Important notes of Maths 1 + 2.

diff $\begin{cases} \cos x \rightarrow -\sin x \\ \sin x \rightarrow \cos x \end{cases}$

$\int \cos x \rightarrow \sin x$

$\int \sin x \rightarrow -\cos x$

- $e^{x^2+6x} = e^{x^2+6x} \cdot (2x+6)$
- $\ln(x^2-4x) = \dfrac{1}{x^2-4x} \cdot (2x-4)$

$\sin(x^2+4x)^6 = \cos(x^2+4x)^6 \cdot 6(x^2+4x)^5 \cdot (2x+4)$

- $\tan^{-1} 8x = \dfrac{1}{1+(8x)^2} \cdot 8$
- $8^{x^2+2x} = 8^{x^2+2x} \cdot 2x+2 \cdot \ln 8$
- $\sin^{-1}(6x^2+3) = \dfrac{1}{\sqrt{1-(6x^2+3)^2}} \cdot 12x$
- $\left(\dfrac{f}{g}\right)' = \dfrac{g \cdot f' - f \cdot g'}{g^2}$
- $(fg)' = f \cdot g' + g \cdot f'$
- $\sqrt{x} = \dfrac{1}{2\sqrt{x}} \cdot 1^{\rightarrow x'}$
- $y^{\sin 8x} = y^{\sin 8x} \cdot \cos 8x \cdot 8 \cdot \ln y$
- $\dfrac{\sqrt{3}}{1} = \dfrac{3}{\sqrt{3}}$
- $\tan x \rightarrow \sec^2 x$
- $\cos Z \rightarrow \sin Z \cdot Z_x$ or $\sin Z \cdot \dfrac{\partial Z}{\partial x}$
- $Z_{xx} \rightarrow Z_{xy}$
- $Z_x \rightarrow Z_y$
- $\int u\,dv = uv - \int v\,du$ $\quad \begin{matrix} u & dv \\ du & v \end{matrix}$
- $\int \dfrac{1}{x^2+1}\,dx = \tan^{-1} x$
- $\int \dfrac{1}{\sqrt{16-x^2}}\,dx = \sin^{-1} \dfrac{x}{4}$
- $\int \dfrac{1}{x^2+9}\,dx = \dfrac{1}{3} \tan^{-1} \dfrac{x}{3}$
- $\int \dfrac{1}{x^2+5}\,dx = \dfrac{1}{\sqrt{5}} \tan^{-1} \dfrac{x}{\sqrt{5}}$
- $\int e^{8x} = \dfrac{e^{8x}}{8}$
- $e^{\infty} \rightarrow \infty$ $\quad$ $e^{-\infty} \rightarrow 0$ $\quad$ $e^{0} \rightarrow 1$
- $\tan\theta \rightarrow \sec^2\theta$ $\quad$ • $\sec\theta \rightarrow \sec\theta \tan\theta$
- polar form : $r = e^{\theta}$
  cartesian : (x, y)
- $x = r\cos\theta$
  $y = r\sin\theta$

- $f_x = \dfrac{\partial f}{\partial x}$
- $f_{xx} = \dfrac{\partial^2 f}{\partial x^2}$ $\quad$ 2^{nd} derivative.
- f_{xxx} $\quad$ 3^{rd} derivative.
- $f_{xy} = f_{yx}$ $\quad$ differentiate x W.R.T y or y W.R.T x
  ↳ 2^{nd} derivative
- $\dfrac{dy}{dx} = -\dfrac{f_x}{f_y}$ $\quad$ after putting equation $\underset{=0}{\text{one}}$ side
- لو اللى هنعمله differentiate تحت هنحل ال power ال -1
  بس لو تحت هنحل $\ln$
- $\dfrac{\partial x}{\partial y} = -\dfrac{f_y}{f_x}$ $\rightsquigarrow$ سؤال -1
- $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial y}{\partial t}$
- homogeneous or not: $x \rightarrow tx$ $\quad y \rightarrow ty$
  ال variables اللى على نفس السطر يبقوا قد بعض فى ال degree
- Euler: • $x f_x + y f_y = kf$
  • $x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = k(k-1)f$
- differentiation under integral sign: $\dfrac{df}{d\,\text{variable}} =$
  Substitute with upper limit $\times$ diff $-$ lower $\times$ diff $+ \int$ dif W.R.T variable ال
- Envelopes: (1) $f(x, y, \lambda) = 0$
  (2) $\dfrac{\partial f}{\partial \lambda} = 0$
  (3) eliminate λ (1), (2) we get envelope
  ↳ in the question
- $f_{xx} + f_{yy} = 0$
  $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = \nabla^2 f = 0$] la place equations
- transformation = chain rule

extrema: $f_x, f_y \rightarrow$ equals 0 to get the points

$\Delta : f_{xx} f_{yy} - f(xy)^2$ ↳ solve simultaniously

$\Delta > 0 \rightarrow$ min: $f_{xx} > 0$ & $f_{yy} > 0$

max: $f_{xx} < 0$ & $f_{yy} < 0$

- $y = e^{ax}$ $\quad \frac{\partial y}{\partial a} = x e^{ax}$
- $dv = \sin bx$ $\quad v = \frac{-\cos bx}{b}$
- $\operatorname{cosec}\theta \rightarrow \operatorname{cosec}\theta \cot\theta$
- $(y''')^4 \rightarrow$ 3rd order & 4th degree
- $M\,dx + N\,dy = 0$
- general Solution
- particular Solution $\quad y(0) = 1 : x = 0 \quad y = 1$ ↳ to get & substituted with c
- $e^{x+y} = e^x . e^y$
- x, y $\quad \cdot$ or $\div$ $\quad$ Seprable

x, y $\quad$ + or − $\quad$ not seprable

- $\int \frac{f'}{\sqrt{f}} = 2\sqrt{F} + c$
- $\int \frac{f'}{f} = \ln f$
- reduced to seprable (let)
- $\sin^2\alpha = \frac{1}{2}[1 - \cos 2\alpha]$

$\cos^2\alpha = \frac{1}{2}[1 + \cos 2\alpha]$

- H.D.EQ: $y = v.x$

$$\frac{dy}{dx} = v.1 + x\frac{dv}{dx} \qquad R = 0$$

ميتفعش يظهر معايا ال y خالص! وبعدها ال x تختفي

- Linear D.EQ: $1\frac{dy}{dx} + p(x).y = q(x)$

مفيش y هنا

(1) I. factor $\mu(x) = e^{\int p(x)dx}$

(2) $\mu(x).y = \int \mu(x) q(x)\,dx + c$

- Bernoly. D.EQ

$$\frac{dy}{dx} + p(x).y = q(x).y^n$$

$n = 1$ Sep

$n = 0$ L.DEQ

- $M\,dx + N\,dy = 0$ $\quad$ [ ]

$M_y = N_x$ $\quad$ Exact $\quad$ integrate $\quad$ Term ال x في ال zero ب التانية

$M_y \neq N_x$ $\quad$ Not exact

$\frac{M_y - N_x}{N} = f(x)$ $\qquad$ $\frac{M_y - N_x}{M} = g(y)$

*شوف ال Sign علشان تفارن

$\mu = e^{\int f(x)dx}$ $\qquad$ $\mu = e^{-\int g(y)dy}$

*هنطلع exact بعد كده، متحلهاش prove تاني!

orthognal Trajectory:

(1) $y = f(x)$ $\qquad$ (3) $\frac{-1}{m}$

(2) $m = \frac{dy}{dx}$ $\qquad$ (4) O.T

- $\frac{dy}{dx} = \frac{-1}{\frac{dy}{dx}} = -\frac{dx}{dy}$
- $D \rightarrow$ diff. operator $= \frac{d}{dx}$

$D \rightarrow m$

- case 1: real, unequal

$y_H = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

case 2: equal

$y_H = 1c_1 e^{mx} + x c_2 e^{mx} + x^2 c_3 e^{mx}$

↳ $[c_1 + c_2 x + c_3 x^2] e^{mx}$

case 3: $m = a \pm ib$

$y_H = e^{ax}[c_1 \cos bx + c_2 \sin bx]$

$\int f^n f' = \frac{f^{n+1}}{n+1}$

$\int \frac{f'}{f} = \ln f$ $\qquad$ • $\frac{1}{2}\int \frac{2x}{\sqrt{x^2+4}}$

$\int \frac{f'}{\sqrt{f}} = 2\sqrt{F}$ $\qquad$ $= \frac{1}{2} . 2\sqrt{x^2+4}$

diff: $\sin \rightarrow \cos$ $\qquad$ $\cos \rightarrow -\sin$

int: $\int \sin \rightarrow -\cos$ $\qquad$ $\int \cos \rightarrow \sin$

$\cos x = \frac{1}{\sec x}$

$\sin x = \frac{1}{\operatorname{cosec} x}$ $\qquad$ $\int \sec^2 \rightarrow \tan$

$\int \sec x : \ln(\sec x + \tan x)$

d order differential equation
constant coeffient
non homogeneous
linear

$y_G = y_{c.f} + y_{P.I}$

$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$
↳ $\neq 0$

$\frac{1}{f(D)} c = \frac{1}{f(0)} c$

$\frac{1}{f(D^2)} \begin{cases} \cos ax \\ \sin ax \end{cases} = \frac{1}{f(-a^2)} \begin{cases} \cos ax \\ \sin ax \end{cases}$

لو cosh أو Sinh هنشيل الـ شـارة

$\frac{1}{1-\alpha} = (1-\alpha)^{-1} = 1+\alpha+\alpha^2+\alpha^3+\dots$

$(1+\alpha)^{-1} = 1-\alpha+\alpha^2-\alpha^3+\dots$

لو power 2 يبقى أكبر منها بـ 1 (3 يعني)

$\frac{1}{f(D)} e^{ax} \cdot g(x) = e^{ax} \frac{1}{f(D+a)} g(x)$
↳ Shift rule $D \to D+a$

• الضرب في الـ conjuget

• $\frac{1}{f(D)} (ax^2+bx+c)$ & $\frac{1}{f(D)} x^n$

• D → differentiation

$\frac{1}{D} = x$, $\frac{1}{D^2} = \frac{x^2}{2}$, $\frac{1}{D^3} = \frac{x^3}{6}$

• e^{-x} or c ⇝ shift rule
↳ $= 0$

• $\frac{1}{D^2+a^2} \begin{cases} \cos ax \\ \sin ax \end{cases} = \begin{cases} \frac{x}{2a} \sin ax \\ -\frac{x}{2a} \cos ax \end{cases}$
↳ $= 0$

• $\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$
$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$
$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

• $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Euler D.E.Q:

$x = e^t$ $\ln x = t$

$xD = \theta$ $x^3D^3 = \theta(\theta-1)(\theta-2)$

$x^2D^2 = \theta(\theta-1)$

unit step:

$L[u(t \mp a)] = \frac{e^{-as}}{s} = \frac{1}{s} \cdot e^{\mp as}$

$L[f(t-a)u(t-a)] = F(s) \cdot e^{-as}$

• $L[f'(t)] = s^1 F(s) - f(0)$
$L[f''(t)] = s^2 F(s) - s^1 f(0) - f'(0)$
$L[f'''(t)] = s^3 F(s) - s^2 f(0) - s^1 f'(0) - f''(0)$

• $L[t f(t)] = -1 F'(s)$

• $L[t^2 f(t)] = (-1)^2 F''(s)$

• $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\, ds$

• $L \int_0^t f(t)\, dt = \frac{F(s)}{s}$ $\therefore \frac{1}{s^2} \to \int\int$

• $L \frac{d}{dt} f(t) = sF(s)$
$\frac{d^2}{dt^2} f(t) = s^2 F(s)$

• $f(t) * g(t) = g(t) * f(t)$
$= \int_0^t f(\lambda) \cdot g(t-\lambda)\, d\lambda$
$= \int_0^t g(\lambda) \cdot f(t-\lambda)\, d\lambda$

$L[f(t) * g(t)] = F(s) \cdot G(s)$

• $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$
• $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
• $\cos n\pi = (-1)^n$
• $\sin n\pi = 0$

• $dA = dx . dy$ or $dy . dx$

$\int \cos f(x) . f'(x)\, dy$

$= \sin f(x)$

• $\iint 1 \; dA$ = area $\quad \iiint 1 \; dV = V$

→ density

• $\iint \sigma \; dA$ = mass

• $x^2 + y^2 = r^2$; $dx\, dy \longrightarrow r\, dr\, d\theta$

$x^2 + y^2 = a^2 \longrightarrow$ circle

• $x = r\cos\theta$, $y = r\sin\theta$

$\underset{dV}{V} = \iiint dZ . dA \longrightarrow$ $dZ \times dx \times dy$

↳ volume

• green theorem: $\int_a^b M(x,y)\, dx + N(x,y)\, dy$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then integral independent of path

• $\iint f(x,y,z)\, dy\, dz + g(x,y,z)\, dx\, dz + h(x,y,z)\, dx\, dy$

$= \iiint_V \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}\right) dZ . dA$

$x^2 + y^2 + z^2 = 16$ sphere's volume $(0,0,0)$ $r = 4$

$V = \frac{4}{3}\pi a^3$ $\quad Z = \sqrt{a^2 - x^2 - y^2}$ $\quad Z = 0 \rightsquigarrow \frac{1}{2}$ sphere

$V = \pi r^2 h$ cylinder's volume

* Straight line: (1) get m (2) y interms of x (3) Substitute (4) $dx = dy$
* Broken line: (1) C_1 is with y & dy (commonly) ↳ 1st point (2) C_2 is with x & dx (commonly x) ↳ 2nd point (3) $I = I_1 + I_2$
* curve: (1) $\overbrace{y \,\&\, x}^{1st} \,\&\, \underbrace{dy \,\&\, dx}_{1st}$ ↳ get the limits (2) Substitute (3) get limits of t
  - ↳ if in terms of x, put x's limits
  - ↳ ~ ~ ~ y, ~ y's ~

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Def

$$\boxed{F(s) = \mathcal{L}[f(t)] = \int_0^{\infty} f(t)\, e^{-st}\, dt}$$

$\longleftrightarrow$

T_1	$f(t)$	$F(s)$
	c	$\frac{c}{s}$
	e^{+at}	$\frac{1}{s - a}$
	e^{-at}	$\frac{1}{s + a}$
	$\cosh at$	$\frac{s}{s^2 - a^2}$
	$\sinh at$	$\frac{a}{s^2 - a^2}$
	$\cos at$	$\frac{s}{s^2 + a^2}$
	$\sin at$	$\frac{a}{s^2 + a^2}$
	+ integer t^{n}	$\frac{n!}{s^{n+1}}$

T_2 $f(t)$ $F(s)$

$f(t)$	$F(s)$
$e^{at} \cdot \sinh bt$	$\frac{b}{(s-a)^2 - b^2}$
$e^{at} \cosh bt$	$\frac{s-a}{(s-a)^2 - b^2}$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
$e^{at} t^n$	$\frac{n!}{(s-a)^{n+1}}$

$L[e^{at} f(t)] = F(s-a)$