

*CheckEG* that adds  $\mathbf{EG} f_1$  to  $\text{label}(s)$  for every  $s$  that satisfies  $\mathbf{EG} f_1$ , assuming that  $f_1$  has already been processed correctly.

In order to handle an arbitrary CTL formula  $f$ , we successively apply the state-labeling algorithm to the subformulas of  $f$ , starting with the shortest, most deeply nested, and work outward to include all of  $f$ . By proceeding in this manner we guarantee that whenever we process a subformula of  $f$  all its subformulas have already been processed. Since each pass takes time  $O(|S| + |R|)$  and since  $f$  has at most  $|f|$  different subformulas, the entire algorithm requires time  $O(|f| \cdot (|S| + |R|))$ .

**THEOREM 1** There is an algorithm for determining whether a CTL formula  $f$  is true in a state  $s$  of the structure  $M = (S, R, L)$  that runs in time  $O(|f| \cdot (|S| + |R|))$ .

We will illustrate the model checking algorithm for CTL on a small example that describes the behavior of a microwave oven. Figure 4.3 gives the Kripke structure for the oven. For clarity, each state is labeled with both the atomic propositions that are true in the state and the negations of the propositions that are false in the state. The labels on the arcs indicate the actions that cause transitions and are not part of the Kripke structure.

We check the CTL formula  $\mathbf{AG}(\text{Start} \rightarrow \mathbf{AF} \text{Heat})$  which is equivalent to the formula  $\neg \mathbf{EF}(\text{Start} \wedge \mathbf{EG} \neg \text{Heat})$  (here, we use  $\mathbf{EF} f$  as an abbreviation for  $\mathbf{E}[\text{true} \mathbf{U} f]$ ). We start by computing the set of states that satisfy the atomic formulas and proceed to more complicated subformulas. Let  $S(g)$  denote the set of all states labeled by the subformula  $g$ . Note that, with a suitable data structure, the computation of  $S(p)$  for all  $p \in AP$  requires time  $O(|S| + |R|)$ .

$$S(\text{Start}) = \{2, 5, 6, 7\}.$$

$$S(\neg \text{Heat}) = \{1, 2, 3, 5, 6\}.$$

In order to compute  $S(\mathbf{EG} \neg \text{Heat})$  we first find the set of nontrivial strongly connected components in  $S' = S(\neg \text{Heat})$ .  $\text{SCC} = \{\{1, 2, 3, 5\}\}$ . We proceed by setting  $T$ , the set of all states that should be labeled by  $\mathbf{EG} \neg \text{Heat}$  to be the union over the elements of  $\text{SCC}$ , that is, initially  $T = \{1, 2, 3, 5\}$ . No other state in  $S'$  can reach a state in  $T$  along a path in  $S'$ . Thus, the computation terminates with

$$S(\mathbf{EG} \neg \text{Heat}) = \{1, 2, 3, 5\}.$$

Next we compute

$$S(\text{Start} \wedge \mathbf{EG} \neg \text{Heat}) = \{2, 5\}.$$

When computing  $S(\mathbf{EF}(\text{Start} \wedge \mathbf{EG} \neg \text{Heat}))$ , we start by setting  $T = S(\text{Start} \wedge$

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