Discrete Wavelet Transform for Multiresolution Analysis

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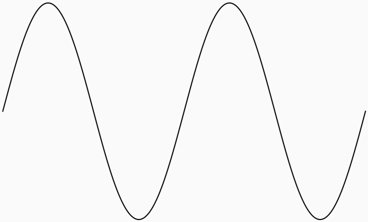
Introduction

What is a signal?

Is this a signal?



Introduction

Maybe this is a signal?

Introduction

Both of them are signals!

Introduction

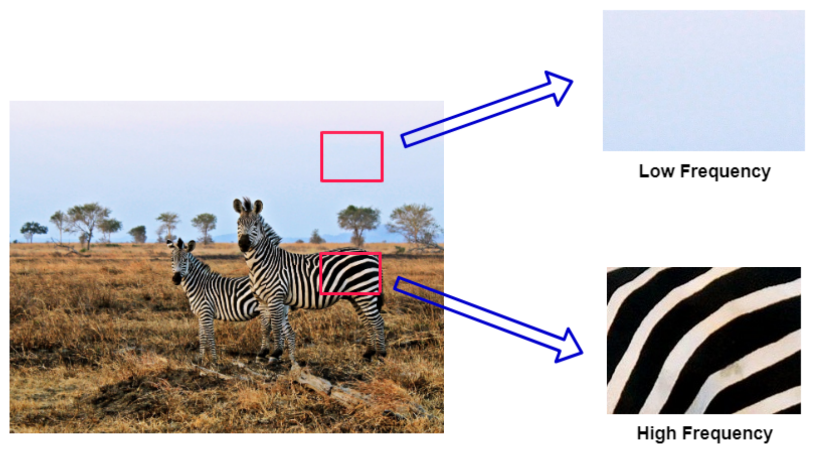
Both of them can be compressed!

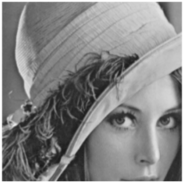
Introduction

Reasons for compression

- Faster image transmission
- Space preserving
- Multimedia communication
- Medical imaging

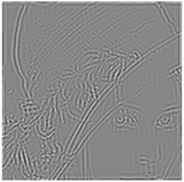
Introduction



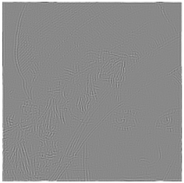


(a) Original image

(b) Low frequency component



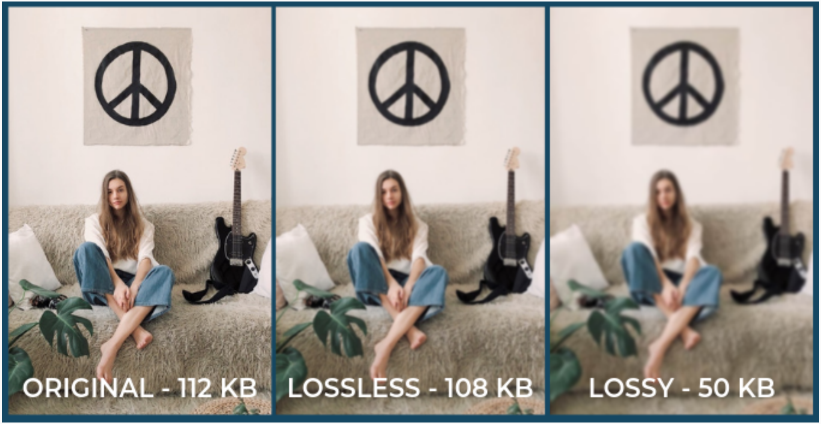
(c) Middle frequency component



(d) High frequency component

Introduction

Image Compression Example



Methods

The most popular Image Compression methods are:

- Scalar Quantization
- Vector Quantization
- Discrete Cosine Transform (DCT)
- Wavelet Transform

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Task Description

- Check existing approaches to compress images
- Compare different methods of compression
- State the advantages and disadvantages of each method

Theory: CWT

$$\mathbf{x}(t) \in L^2(\mathbb{R})$$

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$$\psi(t) \in L^2(\mathbb{R})$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

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Theory: CWT

$$W_\psi[\mathbf{x}](a,b) = \langle \mathbf{x}, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} \mathbf{x}(t)\overline{\psi}_{a,b}(t)dt$$

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Admissibility Condition:

$$C_\psi = \int\limits_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

where $\hat{\psi}$ is the Fourier transform of ψ.

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$$\mathbf{x}(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{1}{a^2} W_\psi[\mathbf{x}](a,b)\psi_{a,b}(t)\,da\,db$$

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Theory: DWT

Critical Sampling:

$$a = 2^{-j},\ b = k2^{-j},\ j, k \in \mathbb{Z}$$

Basis

$$\{\psi_{jk}(t) = 2^{j/2}\psi(2^j t - k) \mid j, k \in \mathbb{Z}\}$$

Theory: DWT

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Theory: DWT

$$\langle \psi_{jk}, \psi_{lm} \rangle = \int\limits_{-\infty}^{\infty} \psi_{jk}(t)\overline{\psi_{lm}}(t)dt = \delta_{jl}\delta_{km}$$

Theory: DWT

$$\mathbf{x}(t) = \sum_{j,k=-\infty}^{\infty} c_{jk}\psi_{jk}(x)$$

where coefficients c_{jk} is given by $W_\psi[\mathbf{x}](a, b)$.

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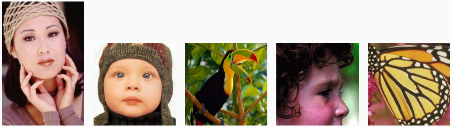
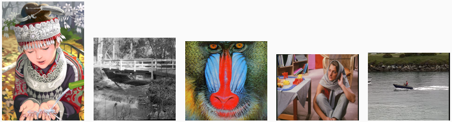
Datasets

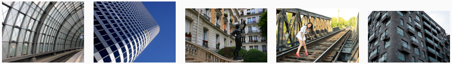
Set5



Set14



Urban100



BSDS100

Manga109

Metrics

- Compression Time
- Compression Factor
- Peak signal-to-noise ratio
- Structural similarity index measure

Metrics: CF

$$\text{Compression Factor} = \frac{\text{original image size}}{\text{compressed image size}}$$

Metrics: PSNR

$$\text{PSNR} = 20 \cdot \log_{10}\left(\frac{\text{MAX}}{\sqrt{MSE}}\right)$$

where:

- MAX - maximum intensity value.
- MSE - MSE between the original and the compressed image.

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Metrics: SSIM

$$\text{SSIM}(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$

where:

- μ_x, μ_y - pixel sample means
- σ_x, σ_y - pixel sample variances
- σ_{xy} - covariance
- $c_1 = (k_1 L)^2$, $c_2 = (k_2 L)^2$
- $L = 2^{\text{bits per pixel}} - 1$, $k_1 = 0.01$, $k_2 = 0.03$

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Scalar Quantization

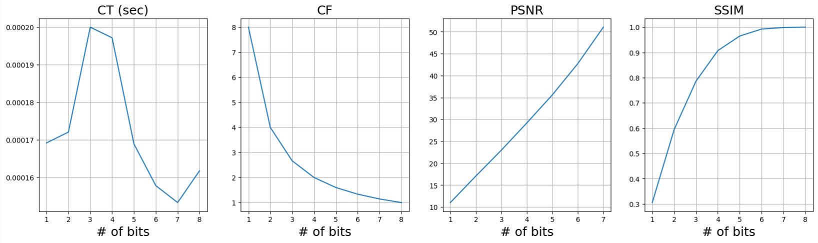


Figure 1: Experimental Results of SQ Image Compression

Scalar Quantization

(a) int8 (CF: 1) **(b)** int4 (CF: 2) **(c)** int2 (CF: 4) **(d)** int1 (CF: 8)

Figure 2: Examples of SQ Image Compression

Vector Quantization

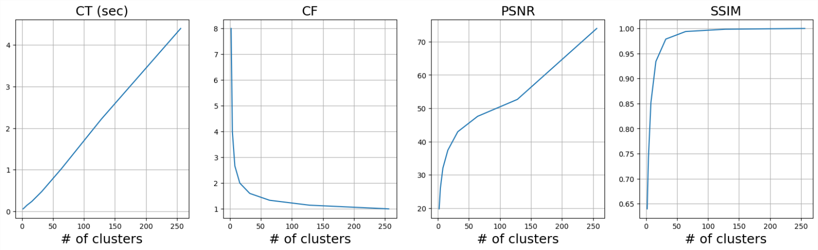


Figure 3: Experimental Results of VQ Image Compression

Vector Quantization

(a) K: 256 (CF: 1) **(b)** K: 16 (CF: 2) **(c)** K: 4 (CF: 4) **(d)** K: 2 (CF: 8)

Figure 4: Examples of VQ Image Compression

Discrete Cosine Transform

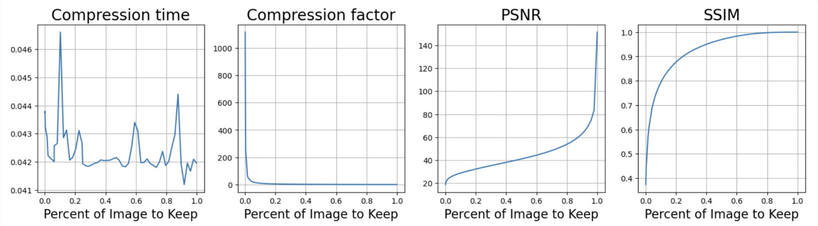
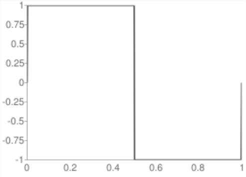


Figure 5: Experimental Results of DCT Image Compression

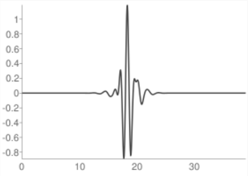
Discrete Cosine Transform

(a) CF: 1 **(b)** CF: 4 **(c)** CF: 64 **(d)** CF: 256

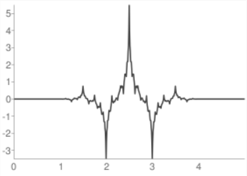
Figure 6: Examples of DCT Image Compression



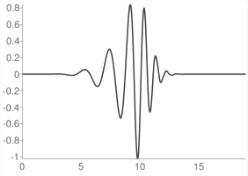
Haar



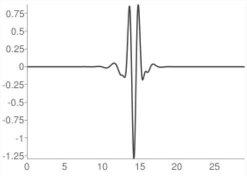
Symlet 20



Biorthogonal 2.2



Daubechi 10



Coiflet 5

Figure 7: Observed Wavelets Examples

Discrete Wavelet Transform

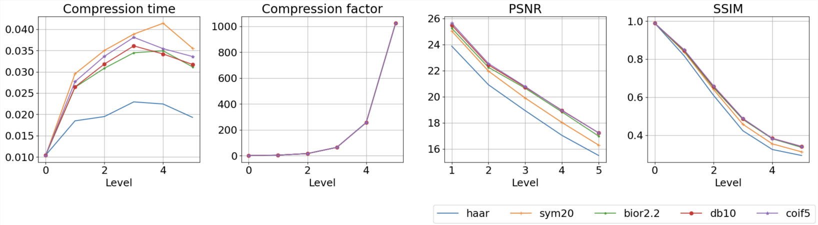


Figure 8: Experimental Results of DWT Image Compression

Discrete Wavelet Transform

(a) lvl: 0, CF: 1 **(b)** lvl: 1, CF: 4 **(c)** lvl: 3, CF: 64 **(d)** lvl: 4, CF: 256

Figure 9: Examples of DWT Image Compression with Haar Wavelet

Discrete Wavelet Transform

(a) lvl: 0, CF: 1 **(b)** lvl: 1, CF: 4 **(c)** lvl: 3, CF: 64 **(d)** lvl: 4, CF: 256

Figure 10: Examples of DWT Image Compression with Symlet 20

Conclusions

- Explored and compared various image compression techniques.
- Scalar quantization and vector quantization provide the fastest compression times, but lowest PSNR and SSIM.
- The DCT method maintains reasonable visual quality.
- The DWT method achieves very high compression factors while maintaining acceptable visual quality.
- The DWT method is a powerful and flexible tool for image compression.

Questions/Answers