### **New Tricks**

## Analysing response time distributions with the ex-Gaussian and shifted Wald

### New Tricks Internal Seminar

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## Outline

#### **New Tricks**

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### Outline

Response Time (RT)

Problems With Mean RT

Mean RT
Statistical

ex-Gaussia Shifted Wal

Fitting
PDF Functions
Objective Function
Search Algorithm

Using RT-Distrib-Fit

Application Example

- Response time (RT) as a dependent measure
- Problems with conventional analysis of mean RT
- Probability functions for representing RT distributions:
  - ex-Gaussian
  - Shifted Wald
- How to fit probability functions to RT data:
  - conceptual overview
  - RT-Distrib-Fit program for MATLAB
- Application example
- Caveats regarding interpretation of distribution parameters



## Response Times (RT)

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#### Outline

Response Time (RT)

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Application Example

- Response time (RT) is a popular dependent measure in cognitive psychology
- Long history and rich tradition (e.g., Donders, 1868; Luce, 1986; Ratcliff, 1978; Sternberg, 1969; Wundt, 1880)
- Spawned several sophisticated sequential sampling models of choice RT (e.g., Ratcliff, 1978; Brown & Heathcote, 2005, 2008; Logan et al., 2014; Usher & McClelland, 2001)
- Detailed analyses of empirical RTs provide powerful constraints for choosing between cognitive models (Farrell & Lewandowsky, 2004; Hurlstone & Hitch, 2015, 2018)

## Mean RT $(M_{RT})$

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Application Example

- In empirical studies of RT, mean RT,  $M_{RT}$ , is the dominant measure of performance
- Faster M<sub>RT</sub> in condition A than condition B implies more efficient cognitive processing in condition A
- Thus, M<sub>RT</sub> is a measure of performance—lower M<sub>RT</sub> implies better performance
- However, some researchers have abandoned this approach in favour of more detailed distributional analyses
- Motivated by many problems associated with analysis of  $M_{RT}$  (e.g., Heathcote et al., 1991)

## Problems With $M_{RT}$ : Skewed Data

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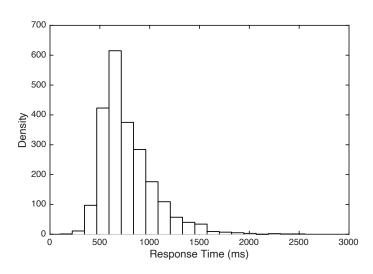
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Application

Cavea





## Problems With $M_{RT}$ : Skewed Data

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Application Example

- Two possible implications of skewed data (Heathcote et al., 1991):
  - 1 the cognitive process of interest yields skewed data
  - 2 the cognitive process of interest yields symmetrical data—skew reflects nuisance variables
- If (1) is true, then an analysis of distribution shape, not M<sub>RT</sub>, is required
- If (2) is true, then nuisance scores must be removed
- Most researchers assume (2) is true and trim or transform their data



## Problems With $M_{RT}$

### **New Tricks**

### Problems With

### Mean RT

- Data trimming
- 2 Data transformation
- 3 Data representation

## Problems With $M_{RT}$

### **New Tricks**

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## Problems With $M_{RT}$ : Data Trimming

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- Addressing skew by trimming data to eliminate extreme (presumed to be nuisance) values
  - removing trials with an RT above a fixed value
  - removing trials with an RT more than a fixed number of standard deviations (typically 2.5) from the mean
- Creates a distribution closer to normal
- It is reasonable to trim post-error trials and when a participant is known to have been distracted
- But trimming on basis of a trial's value is a brutal response to managing skew that "risks throwing the baby out with the bath water" (Heathcote et al., 1991, p.341)

## Problems With $M_{RT}$

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- Data trimming
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## Problems With $M_{RT}$

### **New Tricks**

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- 3 Data representation

## Problems With $M_{RT}$ : Data Transformation

#### **New Tricks**

### Problems With Mean RT

- Addressing skew using a data transformation (e.g., logarithm) of RT)
- Normalises the distribution by discounting extreme (presumed to be nuisance) values
- If skew is produced by a nuisance process, discounting must be done in proportion to the N data points produced by that process
- Failure to do so means rescaled data may misrepresent the cognitive process of interest
- Transformations are misleading and discard valuable information



## Problems With $M_{RT}$

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## Problems With $M_{RT}$

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- 2 Data transformation
- 3 Data representation

## Problems With $M_{RT}$ : Data Representation

#### **New Tricks**

### Problems With Mean RT

- When a distribution is skewed, the mean misrepresents central tendency—it gives extreme values too much weight
- A partial solution is to use median RT instead
- But when data are skewed, the mean, median, and mode do not converge—the concept of central tendency is ambiguous
- Central tendency is only meaningful for symmetrical distributions
- The analysis of means is misleading

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Application Example

- The analysis of entire distributions of RT solves the problems with  $M_{RT}$
- Preserves all information and provides clear description of behaviour
- Avoids mischaracterising central tendency
- Can detect changes across manipulations not possible with M<sub>RT</sub> (e.g., an increase in skew or a shift in the distribution)
- But we need a statistical model to describe the distribution

### **New Tricks**

### Statistical Models

- The two statistical models that have proved most popular are:
  - ex-Gaussian
  - Shifted Wald

### **New Tricks**

ex-Gaussian

- The two statistical models that have proved most popular are:
  - ex-Gaussian
  - Shifted Wald

#### **New Tricks**

ex-Gaussian

- Convolution of a Gaussian and an exponential distribution
- Three parameters:
  - $\mu$  and  $\sigma$ , the mean and standard deviation of the Gaussian component
  - τ, the mean of the exponential component
- Roughly,  $\mu$  and  $\sigma$  reflect the leading edge of the distribution
- au reflects the upper tail
- Has a positively skewed unimodal shape
- Provides excellent fit to BT distributions

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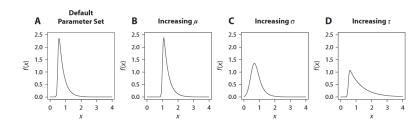
Models ex-Gaussian

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Application Example



Taken from Matzke & Wagenmakers (2009)

### **New Tricks**

ex-Gaussian

 The probability density function (henceforth, 'PDF') of the ex-Gaussian is given by:

$$f(x|\tau,\mu,\sigma) = \frac{1}{\tau} \exp\left(\frac{\mu}{\tau} + \frac{\sigma^2}{2\tau^2} - \frac{x}{\tau}\right) \Phi\left(\frac{x - \mu - \sigma^2/\tau}{\sigma}\right), \tag{1}$$

- where Φ is the cumulative density of the Gaussian component
- Its mean and variance are:

$$E(x) = \mu + \tau \tag{2}$$

and

$$Var(x) = \sigma^2 + \tau^2 \tag{3}$$

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Application Example

- Hohle (1965) proposed the ex-Gaussian reflects the duration of two successive components of cognitive processing
- Gaussian component reflects "the time required for organization and execution of the motor response" (transduction component)
- Exponential component reflects "the decision and perceptual portion of an RT" (decision component)
- This interpretation has been challenged repeatedly (see e.g., Luce, 1965; Matzke & Wagenmakers, 2009; McGill & Gibbon, 1965)

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Application Example

- The ex-Gaussian does not have a plausible theoretical rationale
- The Gaussian component assigns positive probabilities to negative RTs
- It does not correspond to a plausible cognitive process model
- "Although the ex-Gaussian model describes RT data successfully, it does so without the benefit of an underlying theory" (Heathcote et al., 1991, p.346)

### **New Tricks**

ex-Gaussian

- The two statistical models that have proved most popular are:
  - ex-Gaussian
  - Shifted Wald

### **New Tricks**

ex-Gaussian

- The two statistical models that have proved most popular are:
  - ex-Gaussian
  - 2 Shifted Wald

### **New Tricks**

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#### Outline

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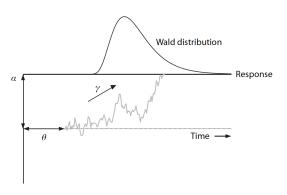
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Application Example

Caveats

 The Wald (1947) distribution is the finishing time distribution of a Wiener diffusion process towards a boundary



Taken from Matzke & Wagenmakers (2009)



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Application Example

- The Wald distribution has two parameters:
  - $\gamma$ , reflecting the drift rate of the diffusion process
  - $\alpha$ , reflecting the separation between the diffusion starting point and boundary
- In the RT context, a third parameter,  $\theta$  is included that shifts the location of the distribution
- Has a positively skewed unimodal shape
- Provides excellent fit to RT distributions

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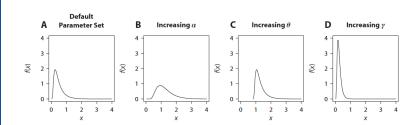
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Taken from Matzke & Wagenmakers (2009)

### **New Tricks**

Shifted Wald

The PDF of the shifted Wald is given by:

$$f(x|\alpha, \theta, \gamma) = \frac{\alpha}{\sqrt{2\pi(x-\theta)^3}}$$

$$\exp\left(-\frac{[\alpha - \gamma(x-\theta)]^2}{2(x-\theta)}\right),$$
(4)

where  $x > \theta$ , its mean and variance are:

$$E(x) = \theta + \alpha/\gamma \tag{5}$$

and

$$Var(x) = \alpha/\gamma^3 \tag{6}$$



### New Tricks

Shifted Wald

- Shifted Wald has a cognitive interpretation
- People accumulate noisy information from the environment until a threshold amount is reached and a response initiated
- Drift rate  $\gamma$  reflects task difficulty or participant ability
- Response criterion  $\alpha$  reflects response caution
- Shift parameter  $\theta$  reflects nondecision time

# Fitting the ex-Gaussian and Shifted Wald to RT Distributions

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# Fitting PDF Functions Objective Function Search Algorithm

Using RT-Distrib-F

Application Example

Caveat

### Next ...

- Guide to how to fit probability functions to RT distributions
- RT-Ditrib-FIT: a MATLAB toolbox for fitting the ex-Gaussian and shifted Wald
- https://github.com/mark-hurlstone/RT-Distrib-Fit
- R toolbox forthcoming

# Fitting the ex-Gaussian and Shifted Wald to RT Distributions

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Application Example

- Fitting probability functions to RT distributions requires at least three functions:
  - 1 functions implementing the ex-Gaussian PDF and shifted Wald PDF
  - a function implementing the computation of the objective function
  - 3 a search algorithm to find best-fitting parameter values

## Fitting the ex-Gaussian and Shifted Wald to RT Distributions

#### **New Tricks**

PDF Functions

 Fitting probability functions to RT distributions requires at least three functions:

- functions implementing the ex-Gaussian PDF and shifted Wald PDF
- a function implementing the computation of the objective function
- a search algorithm to find best-fitting parameter values

## Distribution Functions

#### **New Tricks**

PDF Functions

- Functions are required for implementing the ex-Gaussian (equation 1), and shifted Wald (equation 4) PDFs
- RT-Ditrib-Fit contains two PDF functions:
  - f = exGaussPdf(parms,x)
  - f = shiftWaldPdf(parms,x)
- where f returns the PDF of the relevant distribution, parms is a vector of distribution paramater values  $(\tau,\mu,\sigma \mid \alpha,\theta,\gamma)$ , and x is a data vector of empirical RTs

## Fitting the ex-Gaussian and Shifted Wald to RT Distributions

#### **New Tricks**

PDF Functions

 Fitting probability functions to RT distributions requires at least three functions:

- functions implementing the ex-Gaussian PDF and shifted Wald PDF
- a function implementing the computation of the objective function
- a search algorithm to find best-fitting parameter values

## Fitting the ex-Gaussian and Shifted Wald to RT Distributions

#### **New Tricks**

PDF Functions

- Fitting probability functions to RT distributions requires at least three functions:
  - 1 functions implementing the ex-Gaussian PDF and shifted Wald PDF
  - 2 a function implementing the computation of the objective function
  - a search algorithm to find best-fitting parameter values

## **Objective Function**

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Application Example  An objective function is required that returns the goodness-of-fit of the theoretical PDF— given the supplied parameters—to the empirical data

- Several possibilities:
  - chi-square goodness-of-fit (Smith, 1995)
  - continuous maximum likelihood (Heathcote, 1991)
  - quantile maximum probability (Brown & Heathcote, 2003)
- RT-Distrib-Fit uses continuous maximum likelihood estimation



## Likelihood Function

#### **New Tricks**

Objective Function

• Given a PDF  $f(x|\theta)$  with k parameters,  $\theta = [\theta_1, \theta_2, ..., \theta_k]$ and a set of data containing N observations,  $x_i$ , i = 1,...N, the likelihood function is:

$$L(\theta|x) = \prod_{i=1}^{N} f(x_i|\theta), \tag{7}$$

- where ∏ is the product operator
- Problem: can return values close to zero producing overflow errors

## Log-Likelihood Function

#### **New Tricks**

Objective Function

 Overflow errors can be avoided by using the log of the likelihood

 Substitutes the sum operator with the product operator, which is less likely to produce overflow errors:

$$lnL(\theta|x) = -\sum_{i=1}^{N} ln [f(x_i|\theta)],$$
(8)

- where In is the natural logarithm
- Search algorithms (next) typically use minimisation procedures, so it is customary to minimise the negative log-likelihood instead of maximising the log-likelihood

## Log-Likelihood Function

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- RT-Distrib-Fit computes the log-likelihood via the function, lnL = logMaxLikelihood(parms)
- Nested within the function wrapperLoopFmin, described next

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Application Example  A search algorithm is needed that systematically adjusts the parameters of the to-be-fitted probability distribution to minimise the objective function (maximise the log-likelihood)

- The SIMPLEX algorithm (Nelder & Mead, 1965) is a robust and widely used parameter estimation method
- Invoked in MATLAB using the inbuilt fminsearch function (invoked via the optim function in R)
- RT-Distrib-Fit uses the function fminSearchBnd—version of SIMPLEX with reflection boundaries for to-be-estimated parameters

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Application Example

- SIMPLEX algorithm requires starting parameter values to initiate the search
- The closer these starting points are to the true parameter values, the better the performance of SIMPLEX
- Heuristic starting points are available for the ex-Gaussian (Lacouture & Cousineau, 2008)
- Sensible starting parameters can also be found for the shifted Wald (Heathcote, 2004)
- To avoid local minima problems, it is imperative that the search is conducted with multiple starting parameter values

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Application Example

- RT-Distrib-Fit contains a function that generates starting parameter values and reflection boundaries for parameters
- [startVec1,startVec2,startVec3,IB,uB] = ... genStartingParameters(data,chooseDistrib)
- where data is a data vector of empirical RTs and chooseDistrib is the distribution being fitted (0 = ex-Gaussian, 1 = shifted Wald)
- startvec1, startVec2, and startVec3 are vectors of starting parameter values  $(\tau,\mu,\sigma\mid\alpha,\theta,\gamma)$ , and IB and UB are vectors of lower and upper boundaries on parameter values  $(\tau,\mu,\sigma\mid\alpha,\theta,\gamma)$

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Application Example

- Yields three starting values for each distribution parameter
- Starting values and reflection boundaries are input arguments to another function wrapperLoopFmin
- bestX = wrapper-LoopFmin(parms,data,startVec1,startVec2,startVec3,IB,uB, chooseDistrib)
- Runs SIMPLEX with 27 different starting parameter combinations
- Returns bestX, a vector of the best-fitting parameter estimates (τ,μ,σ | α,θ,γ)

## Using RT-Distrib-Fit

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Application Example

- A front-end script, rtDistribFitScript, controls the fitting
- To-be-fitted data should be stored as text files in the RT-Distrib-Fit MATLAB directory
- Naming convention: Participant\_1.txt, Participant\_2.txt, Participant\_3.txt ...
- Each row represents an RT, each column represents a condition
- Choose what distribution you want to fit by setting the parameter chooseDistrib (0 = exGaussian, 1 = shifted Wald) then hit F5 to run

## Using RT-Distrib-Fit

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#### **Dutline**

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Application Example **1** Retrieves data for participant p and condition c

- Sort's the data and removes any missing values (coded as NaN in data files)
- 3 Generates starting parameter values based on the participant's data
- 4 Fits the data using 27 different starting points
- **5** Records best-fitting parameters, lnL,  $\chi^2$ , and KS tests of data RT distributions
- 6 Iterate until all participant data has been fit
- z-Transform participant RTs, rescale, then fit group distribution (see Rouder, 2014; cf. Vincent averaging)
- 8 Results written to text files (last row is group fit)
- Generate histogram plots with best-fitting PDF overlaid

## Some Considerations

#### **New Tricks**

Using RT-Distrib-Fit

- You need at least 100 RT observations per condition to obtain stable maximum likelihood estimates (Heathcote et al., 1991)
- Ignore fits to individual participants if there are less than 100 observations each—use group fits instead
- Reminder: these are contained in the final row of the output files

# Application Example

#### **New Tricks**

**Application** Example

- Artificial RT data set generated using shifted log-normal distribution (for lack of simple RT data set)
- Another distribution that provides an excellent fit to empirical RT distributions (Ratcliff & Murdock, 1976)
- 15 artificial participants, 3 treatments, 150 RTs each
- Uniform random sampling of parameters  $(\mu, \sigma, \text{ and } \theta)$ with different expected values across treatments
- Fit ex-Gaussian and shifted Wald to resulting RT distributions

## M<sub>RT</sub> By Treatment

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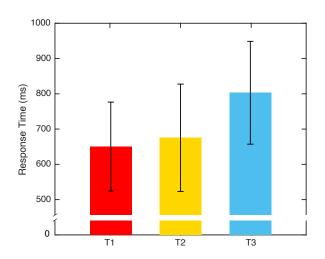
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### PDF Functions Objective Function

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# PDF Histograms With Fitted ex-Gaussian Functions

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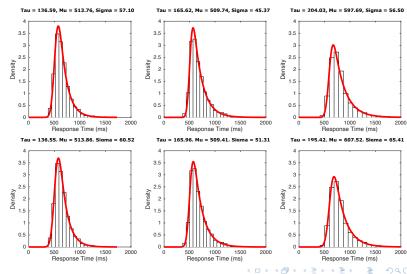
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# PDF Histograms With Fitted Shifted Wald Functions

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## Model

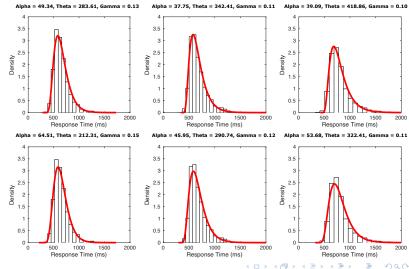
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## Chi-Square Goodness-of-Fits

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	Distribution					
	ex-Gaussian			Shifted Wald		
Fit Type	$T_1$	$T_2$	$T_3$	$T_1$	$T_2$	$T_3$
Individuals	18.76	14.49	27.47	20.99	16.45	21.97
Group	19.55	12.22	30.60	68.88	72.33	99.24

## Summary

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Application Example

- Fits of ex-Gaussian and shifted Wald show the (hypothetical) manipulation caused an increase in skew of RT distribution, but not a shift in location
- Not discernible from analysis of M<sub>RT</sub>
- Both ex-Gaussian and shifted Wald provided excellent fits to individual participant RT distributions
- ex-Gaussian also provided an excellent fit to group data, whereas shifted Wald performed less well
- Parameter averaging recommended for both distributions (shifted Wald perhaps more so) where possible (cf. Rouder & Speckman, 2004)

## How To Report a Distributional Analysis

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Application Example

- Conventional analysis of M<sub>RT</sub>
- Distributional analysis:
  - KS tests of empirical RT distributions with example density histograms for one or more participants
  - Group density histograms per condition, with best fitting probability function overlaid
  - Table or plot of estimated distribution parameters by condition
  - Inferential statistics (e.g., ANOVA) performed on distribution parameters (for fits to individual participants)
- See Heathcote et al. (1991) for guidelines and an example



## Caveats

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Application Example

- Should you interpret changes in distribution parameters in terms of components of cognitive processing?
- Matzke and Wagenmakers (2009) fit the Ratcliff (1978) diffusion model to ex-Gaussian and shifted Wald probability distributions
- Diffusion model contains parameters that are known to map onto specific cognitive processes
- If ex-Gaussian and shifted Wald parameters represent components of cognitive processing, they should relate to parameters of the diffusion model

## Caveats

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Application Example

- Matzke and Wagenmakers (2009) find no one-to-one mapping of diffusion model parameter estimates with ex-Gaussian and shifted Wald parameter values
- We conclude that researchers should resist the temptation to interpret changes in the ex-Gaussian and shifted Wald parameters in terms of cognitive processes (Matzke & Wagenmakers, 2009, p.798)
- Use these distributions as descriptive, rather than inferential, tools

## Fin!

New Tricks

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Outline

Response Time (RT)

Problems With

Mean RT

Models

ex-Gaussia

Shifted Wald

PDF Functions
Objective Function
Search Algorithm

Using RT-Distrib-Fi

Applicatio Example

Caveats

Thanks for listening!

## Recommended Reading and References

### **New Tricks**

Caveats

To be added shortly ...

