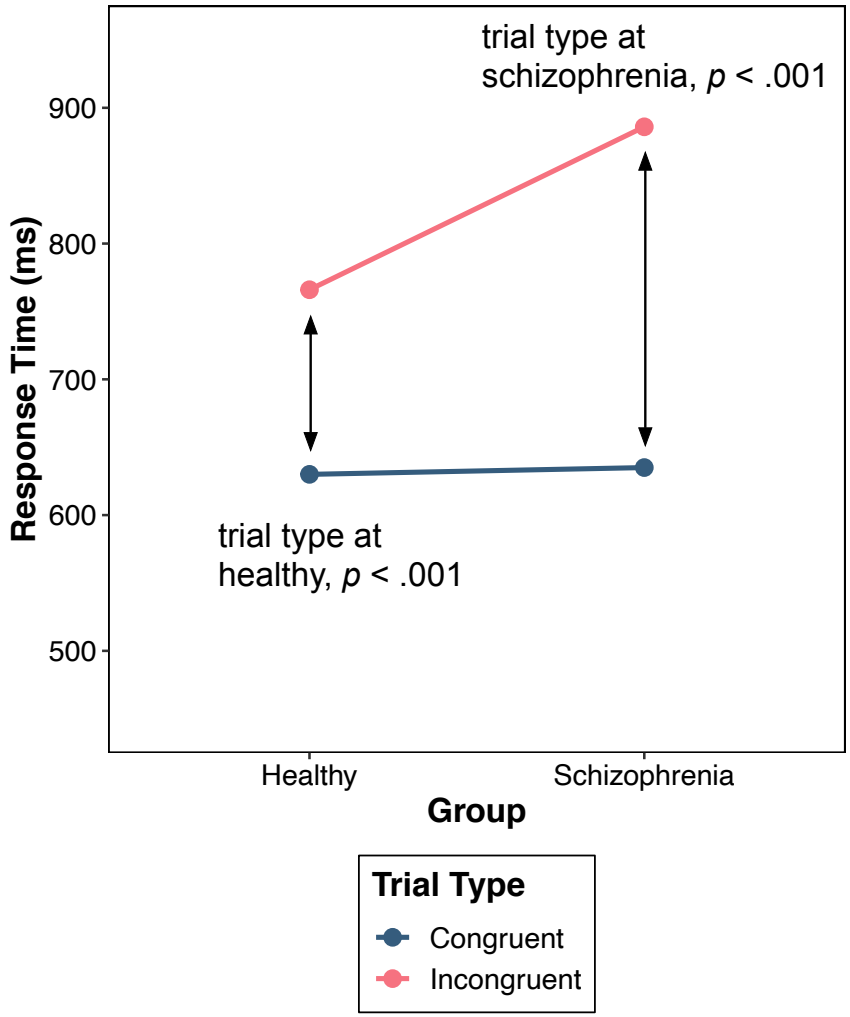

Simple Main Effects Table For Mixed-Design Stroop Experiment

- Once you have a graph and have calculated the simple main effects, write out the various effects as you were shown in the Week 7 lab session
- This involves reporting the *F* values for each simple main effect and stating the direction of the significant differences
- Once the significant effects have been identified, they must be interpreted
- Write a couple of sentences to describe the nature of the interaction (see the Week 7 lab session for an example)

What If The Design Has Three or More Levels In Either Factor?

- If the interaction is not significant, any significant main effects for factors with three or more levels will need to be followed up with planned comparisons (*t*-tests) or post-hoc tests (Tukey test)
- When the interaction is significant, the simple main effects for a factor with three or more levels will need to be followed up with planned comparisons or post-hoc tests
- In both circumstances, planned comparisons will often be preferable
- Make sure you use the right type (independent samples *vs.* repeated measures) for the effect you are testing

Calculating *F* ratios

- The book chapter in the recommended reading includes a demonstration of how to calculate the *F* ratios for a mixed design by hand
- Only study this if you are curious, it is not something you will be assessed upon
- The procedure is very similar to that used when we calculated *F* ratios for a two-factor between-participant design—it uses the same basic ratios (plus one new ratio)

A Two-Factor Fully Within-Participants Design

- A researcher wants to know if the size of the Stroop effect decreases with practice
- She employs a 2×3 fully within-participants design:
  - trial type: congruent *vs.* incongruent
  - block: 1 *vs.* 2 *vs.* 3
- Making *trial type* within-participants means we can establish each participant's susceptibility to the Stroop effect
- *block* must necessarily be a within-participants factor, as it requires experience with the task
- There are $2 \times 3 = 6$ conditions; a single group of participants completes each condition

Hypothetical Data For Fully Within-Participants Design Stroop Experiment

	A_1 congruent			A_2 incongruent		
	B_1 block 1	B_2 block 2	B_3 block 3	B_1 block 1	B_2 block 2	B_3 block 3
P_1	700	600	550	860	700	625
P_1	600	550	575	750	650	650
P_1	480	590	693	720	685	743
P_1	630	690	597	770	790	672
P_1	720	730	650	780	830	725
Means	626	632	613	776	731	683

ANOVA Table For Fully Within-Participants Design Stroop Experiment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (trial type)	84800.833	1	84800.833	173.654	$< .001$
Error $A \times P$	1953.333	4	488.333		
B (block)	14371.667	2	7185.833	0.878	0.452
Error $B \times P$	65457.333	8	8182.167		
$A \times B$	8201.667	2	4100.833	5.042	0.038
Error $A \times B \times P$	6506.667	8	813.333		
P (participants)	42098.667	4	10524.667		

- Each effect has its own error term directly underneath it, which makes locating the degrees of freedom easier

ANOVA Table For Fully Within-Participants Design Stroop Experiment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (trial type)	84800.833	1	84800.833	173.654	$< .001$
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Simple Main Effects Table For Fully Within-Participants Design Stroop Experiment

- To test the simple main effects, we calculate the between-group variances as we did in our Week 6 lecture
- The error terms to use are those from the original ANOVA table
- Thus, for the simple main effects of factor *A* (trial type) at B_1, B_2, and B_3 the error term for testing factor *A* could be used (*Error A* $\times$ *P)*
- For the simple main effects of factor *B* (block) at A_1 and A_2 the error term for testing factor *B* could be used (*Error B* $\times$ *P)*

Simple Main Effects Table For Fully Within-Participants Design Stroop Experiment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
Trial type at					
block 1	56250.000	1	56250.000	115.188	< .001
block 2	24502.500	1	24502.500	50.176	0.002
block 3	12250.00	1	12250.000	25.085	0.007
Error term	1953.333	4	488.333		
Block at					
congruent	943.333	2	471.667	0.058	0.944
incongruent	21630.000	2	10815.000	1.322	0.319
Error term	65457.333	8	8182.167		

Simple Main Effects Table For Fully Within-Participants Design Stroop Experiment

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Trial type at					
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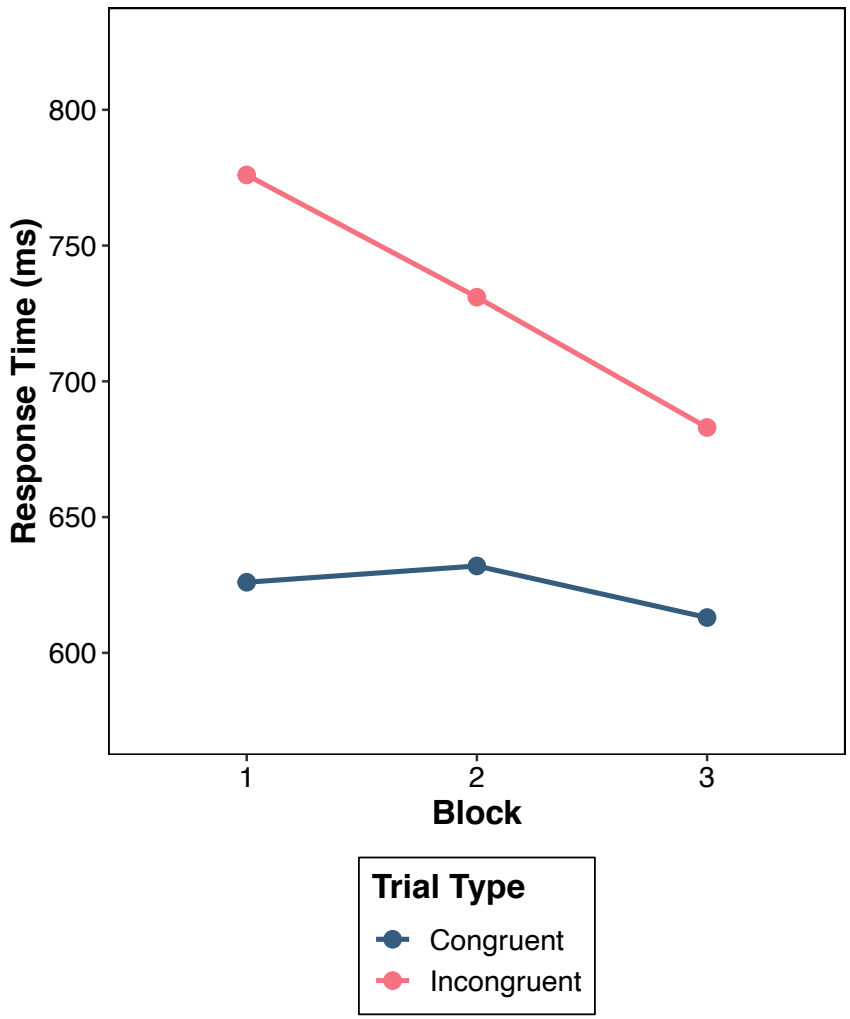
Interaction Plot



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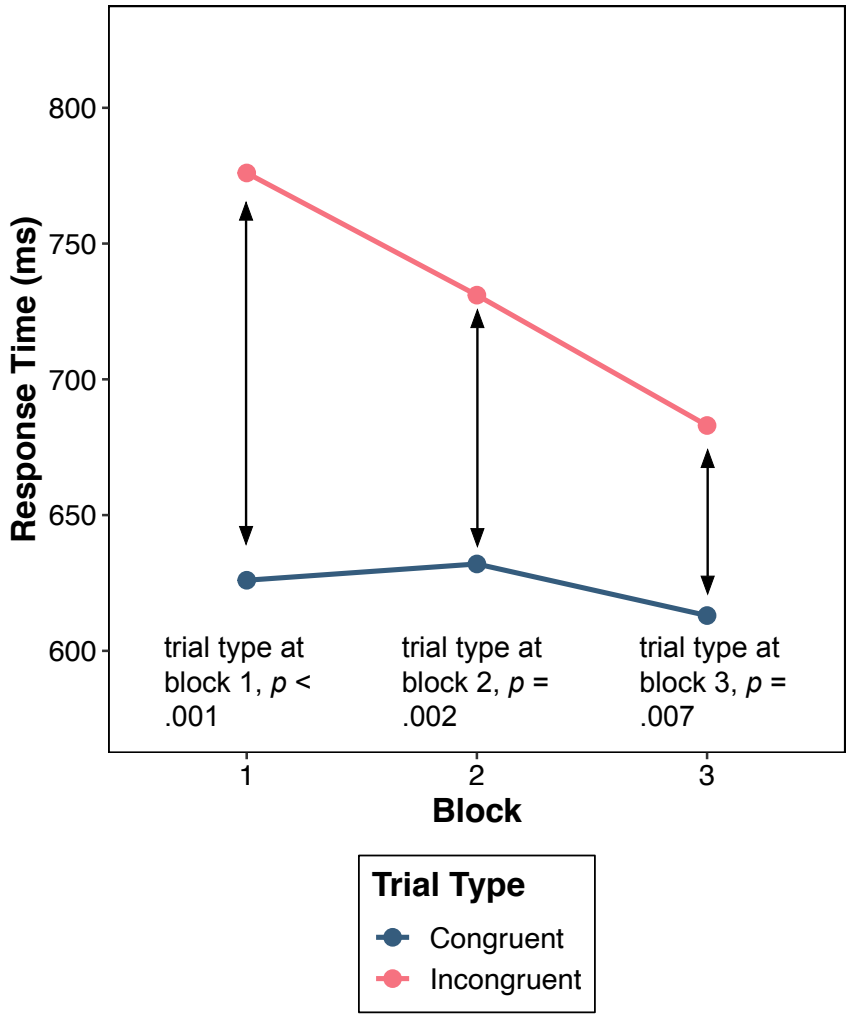

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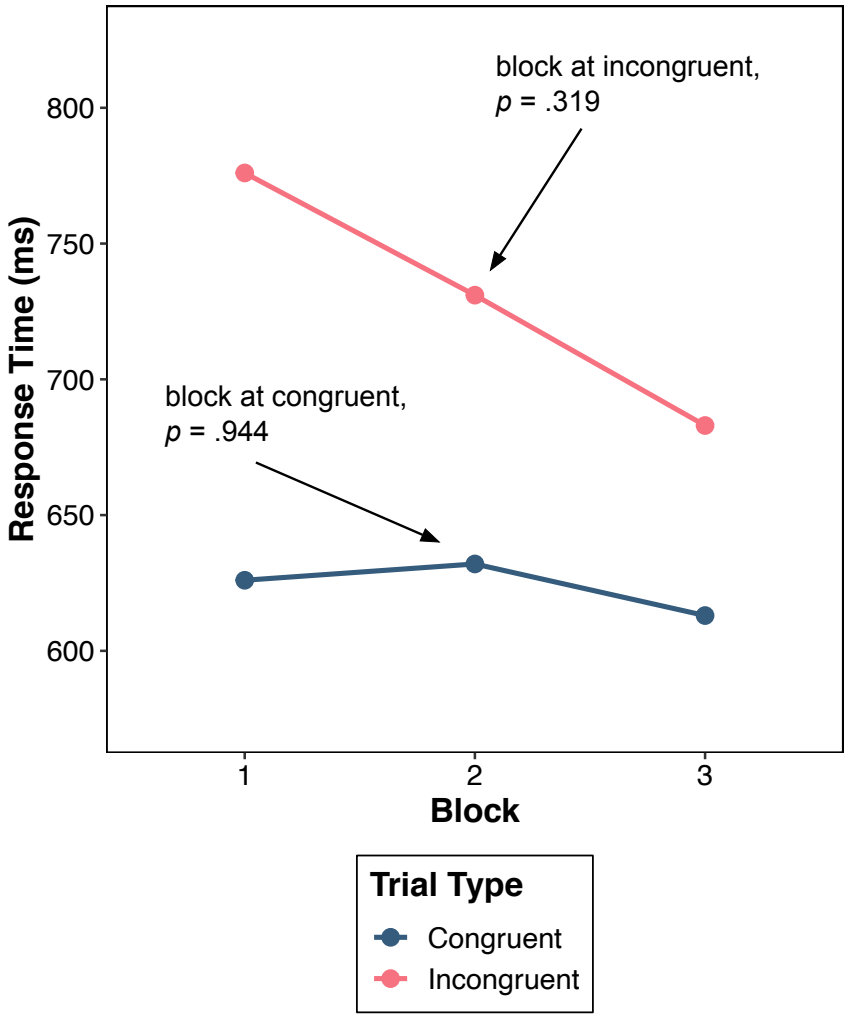

Follow Up Tests For Simple Main effects Of Factors With Three Or More Levels

- In this instance, the simple main effects of our factor with three levels (block) were both non significant
- If one or both of these simple main effects had been significant, we would need to perform follow up tests (planned comparisons or post-hoc tests)
- We again recommend planned comparisons, as they are more simple and straightforward
- Suppose the simple main effect of block at incongruent trials been significant, we could perform repeated-measures *t*-tests comparing block 1 *vs.* block 2 and block 2 *vs.* block 3 for incongruent trials only (ignoring congruent trials)

Calculating *F* ratios

- The book chapter in the recommended reading includes a demonstration of how to calculate the *F* ratios for a mixed design by hand
- Only study this if you are curious, it is not something you will be assessed upon
- The procedure is very similar to that used when we calculated *F* ratios for a two-factor between-participant design—it uses the same basic ratios (plus one new ratio)

A Note On The Sphericity Assumption

- The sphericity assumption extends to within-participants factorial designs with factors containing three or more levels
- It also applies to within-participant factors with three or more levels in mixed designs
- R will apply the Greenhouse and Geisser correction if the sphericity assumption is violated
- We'll cover this in more detail in next week's lab

Additional Resources

- The R code for all plots generated in this lecture (minus annotations) has been uploaded with these slides to the Week 8 lecture folder (R Plots For Lecture 8.R)

In Next Week's Lab ...

- Running a 2×3 mixed/within-participants ANOVA in R
- Follow-up tests for factors with more than two levels

References

Roberts, M. J., & Russo, R. (1999, Chapter 11). *A student's guide to Analysis of Variance*. Routledge: London.