Factorial Designs

PSYC204: Experimental Research Methods

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Week 4

Learning Objectives

- Introduction to factorial designs
- Main effects and interactions
  - Identifying interactions
  - Interpreting main effects and interactions
  - Independence of main effects and interactions
- Types of factorial designs
  - Between- and within-participants designs
  - Mixed designs
  - Pre- and post-test designs
  - Higher-order factorial designs
- Statistical analysis of factorial designs

Factorial Designs

- So far, we have focused on situations involving one independent variable and one dependent variable
- However, variables rarely exist in isolation
- Behaviour is usually influenced by a variety of different variables acting and interacting simultaneously
- To examine these more complex situations, we often design studies that include more than one independent variable

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Today:

Introduction to **factorial designs**

Designs incorporating two or more independent variables

Example: Ackerman & Goldsmith (2011)

- Compared the effectiveness of presentation format and study time on retention of information presented in text
- Two independent variables:
    1. presentation format
        - on paper vs. on screen
    2. study time
        - fixed vs. self regulated
- One dependent variable:
    - exam scores a subsequent multiple choice test of the studied material

Matrix Representation of Conditions

Terminology of Factorial Designs

- When two or more independent variables are combined in a study, the independent variables are called **factors**
- In the study of Ackerman and Goldsmith (2011), there are two factors; presentation format and study time
- A study involving two or more factors is called a **factorial design**
- This type of design is often referenced by the number of its factors (e.g., two-factor design, three-factor design etc.)
- The Ackerman and Goldsmith (2011) study is a **two-factor design**
- A study with a single independent variable is often called a **single-factor design**

Terminology of Factorial Designs

- Factors are referenced by their name (e.g., A, B, C)
- A notation system is used to indicate the number of **levels** of each factor
- Our example study has two levels for the presentation-format factor, and two levels for the study-time factor
- It can be described as a 2×2 (read as "two by two") factorial design
- The total number of treatment conditions can be determined by multiplying the levels for each factor
- A $2 \times 3 \times 2$ design is a **three-factor design** with two, three, and two levels of each of the three factors (12 conditions in total)

Main Effects and Interactions

- A factorial design allows researchers to examine how unique combinations of factors acting together influence behaviour
- We illustrate this using the simplest possible factorial design, the two-factor design
- The data from a factorial study generates two sources of information:
    1. Main effects
    2. Interaction between factors

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Main Effects

- The mean differences among the levels of one factor are called the **main effect** of that factor
- Main effects provide information about the independent effects of each factor
- A two factor study has two main effects, one for each of the two factors
- When a study is represented as a matrix:
  - mean differences among the *columns* define the main effect one factor
  - mean differences among the *rows* define the main effect for the second factor

Main Effects

- We consider some hypothetical data for the paper/on-screen study to illustrate main effects

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Interactions

- In the previous example, the effects of one factor were independent of the levels of the second factor
- Neither factor had a direct influence on the other
- The difference between paper versus on-screen presentation did not depend on how study time was controlled
- The main effect for presentation mode (the 4-point difference in test scores) applied equally to both study-time conditions
- There was a 4-point difference between paper and on-screen in the top row (fixed) and in the bottom row (self regulated)

Interactions

- Sometimes, one factor has a direct influence on the effect of a second factor, producing an **interaction between factors**
- An **interaction** occurs whenever two factors, acting together, produce mean differences not explained by the main effects of the two factors
- If the main effect for either factor applies equally across all levels of the second factor, then the two factors are independent, and there is *no interaction*

Interactions

- We illustrate an interaction between factors using a new data set for the paper/on-screen study
- These reflect the actual pattern of results observed in the original study by Ackerman and Goldsmith (2011)

Interactions

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- Earlier, we defined an interaction as *unique mean differences not explained by the main effects*
- However, there is an alternative, more common, definition
- Specifically, an **interaction** exists *when the effects of one factor depend on the different levels of a second factor*
- For the data just examined, in the fixed-time condition, there is no difference between the two presentation formats
- However, in the self-regulated condition, the paper group scores an average of 8 points higher on the test
- Thus, the effect of one factor (presentation format) depends on the levels of the second (study time), which indicates an interaction

Identifying Interactions in a Data Matrix

- To identify an interaction in a data matrix, we compare the mean differences in any individual row (or column) with the mean differences in other rows (or columns)
- If the size and direction of differences in one row (or column) are the same as the corresponding differences in other rows (or columns) there is no interaction
- If the differences change from one row (or column) to another, there is evidence of an interaction
- For example, in the data just examined, the two means in the top row are 20 and 20, whereas in the bottom row they are 20 and 12
- As the mean difference changes from the top to the bottom row, these data indicate the presence of an interaction

Identifying Interactions in Graphs

- Typically, it is easier to detect the presence or absence of an interaction by plotting the data visually as a line graph
- For a two-factor study, one factor is chosen as the independent variable to appear on the horizontal axis
- Different lines are then plotted, each representing a different level of the second independent variable
- When the results of a two-factor study are graphed, the existence of *nonparallel* lines (lines that cross or converge) is an indication of an interaction between factors
- Let's illustrate using the hypothetical and real data for the paper/on-screen study …

Identifying Interactions in Graphs

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Interpreting Main Effects and Interactions

- In a two-factor study, mean differences between columns and between rows describe the main effects; mean differences between cells describe the interaction
- However, these mean differences are merely descriptive
- They must be evaluated by a statistical test (discussed later) before they can be considered significant
- Until the data are analysed by statistical test, you should exert caution interpreting the results of a factorial study

Interpreting Main Effects and Interactions

- Even if a statistical analysis reveals significant effects, you must still interpret data cautiously
- In particular, if the analysis yields a significant interaction, then the main effects, whether significant or not, may not present an accurate picture of the data
- Remember, the main effect for one factor is obtained by averaging all the different levels of the second factor
- Since each main effect is an average, it may not accurately represent any of the individual effects used to compute that average

Interpreting Main Effects and Interactions

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Independence of Main Effects and Interactions

- A two-factor study allows us to evaluate three separate sets of mean differences:
    1. Mean differences from the main effect of factor A
    2. Mean differences from the main effect of factor B
    3. Mean differences from the interaction between factors
- The three sets of mean differences are separate and completely independent
- A two-factor study may therefore yield *any* possible combination of main effects and interaction

Three Possible Combinations of Main Effects and Interactions

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Types of Factorial Designs

- There are different types of factorial designs:
    1. Between- and within-participants designs
    2. Mixed designs
    3. Pretest–posttest designs
    4. Higher-order factorial designs

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Between- and Within-Participants Designs

- A factorial study can be constructed that is purely a between-participants design
- The advantages and disadvantages of such a design are the same as those highlighted in previous lectures
- One disadvantage merits further comment; specifically, between-participants designs require a large number of participants
- In factorial designs, this problem is often worsened because a multi-factor study typically has more treatment conditions than a single-factor study
- For example, with 30 participants per treatment group a 2×4 factorial design has 8 treatment conditions and requires a total of 240 (8×30) participants

Between- and Within-Participants Designs

- Another disadvantage of between-participants designs is that individual differences can become confounding variables and increase the variance of scores
- On the positive side, a between-participants design is not subject to order effects
- Such designs are best suited to when lots of participants are available, individual differences are small, and order effects are likely

Between- and Within-Participants Designs

- A factorial study can also be constructed that is purely a within-participants design
- The advantages and disadvantages of such a design are the same as those highlighted in previous lectures
- A particular disadvantage for a factorial study is the number of treatment conditions a participant must undergo
- In a 2×4 factorial study, for example, each participant must complete 8 different treatment conditions
- This can be time-consuming, introduce testing effects (e.g., fatigue or practice effects), and make it more difficult to counterbalance the design to control for order effects

Between- and Within-Participants Designs

- On the positive side, within-participants designs require fewer participants and reduce problems associated with individual differences
- Such designs are best suited to situations in which individual differences are large, and there is little reason to expect order effects to be large and disruptive

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Mixed Designs

- Sometimes the advantages of a between-participants design apply to one factor, whereas the advantages of a within-participants design apply to another factor
- For example, one might want to use a within-participants design to take maximum advantage of a small group of participants
- However, if one factor is expected to produce large order effects, then a between-participants design should be used for that factor
- A **mixed design** is a factorial design with one between-participants factor and one within-participants factor

Example: Durso, Luttrell, and Way (2015)

- Examined the effect of acetominophen on experience of pleasure and pain
- Half the participants were given a 1000 mg dose of acetominophen and half were given a placebo (between-participants factor)
- Participants then saw a series of 40 photographs, some containing highly positive images and some containing highly negative images (within-participants factor)
- Participants were required to rate the pleasantness/unpleasantness of each photo

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Pre- and Post-Test Control Group Designs

- A pretest–posttest design involves (at least) two groups of participants
- One group, the treatment group, is measured before and after receiving a treatment
- A second group, the control group, is also measured twice (pretest and posttest) but does not receive any treatment between the two measurements
- This design can be represented as follows:

R	O	X	O	(treatment group)
R	O		O	(nonequivalent control group)

- Where O represents a measurement, X represents a treatment, and R represents random assignment

Pre- and Post-Test Control Group Designs

Pre- and Post-Test Control Group Designs

- This design is an example of a two-factor mixed design
- One factor, treatment/control, is a between-participants factor
- The other factor, pretest-posttest, is a within-participants factor

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Higher-Order Factorial Designs

- It is possible, and common, for factorial designs to incorporate three or more factors
- Such designs are known as **higher-order factorial designs**
- Although powerful, such designs introduce additional complexity
- For example, a three-factor design has three factors (A, B, & C) and produces three main effects
- It also generates three two-way interactions $A \times B$, $B \times C$, $A \times C$
- Additionally, the extra factor introduces the potential for a three-way interaction: $A \times B \times C$

Higher-Order Factorial Designs

- A two-way interaction, such as A $\times$ B, indicates that the effect of factor A depends on the levels of factor B
- The A $\times$ B $\times$ C three-way interaction indicates that the two-way interaction between A and B depends on the levels of factor C
- A three-way interaction can be a challenge to interpret, especially if there are more than two levels within a factor
- It is much harder to interpret a four-way (or higher) interaction
- Although it is possible to add factors to a study without limit, studies incorporating more than three factors can yield complex results that are difficult to interpret

Statistical Analysis of Factorial Designs

- The analysis of a factorial design is undertaken using factorial ANOVA
- The version used depends on whether the design is between-participants, within-participants, or mixed
- The two-factor ANOVA conducts three separate hypothesis tests:
  - one to evaluate the main effect of factor A
  - one to evaluate the main effect of factor B
  - one to evaluate the interaction
- The test uses an F-ratio to determine whether the actual mean differences in the data are significantly larger than expected by chance

Statistical Analysis of Factorial Designs

- If the interaction is significant, then the data should be interpreted in terms of the interaction and not the main effects
- A significant interaction tells us that there are differences in the cell means in the data matrix that cannot be explained in terms of the main effects
- However, it does not tell us which of those cell means differ significantly from one another and which do not
- In a two-factor study, this is typically assessed using a **simple effects analysis**

Statistical Analysis of Factorial Designs

- In a simple effects analysis, we conduct F-tests comparing the differences between the levels of one factor at each level of the second factor:
  - for level 1 of factor A, compare the cell means for level 1 and 2 of factor B
  - for level 2 of factor A, compare the cell means for level 1 and 2 of factor B
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