```
procedure CheckEU(f1, f2)
    T := { s | f2 ∈ label(s) };
    for all s ∈ T do label(s) := label(s) ∪ { E[f1 U f2] };
    while T ≠ ∅ do
        choose s ∈ T;
        T := T \ {s};
        for all t such that R(t, s) do
            if E[f1 U f2] ∉ label(t) and f1 ∈ label(t) then
                label(t) := label(t) ∪ { E[f1 U f2] };
                T := T ∪ {t};
            end if;
        end for all;
    end while;
end procedure
```

Figure 4.1
Procedure for labeling the states satisfying $\mathbf{E}[f_1 \mathbf{\ U\ } f_2]$.

The case in which $g = \mathbf{EG}\ f_1$ is slightly more complicated. It is based on the decomposition of the graph into nontrivial strongly connected components. A *strongly connected component* (SCC) C is a *maximal* subgraph such that every node in C is reachable from every other node in C along a directed path entirely contained within C. C is *nontrivial* iff either it has more than one node or it contains one node with a self-loop.

Let M' be obtained from M by deleting from S all of those states at which f_1 does not hold and restricting R and L accordingly. Thus, $M' = (S', R', L')$ where $S' = \{ s \in S \mid M, s \models f_1 \}$, $R' = R|_{S' \times S'}$, and $L' = L\,|_{S'}$. Note that R' may not be total in this case. The states with no outgoing transitions may be eliminated, but this is not essential for the correctness of our algorithm. The algorithm depends on the following observation.

LEMMA 1 $M, s \models \mathbf{EG}\ f_1$ iff the following two conditions are satisfied:

1. $s \in S'$.

2. There exists a path in M' that leads from s to some node t in a nontrivial strongly connected component C of the graph (S', R').

Proof Assume that $M, s \models \mathbf{EG}\ f_1$. Clearly $s \in S'$. Let π be an infinite path starting at s such that f_1 holds at each state on π. Since M is finite, it must be possible to write π as $\pi = \pi_0\pi_1$ where π_0 is a finite initial segment and π_1 is an infinite suffix of π with the property that each state on π_1 occurs infinitely often. Then, π_0 is contained in S'. Let C be the set of states in π_1. Clearly, C is contained in S'. We now show that there