Faraday's Law: Changing Flux Through A Coil

1. A coil with two turns has a resistance of $0.036\,\Omega$ and a 0.040 m radius. A uniform field with an initial strength of 1.4 T passes through the coil. The field lines are perpendicular to the open ends of the coil (Fig. 1.A). A steady increase to 2.8 T induces a current of 0.20 A in the coil. The ultimate questions are these: what is the direction of the current and how long does the change in field strength take?

Prepare

Tactics and Strategies

The following tactics and strategies will be useful and are attached, at the end of this booklet (pg.5).

TB 24.1, pg. 771: Right-hand Rule For Fields
TB 25.1, pg. 813: Using Lenz's Law
PSS 25.1, pg. 815: Electromagnetic Induction

Simplify

(a) Which of the following contibute to magnetic flux? Put an "X" in the appropriate boxes: ⊠ field strength: B, ⊠ the area of the coil: A, ⊠ coil orientation: θ.

(b) Induction requires a *change* in flux. To simplify, treat the coil as stationary, with contant position and orientation. Note that the coil has a fixed radius. For these conditions, which of the following contribute to *changing* flux for the magnetic field of the earth? Put an "X" in the appropriate boxes: □ B, □ A, □ θ.

(c) Does the magnetic field of the earth contribute to induction for a stationary coil? Put an "X" in the appropriate box: □ yes, ⊠ no.

Diagram

(d) Establish coordinates. Add a "•" or an "×" to the z-axis in the end view (Fig. 1.A).

(e) Subscripts A and I refer to the applied and induced fields, respectively, while subscripts i and f refer to initial and final fields, respectively. Compare the initial applied field, $(\vec{B}_A)_i$ to the axes in the top and side views. Label the axes in the side view (Fig. 1).

(f) Choose a loop axis. It must be perpendicular to the area of the coil but that leaves two possible directions. Choose the direction that gives the smallest angle, θ_A between the applied field, $\vec{B}_A$, and the axis. Add a a "•" or an "×" to the loop axis (LA) in the initial end view (Fig. 1.A). Draw and label an arrow for the loop axis in the initial side view (Fig. 1.B).

(g) From the statement of the problem, is the applied magnetic field increasing of decreasing? Does the magnetic field of the earth contribute to induction for a stationary coil? Put an "X" in the appropriate box: ⊠ increasing, □ decreasing.

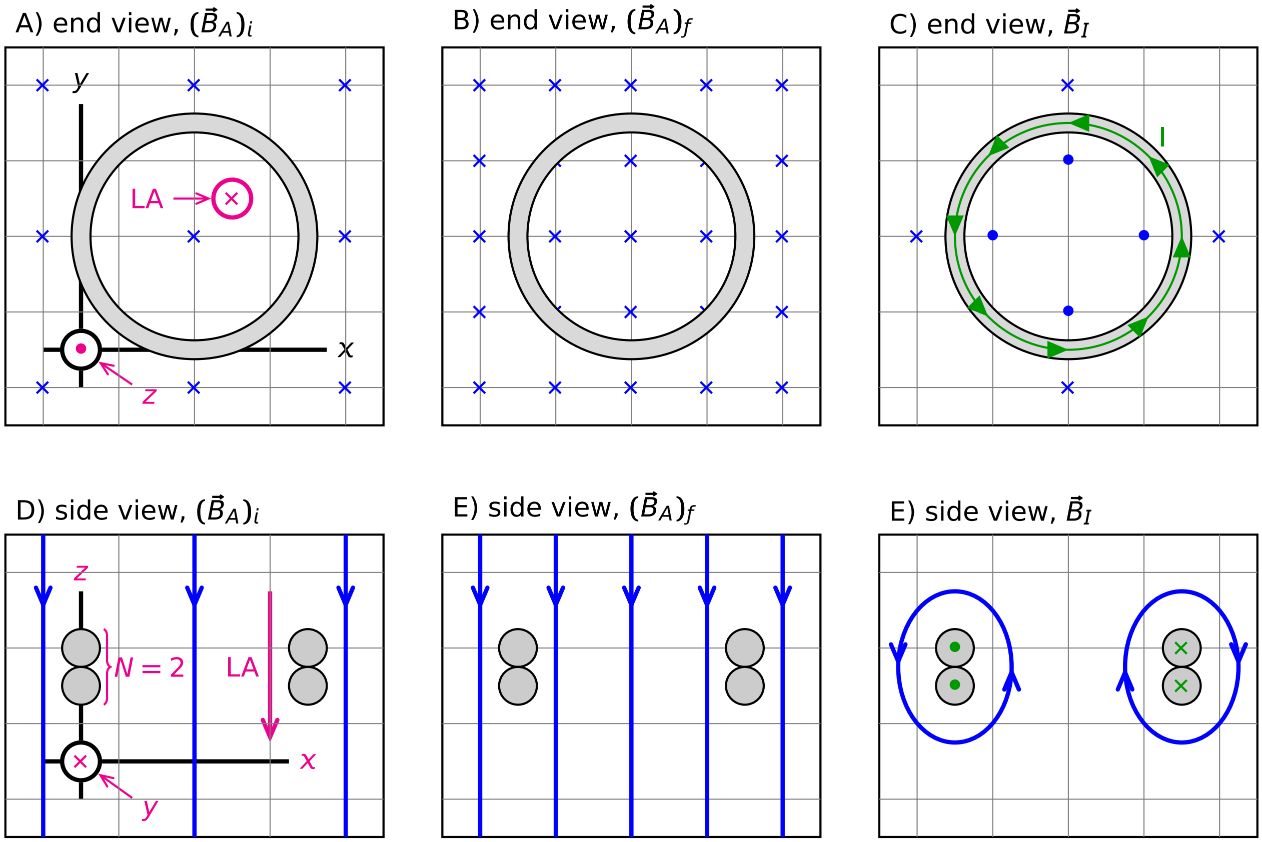


Figure 1: Applied and induced magnetic fileds for a coil, seen from the end and the side.

(h) Based on part (g), sketch an increased or decreased density of field lines on the final side view (Fig. 1.E). It will be a pain to draw all the lines on the end view (Fig. 1.B) so maybe skip this or just draw lines for the middle row of the grid.

(i) Based on part (g) and Lenz's Law (TB 25.1.3), which of the following is true? Put an "X" in the appropriate box:

- ☒ $\vec{B}_A$ and $\vec{B}_I$ point in opposite directions
- ☐ $\vec{B}_A$ and $\vec{B}_I$ point in the same direction
- ☐ there is no induced magnetic field

(j) Based on part (i), sketch *only* the induced field *inside* the coil, for the *end* view (Fig. 1.C). The side view and the field outside the coil are tricky.

(k) Based on part (j) and the right-hand rule for fields (TB 24.1), sketch the induced current in the end view (Fig. 1.C) then use "•" and "×" to sketch the induced current in the side view (Fig. 1.C).

(l) Based on the current from part (k), and the right-hand rule for fields (TB 24.1), sketch the induced field *outside* the coil. Use closed loops for the side view (Fig. 1.E) and "•" or "×" for the end view (Fig. 1.C).

Define Symbols

(m) Which of the following symbols represent *given* numbers, from the statement of the problem (*not* calculated values)? Put an "X" in the appropriate boxes:

- ☐ coil area: $A = \pi r^2$
- ☒ initial applied field: $(B_A)_i$
- ☒ final applied field: $(B_A)_f$
- ☐ induced field: B_I
- ☐ change in flux: $\Delta\Phi_A$
- ☐ elapsed time: Δt
- ☒ induced current: I
- ☒ coil radius: r
- ☒ coil resistance: R
- ☐ applied filed direction: θ_A
- ☐ initial time: t_i
- ☐ final time: t_f

(n) Which symbols in part (m) represents *needed* values? Circle the appropriate boxes. At this point, the direction of the induced current is already known, so it will not appear in the list.

Solve

(o) Based on part (f), write values for θ_A and $\cos\theta_A$ in the spaces provided.

1.(o)-1	In degrees: $\theta_A = 0^\circ$	In numbers: $\cos\theta_A = 1$	1.(o)-2

For simplicity, plug these values into subsequent formulæ.

(p) In the space provided, use part (o) to write the initial flux through the coil in terms of symbols for given values, from part (m). The final flux will have the same form. Write the *change* in flux, in the space provided.

1.(p)-1	In symbols: $\Phi_i = N\pi r^2 (B_A)_i$	In symbols: $\Delta\Phi = \big((B_A)_f - (B_A)_i\big) N\pi r^2$	1.(p)-2

(q) In the coil, $\mathcal{E}$ plays the role of ΔV in Ohm's Law so, Ohm's Law and Faraday's Law give two different formulæ for $\mathcal{E}$ (TB 25.1). Write those formulæin the space provided.

1.(q)-1	In symbols, Faraday's Law: $\mathcal{E} = \left\lvert \dfrac{\Delta\Phi}{\Delta t} \right\rvert$	In symbols, Ohm's Law: $\mathcal{E} = IR$	1.(q)-2

(r) There is a standard trick for dealing with absolute values. For this problem, which of the following is true? Put an "X" in the appropriate boxes:

$$\boxtimes\ (B_A)_f > (B_A)_i \implies \Delta\Phi > 0 \implies \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{\Delta\Phi}{\Delta t} \text{ since } \Delta\Phi \text{ and } \Delta t \text{ are positive}$$

$$\square\ (B_A)_f < (B_A)_i \implies \Delta\Phi < 0 \implies \left|\frac{\Delta\Phi}{\Delta t}\right| = -\frac{\Delta\Phi}{\Delta t} \text{ since } -\Delta\Phi \text{ and } \Delta t \text{ are positive}$$

(s) Combine parts (p), (r) and both equations from part (q) and solve for the unknown from part (n). Express the answer in terms of given symbols form part (m). Then plug in numbers.

1.(s)-1	In symbols: $\Delta t = \left(\frac{(B_A)_f - (B_A)_i}{IR}\right) N\pi r^2$	In numbers: $\Delta t = 1.95\,\mathrm{s}$	1.(s)-2

Assess

(t) For a given loop, the direction of all fields must be measured from the same axis. For the loop axis from part (f), which of the following is true for the *induced* field? Put an "X" in the appropriate box:

$$\square\ \theta_I = 0^\circ \implies \Phi_I = AB_I\cos(0^\circ) = AB_I \qquad \boxtimes\ \theta_I = 180^\circ \implies \Phi_I = AB_I\cos(180^\circ) = -AB_I$$

(u) Is the result from part consistent with Lenz's Law and the increasing primary field in this problem. Explain. Write your assessment in the space provided.

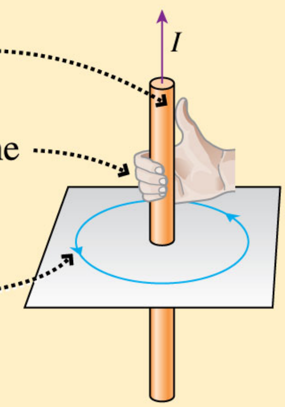
The increasing primary field adds flux which a negative induced flux counteracts, as required by Lenz's Law.	1.(u)

Appendix: Tactics and Strategies

The following tactics and strategies are from *College Physics: A Strategic Approach (3rd Edition)* by R. D. Knight, B. Jones and S. Field (Pearson, 2014).

TACTICS BOX 24.1 **Right-hand rule for fields**

1. Point your *right* thumb in the direction of the current.
2. Wrap your fingers around the wire to indicate a circle.
3. Your fingers curl in the direction of the magnetic field lines around the wire.



Exercises 6–11

TACTICS BOX 25.1 **Using Lenz's law**

1. Determine the direction of the applied magnetic field. The field must pass through the loop.
2. Determine how the flux is changing. Is it increasing, decreasing, or staying the same?
3. Determine the direction of an induced magnetic field that will oppose the *change* in the flux:
   - Increasing flux: The induced magnetic field points opposite the applied magnetic field.
   - Decreasing flux: The induced magnetic field points in the same direction as the applied magnetic field.
   - Steady flux: There is no induced magnetic field.
4. Determine the direction of the induced current. Use the right-hand rule to determine the current direction in the loop that generates the induced magnetic field you found in step 3.

Exercises 9–11

PROBLEM-SOLVING STRATEGY 25.1

Electromagnetic induction

Faraday's law allows us to find the *magnitude* of induced emfs and currents; Lenz's law allows us to determine the *direction.*

PREPARE Make simplifying assumptions about wires and magnetic fields. Draw a picture or a circuit diagram. Use Lenz's law to determine the direction of the induced current.

SOLVE The mathematical representation is based on Faraday's law

$$\mathcal{E} = \left|\frac{\Delta\Phi}{\Delta t}\right|$$

For an N-turn coil, multiply by N. The size of the induced current is $I = \mathcal{E}/R$.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 16