Introduction To Factorial Designs and Interactions

PSYC214: Statistics For Group Comparisons

Mark Hurlstone
Lancaster University

Week 6

Learning Objectives

- Introduction to factorial designs
  - Two-factor designs
  - Three-factor designs
- Outcomes of factorial designs
  - Main effects
  - Simple main effects
  - Interaction
- Why do we need factorial designs?
- Planning factorial designs
- Analysing factorial designs

Beyond Single Factor Designs

- The **single factor design** forms a minority in psychology:
  - too simple to address complex questions
  - can give a false impression of importance of a factor
- In a **factorial design**, two or more factors are varied simultaneously:
  - common in cognitive and social psychology
  - can address more complicated research questions
  - can identify interactions between factors
- Couldn't we just use multiple t-tests?
  - inflation of familywise Type I error rate

Language of Factorial Designs

- A factorial design is referenced by the number of its factors (e.g., two-factor design, three-factor design etc.)
- Factors are referenced by name (e.g., A, B, C)
- Levels of a factor are referenced by subscripts (e.g., A_1, A_2, B_1, B_2)
- A design with two-factors, each with two levels, is described as a 2 $\times$ 2 (read as "two–by–two") factorial design
- The total number of treatment conditions is calculated by multiplying the levels of each factor
- A 2 $\times$ 2 $\times$ 2 design is a **three-factor design** where each factor has two levels (8 conditions in total)

Language of Factorial Designs

- **Fully between-participants factorial design:**
  - a design containing factors that are all manipulated between-participants
- **Fully within-participants factorial design :**
  - a design containing factors that are all manipulated within-participants
- **Mixed factorial design:**
  - a design containing a mixture of factors that are manipulated between- or within-participants

Example: Fear Appeals and COVID-19 Vaccination Intentions

- Does exposure to a fear-based message—known as a “fear appeal”—increase people’s intention to get vaccinated against COVID-19?
- Does exposure to a self-efficacy message increase people’s intention to get vaccinated against COVID-19?
- A 2 $\times$ 2 fully between-participants design:
  1. Fear: no fear appeal *vs.* fear appeal
  2. Efficacy: no efficacy message *vs.* efficacy message
- One dependent variable:
  - Likelihood of vaccinating against COVID-19 measured on a 0 (Very Unlikely) to 10 (Very Likely) scale

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear	
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*
Factor *B*:	Level B_1 no efficacy message	Vaccination intention scores for a group of participants who received no fear appeal and no efficacy message	Vaccination intention scores for a group of participants who received a fear appeal but no efficacy message
Efficacy	Level B_2 efficacy message	Vaccination intention scores for a group of participants who received no fear appeal but did receive an efficacy message	Vaccination intention scores for a group of participants who received both a fear appeal and an efficacy message

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear	
		Level A_1	*Level A_2*
		no fear appeal	*fear appeal*
Factor B:	Level B_1 no efficacy message	Vaccination intention scores for a group of participants who received no fear appeal and no efficacy message	Vaccination intention scores for a group of participants who received a fear appeal but no efficacy message
Efficacy	Level B_2 efficacy message	Vaccination intention scores for a group of participants who received no fear appeal but did receive an efficacy message	Vaccination intention scores for a group of participants who received both a fear appeal and an efficacy message

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear	
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*
Factor *B*:	Level B_1 no efficacy message	Vaccination intention scores for a group of participants who received no fear appeal and no efficacy message	Vaccination intention scores for a group of participants who received a fear appeal but no efficacy message
Efficacy	Level B_2 efficacy message	Vaccination intention scores for a group of participants who received no fear appeal but did receive an efficacy message	Vaccination intention scores for a group of participants who received both a fear appeal and an efficacy message

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear	
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*
Factor *B*:	Level B_1 no efficacy message	Vaccination intention scores for a group of participants who received no fear appeal and no efficacy message	Vaccination intention scores for a group of participants who received a fear appeal but no efficacy message
Efficacy	Level B_2 efficacy message	Vaccination intention scores for a group of participants who received no fear appeal but did receive an efficacy message	Vaccination intention scores for a group of participants who received both a fear appeal and an efficacy message

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear	
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*
Factor *B*:	Level B_1 no efficacy message	Vaccination intention scores for a group of participants who received no fear appeal and no efficacy message	Vaccination intention scores for a group of participants who received a fear appeal but no efficacy message
Efficacy	Level B_2 efficacy message	Vaccination intention scores for a group of participants who received no fear appeal but did receive an efficacy message	Vaccination intention scores for a group of participants who received both a fear appeal and an efficacy message

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear		
		Level A_1	*Level A_2*	
		no fear appeal	*fear appeal*	
Factor B:	Level B_1 no efficacy message	A_1B_1	A_2B_1	Mean B_1
Efficacy	Level B_2 efficacy message	A_1B_2	A_2B_2	Mean B_2
		Mean A_1	Mean A_2	Grand Mean

A 2 × 2 Factorial Design

Table: A 2 × 2 factorial design

		Factor A: Fear	
		Level A_1	*Level A_2*
		no fear appeal	*fear appeal*
Factor *B*:	Level B_1 no efficacy message	1/4 of participants	1/4 of participants
Efficacy	Level B_2 efficacy message	1/4 of participants	1/4 of participants

Factors Can Have More Than Two Levels

- Just as there is no limit on the number of factors in a factorial design, there is no limit on the number of levels in a factor
- Sticking with our COVID-19 example, suppose we want to know if the amount of fear depicted in the fear appeal influences COVID-19 vaccination intentions
- We could adopt a 3 $\times$ 2 fully between-participants design:
    1. Fear: low fear *vs.* medium fear *vs.* high fear
    2. Efficacy: no efficacy message *vs.* efficacy message
- As before, we measure likelihood of vaccinating against COVID-19 on a 0 (Very Unlikely) to 10 (Very Likely) scale

A 3 × 2 Factorial Design

Table: A 3 × 2 factorial design

		Factor A: Fear			
		Level A_1 *low fear*	*Level A_2* *medium fear*	*Level A_3* *high fear*	
Factor B:	Level B_1 no efficacy message	A_1B_1	A_2B_1	A_3B_1	Mean B_1
Efficacy	Level B_2 efficacy message	A_1B_2	A_2B_2	A_3B_2	Mean B_2
		Mean A_1	Mean A_2	Mean A_3	Grand Mean

Examples of Fully Within-Participants and Mixed Designs

- In the Stroop task, participants must name the ink colour of a colour word as quickly as possible:
    - on *congruent trials*, the ink colour and colour name are consistent
    - on *incongruent trials*, the ink colour and colour name are inconsistent
- Stroop effect = longer RTs for incongruent, compared to congruent, trials
- A measure of **response inhibition**

Congruent Trials

RED
GREEN

Incongruent Trials

RED
GREEN

Example of A Fully Within-Participants Design

Research Methods I

m.hurlstone@lancaster.ac.uk

Factorial Designs
Two-Factor Designs
Three-Factor Designs

Outcomes of Factorial Designs
Main Effects
Simple Main Effects
Interaction

Why Factorial Designs?

Planning Factorial Designs

Analysing Factorial Designs

References

- A researcher wants to know if the size of the Stroop effect decreases with practice
- She employs a 2 $\times$ 3 fully within-participants design:
  - trial type: congruent *vs.* incongruent
  - trial block: 1 *vs.* 2 *vs.* 3
- Making *trial type* within-participants means we can establish each participant's susceptibility to the Stroop effect
- *trial block* must necessarily be a within-participants factor, as it requires experience with the task
- There are 2 $\times$ 3 = 6 conditions; a single group of participants completes each condition

Example of A Mixed Design

- A researcher wants to know if response inhibition is impaired in patients with Schizophrenia using the Stroop task
- She employs a 2×2 mixed design:
  - trial type: congruent *vs.* incongruent
  - patient group: healthy *vs.* Schizophrenia
- *trial type* is once again a within-participants factor
- *patient group* must necessarily be a between-participants factor
- There are $2 \times 2 = 4$ conditions; two groups of participants (healthy *vs.* Schizophrenia) each complete two conditions of the experiment (congruent *vs.* incongruent trials)

Three-Factor Designs

- The simplest possible three-factor design is a 2 $\times$ 2 $\times$ 2 design
- This yields 2 $\times$ 2 $\times$ 2 = 8 conditions, or cells
- Any combination of between- and within-participant factors is possible:
  - fully between-participants factors
  - fully within-participants factors
  - one between-participants factor and two within-participants factors (*or* vice versa)
- We have to split the table by one of the factors to display the cell means

A 2 × 2 × 2 Factorial Design

Table: A 2 × 2 × 2 factorial design

Factor A		*Level A_1*		*Level A_2*	
Factor B		*Level B_1*	*Level B_2*	*Level B_1*	*Level B_2*
Factor C	Level C_1	$A_1B_1C_1$	$A_1B_2C_1$	$A_2B_1C_1$	$A_2B_2C_1$
	Level C_2	$A_1B_1C_2$	$A_1B_2C_2$	$A_2B_1C_2$	$A_2B_2C_2$

Three-Factor Designs

- Three-factor designs are not for the feint hearted!
- We won't touch upon them again until our final lecture ☺

Outcomes of Factorial Designs

- In a factorial experiment, various different outcomes are possible:
  - Main effects
  - Simple main effects
  - Interaction

Outcomes of Factorial Designs

- In a factorial experiment, various different outcomes are possible:
  - Main effects
  - Simple main effects
  - Interaction

Outcomes of Factorial Designs: Main Effects

- The simplest outcomes are the **main effects**:
  - represent the overall difference in means of one factor, ignoring the other(s)
- If people given a fear appeal have higher vaccination intentions than those that weren't overall, there is a *significant main effect of fear*
- If people given a self-efficacy message have higher vaccination intentions than those that weren't overall, there is a *significant main effect of efficacy*

Hypothetical Data Table

Table: A 2 $\times$ 2 factorial design

		Factor A: Fear		
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*	Mean
Factor B:	Level B_1 no efficacy message	4	4	4
Efficacy	Level B_2 efficacy message	4	9	6.5
	Mean	4	6.5	5.25

Hypothetical Data Table

Table: A 2 $\times$ 2 factorial design

		Factor A: Fear		
		Level A_1	*Level A_2*	
		no fear appeal	*fear appeal*	Mean
Factor *B*:	Level B_1 no efficacy message	4	4	4
Efficacy	Level B_2 efficacy message	4	9	6.5
	Mean	4	6.5	5.25

Hypothetical Data Table

Table: A 2 $\times$ 2 factorial design

		Factor A: Fear		
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*	Mean
Factor *B*:	Level B_1 no efficacy message	4	4	4
Efficacy	Level B_2 efficacy message	4	9	6.5
	Mean	4	6.5	5.25

Hypothetical Data Table

Table: A 2 $\times$ 2 factorial design

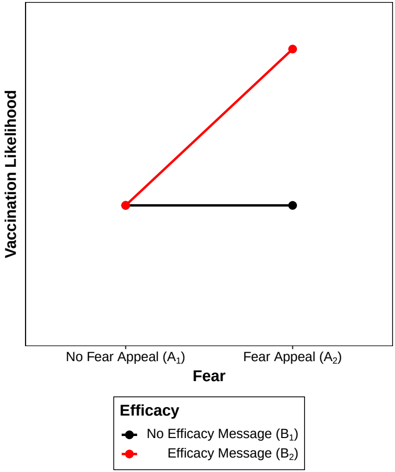
		Factor A: Fear		
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*	Mean
Factor *B*: Efficacy	Level B_1 no efficacy message	4	4	4
	Level B_2 efficacy message	4	9	6.5
	Mean	4	6.5	5.25

Hypothetical Data Table

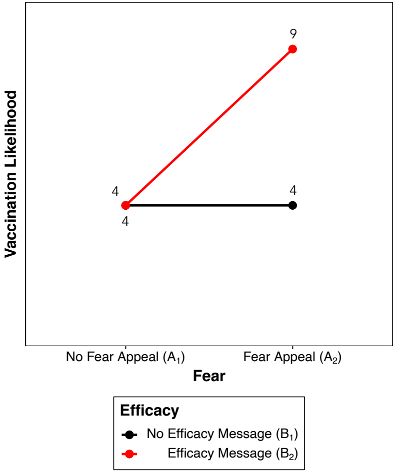
Table: A 2 $\times$ 2 factorial design

		Factor *A*: Fear		
		Level A_1 *no fear appeal*	*Level A_2* *fear appeal*	Mean
Factor *B*:	Level B_1 no efficacy message	4	4	4
Efficacy	Level B_2 efficacy message	4	9	6.5
	Mean	4	6.5	5.25

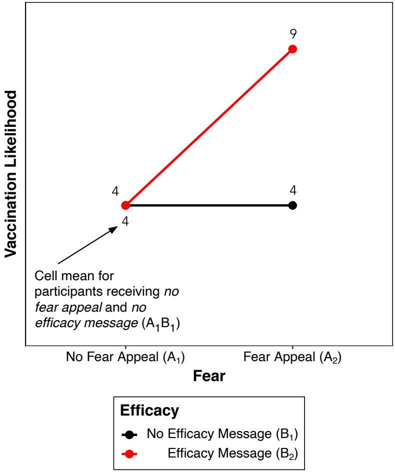
Hypothetical Data Plots



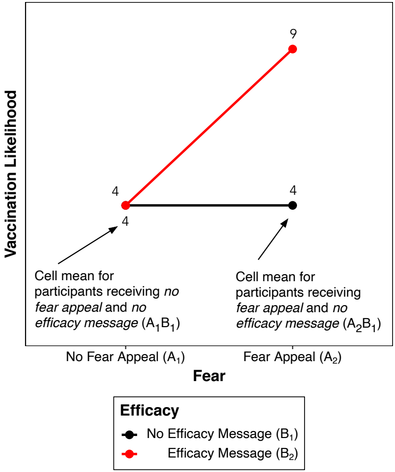
Hypothetical Data Plots



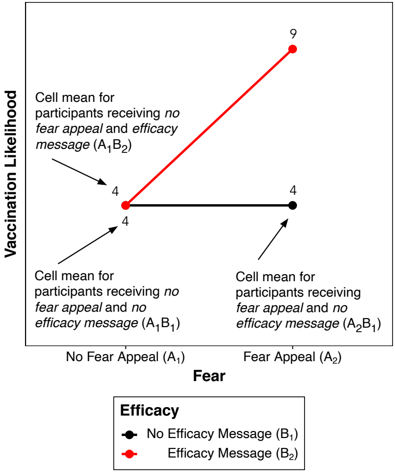
Hypothetical Data Plots



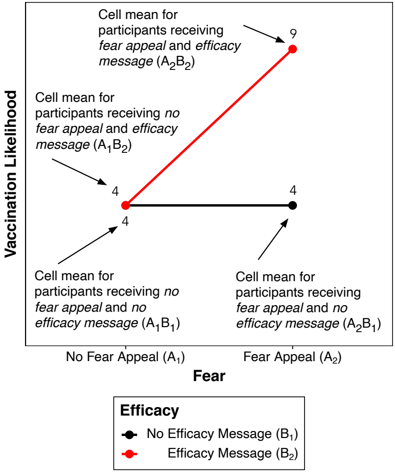
Hypothetical Data Plots



Hypothetical Data Plots



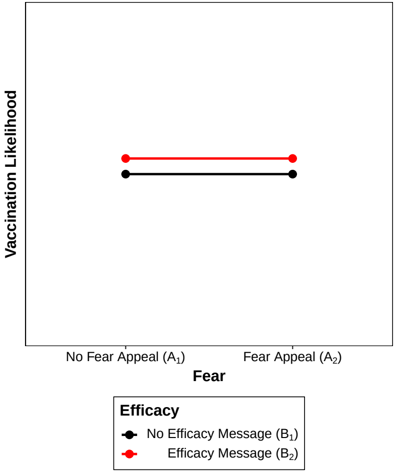
Hypothetical Data Plots



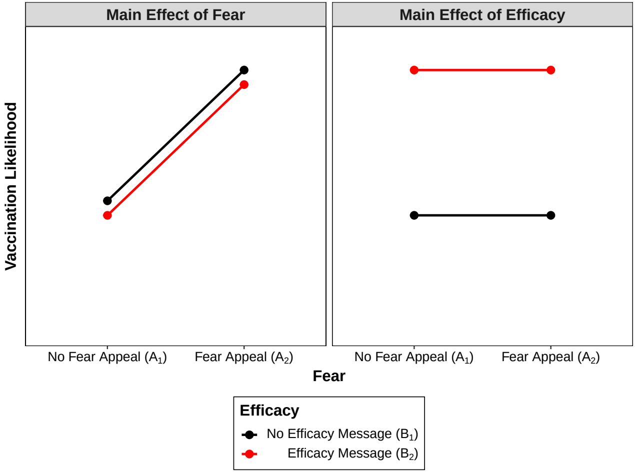
Possible Outcomes For Main Effects

- In a two-factor design, there are three possible outcomes in terms of the main effects:
  1. No significant main effects
  2. One significant main effect
  3. Two significant main effects

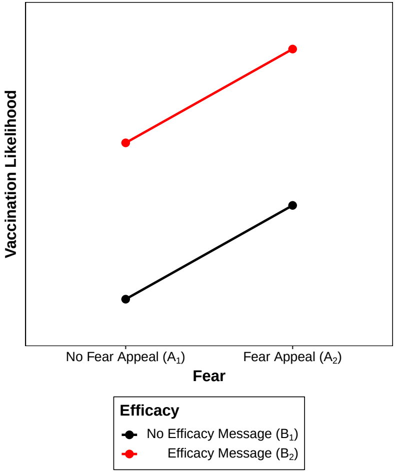
1. No Significant Main Effects



2. One Significant Main Effect



3. Two Significant Main Effects



Outcomes of Factorial Designs

- In a factorial experiment, various different outcomes are possible:
  - Main effects
  - Simple main effects
  - Interaction

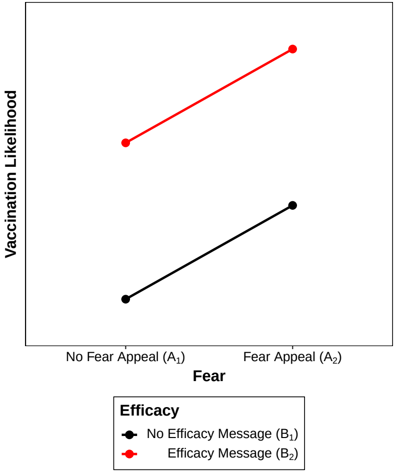
Outcomes of Factorial Designs

- In a factorial experiment, various different outcomes are possible:
  - Main effects
  - Simple main effects
  - Interaction

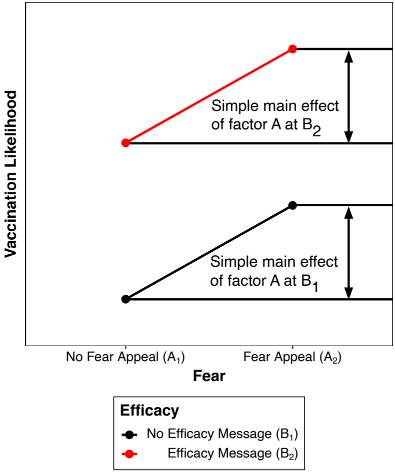
Simple Main Effects

- **Simple main effects** break down main effects into their component parts:
  1. simple main effect of factor A (no fear appeal *vs.* fear appeal) at level B_1 (no efficacy message) of factor B
  2. simple main effect of factor A (no fear appeal *vs.* fear appeal) at level B_2 (efficacy message) of factor B
  3. simple main effect of factor B (no efficacy message *vs.* efficacy message) at level A_1 (no fear appeal) of factor A
  4. simple main effect of factor B (no efficacy message *vs.* efficacy message) at level A_2 (fear appeal) of factor A
- *Let's look at these effects visually ...*

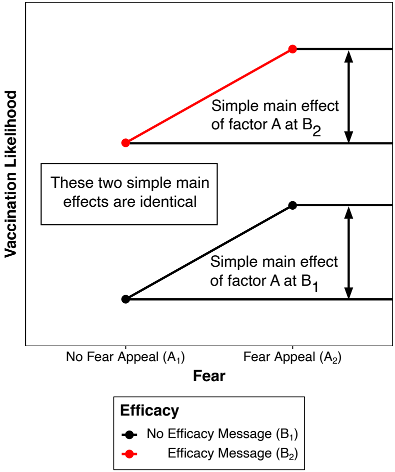
Simple Main Effects



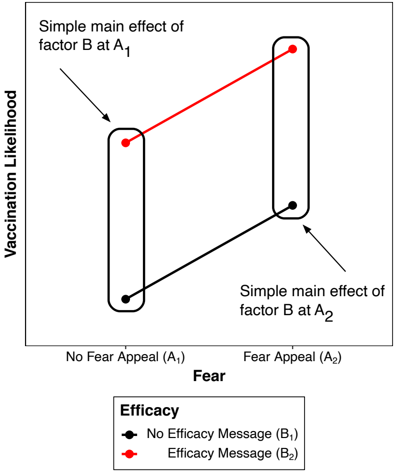
Simple Main Effects



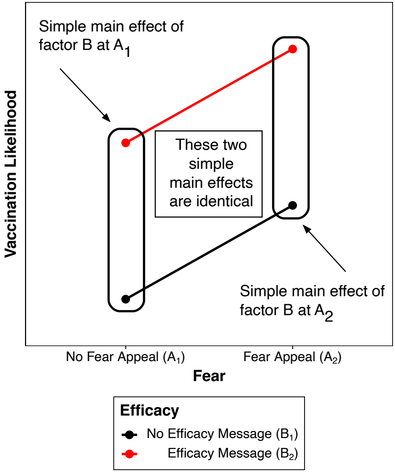
Simple Main Effects



Simple Main Effects



Simple Main Effects



Simple Main Effects

Research Methods I

m.hurlstone@lancaster.ac.uk

Factorial Designs
Two-Factor Designs
Three-Factor Designs
Outcomes of Factorial Designs
Main Effects
Simple Main Effects
Interaction
Why Factorial Designs?
Planning Factorial Designs
Analysing Factorial Designs
References

- In the preceding example, the two factors had independent effects on the dependent measure
- The two simple effects for each factor were identical to the overall main effect from which they were obtained
- *Vaccination intention scores were higher with vs. without a fear appeal, regardless of whether or not participants received an efficacy message*
- *Vaccination intention scores were higher with vs. without an efficacy message, regardless of whether or not participants received a fear appeal*

Simple Main Effects

- Sometimes the simple main effects of one factor will be different at different levels of the second factor
- *In other words, the way one factor is related to the dependent variable may depend on the level of the second factor*
- When this happens, we have an **interaction**
- When there is an interaction, you cannot interpret the results in terms of the main effects
- Instead, you must determine how the factors are *combining* to influence the dependent variable by looking at the simple main effects

Outcomes of Factorial Designs

- In a factorial experiment, various different outcomes are possible:
  - Main effects
  - Simple main effects
  - Interaction

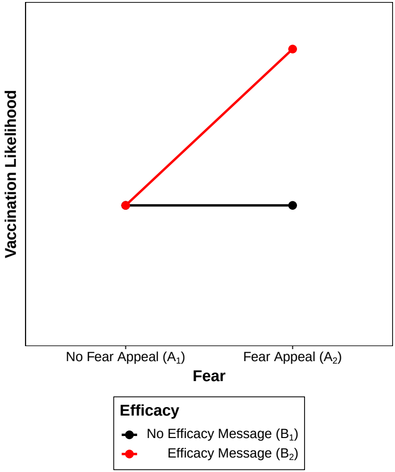
Outcomes of Factorial Designs

- In a factorial experiment, various different outcomes are possible:
  - Main effects
  - Simple main effects
  - Interaction

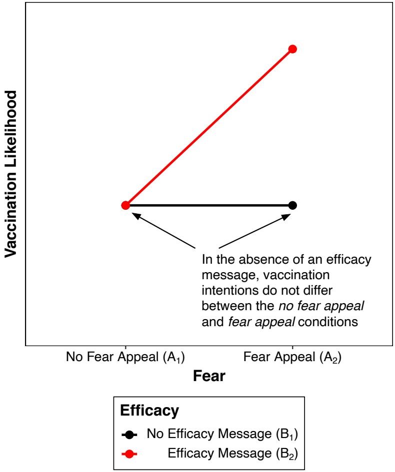
Significant Interaction

- You may now have realised that the hypothetical data presented earlier are an example of an interaction
- *Let's revisit those data ...*

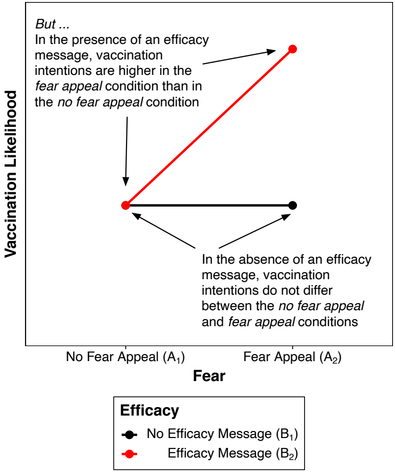
Significant Interaction



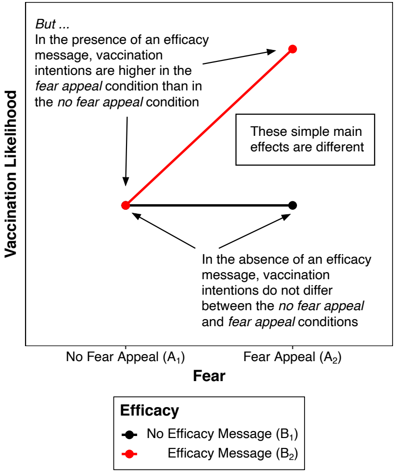
Significant Interaction



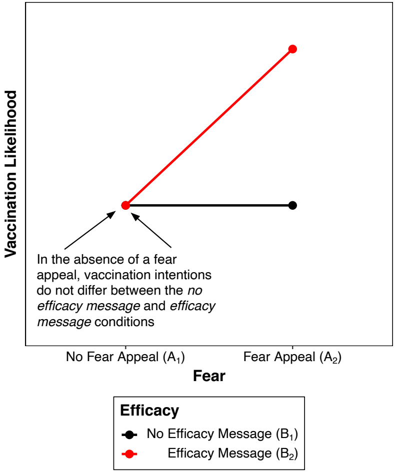
Significant Interaction



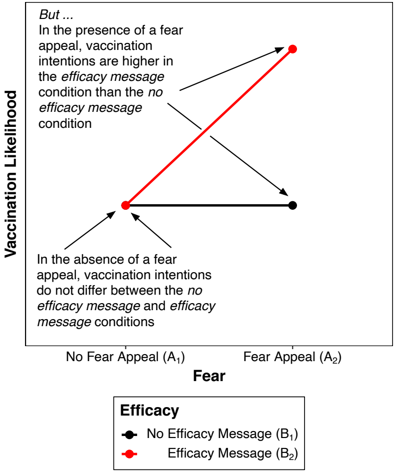
Significant Interaction



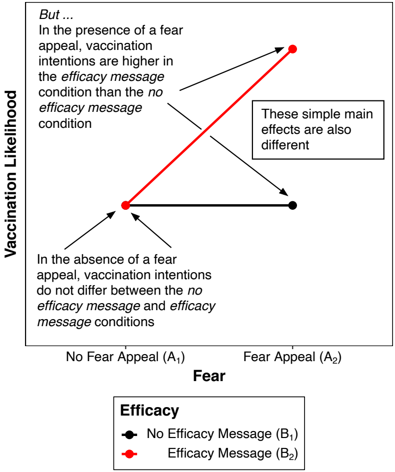
Significant Interaction



Significant Interaction



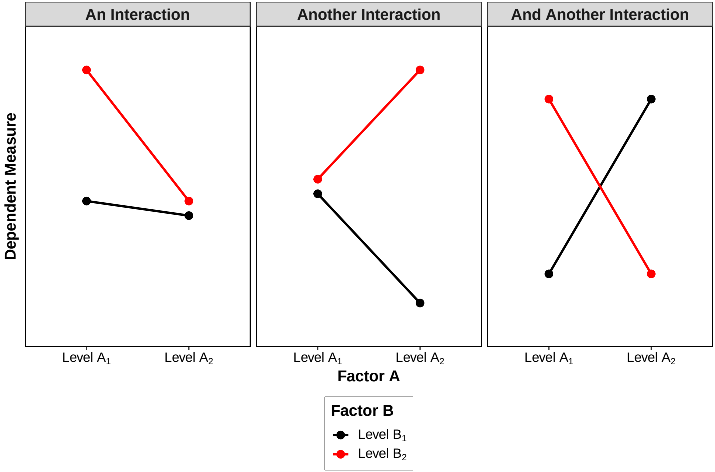
Significant Interaction



How To Spot An Interaction

- If a line plot of the data (also known as an *interaction plot*) has *non-parallel lines*, then this is indicative of the presence of an interaction
- This is the case for the hypothetical data we just considered
- *Here are some additional examples ...*

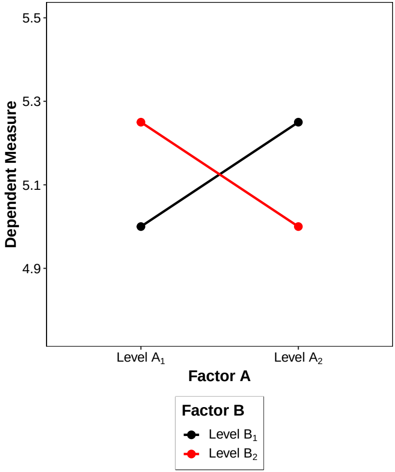
Examples of Interactions: All Have Non-Parallel Lines



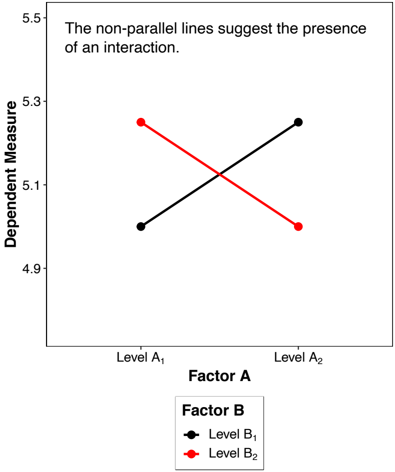
How To Spot An Interaction

- When inspecting interaction plots, check the scale limits on the y-axis
- A tightly compressed scale can create the "illusion of an interaction"

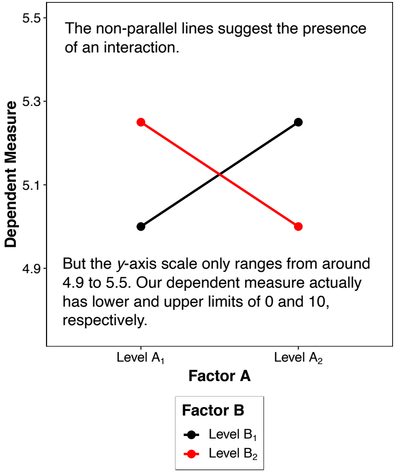
Example of "Illusory Interaction" Due to Scale Compression



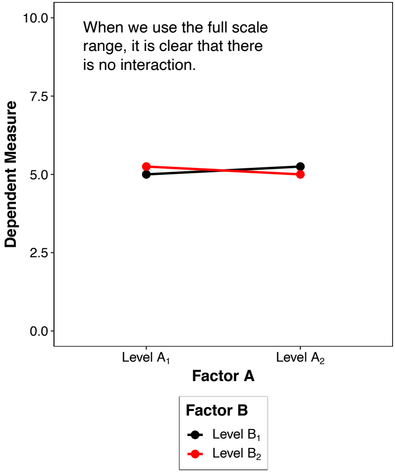
Example of "Illusory Interaction" Due to Scale Compression



Example of "Illusory Interaction" Due to Scale Compression



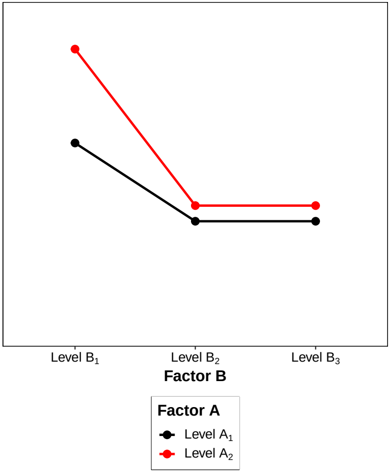
Example of “Illusory Interaction” Due to Scale Compression



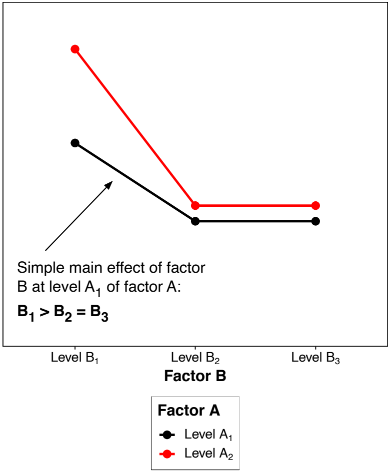
Independence of Sets of Simple Main Effects

- Remember, if there is a significant interaction we must examine the simple main effects
- Keep in mind that sets of simple main effects are *independent*:
  - some simple main effects of one factor may be significant and others not ...
  - ... but this does not mean that some simple main effects of the other factor will also be significant and others not
- *Here's an example using a 2 $\times$ 3 design ...*

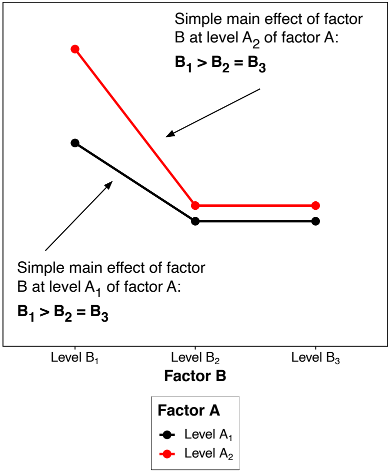
Independence of Sets of Simple Main Effects



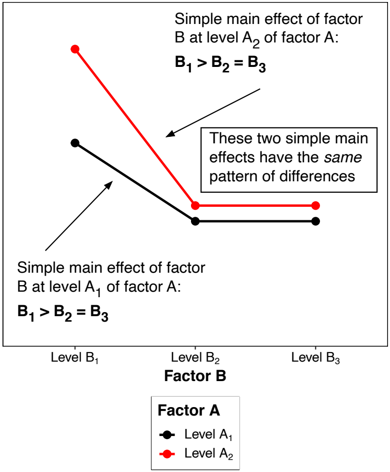
Independence of Sets of Simple Main Effects



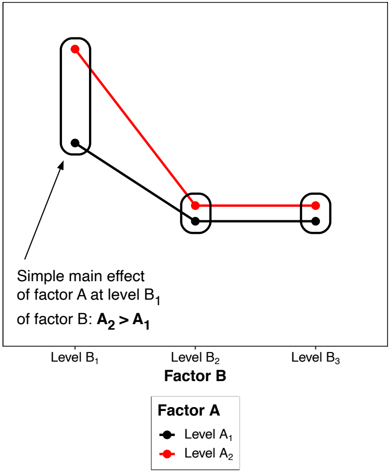
Independence of Sets of Simple Main Effects



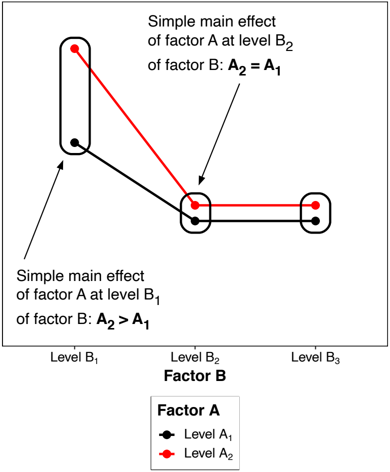
Independence of Sets of Simple Main Effects



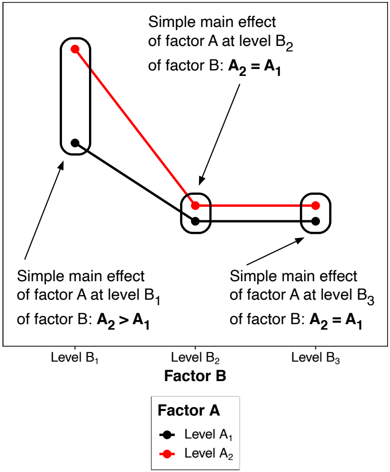
Independence of Sets of Simple Main Effects



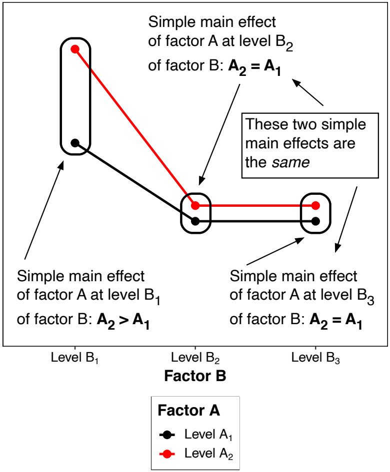
Independence of Sets of Simple Main Effects



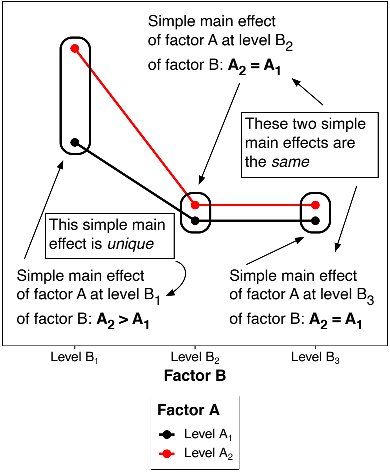
Independence of Sets of Simple Main Effects



Independence of Sets of Simple Main Effects



Independence of Sets of Simple Main Effects



Why Factorial Designs?

- The effect of a factor in a single-factor design can be misleading and conceal a potential interaction
- If we just compare COVID-19 vaccination intentions in the absence and presence of a fear appeal, we would conclude the fear appeal has no effect
- We would dismiss as ineffective the use of fear-based messages to increase COVID-19 vaccination rates
- However, we know from our factorial experiment example that this result is misleading
- When a fear appeal is combined with a self-efficacy enhancing message, the fear-based message has a positive effect on COVID-19 vaccination intentions

Why Factorial Designs?

- In PSYC204 (Week 4), we considered the TV viewing habits of children and their future High-School grades
- When viewing habits are ignored, time watching TV (small *vs.* large amount) as a child has no effect on grades
- When viewing habits are factored into account, there is an interaction:
    - for educational content, High-School grades *increase* with time watching TV
    - for noneducational content, High-School grades *decrease* with time watching TV
- *In both of these examples, a factorial design was required to reach an appropriate conclusion*

Planning Factorial Designs

- Fully between-participants designs are generally easier to interpret but require more participants
- Make sure you have adequate sample size per cell ($\approx$ 20) to protect against Type II errors
- There are tradeoffs between the complexity of a design, how practical it is to run, and the interpretability of its results
- Try to avoid designing studies with more than three factors
- Ideally, no factor should have more than two levels

Analysing Factorial Designs

- We cannot know for certain from "eyeballing" our data what outcomes are significant or not
- A factorial ANOVA produces an *F*-ratio and *p* value for each main effect and interaction
- In a two-factor design, this means:
  - an *F*-ratio and *p* value for the main effect of factor A
  - an *F*-ratio and *p* value for the main effect of factor B
  - an *F*-ratio and *p* value for the A $\times$ B interaction
- Each simple main effect also has an *F*-ratio and *p* value, but we only generate these if the interaction is significant
- Follow up tests will be required for simple main effects with three or more levels

Additional Resources

- The R code for all plots generated in this lecture (minus annotations) has been uploaded with these slides to the Week 6 lecture folder (R Plots For Lecture 6.R)

In Next Week's Lab ...

- Producing line plots and bar graphs for factorial studies
- Interpreting simple main effects
- Generating simulated data

References

Roberts, M. J., & Russo, R. (1999, Chapter 8). *A student's guide to Analysis of Variance*. Routledge: London.