Three-Factor ANOVA

PSYC214: Statistics For Group Comparisons

Mark Hurlstone
Lancaster University

Week 9

Learning Objectives

- Procedures for analysing and interpreting three-factor ANOVA
- How to decompose a three-way interaction:
  - splitting the design and analysing it as a series of two-factor ANOVAs
- Examples:
  - $2 \times 2 \times 2$ fully within-participants ANOVA
  - $2 \times 2 \times 2$ mixed ANOVA
- General things to consider

Three-Factor ANOVA

- Three-factor ANOVAs are common in psychology
- In such designs, there are three possible two-way interactions:
  - A $\times$ B
  - A $\times$ C
  - B $\times$ C
- There is also the possibility of a *three-way interaction*:
  - A $\times$ B $\times$ C
- Complexity of interpreting these designs arises when the three-way interaction is significant

Three-Factor ANOVA

- Basic design principles of earlier lectures still apply
- A between-participants design is still relatively simple, with only a single error term for all effects
- However, a $2 \times 2 \times 2$ design would require at least 160 participants (obeying our maxim of $N = 20$ per cell)
- If resources are available, the reward is a design that is straightforward to analyse
- Problems with fully within-participants and mixed designs apply equally to three-factor designs
- Try to avoid exceeding two levels per factor where possible

Three-Factor ANOVA

- The most straightforward outcome is when the three-way interaction is not significant
- Where this occurs, one or more of the two-way interactions may be significant
- In which case, each significant two-way interaction should be investigated separately of the others
- The procedures for interpreting each interaction are the same as those discussed in previous lectures
- For example, if the A $\times$ B two-way interaction is significant, the simple main effects of factor *A* at *B* and *B* at *A* can be investigated

Three-Factor ANOVA

- The simplest case arises when none of the interactions are significant
- In this case, the outcome must be interpreted in terms of the main effects, if any of these are significant
- If nothing is significant, then unless specific pairwise comparisons are planned, the analysis is complete

Dealing With A Significant Three-Way Interaction

- A significant three-way interaction occurs when there are different two-way interactions between two of the factors according to the levels of the third factor
- The simplest way to analyse a significant three-way interaction is to reanalyse it as a series of two-factor ANOVAs, e.g:
  1. a 2 (factor A: level A_1 vs. level A_2) $\times$ 2 (factor B: level B_1 vs. level B_2) ANOVA at level C_1 of factor C
  2. a 2 (factor A: level A_1 vs. level A_2) $\times$ 2 (factor B: level B_1 vs. level B_2) ANOVA at level C_2 of factor C
- Any significant interactions would be followed up with a simple main effects analysis

Memory and Context: A $2 \times 2 \times 2$ Fully Within-Participants Design

- A memory researcher wants to know if memory is better when material is tested in the same context it was learned in
- The researcher also want to know whether recall and recognition memory are equally context dependent
- The researcher manipulates three factors in a $2 \times 2 \times 2$ fully within-participants design:
  1. memory test (recall *vs.* recognition)
  2. learning context (learn under water *vs.* learn land)
  3. testing context (test under water *vs.* test land)
- Participants given words to remember in a learning context $\rightarrow$ memory for the words tested via recall or recognition
- Dependent measure is the number of words remembered correctly

Raw Data For Memory and Context Study

Factor A: Task	*Level A_1 recall*				*Level A_2 recognition*			
Factor B: Learning	*Level B_1 under*		*Level B_2 land*		*Level B_1 under*		*Level B_2 land*	
Factor C: Testing	*C_1 under*	*C_2 land*	*C_1 under*	*C_2 land*	*C_1 under*	*C_2 land*	*C_1 under*	*C_2 land*
P_1	8	5	3	7	5	5	7	6
P_2	9	6	3	8	7	6	5	8
P_3	7	5	4	6	6	7	5	6
P_4	8	4	4	5	7	5	6	5
P_5	6	3	3	8	5	4	6	4

Aggregate Data For Memory and Context Study

	Level A_1 recall task		
	Level B_1 *under water*	*Level B_2* *land*	*Overall*
Level C_1 under water	7.6 (same context)	3.4	5.5
Level C_2 land	4.6	6.8 (same context)	5.7
Overall	6.1	5.1	5.6

	Level A_2 recognition task		
	Level B_1 *under water*	*Level B_2* *land*	*Overall*
Level C_1 under water	6 (same context)	5.8	5.9
Level C_2 land	5.4	5.8 (same context)	5.6
Overall	5.7	5.8	5.8

ANOVA Table For Memory and Context Study

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (memory task)	0.23	1	0.23	1.00	0.374
Error $A \times P$	0.90	4	0.23		
B (learning context)	2.03	1	2.03	1.59	0.276
Error $B \times P$	5.10	4	1.28		
C (testing context)	0.03	1	0.03	0.01	0.911
Error $C \times P$	7.10	4	1.78		
$A \times B$	3.03	1	3.03	2.95	0.161
Error $A \times B \times P$	4.10	4	1.03		
$A \times C$	0.63	1	0.63	0.71	0.446
Error $A \times C \times P$	3.50	4	0.88		
$B \times C$	30.63	1	30.63	27.22	0.006
Error $B \times C \times P$	4.50	4	1.13		
$A \times B \times C$	21.03	1	21.03	27.13	0.007
Error $A \times B \times C \times P$	3.10	4	0.78		
P (participants)	10.90	4	2.73		

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Interpreting The Significant Three-Way Interaction

- To decompose our significant three-way interaction, we first need to decide which factor to split our design by
- The obvious choice is factor *A* (memory task: recall *vs.* recognition)
- Next, we perform two two-factor ANOVAs:
  1. 2 (learning context: learn under water *vs.* learn land) $\times$ 2 (testing context: test under water *vs.* test land) ANOVA for the recall memory test condition only
  2. 2 (learning context: learn under water *vs.* learn land) $\times$ 2 (testing context: test under water *vs.* test land) ANOVA for the recognition memory test condition only

ANOVA Table For *Recall* Memory Task

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
B (learning context)	5.0	1	5.00	3.64	0.129
Error $B \times P$	5.5	4	2.38		
C (testing context)	0.20	1	0.20	0.19	0.688
Error $C \times P$	4.30	4	1.08		
$B \times C$	51.20	1	51.20	62.06	0.001
Error $B \times C \times P$	3.30	4	0.82		
P (participants)	5.30	4	1.33		

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A (learning context)	5.0	1	5.00	3.64	0.129
Error $A \times P$	5.5	4	2.38		
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$A \times B$	51.20	1	51.20	62.06	0.001
Error $A \times B \times P$	3.30	4	0.82		
P (participants)	5.30	4	1.33		

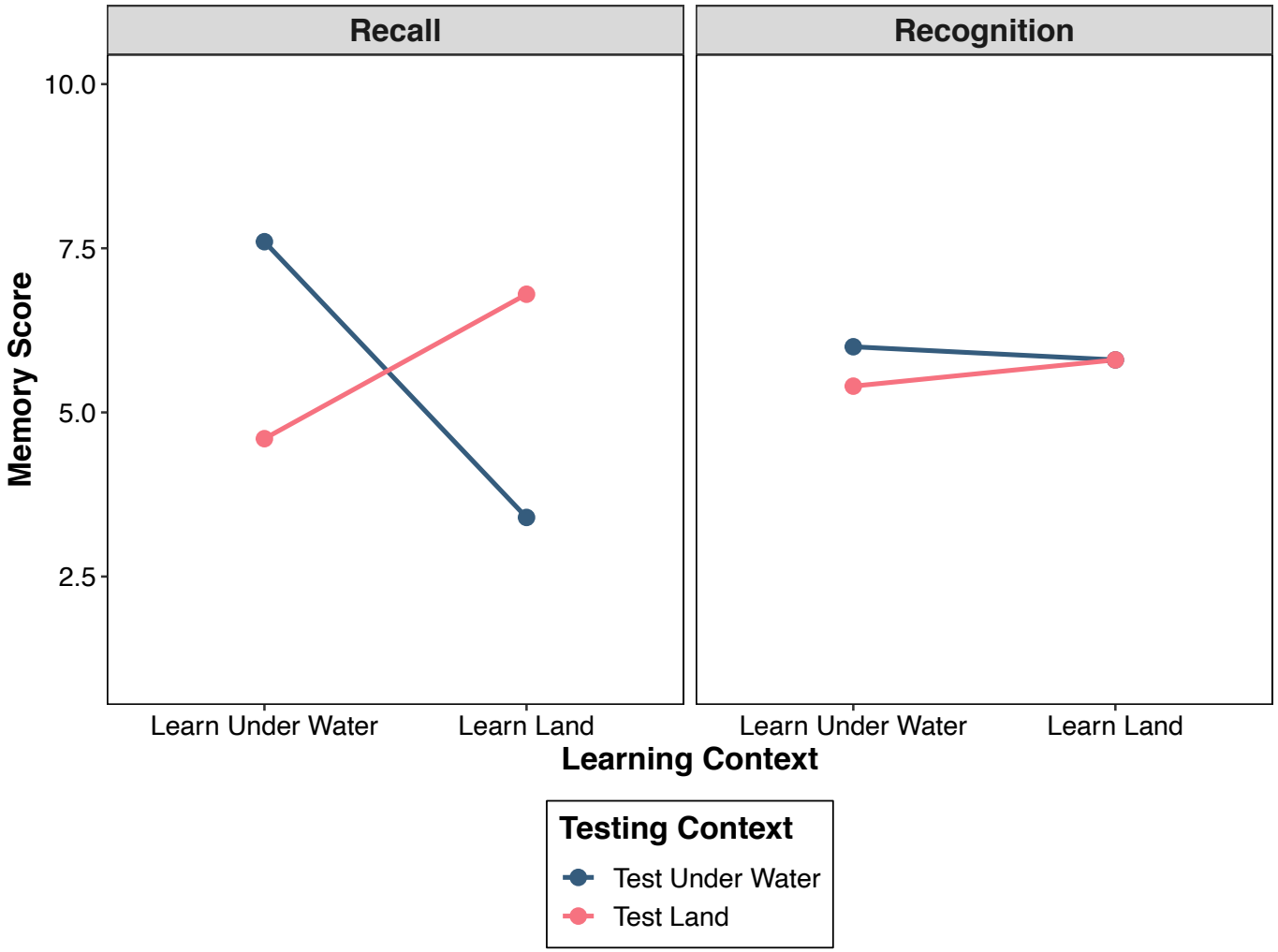
Simple Main Effects Table For *Recall* Memory Task

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
Learning context at					
under water	44.100	1	44.100	32.073	0.005
land	12.100	1	12.100	8.800	0.041
Error term	5.50	4	1.375		
Testing context at					
under water	22.500	1	22.500	20.930	0.010
land	28.900	1	28.900	26.884	0.007
Error term	4.300	4	1.075		

ANOVA Table For *Recognition* Memory Task

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (learning context)	0.05	1	0.05	0.05	0.828
Error $A \times P$	3.70	4	0.93		
B (testing context)	0.45	1	0.45	0.29	0.621
Error $B \times P$	6.30	4	1.58		
$A \times B$	0.45	1	0.45	0.42	0.553
Error $A \times B \times P$	4.30	4	1.08		
P (participants)	6.50	4	1.63		

Interaction Plots For Memory and Context Study



Learning To Pronounce Irregular Words: A 2 × 2 × 2 Mixed Design

- A researcher wants to investigate the development in children's ability to pronounce regular and irregular words
- The researcher adopts a 2 × 2 × 2 mixed design:
  1. age (7 years old *vs.* 9 years old) is between-participants
  2. word frequency (low *vs.* high) is within-participants
  3. word type (regular *vs.* irregular) is within-participants
- Participants are given 10 words to pronounce in each category (40 words in total)
- Dependent measure is the number of pronunciation errors

Raw Data For Word Pronunciation Study

Factor A: Age	*Level A_1 7-years-old*					*Level A_2 9-years-old*			
Factor B: Frequency	*Level B_1 high*		*Level B_2 low*			*Level B_1 high*		*Level B_2 low*	
Factor C: Word type	*C_1 reg*	*C_2 irr*	*C_1 reg*	*C_2 irr*		*C_1 reg*	*C_2 irr*	*C_1 reg*	*C_2 irr*
P_1	6	7	5	6	P_6	4	4	3	6
P_2	7	5	6	7	P_7	3	4	4	7
P_3	5	6	7	6	P_8	4	3	5	9
P_4	6	7	5	7	P_9	5	5	3	8
P_5	6	6	5	7	P_{10}	3	4	3	7

Aggregate Data For Word Pronunciation Study

	Level A_1 7-years-old		
	Level B_1 high	*Level B_2 low*	*Overall*
Level C_1 regular	6	5.6	5.8
Level C_2 irregular	6.2	6.6	6.4
Overall	6.1	6.1	6.1

	Level A_2 9-years-old		
	Level B_1 high	*Level B_2 low*	*Overall*
Level C_1 regular	3.8	3.6	3.7
Level C_2 irregular	4.0	7.4	5.7
Overall	3.9	5.5	4.7

ANOVA Table For Word Pronunciation Study

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (age)	19.600	1	19.600	34.844	< .001
Between error S/A	4.500	8	0.562		
B (frequency)	6.400	1	6.400	5.885	0.042
Error B × *S/A*	8.700	8	1.087		
C (word type)	16.900	1	16.900	36.541	< .001
Error C × *S/A*	3.700	8	0.462		
A × *B*	6.400	1	6.400	5.885	0.042
Error B × *S/A*	8.700	8	1.087		
A × *C*	4.900	1	4.900	10.595	0.012
Error C × *S/A*	3.700	8	0.462		
B × *C*	12.100	1	12.100	17.600	0.003
Error B × *C* × *S/A*	5.500	8	0.688		
A × *B* × *C*	4.900	1	4.900	7.127	0.028
Error B × *C* × *S/A*	5.500	8	0.688		

ANOVA Table For Word Pronunciation Study

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (age)	19.600	1	19.600	34.844	< .001
Between error S/A	4.500	8	0.562		
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Interpreting The Significant Three-Way Interaction

- To decompose our significant three-way interaction, we first need to decide which factor to split our design by
- The obvious choice is our between-participants factor *A* (age: 7 year olds *vs.* 9 year olds)
- Next, we perform two two-factor ANOVAs:
  1. 2 (frequency: low *vs.* high) $\times$ 2 (word type: regular *vs.* irregular) ANOVA for the 7 year olds only
  2. 2 (frequency: low *vs.* high) $\times$ 2 (word type: regular *vs.* irregular) ANOVA for the 9 year olds only

ANOVA Table For *7 Year Olds*

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (frequency)	0.000	1	0.000	0.000	1.000
Error $A \times P$	2.500	4	0.625		
B (word type)	1.800	1	1.800	5.885	0.178
Error $B \times P$	2.700	4	0.675		
$A \times B$	0.800	1	0.800	5.885	0.405
Error $A \times B \times P$	3.700	4	0.925		
P (participants)	0.300	4	0.075		

ANOVA Table For *9 Year Olds*

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (frequency)	12.800	1	12.800	8.258	0.045
Error A $\times$ *P*	6.200	4	1.550		
B (word type)	20.000	1	20.000	80.000	$< .001$
Error B $\times$ *P*	1.000	4	0.250		
A $\times$ *B*	16.200	1	16.200	36.000	0.004
Error A $\times$ *B* $\times$ *P*	1.800	4	0.450		
P (participants)	4.200	4	1.050		

ANOVA Table For *9 Year Olds*

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (frequency)	12.800	1	12.800	8.258	0.045
Error A $\times$ *P*	6.200	4	1.550		
B (word type)	20.000	1	20.000	80.000	< .001
Error B $\times$ *P*	1.000	4	0.250		
A $\times$ *B*	16.200	1	16.200	36.000	0.004
Error A $\times$ *B* $\times$ *P*	1.800	4	0.450		
P (participants)	4.200	4	1.050		

Simple Main Effects Table For *9 Year Olds*

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
word frequency at					
regular words	0.100	1	0.100	0.065	0.812
irregular words	28.900	1	28.900	18.645	0.013
Error term	6.200	4	1.550		
Word type at					
low frequency	36.100	1	36.100	144.400	< .001
high frequency	0.100	1	0.100	0.400	0.561
Error term	1.000	4	0.250		

Simple Main Effects Table For *9 Year Olds*

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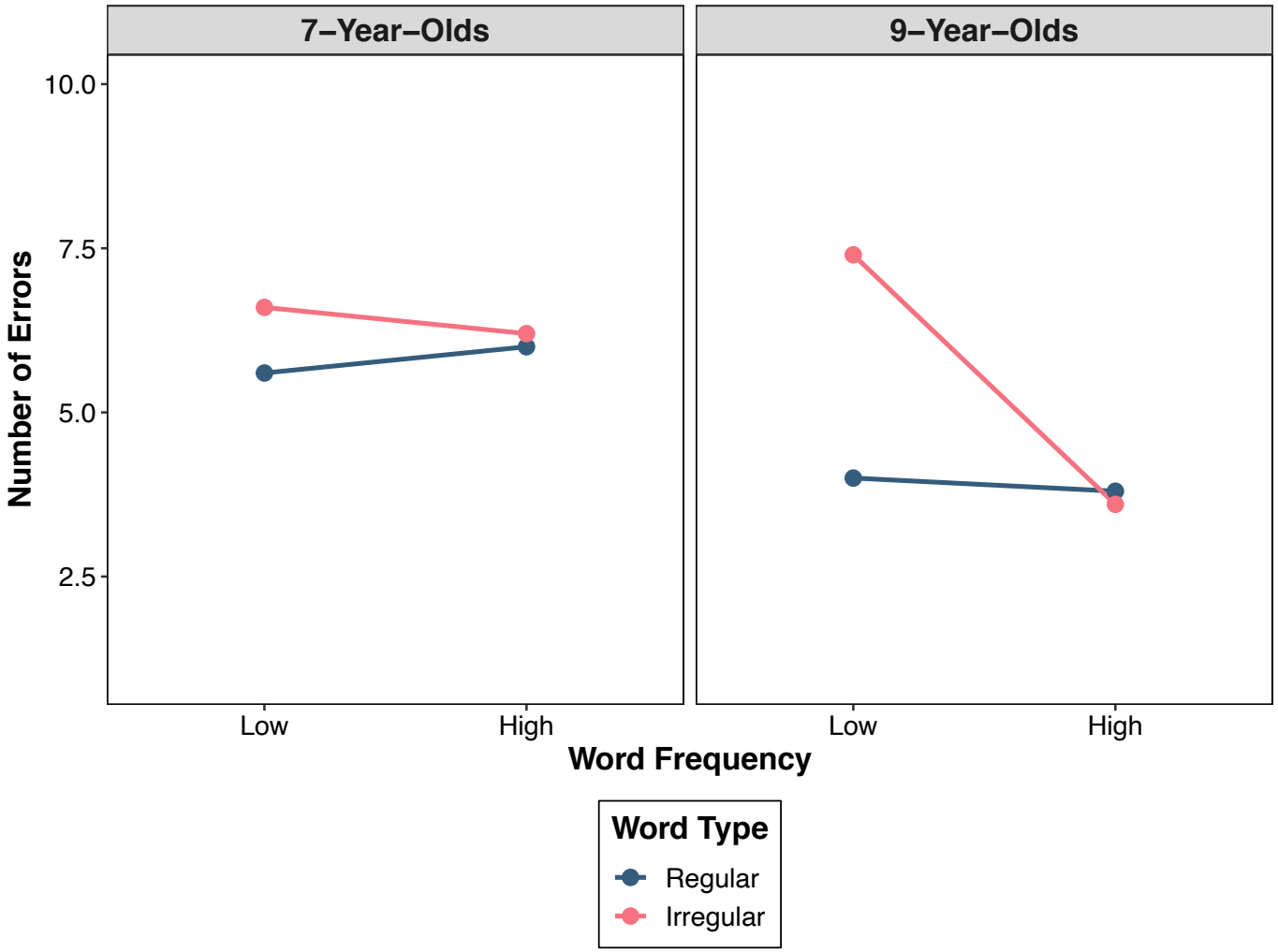
word frequency at					
regular words	0.100	1	0.100	0.065	0.812
irregular words	28.900	1	28.900	18.645	0.013
Error term	6.200	4	1.550		
Word type at					
low frequency	36.100	1	36.100	144.400	< .001
high frequency	0.100	1	0.100	0.400	0.561
Error term	1.000	4	0.250		

Interaction Plots For Word Pronunciation Study



A Final Note On Interpreting Three-way Interactions

- In both of these examples, one of the two-factor ANOVAs returned a significant interaction, whereas the other returned a non-significant interaction
- This will **not** always be the case
- Sometimes the interaction for each two-factor ANOVA will be significant and both will need to be followed up with a simple main effects analysis
- Under these conditions, the simple main effects for the two interactions will differ in direction and/or size of their trends

General Points

- As always, start at the bottom of the ANOVA table and work your way up
- If the three-way interaction is significant, then this must be analysed
- If not, then each of the significant two-way interactions should be analysed independently
- If none of the two-way interactions is significant, the ANOVA results may be described in terms of the main effects, with follow-up tests for any factors with three or more levels

In Next Week's Lab ...

- Running a three-factor (fully within-participants and mixed) ANOVA in R

References

Roberts, M. J., & Russo, R. (1999, Chapter 12). *A student's guide to Analysis of Variance*. Routledge: London.