Lowering the Bar

1. A horizontal metal cylinder is free to slide on vertical metal poles (Fig. 1). The cylinder has a resistance of $0.50\,\Omega$, a mass of $0.050\,\mathrm{kg}$ and it is $0.12\,\mathrm{m}$ in length. The cylinder is falling. The poles are connected, at the top by an identical cylinder. A uniform field of $0.060\,\mathrm{T}$ is perpendicular to the plane of the poles. Eventually, the falling cylinder reaches constant speed. The ultimate questions are these: what is maximum speed of the cylinder and what current flows in the cylinder, once that speed is reached?

Prepare

Tactics and Strategies

The following tactics and strategies will be useful and are attached, at the end of this booklet (pg.5).

TB 24.1, pg. 771: Right-hand Rule For Fields
PSS 24.1, pg. 786: Magnetic Force Problems
TB 25.1, pg. 813: Using Lenz's Law
PSS 25.1, pg. 815: Electromagnetic Induction

The logic for this problem is similar to (25.6.2), from the notes.

Simplify

1. Neglect the magnetic field of the earth (about $5 \times 10^{-5}\,\mathrm{T}$).
2. Neglect any complications with connecting the moving cylinder to the poles, including friction and resistance in the connecters.
3. Neglect resistance in the poles (since resistance in the poles is equivalent to increased resistance in the cylinders, whit are in series).
4. In the diagram (Fig. 1), there is a gap between the end of each cylinder and the nearest pole. Even so, treat the length of both cylinders as equal to the distance between the poles since, by comparison, the gaps look negligible.

Diagram

(a) The induced current will not change direction, over time. The direction is given for the initial end view (Fig. 1.B) and the final side view (Fig. 1.C). The axes for the side view are labeled. Label the axes for the end view.

(b) Based on given directions for the induced current, sketch the induced field. Add one "•" and one "×" near each horizontal cylinder, in the final side view (Fig. 1.C). Draw field lines, with direction arrows, in the final end view (Fig. 1.D).

(c) It would be tedious to draw the applied field at every point. Instead, based on the induced field from part (b), sketch the applied field by adding one "•" or one "×" to the initial side view (Fig. 1.A). Likewise, add one field line, with an arrow, to the initial end view (Fig. 1.B).

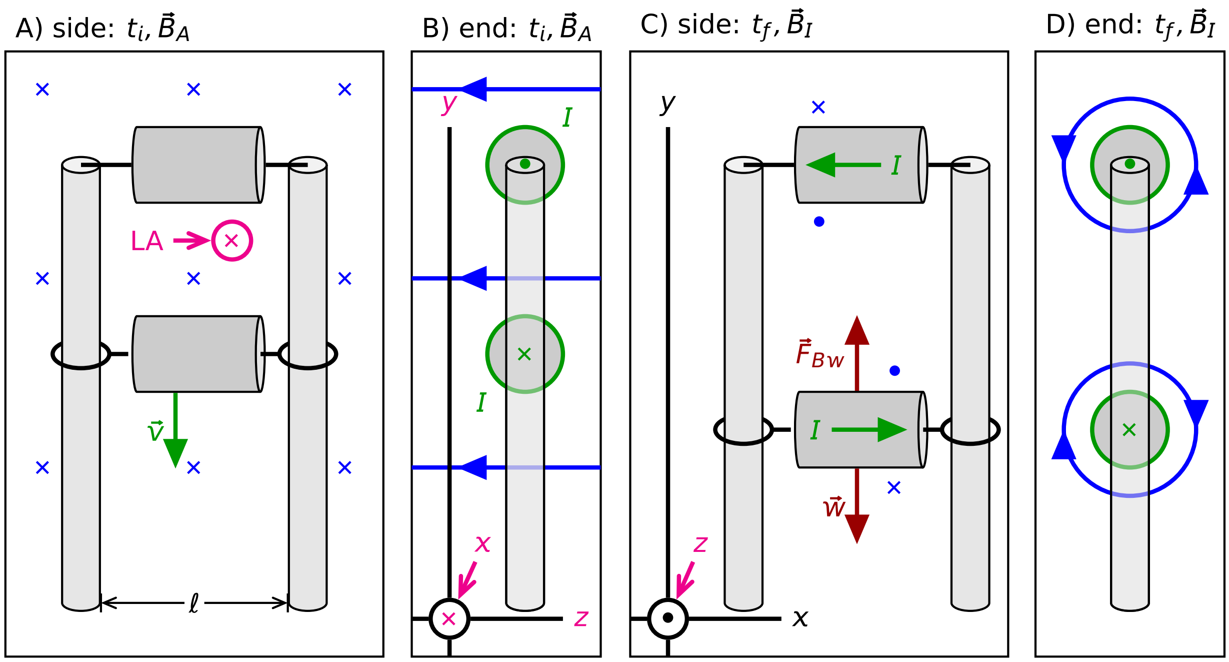


Fig. 1

Figure 1: The induced field (C,D) as a conducting cylinder falls through a constant applied field (A,B). Note that the force on the cylinder is by the *applied* field (since, as with any object, the cylinder can't exert force on itself).

(d) Chose a loop axis that makes the smallest possible angle with the applied field. Based on part (c), add "•" or "×" to the loop axis (LA) in the initial side view (Fig. 1.A). Draw and label an arrow for the loop axis in the initial end view (Fig. 1.B).

(e) On the initial side view (Fig. 1.B), sketch a velocity vector to give a sense of the motion.

(f) On the final side view (Fig. 1.C), sketch the forces on the falling crylinder using $\vec{F}_{Bw}$ for the force by the applied field and $\vec{w}$ for the weight.

Vector Components

(g) Based on part (f), how are components and magnitudes of vectors $\vec{F}_{Bw}$ and $\vec{w}$ related? Put an "X" in the appropriate boxes:

☐ $(F_{Bw})_y = -F_{Bw}$ ☒ $w_y = -w = -mg$ ☒ $(F_{Bw})_y = F_{Bw}$ ☐ $w_y = w = mg$

Solve

Flux

(h) On the y-axis, in the initial end view (Fig. 1.B), add h for the height of the poles and y_i as the initial height of the falling cylinder. The initial side view shows ℓ as the length of the falling cylinder. In the final side view (Fig. 1.C), add y_f as the final height of the falling cylinder. The vertical poles and the horizontal cylinders form a closed loop. In terms of ℓ, y_i and y_f, write initial and final loop areas in the space provided.

1.(h)-1 Initial area, in symbols: $A_i = (h - y_i)\ell$	Final area, in symbols: $A_f = (h - y_f)\ell$ 1.(h)-2	

(i) As always, θ_A is the angle between a applied field and the chosen loop axis. Based on part (d), write values for θ_A and $\cos\theta_A$ in the spaces provided. For simplicity, plug these values into subsequent formulæ.

1.(i)-1 In degrees: $\theta_A = 0^\circ$	In numbers: $\cos\theta_A = 1$ 1.(i)-2

(j) Using parts (h) and (i), write down final flux and the *change* in flux, with B_A as the applied field strength. Notice that h cancels in $\Delta\Phi$ and use $(y_i - y_f) = -\Delta y$ to simplify.

1.(j)-1 In symbols: $\Phi_f = (h - y_f)\ell B_A$	In symbols: $\Delta\Phi = -(y_f - y_i)\ell B_A = -\ell B_A \Delta y$ 1.(j)-2

Induced Current

(k) In the coil, $\mathcal{E}$ plays the role of ΔV in Ohm's Law so, Ohm's Law and Faraday's Law give two different formulæ for $\mathcal{E}$ (TB 25.1). Write those formulæin the space provided.

1.(k)-1 In symbols, Faraday's Law: $\mathcal{E} = \left\lvert \dfrac{\Delta\Phi}{\Delta t} \right\rvert$	In symbols, Ohm's Law: $\mathcal{E} = IR$ 1.(k)-2

(l)) Combine part (j) and both equations from part (k), with $v = \Delta y/\Delta t$ as the speed of the falling bar. Solve for induced current, using B_A, ℓ, R and v as the only symbols.

In symbols: $I = \dfrac{v\ell B_A}{R}$ 1.(l)

Magnetic Force

(m) As alway, α is the angle between the magnetic field and the current (or the velocity of a moving charge). Based on either sketch, write values for α and $\sin\alpha$ in the spaces provided. For simplicity, plug these values into subsequent formulæ.

1.(m)-1 In degrees: $\alpha = 90^\circ$	In numbers: $\sin\alpha = 1$ 1.(m)-2

(n) Consult equation (24.9) on pg. 784 of the book. Write down the magnetic force on the wire using B_A, ℓ, R and v from part (l) as the only symbols (some squared).

In symbols: $F_{Bw} = \dfrac{v(\ell B_A)^2}{R}$	1.(n)

Equilibrium

(o) Once it reaches constant speed, the falling cylinder is in (dynamic) equilibrium. Write down a sum of the vector components. Solve for $(F_{Bw})_y$ to determine how $(F_{Bw})_y$ and w_y are related. Be careful about signs.

1.(o)-1	Total Force: $\sum(F)_y = (F_{Bw})_y + w_y = 0$	$(F_{Bw})_y = -w_y$	1.(o)-2

Terminal Speed

(p) Combine parts (g), (n) and (o) to solve for v, with B_A, g ℓ, m and R as the only symbols. Then plug in numbers. Notice that the cylinders are in series and use $R = 1.0\,\Omega$ as the equivalent resistance.

1.(p)-1	In symbols: $v = \dfrac{mgR}{(\ell B_A)^2}$	In numbers: $v = 9452\,\text{m/s} = 9500\,\text{m/s}$	1.(p)-2

Assess

Rules of Thumb

(q) Are the directions of $\vec{F}_{Bw}$ and $\vec{B}$ consistent? Explain.

As expected, $\vec{F}_{Bw}$ and $\vec{B}$ are perpendicular.	1.(q)

(r) Are the directions of $\vec{F}_{Bw}$ and I consistent? Explain.

As expected, $\vec{F}_{Bw}$ and I are perpendicular.	1.(r)

Justify Simplifications

(s) Drag force increases with speed. Based on part (p) does neglecting drag seem a realistic simplification?

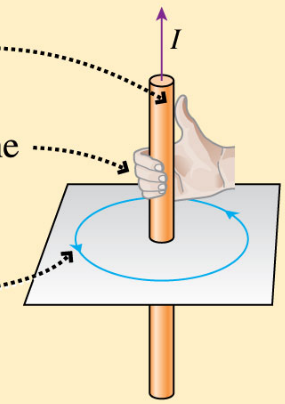
At such high speeds, neglecting drag seems unrealistic, even though the falling cylinder is somewhat aerodynamic.	1.(s)

Appendix: Tactics and Strategies

The following tactics and strategies are from *College Physics: A Strategic Approach (3rd Edition)* by R. D. Knight, B. Jones and S. Field (Pearson, 2014).

TACTICS BOX 24.1 **Right-hand rule for fields**

1. Point your *right* thumb in the direction of the current.
2. Wrap your fingers around the wire to indicate a circle.
3. Your fingers curl in the direction of the magnetic field lines around the wire.



Exercises 6–11

PROBLEM-SOLVING STRATEGY 24.1

Magnetic force problems

Magnetic fields exert forces on both moving charges and current-carrying wires, but in both cases the force can be found using the same strategy.

PREPARE There are two key factors to identify in magnetic force problems:

- The source of the magnetic field.
- The charges or currents that feel a force due to this magnetic field.

SOLVE First, determine the magnitude and direction of the magnetic field at the position of the charges or currents that are of interest.

- In some problems the magnetic field is given.
- If the field is due to a current, use the right-hand rule for fields to determine the field direction and the appropriate equation to determine its magnitude.

Next, determine the force this field produces. Working with the charges or currents you identified previously,

- Use the right-hand rule for forces to determine the direction of the force on any moving charge or current.
- Use the appropriate equation to determine the magnitude of the force on any moving charge or current.

ASSESS Are the forces you determine perpendicular to velocities of moving charges and to currents? Are the forces perpendicular to the fields? Do the magnitudes of the forces seem reasonable?

Exercise 34

TACTICS BOX 25.1 **Using Lenz's law**

1. Determine the direction of the applied magnetic field. The field must pass through the loop.
2. Determine how the flux is changing. Is it increasing, decreasing, or staying the same?
3. Determine the direction of an induced magnetic field that will oppose the *change* in the flux:
   - Increasing flux: The induced magnetic field points opposite the applied magnetic field.
   - Decreasing flux: The induced magnetic field points in the same direction as the applied magnetic field.
   - Steady flux: There is no induced magnetic field.
4. Determine the direction of the induced current. Use the right-hand rule to determine the current direction in the loop that generates the induced magnetic field you found in step 3.

Exercises 9–11

PROBLEM-SOLVING STRATEGY 25.1 **Electromagnetic induction**

Faraday's law allows us to find the *magnitude* of induced emfs and currents; Lenz's law allows us to determine the *direction*.

PREPARE Make simplifying assumptions about wires and magnetic fields. Draw a picture or a circuit diagram. Use Lenz's law to determine the direction of the induced current.

SOLVE The mathematical representation is based on Faraday's law

$$\mathcal{E} = \left|\frac{\Delta\Phi}{\Delta t}\right|$$

For an N-turn coil, multiply by N. The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 16