The LANS-alpha turbulence model in primitive-equation ocean modeling

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The Lagrangian-Averaged Navier-Stokes alpha (LANS-α) and Leray turbulence parameterizations are demonstrated in a primitive-equation ocean model using an idealized channel domain. For LANS-α, turbulence statistics such as kinetic energy, eddy kinetic energy, and temperature profiles resemble doubled-resolution statistics with the standard model. In a North Atlantic domain with realistic topography, the Leray model increases eddy activity. The LANS-α and Leray models show great promise to improve heat transport and temperature distributions in global ocean-climate simulations, as these processes depend on better resolution of eddies near the grid-scale.

Global ocean-climate modeling has reached a critical turning point. These models can now be run at so-called eddy permitting resolution of 0.1^o in latitude and longitude for ocean-only simulations. Ocean eddies transport momentum and heat, change temperature profiles through the baroclinic instability, and affect the position of large-scale currents [1, 2], but these important effects are missing in current climate simulations, which must be run at low resolutions (typically 1°) because they are centuries-long and coupled with atmosphere, sea-ice, and land-surface models. The LANS-α and Leray models increase eddy activity, and therefore are useful to produce results similar to higher resolution simulations.

The LANS-α model improves turbulence statistics due to a smoothed advecting velocity and an additional nonlinear term. It had previously been implemented in turbulent pipe flow, large eddy simulations, shallow water models, and quasigeostrophic models, all with positive results (see review in [3]). In brief, the primitive equations are modified as

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{v} + u_3 \partial_z \mathbf{v} + v_j \nabla u_j + \mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_0} \nabla \pi + \mathcal{F}(\mathbf{v}), \tag{1}$$

$$\partial_z \pi + v_j \partial_z u_j = -\rho g, \qquad \pi = p - \frac{1}{2}|\mathbf{u}|^2 - \frac{\alpha^2}{2}|\nabla \mathbf{u}|^2, \tag{2}$$

$$\nabla \cdot \mathbf{u} + \partial_z u_3 = 0, \qquad \mathbf{u} = \left(1 - \alpha^2 \nabla^2\right)^{-1} \mathbf{v}, \tag{3}$$

where π is a modified pressure and $\mathcal{F}$ is a diffusion operator. Note there are two velocities in the ocean model, a smooth velocity $\mathbf{u}$, only containing scales greater than α, which performs the transport, and a rough velocity $\mathbf{v}$ that is transported by the flow. These two velocities are related through the Helmholtz operation of Eqn. 3. The extra nonlinear term $v_j \nabla u_j$ is included in LANS-α but not the Leray model, and is critical to potential vorticity conservation: LANS-α conserves potential vorticity, while the Leray model does not.

The LANS-α and Leray turbulence models have been implemented in the POP primitive-equation ocean-climate model. POP, developed at Los Alamos National Laboratory, is the ocean component of NCAR's Community Climate System Model, and is used for twenty-first century climate simulations by the Intergovernmental Panel on Climate Change (IPCC). POP uses finite differences to discretize conservation of momentum, mass, energy, and tracers. The proper implementation of LANS-α in POP raised several new issues, including proper use of the split barotropic/baroclinic timestepping [4], and the choice of efficient local filters rather than the Helmholtz inversion (3) to smooth the rough velocity [5].

The POP-α algorithms and smoothing methods were tested using an idealized channel configuration that induces the baroclinic instability, as described in [4, 5]. The domain is a zonally periodic channel with a meridional deep-sea ridge, westerly wind forcing, and surface thermal forcing that is warm in the north and cool in the south. These conditions are similar to those in the Southern Ocean, and cause the isopycnals to tilt downward from south to north. In the real ocean, mesoscale eddies transfer heat and flatten the isopycnals, thereby converting the potential energy of tilted isopycnals to the kinetic energy of the eddies themselves. In ocean models, this only occurs if the resolution is sufficiently high for eddies to appear in the simulations. Thus the slope of the isopycnals is an additional diagnostic to measure eddy activity in this test problem.

The kinetic energy (KE), eddy kinetic energy (EKE), and isopycnals using POP-α resemble doubled-resolution POP simulations (Fig. 1, 2). This shows that lower-resolution POP-α simulations actually have more eddy activity, as measured by EKE, and that these eddies produce the correct thermal effect in baroclinic instability, namely the flattening of isotherms and cooling of deep water. Qualitative comparisons of velocity fields often show eddies that are stronger and larger in number in POP-α simulations, when compared to POP simulations of the same resolution [6].

The Leray model produces weaker turbulence statistics than the LANS-α model (Fig. 1, 2). Figure 1 shows energy in the uv norm for both LANS-α and Leray. The conserved quantities (uv norm for inviscid LANS-α and v^2 norm for Leray) are

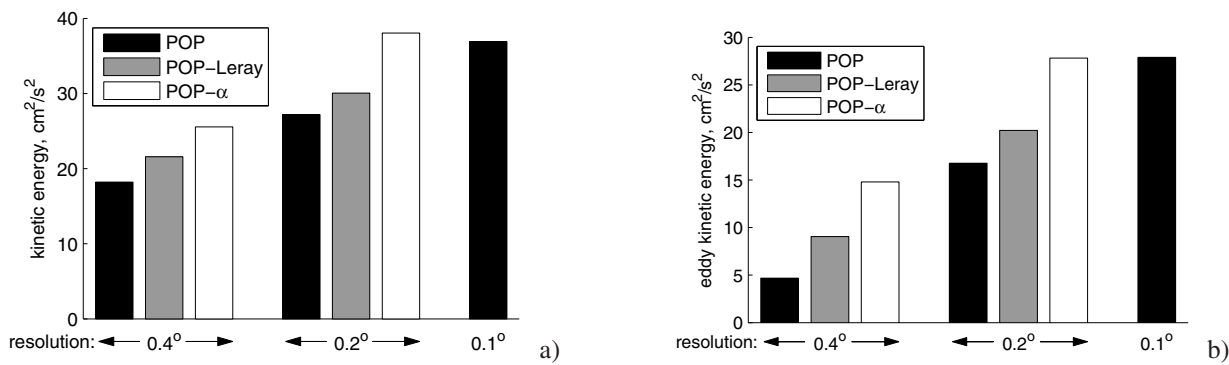* Corresponding author: e-mail: mpetersen@lanl.gov, Phone: 505-661-2587

Fig. 1 Comparison of the LANS-α and Leray models in the ocean channel domain: **(a)** kinetic energy and **(b)** eddy kinetic energy. POP-α resembles the doubled-resolution POP statistics in both measures, indicating that the LANS-α model increases eddy activity. POP-Leray is less energetic than POP-α at the same resolution.
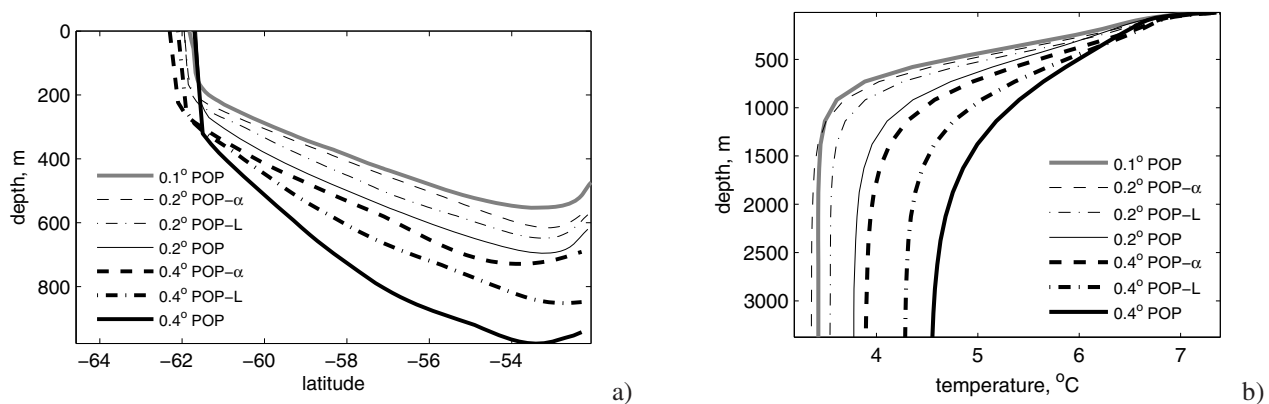

Fig. 2 Comparison of temperature distributions in the channel domain using POP, POP-α, and POP-Leray, showing: **(a)** depth of 6^oC isotherm of potential temperature and **(b)** horizontally averaged potential temperature versus depth. At each resolution, the curve for POP-α is comparable to that of the doubled-resolution POP case. This shows that POP-α is correctly capturing the effects of baroclinic instability, where **(a)** eddy activity flattens isotherms and thus **(b)** cools the deep water and creates a sharper thermocline. POP-Leray correctly captures this same process, but to a lesser degree than POP-α

usually equal (see discussion in [6]). Because of this question of which norm to use when comparing models, the temperature distribution provides a fairer comparison (Fig. 2). Here it is clear that POP-α produces effects near doubled-resolution POP; POP-Leray produces the same trends, but to a lesser degree.

Simulations with the Leray turbulence parameterization were conducted in the North Atlantic domain at 0.2^o resolution, following [1, 2]. As in the channel domain, these simulations produce more eddy activity: standard POP averaged 17.3 and 11.1 cm^2/s^2 in KE and EKE, while POP-Leray averaged 22.1 and 13.1 cm^2/s^2 (in the uv norm) [6].

The LANS-α and Leray turbulence parameterizations have shown a great deal of promise in both an idealized channel model domain and the realistic bathymetry of the North Atlantic. By increasing eddy activity, they can produce turbulence statistics that resemble higher resolution simulations. Current work focuses on improving the filtering technique at the boundary, and combining LANS-α with the Gent-McWilliams tracer isopycnal diffusion model.

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