Pareto Improving Fiscal and Monetary Policies: Samuelson in the New Keynesian Model[*]

Mark Aguiar

Princeton University,
and NBER

Manuel Amador

University of Minnesota,
Federal Reserve Bank of
Minneapolis, and NBER

Cristina Arellano

Federal Reserve Bank
of Minneapolis

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Abstract

This paper explores the normative and positive consequences of government bond issuances in a New Keynesian model with heterogeneous agents, focusing on how the stock of government bonds affects the cross-sectional allocation of resources in the spirit of Samuelson (1958). We characterize the Pareto optimal levels of government bonds and the associated monetary policy adjustments that should accompany Pareto-improving bond issuances. The paper introduces a simple phase diagram to analyze the global equilibrium dynamics of inflation, interest rates, and consumption in response to changes in the stock of government debt. It provides a tractable tool to explore the use of fiscal policy to escape the Effective Lower Bound (ELB) on nominal interest rates and the resolution of the "forward guidance puzzle." We show how the fiscal policy stance matters for the effectiveness of the latter. A common theme throughout is that following the monetary policy guidance from the standard Ricardian framework leads to excess fluctuations in inflation and consumption.

1 Introduction

It is well known, going back to at least Samuelson (1958), that the introduction of outside assets can (Pareto) improve the allocation of consumption in models with heterogeneity. The role of government bonds as a safe store of value, in particular, has played an important role in welfare analyses of both overlapping generations (OLG) and Bewley-Huggett-Aiyagari incomplete market economies. We revisit this idea in a tractable OLG version of the standard New Keynesian model and focus on Pareto improvements for the welfare metric. In particular, we analyze the normative and positive consequences of government bond issuances, and how these depend on the monetary policy rule. The main tool of analysis is a phase diagram that allows easy analysis of the economy's response to a variety of shocks, without recourse to local approximations. We show that the standard monetary rule that is derived in the Ricardian setting generates unnecessary fluctuations in consumption, markups, and inflation in response to a potentially welfare-improving bond issuance. The focus on Pareto improvements complements the explicitly redistributional policies that have been the focus of the recent policy analyses in heterogeneous agent New Keynesian (HANK) models.[1]

A few distinguishing characteristics of our approach should be flagged at the outset. The analysis abstracts from the direct link between monetary policy and the government budget constraint studied in the classic paper of Sargent and Wallace (1981) and the more recent work on the fiscal theory of the price level (e.g., Cochrane, 2023). To make the distinction from the latter literature crystal clear, we model the government as issuing real bonds, although this is done for expositional reasons rather than as a necessary component of the analysis. To contrast with work of Sargent and Wallace (1981), we explore a cashless economy with zero seigniorage revenue.

We build on two standard platforms. To generate a link between government debt and the real economy, we break Ricardian Equivalence using the perpetual youth framework of Blanchard (1985) and Yaari (1965), augmented to include endogenous labor supply but without physical capital. In this framework, we embed a textbook New Keynesian (NK) model of nominal rigidities, as in Rotemberg (1982), Galí (2015), and Woodford (2004). This is a combination that has been used several times in the literature, which we review below. It also generates heterogeneity in a tractable manner, providing clean insights that are applicable to richer heterogeneous agents New Keynesian models now popular in the quantitative literature.

One useful assumption we make is that the individuals supplying labor ("workers") save (in aggregate) in the government bond, while firm owners ("entrepreneurs") save in equity. This

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[1]Recent examples of optimal policy in HANK models using a utilitarian criteria include Bhandari et al. (2020), Nuno and Thonmas (2021), Dávilla and Schaab (2022), Acharya, Challe, and Dogra (2020), Bilbiie and Ragot (2021), and McKay and Wolf (2022) and Yang (2022). LeGrand, Martin-Baillon, and Ragot (2021) studies optimal policy using a social welfare function with empirically derived weights.

dichotomy echoes the reality that many households whose primary income consists of labor earnings do not own shares of firms. As a modeling choice, the key implication is that price setters and holders of government bonds value inter-temporal tradeoffs differently, as there may be a wedge between the equilibrium return on government bonds and the return on equity. While stark, this segmentation proves extremely convenient when analyzing dynamics.

From the workers' problem, we obtain an aggregate Euler equation. The key distinction between this Euler equation and that of the standard model is the presence of government bonds, reflecting that these are seen as wealth by non-Ricardian households. In the product market, prices are set in a monopolistically competitive fashion subject to quadratic adjustment costs, generating a standard NK Phillips curve that relates anticipated inflation, current inflation, and the inverse markup, which in equilibrium is proportional to worker consumption. Finally, monetary policy is set through a nominal interest rate rule subject to an effective/zero lower bound (ELB). We will study the implications of a standard Taylor rule, where the nominal interest rate is a linear function of deviations from the inflation target, with a slope coefficient strictly greater than one (i.e., the Taylor Principle holds), and an augmented rule where the nominal interest rate also responds to the level of government debt.

Given these equilibrium restrictions, we characterize the economy as a system of two non-linear ordinary differential equations (ODEs). We do so in terms of inflation and aggregate worker consumption, using equilibrium conditions to substitute out the remaining macro aggregates. The system of ODEs is amenable to analysis using a simple phase diagram, which we use to characterize global dynamics and the economy's response to alternative fiscal and monetary policies.

Our benchmark experiment focuses on a debt-financed fiscal transfer. This exercise is of interest for three reasons. The first is that it is a common real world occurence; for example, US Presidents Ronald Reagan, George W. Bush, Donald Trump, and Joseph Biden all implemented either tax cuts and/or additional transfers financed by increased government borrowing. Another less successful attempt of a similar policy was UK Prime Minister Elizabeth Truss' "mini-budget" of September 2022.[2] Second, as noted above, an expansion of government bonds has the potential to be welfare enhancing in a heterogenoeus agent environment. Finally, the equilibrium dynamics that occur in anticipation of and simultaneously with a debt issuance depend crucially on the monetary policy rule, making this an ideal laboratory to understand how rules designed for Ricardian environments (mis)behave in non-Ricardian settings.

In particular, consider a surprise announcement at $t = 0$ of a one-time debt-financed tax cut (or transfer) that will take place in $t' > 0$ periods. This is equivalent to a "helicopter" distribution of government bonds to taxpayers at t'. For households to hold these bonds in equilibrium requires

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[2]On the other hand, Presidents George H.W. Bush and William Clinton raised taxes and reduced debt, the mirror image of the expansive policy.

some combination of a higher real interest rate and/or increased income.

First, consider a "standard Taylor rule" in which the monetary policy's reaction function depends on inflation but does not depend directly on the amount of government debt outstanding, as would be appropriate in a model with Ricardian agents.[3] We show that if t' is not too distant, inflation, both the nominal and real interest rates, and worker consumption will jump up on the announcement. After the initial impact, inflation, interest rates, and consumption will all continue to increase, at an accelerating rate, reaching the new steady state at precisely $t = t'$.[4] At this new steady state, the economy has "moved along" the Phillips curve, and inflation, both nominal and real interest rates, and workers' consumption are all higher. The monetary policy rule selects this particular combination of higher income and real rates to clear the bond market. Anticipating this, as $t \to t'$ the economy experiences a prolonged boom with elevated inflation.

For the same fiscal experiment, the dynamics are markedly different if monetary policy instead follows an augmented interest rate rule that explicitly responds to the increase in government debt. In this case, we assume monetary policy increases the nominal rate simultaensously with the debt issuance; specifically, the nominal rate is increased to the level associated with the change in the real rate that would occur in a flexible price economy. Under this policy, the increase in the real rate alone clears the bond market without any increase in workers' income. Anticipating this, there are no dynamics in anticipation of the debt issuance for $t < t'$.

The analysis highlights the importance of adapting policy for non-Ricardian environments. If the central bank were to follow the policy advice obtained from a standard Taylor rule, it would induce unnecessary fluctuations in both consumption and inflation in response to fiscal deficits. In particular, the central bank would appear to be conscientiously fighting the inflation apparently caused by the lax fiscal authority, when in reality the rigidity of the monetary policy rule is equally to blame for the adverse inflationary consequences.

With the positive analysis of a fiscal expansion in hand, we turn to the welfare consequences of changes in the stock of government debt. This question has been the focus of much work using real models in the tradition of Aiyagari (1994) and also for the classic insights of Samuelson (1958) and Balasko and Shell (1980). Building on our previous work (Aguiar, Amador, and Arellano, 2022), we start our normative analysis studying the feasibility of robust Pareto improvements (RPI), which are policies that induce changes in prices and taxes that expand the budget sets of all agents. Our previous work established that a key condition in the feasibility of an RPI is that

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[3]More precisely, in Ricardian models the long-run real interest rate is pinned down independently of debt, and hence bond issuances do not alter the long-run target nominal interest rate. This is separate from secular shifts in preferences (discount factors), openness, or demographic changes that may play a role in long-run trends in the real interest rate.

[4]It is not obvious whether or why the economy must be at the new steady state at t', but we postpone a detailed discussion of this to the body of the paper.

savers are willing to hold additional wealth without large increases in the real interest rate; that is, that the aggregate savings supply schedule is elastic. We show that the same condition holds in a flexible price version of the current environment. With nominal rigidities, the monetary authority can induce a greater savings elasticity with respect to the interest rate by exploiting the fact that higher income implies the bond market can clear at a lower real rate. Unfortunately, this is not a viable avenue to increase the feasibility of Pareto improvements, as higher income is associated with higher inflation via the Philips curve, which decreases entrepreneurs' profits, both due to the deadweight costs of price adjustment and the transfer to workers via higher wages. We show that to implement an RPI, monetary policy needs to replicate the response of the flexible price economy to the fiscal policy change.[5] In this precise sense, monetary and fiscal policy must work in a complementary fashion in order to achieve the Pareto improvement.

While our focus is primarily on how and when government bonds improve allocations in economies with heterogeneity and nominal rigidities, our model also allows for a tractable analysis of two other questions that have been core topics in monetary economics. The first one relates to the forward guidance puzzle of Del Negro, Giannoni, and Patterson (2023) and McKay, Nakamura, and Steinsson (2016). This puzzle stems from the fact that in the standard representative agent New Keynesian model, an anticipated decline in interest rates far in the future has a large impact on current demand. Our phase diagram shows clearly why, in a non-Ricardian environment, there is no such puzzle, providing an additional insight into the resolution of the puzzle proposed by Del Negro, Giannoni, and Patterson (2023) without relying on linear approximations. In a related paper, Farhi and Werning (2019) study a model with incomplete markets and argue that the forward guidance puzzle is not generally resolved. We discuss the key difference in assumptions that lead to this result, and, in doing so, show precisely how alternative fiscal policies alter the effectiveness of forward guidance.

Another extension concerns a transitory decline in the discount rate of savers, a common mechanism for generating an exogenous decline in "aggregate demand." If the ELB does not bind, we show that the economy experiences a decline in inflation and worker consumption on impact, and then a recovery that features rising inflation and potentially non-monotonic dynamics in worker consumption. Again, the intuition can be obtained from bond market equilibrium; the increased desire to save must be accommodated through lower real interest rates, lower consumption, or anticipated consumption growth. From the Taylor rule, lower real interest rates are generated via lower inflation. The Phillips curve then requires a decline in worker consumption.

It may be the case that the increase in patience is so severe that the ELB binds and the economy enters a *liquidity trap*. In this case, we show that fiscal policy provides an alternative to the

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[5]That is, for an RPI, monetary policy should track the real interest rate that will prevail without nominal rigidities but in the presence of the fiscal policy change (i.e., the appropriate Wicksellian interest rate, Woodford, 2004).

traditional "forward guidance" of Krugman (1998), Eggertsson and Woodford (2003), and Werning (2007). In particular, the fiscal authority can reflate the economy by issuing bonds and rebating the proceeds back to taxpayers. By doing so, it increases the flex-price real interest rate, allowing the economy to escape the ELB. The effectiveness of this policy, of course, leverages the non-Ricardian environment.

Related Literature

Our work builds on the literature that has integrated the Blanchard-Yaari perpetual youth model into monetary models. Marini and Ploeg (1988) and Cushing (1999) identify monetary non-neutralities in this set-up in the context of flexible prices. Piergallini (2006) and Nistico (2016) study optimal monetary policy in environments with sticky prices and find that strict inflation targeting might no longer be optimal because of the additional financial wealth effects this framework contains. Galí (2021) and Piergallini (2023) focus on the case of low interest rates, $R < G$, and study the implications for asset pricing bubbles and liquidity trap equilibria.[6] Relative to this work, our contribution focuses on the interactions between monetary and fiscal policy, as in the recent work of Angeletos, Lian, and Wolf (2023). These authors explore how and when fiscal deficits can be "self-financing," either because they generate a boom in output that raises (proportional) taxes, or (with nominal bonds) because they generate inflation. In recent work, Kaplan, Nikolakoudis, and Violante (2023) explores the mechanics of the fiscal theory of the price level in a heterogeneous agent economy. Our focus is not on how deficits can be self-financing or the fiscal theory of the price level, but rather on the interplay of fiscal and monetary policy in improving the cross-sectional allocation of output via government bonds. Moreover, we study global non-linear solutions which contrast with the first-order approximation approach used in these papers.

Michaillat and Saez (2021) study a New Keynesian framework where the natural rate of interest depends on financial factors. They introduce preferences where households obtain utility from their relative wealth with respect to the population. Our phase diagram is similar to the phase diagram they use to analyze several of the anomalies present in the standard New Keynesian model. Their model however features Ricardian equivalence, and thus it cannot address the effect of changes in the level of government debt, which is our focus.

Our paper is to a large degree motivated by the growing literature on Heterogeneous Agents New Keynesian (HANK) models, which highlights the interaction between fiscal and monetary

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[6]See also the contributions of Lepetit (2022), Leith and Wren-Lewis (2000), Nisticò (2012). We have already mentioned the contributions of Del Negro, Giannoni, and Patterson (2023) and Farhi and Werning (2019) that are aso based on a monetary version of the Blanchard-Yaari model.

policies. In HANK models, government debt-financed transfers are non-Ricardian because richer households, who hold a larger share of the debt, have a lower marginal propensity to consume (MPC) than poorer households. Kaplan, Moll, and Violante (2018), for example, shows that the effectiveness of monetary policy depends crucially on the fiscal response. Specifically, monetary expansions are more potent when the government transfers the savings from the interest rate expense back to households. Auclert, Rognlie, and Straub (2018) study fiscal policy multipliers and highlight the importance of the aggregate intertetemporal MPCs, for the magnitude of the multipliers. Other studies, such as McKay, Nakamura, and Steinsson (2016), have examined the impact of forward guidance and the ELB, and found that precautionary savings motives can also temper the power of forward guidance. Our simpler OLG model shares some of the properties of these richer HANK models as in both frameworks households' wealth matters for the determination of interest rates. Our theoretical analysis complements the existing quantitative work and provides insights that can be useful in other applications.

In our framework, government debt matters for the interest rate directly, beyond its effect on aggregate consumption, because the holdings of government debt differ across generations. This is related to findings in Krishnamurthy and Vissing-Jorgensen (2012), that illustrate empirically that government debt carries a convenience yield, which tends to fall with more debt, and that provide a framework to rationalize these findings, where a representative agent values government bonds them in the utility function. Building on this work, Mian, Straub, and Sufi (2022), study how the restrictions from the ELB on monetary policy interact with fiscal policy. Similar to our perpetual youth environment, they find that a low level of government debt can increase the likelihood of a binding ELB equilibrium. However, their emphasis is on the impact of this configuration on the fiscal space of the government rather than on inflation, which never exceeds the target level. Another closely related paper is Bassetto and Sargent (2020), which shares with us the focus on monetary/fiscal interactions. Their example in Section 4.1 showcases the use of the debt Laffer curve, and the result that an optimal policy may stop issuing debt before reaching $r = g + n$ because heterogeneity and distributional concerns. This case fits nicely within our RPI criteria. Finally, a series of papers Caballero, Farhi, and Gourinchas (2017), Caballero and Farhi (2017), and Caballero, Farhi, and Gourinchas (2021) explore the role of government bonds in improving economic outcomes when monetary policy is constrained by the ELB. Our analysis of fiscal policy at the ELB echoes their emphasis on how a shortage of government debt can generate slumps at the ELB and how bonds become an aggregate "demand shifter." We extend the analysis to the impact of debt on global dynamics away from the ELB and to the distribution of resources across heterogeneous agents.

2 Environment

The environment builds closely on the canonical perpetual youth model of Blanchard and Yaari, embedded in the textbook New Keynesian paradigm, as in Galí (2021). Time is continuous and there is no aggregate uncertainty. All announcements will be zero probability "MIT" shocks. A measure of workers supply labor (with a potential lifecycle of earnings), save in a government bond, and are subject to a constant hazard of death, which they insure via annuities. The crucial element for the non-Ricardian aspect of the model is that a new cohort of workers is born every period. On the production side, firms produce differentiated intermediate goods, compete monopolistically, and face quadratic price adjustment costs as in Rotemberg. Workers and the owners of firms (entrepreneurs) are segmented in the sense that workers cannot own shares in firms. This will be useful to separate the return on government bonds from the internal rate of return to private equity. Finally, the government conducts fiscal and monetary policy. In the following subsections, we fill in the details on each block of the model and then characterize the equilibrium.

2.1 Workers

The worker sector closely follows Blanchard (1985) (as well as Buiter, 1988 for the extension to population and technological growth). At any point in time s, a cohort of workers of size $(\lambda + n)e^{ns}$ is born, where n denotes population growth. Each worker faces a constant hazard rate of dying, given by $\lambda > 0$. The expected lifespan of a worker is therefore $1/\lambda$. We require $\lambda + n > 0$.

Letting $\phi(s, t)$ denote the time-t size of the cohort born at $s \leq t$, we have

$$\phi(s, t) = (\lambda + n)e^{ns}e^{-\lambda(t-s)}.$$

The size of the total worker population at time t, denoted m_t, is then

$$m_t = \int_{-\infty}^{t} \phi(s, t)ds = e^{nt}.$$

A (representative) worker born in period s and alive at $t \geq s$ has preferences given by:

$$\int_{t}^{\infty} e^{-(\rho+\lambda)(\tau-t)}u(c(s, \tau), l(s, \tau))d\tau, \tag{1}$$

where $\rho > 0$ is the subjective discount factor; $c(s, t)$ is consumption of the final good; and $l(s, t)$ is the amount of labor supplied. Workers effectively discount the future with the sum of the discount factor and the probability of dying. It will be useful to consider the following functional

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form:

$$u(c, n) = \ln c + \psi \ln (1 - l),$$

with $c \geq 0$ and $l \leq 1$.[7]

A worker's productivity z changes over the life cycle. Specifically, define $z_0 \equiv (\lambda + \alpha + n)/(\lambda + n)$. The productivity of a worker born at time s and alive at time $t \geq s$ is given by:[8]

$$z(s, t) = z_0 e^{gt} e^{-\alpha(t-s)},$$

so z declines exponentially with age at rate $\alpha \geq 0$, and grows with time at rate g, where g captures technological growth. Aggregate productivity at time t is then

$$Z(t) \equiv \int_{-\infty}^{t} \phi(s, t) z(s, t) ds = e^{(g+n)t}.$$

As in Blanchard, workers can perfectly insure their survival risk in spot annuity markets. Let $i(t)$ denote the nominal return on government bonds. For each nominal unit ("dollar") held by a worker in the annuity, they receive $(i(t) + \lambda)dt$ if they survive the next $dt \to 0$ periods. If they die, the insurance intermediaries receive the asset. As λdt workers die, the insurance sector breaks even with probability one.

Let $P(t)$ be the price of the final good at time t and let $W(t)$ denote the nominal wage per efficiency unit of labor. Workers of cohort s at time t pay non-distortionary taxes $P(t)T(s, t)$. We let

$$T(s, t) = z(s, t)T(t), \tag{2}$$

where T is the tax burden per effective unit of labor. Specifically, we have indexed taxes by cohort

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[7]Note that we do not restrict $l \geq 0$. A negative l is equivalent to the worker hiring another individual to assist them in "daily living," removing that unit of time from the production sector, a result that we think is reasonable. Ascari and Rankin (2007) highlighted the possibility of negative labor supply in the perpetual youth framework and proposed using preferences without wealth effects on labor supply to eliminate the possibility.

[8] We assume that all cohorts enjoy productivity growth as they age. Instead of modeling growth as a "time effect," we could have assumed that growth is across cohorts:

$$z(s, t) = z_0 e^{gs} e^{-\alpha(t-s)}.$$

As is well known due to the fact that age, cohort, and time have a linear relationship, the two approaches have a simple correspondence. In particular, let $\tilde{\alpha} \equiv \alpha - g$, we have

$$z(s, t) = z_0 e^{gt} e^{-\alpha(t-s)} = z_0 e^{gs} e^{-\tilde{\alpha}(t-s)}.$$

Thus whether technological growth affects all cohorts equally over time or just cohorts at birth (or any linear combination of the two) is covered by the representation in the text.

in order to allow the tax burden to decline with productivity.[9] Aggregate tax revenue at t is:

$$\overline{T}(t) \equiv \int_{-\infty}^{t} \phi(s, t)T(s, t)ds = Z(t)T(t)$$

Let $P(t)a(s, t)$ denote the nominal asset position of the representative agent from cohort s at time t. The flow budget constraint for cohort s is given by:

$$\frac{d}{dt}[P(t)a(s, t)] = \dot{P}(t)a(s, t) + \dot{a}(s, t)P(t)$$

$$= (i(t) + \lambda)P(t)a(s, t) + W(t)z(s, t)l(s, t) - P(t)c(s, t) - P(t)T(s, t),$$

where a "dot" indicates the derivative with respect to time. Dividing through by $P(t)$, we have

$$\dot{a}(s, t) = (r(t) + \lambda)a(s, t) + w(t)z(s, t)l(s, t) - c(s, t) - T(s, t), \tag{3}$$

where $w(t) \equiv W(t)/P(t)$ is the real wage, $r(t) \equiv i(t) - \pi(t)$ is the real interest rate, and $\pi(t) \equiv \dot{P}(t)/P(t)$ is the rate of inflation.[10] Households are subject to the natural borrowing limit, $a(s, t) \geq \underline{a}(s, t)$, which, combined with the log preferences, ensures an interior consumption sequence at an optimum. Letting

$$R(t, \tau) \equiv e^{-\int_t^{\tau}(r(m)+\lambda)dm}$$

we can integrate the flow budget constraint forward to obtain:

$$a(s, t) = \int_{t}^{\infty} R(t, \tau)\left[c(s, \tau) + T(s, \tau) - w(\tau)z(s, \tau)l(s, \tau)\right]d\tau. \tag{4}$$

Workers are born with zero wealth; that is, $a(s, s) = 0$ for all cohorts s.

Given a sequence of aggregate taxes $T(t)$ and prices $\{w(t), r(t)\}$, a worker born at time s chooses sequences $\{c(s, t), l(s, t)\}_{t \geq s}$ to maximize (1) subject to (4), with $a(s, s) = 0$, as well as the constraints $c(s, t) \geq 0$, $l(s, t) \leq 1$, and $a(s, t) \geq \underline{a}(s, t)$ for all $t \geq s$, where the natural borrowing limit is

$$\underline{a}(s, t) \equiv -\int_{t}^{\infty} R(t, \tau)z(t, \tau)\left(w(t) - T(t)\right)d\tau, \tag{5}$$

which is the present value of maximal labor earnings (i.e., $l = 1$) net of taxes. We will require

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[9]We do this for simplicity as it facilitates aggregation. Note that the tax (or transfer if negative) remains lump-sum and it is not a function of the labor supply choice.

[10]For some policy experiments, we may want to consider an unanticipated lump-sum "helicopter" drop of assets (government) bonds to various cohorts at a fixed time t_0. We will be more explicit about this in Section 2.3, but to streamline the exposition we will suppress this from the notation until needed.

that, in an equilibrium, $\underline{a}(s, t)$ be finite. Note also that $\underline{a}(t, t)$ must be negative, as newborns have no wealth, $a(t, t) = 0$.

The solution to the workers' problem is characterized as follows:

Lemma 1. *Suppose that $\underline{a}(s, t)$ is finite for all (s, t). The following conditions characterize the optimal consumption and labor plan of a worker born at s evaluated at $t \geq s$:*

(i) The Euler equation:

$$\frac{\dot{c}(s, t)}{c(s, t)} = r(t) - \rho; \tag{6}$$

(ii) The static labor-consumption condition:

$$\psi c(s, t) = w(t)z(s, t)\left(1 - l(s, t)\right). \tag{7}$$

(iii) and a consumption function:

$$c(s, t) = \left(\frac{\rho + \lambda}{1 + \psi}\right)\left(a(s, t) + h(s, t) - \mathcal{T}(s, t)\right), \tag{8}$$

where

$$h(s, t) \equiv \int_t^\infty R(t, \tau)z(s, \tau)w(\tau)d\tau$$

represents potential "human wealth" and

$$\mathcal{T}(s, t) \equiv \int_t^\infty R(t, \tau)z(s, \tau)T(\tau)d\tau. \tag{9}$$

represents the present value tax burden.

In the next subsection, we discuss aggregation. We flag a few elements of the individual worker's problem that will be useful. One is that all cohorts that are alive have consumption that grows at the same rate $(r(t) - \rho)$ and the level of consumption is linear in total wealth net of taxes. A second feature is that given wages, labor supply is linear in consumption, with labor income equal to potential income $w(t)z(s, t)$ minus a fraction ψ of consumption. This feature allows us to express consumption as a fraction financial wealth and potential human wealth.

2.1.1 Aggregation

We now characterize the aggregate behavior of the workers, integrating over the various cohorts. Given that the size of a cohort s at time t is $\phi(s, t)$, and letting capital letters indicate aggregate quantities, we can integrate over s to define the aggregates:

$$C^w(t) \equiv \int_{-\infty}^{t} \phi(s, t)c(s, t)ds$$

$$N(t) \equiv \int_{-\infty}^{t} \phi(s, t)z(s, t)l(s, t)ds$$

$$A(t) \equiv \int_{-\infty}^{t} \phi(s, t)a(s, t)ds$$

$$H(t) \equiv \int_{-\infty}^{t} \phi(s, t)h(s, t)ds$$

$$\mathcal{T}(t) \equiv \int_{-\infty}^{t} \phi(s, t)\mathcal{T}(s, t)ds.$$

The following characterizes the aggregate behavior of workers:

Lemma 2. *Given a path $\{w(t), r(t), \overline{T}(t)\}$, worker optimization implies:*

(i) An aggregate Euler equation:

$$\dot{C}^w(t) = (r(t) - \rho + \alpha + n)C^w(t) - \frac{(\rho + \lambda)(\alpha + \lambda + n)}{1 + \psi}A(t); \tag{10}$$

(ii) An aggregate labor supply:

$$\psi C^w(t) = w(t)Z(t) - w(t)N(t); \tag{11}$$

(iii) An aggregate consumption function:

$$C^w(t) = \left(\frac{\rho + \lambda}{1 + \psi}\right)[A(t) + H(t) - \mathcal{T}(t)]; \tag{12}$$

(iv) And an aggregate evolution of financial wealth:

$$\dot{A}(t) = r(t)A(t) + w(t)Z(t) - \overline{\overline{T}}(t) - (1 + \psi)C^w(t). \tag{13}$$

The aggregate labor supply and consumption function, conditions (11) and (12), follow immediately from integrating across cohorts their static decisions. The aggregate dynamics of con-

sumption and financial wealth, however, also have to consider that fraction λ of the population dies every period and a new cohort is born. Those dying in aggregate have average financial wealth $A(t)$ while those being born have zero. The difference in wealth between those dying and those being born matters for the aggregate Euler equation (10) and the evolution of financial wealth (13). Financial wealth $A(t)$ shows up in (10) because richer agents are replaced by poorer agents; the growth of aggregate consumption is lower with high aggregate wealth. The aggregate Euler equation illustrates that the level of household financial wealth A_t matters for aggregate consumption dynamics, in addition to the interest rate, discount rate, and the age profile of productivity. From (13), aggregate worker wealth evolves "as if" all cohorts inelastically supply $Z(t)$ efficiency units of labor while at the same time spending an extra ψ on consumption. This reflects that for each individual, any increase in consumption reduces labor income at the linear rate ψ via the income effect on labor supply. Note also that Lemma 2 holds for an arbitrary distribution of individual financial assets among surviving cohorts.

Given an initial $A(0)$ and a path for $\{w(t), r(t), \overline{T}(t)\}_{t=0}^{\infty}$, equations (11), (12), and (13) completely characterize the aggregates of the household sector, $\{A(t), C^w(t), N(t)\}_{t=0}^{\infty}$.

2.2 Entrepreneurs

The technology side of the model is familiar from the standard textbook New Keynesian model.[11] There is a measure-one continuum of entrepreneurs, each of whom operates a firm that produces a unique intermediate input $j \in [0, 1]$. Intermediate firm technology is given by $y_j(t) = \ell_j(t)$, where ℓ_j are efficiency units of labor. Firms hire these units in a competitive labor market at real wage $w(t)$.

Entrepreneurs sell their output to a competitive final goods sector that combines inputs using a constant-elasticity-of-substitution technology:

$$Y(t) = \left(\int_0^1 y_j(t)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} .$$

Given the constant-returns-to-scale technology and competitive behavior, there is no value added generated by this sector and hence no need to detail the ownership of final-good firms. The price index of the final good is given by

$$P(t) = \left(\int_0^1 p_j(t)^{1-\eta} dj \right)^{\frac{1}{1-\eta}} ,$$

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[11]In particular, this part of the model follows Kaplan, Moll, and Violante (2018) closely. Our Lemma 3 below reproduces their Lemma 1.

where $p_j(t)$ is the price of intermediate $j \in [0, 1]$.

For simplicity, and not crucial for what follows, we assume that there are no demographic dynamics for the entrepreneurs: entrepreneurs live forever. They have linear utility and discount at the rate $\tilde{\rho}$. Entrepreneurial wealth consists of shares of its firm. Shares in firms are (potentially) traded among entrepreneurs, but as mentioned above are not available to workers. Moreover, entrepreneurs do not hold government bonds. This is stated as an assumption but is consistent with any equilibrium in which $r(t) < \tilde{\rho}$ for all t.

The entrepreneur's consumption/savings problem is given by:

$$\max_{\{c(\tau)\}_{\tau=t}^{\infty}} \int_t^{\infty} e^{-\tilde{\rho}(\tau-t)} c(\tau) d\tau \quad \text{s.t.} \quad \int_t^{\infty} e^{-\tilde{\rho}(\tau-t)} c(\tau) d\tau \leq V(t),$$

where V is the value of equity held by the entrepreneur at time t. We already impose in the representative entrepreneur's problem that the internal rate of return to equity is $\tilde{\rho}$, which follows from the linearity of preferences and the requirement that consumption be interior in equilibrium. In a symmetric equilibrium, entrepreneurs do not actively trade shares among themselves, and the value of an individual entrepreneur's shares will be equal to the value of their firm, denoted Q.

Entrepreneurs compete monopolistically and choose a sequence of prices to maximize the value of their firm. We restrict attention to symmetric equilibria in which all firms pursue an identical policy.

Intermediate good firms face a nominal friction when setting prices. Let $p(t)$ be the nominal price of an individual variety, where we drop the j index. Intermediate good firms choose the rate of change in their nominal price, $x(t) \equiv \dot{p}(t)/p(t)$ and pay a cost $f(x)Y$, where:

$$f(x) = \begin{cases} \varphi \underline{\pi} x - \frac{\varphi}{2} \underline{\pi}^2 & \text{if } x < \underline{\pi} \\ \frac{\varphi}{2} x^2 & \text{if } x \in [\underline{\pi}, \overline{\pi}] \\ \varphi \overline{\pi} x - \frac{\varphi}{2} \overline{\pi}^2 & \text{if } x > \overline{\pi}. \end{cases}$$

The costs of price adjustment are weakly convex, continuous, and continuously differentiable. For intermediate inflation rates $x \in [\underline{\pi}, \overline{\pi}]$, for some $\underline{\pi} < 0 < \overline{\pi}$, adjustment costs are quadratic, as in Rotemberg (1982). For extreme rates of change, costs are linear. In the spirit of Nakamura and Steinsson (2010), this captures that for high inflation environments, price setting is different than at moderate inflation rates. We shall see that this induces a vertical Phillips curve at extreme inflation.

The entrepreneur chooses a path of p via control $x = \dot{p}/p$ to maximize the value of the firm:

$$Q(t) = \sup_{\{x(\tau)\}_{\tau \geq t}} \int_t^\infty e^{-\tilde{\rho}(\tau - t)} \left[\Pi(p(\tau), \tau) - f(x)Y(\tau) \right] dt$$

$$\text{subject to: } \dot{p}(t) = x(t)p(t),$$

where $\Pi(p, t)$ are the real flow profits gross of adjustment costs of a firm charging price p at time t:

$$\Pi(p, t) = \left(\frac{p}{P(t)} - w(t) \right) \left(\frac{p}{P(t)} \right)^{-\eta} Y(t).$$

Note that we assume the government does not tax (or subsidize) entrepreneurs. This rules out implicit transfers to workers through the taxation of entrepreneurs to pay interest on the debt held by workers.[12]

The solution to the optimal pricing plan generates the following Phillips curve:

Lemma 3. *Let $g_Y(t) \equiv \dot{Y}(t)/Y(t)$ denote the real growth rate; $w^\star = (\eta - 1)/\eta$ denote the flexible price optimal inverse markup; and $\tilde{\kappa} \equiv \eta/\varphi$. In a symmetric equilibrium with a path for aggregate for real wages $\{w(t)\}$, aggregate inflation $\pi(t) \equiv \dot{P}(t)/P(t)$ satisfies:*

$$\dot{\pi}(t) = (\tilde{\rho} - g_Y(t))\pi(t) + \tilde{\kappa} \left[w^\star - w(t) \right] \qquad \text{if } \pi(t) \in [\underline{\pi}, \overline{\pi}]; \qquad (14)$$

and for $\pi(t) \notin [\underline{\pi}, \overline{\pi}]$, we have:

$$(\tilde{\rho} - g_Y(t)) \, \underline{\pi} = \tilde{\kappa} \left(w(t) - w^\star \right) \qquad \text{if } \pi(t) < \underline{\pi}$$
$$(\tilde{\rho} - g_Y(t)) \, \overline{\pi} = \tilde{\kappa} \left(w(t) - w^\star \right) \qquad \text{if } \pi(t) > \overline{\pi}.$$

For interior inflation, $\pi(t) \in [\underline{\pi}, \overline{\pi}]$, the last term in (14) represents the deviation from the flex-price markup, with a positive value indicating that the markup is higher than the flex-price markup. At the extreme rates of inflation, the real wage is uniquely pinned down for any π, generating a "vertical" Phillips curve in $\pi \times w$ space. Let $\underline{w}$ and $\overline{w}$ denote the associated low and high real wages, respectively.[13]

One loose end is that the firm always has the option to shut down production. Flow profits are negative if $f(\pi(t)) > 1 - w(t)$. We rule out equilibria that violate this condition. In particular, this rules out equilibria in which inflation explodes in either direction.

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[12]This assumption is made for simplicity, and allows for a simple equilibrium value for aggregate output, as we discuss below.

[13]Specifically, $\underline{w} \equiv \tilde{\rho}\underline{\pi}/\tilde{\kappa} - w^\star$ and $\overline{w} \equiv \tilde{\rho}\overline{\pi}/\tilde{\kappa} - w^\star$.

Let $C^e(t) \equiv \int_0^1 c_j(t)dj$ denote the consumption of the entrepreneurial sector. Given paths $\{w(t), \tilde{\rho}\}_{t=0}^{\infty}$, we say $\{C^e(t), Y(t), V(t), Q(t)\}_{t=0}^{\infty}$ and $\{\pi(t)\}_{t=0}^{\infty}$ that solve the entrepreneurs' problem and satisfy Lemma 3 characterize the entrepreneurs' sector.

2.3 Government

The government sets fiscal and monetary policies under full commitment. Fiscal policy consists of a sequence of non-distortionary aggregate taxes, $\overline{T}(t)$, and real debt $B(t)$, subject to the budget constraint

$$\dot{B}(t) = r(t)B(t) - \overline{T}(t). \tag{15}$$

We assume the government borrows in real bonds promising a real return to differentiate our analysis from the fiscal theory of the price level.[14] We assume a vanishing small amount of nominal bonds that carry the nominal rate $i(t)$ in order to ensure the Fisher equation $i = r + \pi$ holds in equilibrium.

Our policy experiments involve a discrete change to the stock of government debt. For example, suppose at time t_0 the government expands government debt by a discrete amount. The fiscal authority rebates the proceeds to workers. Note that this involves a sale of bonds offset by an aggregate transfer of equal amount. We could circumvent this sale-transfer by assuming the government simply "helicopter drops" the new bonds to households. Specifically, let $\xi(s, t)$ denote the cumulative bond transfers to cohort s as of time $t \geq s$. Let $d\xi(s, t) = \xi(s, t) - \lim_{\tau \uparrow t} \xi(s, \tau)$ denote the amount of new bonds transferred to workers of cohort s at time t. Our experiments assume $\xi(s, t) = 0$ for $t < t_0$, and is constant thereafter. Note that ξ can be negative as well as positive, with negative numbers representing an expropriation of assets. Let

$$d\xi(t) \equiv \int_{-\infty}^t d\xi(s, t)ds$$

denote the aggregate change to worker assets.

We then augment (3) to incorporate the helicopter drop to write:

$$da(s, t) = [(r(t) + \lambda)a(s, t) + w(t)z(s, t)l(s, t) - c(s, t) - T(s, t)] \, dt + d\xi(s, t).$$

---

[14]Although in our current model in continuous time with instantaneous bonds and a price level process that cannot jump (given the adjustment costs), the fiscal theory of the price level will remain inoperative even if the government were to issue nominal bonds.

We augment (15) similarly,

$$dB(t) = \left[r(t)B(t) - \overline{T}(t) \right] dt + d\xi(t).$$

As noted already, the evolution of aggregate worker wealth evolves independently of the idiosyncratic distribution of wealth, given a path of prices and taxes (see equation 13). Thus, we focus on the aggregate change in government debt, $d\xi(t_0)$, rather than the specifics of the distribution.

Monetary policy is set via an "augmented" interest rate rule:

$$i(t) = \max \left\{ \bar{\imath} + \theta_\pi \pi(t) + \theta_b \left[\frac{B(t)}{Z(t)} - \frac{B(0)}{Z(0)} \right], 0 \right\}, \tag{16}$$

where 0 is the ELB on nominal interest rates. When $\theta_b = 0$, monetary policy responds only to inflation, as in a standard Taylor rule, and we impose (unless otherwise noted) that the "Taylor principle" $\theta_\pi > 1$ holds. In this case, the intercept $\bar{\imath}$ is a constant that in the baseline equilibrium corresponds to the level of the real interest rate that is realized when inflation is zero. In our analysis, we also consider a monetary policy rule that can change in response to fiscal policy, hence θ_b. As we will see, this term is useful as fiscal policy affects the real rate in the flexible price allocation.[15]

2.4 Definition of Equilibrium

Given a path for fiscal policy $\{B(t), \overline{T}(t)\}$ and monetary policy rule (16), an equilibrium is a path of prices $\{\pi(t) = \dot{P}(t)/P(t)\}$ starting from $P(0) = 1$; real interest rates $\{r(t)\}$ for government bonds; nominal interest rates $\{i(t)\}$; real wages $\{w(t)\}$; firm values $\{Q(t)\}$; and quantities $\{Y(t), C^w(t), C^e(t), N(t), A(t)\}$ such that:

  (i) $C^w(t), N(t)$, and $A(t)$ solve workers' problem given w, r, and T and where $\underline{a}(s, t)$ in (5) is finite for all (s, t);

 (ii) $C^e(t), Y(t), Q(t)$, and $\pi(t)$ solve the entrepreneurs' problem given w;

(iii) the bond market clears $A(t) = B(t)$;

(iv) the resource condition is satisfied $C^w(t) + C^e(t) = (1 - f(\pi)) Y(t) = (1 - f(\pi)) N(t)$;

 (v) arbitrage between nominal and real returns by households implies that the Fisher equation is satisfied, $i(t) = r(t) + \pi(t)$; and

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[15]There are several papers that study adjustments to the Taylor rule in cases where Ricardian equivalence fails. See for example Curdia and Woodford (2010). Closer to our case, Nisticò (2012) studies a Blanchard-Yaari environment where the "Wicksellian" interest rate deviates (due to preference/demand shocks) from the one of the representative agent.

(vi) the monetary policy rule and the government budget constraint are satisfied.

3   Characterizing Equilibrium Dynamics

In this section, we introduce a phase diagram that will be the main tool to analyze equilibrium dynamics. As a first step, we note a possibly surprising aspect of the equilibrium; namely, effective aggregate labor supply, $N(t)$, and output, $Y(t)$, both grow at an exogenous constant rate $g + n$. To see this, let us use the aggregate evolution of financial wealth (13), together with the budget constraint of the government, and the equilibrium condition $A(t) = B(t)$, and we obtain:

$$w(t)Z(t) = (1 + \psi)C^w(t).$$

The static labor-consumption condition (11) states that $\psi C^w(t) = w(t)Z(t) - w(t)N(t)$. Hence:

$$N(t) = \frac{Z(t)}{1 + \psi},$$

and $Y(t) = N(t)$. In equilibrium, the aggregate labor supply curve is "vertical"; that is, independent of other equilibrium outcomes. This result stems from the balanced growth preferences of workers plus the segmentation of bond markets and taxation. In this model, therefore, fiscal and monetary policies can change the share of output consumed, due to their effect on adjustment costs and inflation, and the share of consumption going to workers versus entrepreneurs, but not the total amount of output produced.

From the workers' side, recall the aggregate Euler equation (10). Substituting the asset market-clearing condition $A(t) = B(t)$, we obtain

$$\dot{C}^w(t) = (r(t) - \rho + \alpha + n)C^w(t) - \frac{(\rho + \lambda)(\alpha + \lambda + n)}{1 + \psi}B(t).$$

As $B(t)$ increases, in order to clear the bond market we need some combination of a higher $r(t)$, higher $C^w(t)$, or lower growth rate of $C^w(t)$.

As usual, it is convenient to rewrite aggregates in terms of total effective units of labor. In particular, let

$$c(t) \equiv \frac{C^w(t)}{Z(t)}, \text{ and } b(t) \equiv \frac{B(t)}{Z(t)}.$$

As $Z(t)$ grows at rate $g + n$, we have:

$$\dot{c}(t) = (r(t) - \rho - g + \alpha)\, c(t) - \mu b(t), \tag{17}$$

$$\dot{b}(t) = (r(t) - g - n)\, b(t) - T(t). \tag{18}$$

where

$$\mu \equiv \frac{(\rho + \lambda)(\alpha + \lambda + n)}{1 + \psi}$$

Finally, $(1 + \psi)c(t) = w(t)$, and letting $c^\star \equiv w^\star/(1 + \psi)$, we can rewrite the Phillips curve as:

$$\dot{\pi}(t) = \hat{\rho}\pi(t) + \kappa \left[c^\star - c(t)\right], \tag{19}$$

for $\pi(t) \in [\underline{\pi}, \overline{\pi}]$, and where $\hat{\rho} \equiv \tilde{\rho} - (g + n)$ and $\kappa \equiv (1 + \psi)\tilde{\kappa}$. We postpone the discussion of dynamics for $\pi(t) \notin [\underline{\pi}, \overline{\pi}]$ to below. We will assume that $\tilde{\rho} > (g + n)$, a restriction that guarantees that the firm's pricing problem is well defined.

3.1 Balanced Growth Paths

It will be useful to describe a few properties of a balanced growth path (BGP). In a BGP, all quantities grow at the rate of output, and the real interest rate is constant.

Setting $\dot{c} = 0$, and letting b, c, and r denote the corresponding BGP values, we have:

$$\frac{b}{c} = \frac{(r - \rho - g + \alpha)}{\mu}. \tag{20}$$

Note that the right-hand side is linearly increasing in r. The interest rate in financial autarky, which occurs when $b = 0$, is given by $\rho + g - \alpha$, and for higher levels of financial assets we have $r > \rho + g - \alpha$. In a BGP, as debt relative to worker consumption increases, the real interest rate must also increase for the bond market to clear.

Setting $\dot{b} = 0$, we get from the government sequential budget constraint:

$$T = (r - g - n)b.$$

In a BGP, $w > T$ for the borrowing limit, $\underline{a}(t, t)$ to be negative. That is, $(r - g - n)b = T < (1 + \psi)c$, which requires

$$\frac{(r - g - n)(r - \rho - g + \alpha)}{(1 + \psi)\mu} < 1 \Rightarrow r < \rho + g + n + \lambda,$$

where we used that $r > \rho + g - \alpha$. For $\underline{a}(s, t)$ to be finite for all (s, t), we also require that $r > g - \alpha - \lambda$,

but this is satisfied given $\rho > 0$ and $\lambda \geq 0$ and $r \geq \rho + g - \alpha$. Thus, in a BGP, the interest rate must satisfy:

$$\rho + g - \alpha \leq r < \rho + g + n + \lambda \tag{21}$$

Flexible price interest rates. It will be useful to define the BGP real interest rate associated with the flexible price markup, given debt, $r_\star(b)$ as well as its inverse with the following:

$$r_\star(b) \equiv \rho + g - \alpha + \mu \frac{b}{c^\star} \tag{22}$$

$$b_\star(r) \equiv (r - \rho - g + \alpha) \frac{c^\star}{\mu}. \tag{23}$$

The one remaining element is the determination of the real interest rate $r(t)$, which will depend on the monetary policy rule. We first characterize the case in which monetary policy is not bound by the ELB, and then discuss dynamics in the binding-ELB region of the state space in Section 5.2.

3.2 Dynamics away from the ELB

Let us consider a fiscal policy that maintains a constant $b(t) = b^o$ for all $t \geq 0$. When the ELB is not binding, the Taylor rule (16) becomes

$$i(t) = \bar{\imath} + \theta_\pi \pi(t).$$

Here, we assume the target inflation rate is zero, which requires $\bar{\imath} = r_\star(b^o)$. For the ELB not to bind we require $i(t) > 0$, which implies $\pi(t) > -\bar{\imath}/\theta_\pi$. This will be the range of inflation relevant for this subsection.

The Taylor rule, combined with the Fisher equation implies $r(t) = i(t) - \pi(t) = \bar{\imath} + (\theta_\pi - 1)\pi(t)$. Substituting into (17), we have

$$\dot{c}(t) = (\bar{\imath} + (\theta_\pi - 1)\pi(t) - \rho - g + \alpha)\, c(t) - \mu b^o. \tag{24}$$

Equations (19) and (24) are two ordinary differential equations (ODEs) in $\pi(t)$ and $c(t)$. This system of two equations, combined with the condition that inflation is bounded and $c \geq 0$ in equilibrium, characterize all possible equilibria in which the ELB does not bind.

To analyze the dynamic system, we use the phase diagram in Figure 1. The curve labelled

Figure 1: Phase Diagram: Dynamics Away From the ELB
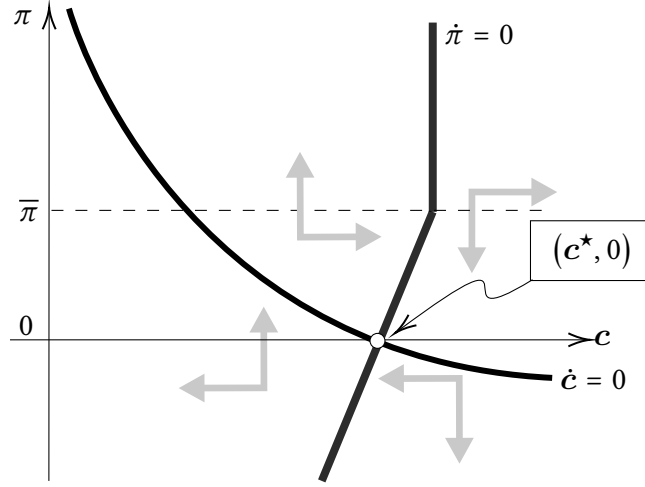
"$\dot{\pi} = 0$" sets $\dot{\pi}$ in (19) to zero:

$$\pi = \frac{\kappa}{\hat{\rho}}\left(c - c^{\star}\right) \qquad \text{for } \pi \in [\underline{\pi}, \overline{\pi}]. \tag{25}$$

As c increases above $c^{\star}$, firms would like to raise their markup, which is counterbalanced by the costs of adjusting prices faster. The stationary point trades off higher π against higher c.

Along the $\dot{\pi} = 0$ curve inflation is constant, at a value that increases with c. Above this locus, $\dot{\pi} > 0$, and below we have $\dot{\pi} < 0$. These dynamics are represented by the arrows pointing up and down in the phase diagram.

Outside of $[\underline{\pi}, \overline{\pi}]$, the lower terms in (14) imply that the Phillips curve is vertical. There is a subtlety when it comes to dynamics for $\pi \notin [\underline{\pi}, \overline{\pi}]$. Along the vertical portion of the $\dot{\pi} = 0$ locus, constant inflation is consistent with profit maximization, but so are movements along the vertical section, as firms are indifferent about the choice of π.

The curve labelled "$\dot{c} = 0$" is the locus of points at which $\dot{c} = 0$. From (24), we have:

$$\pi = -\left(\frac{\bar{\imath} - \rho - g + \alpha}{\theta_\pi - 1}\right) + \left(\frac{\mu}{\theta_\pi - 1}\right)\frac{b^o}{c}. \tag{26}$$

As c increases, the bond market requires a lower real interest rate to clear. Given that $\theta_\pi > 1$, this implies a lower rate of inflation and an even lower nominal interest rate. This generates a negative relationship between c and π in order to keep $\dot{c} = 0$.

From (24), we see that as π increases for a given c relative to the $\dot{c} = 0$ locus, $\dot{c} > 0$. These dynamics are depicted by the horizontal arrows in Figure 1. Note that when $b^o = 0$, the $\dot{c} = 0$

curve would be horizontal with inflation equal to the first term in (26).

Again, care must be taken for $\pi \notin [\underline{\pi}, \overline{\pi}]$. For inflation outside this interval, there are unique levels of consumption, $\underline{c}$ and $\overline{c}$, that are consistent with firm optimization. Thus, there are no consumption dynamics in equilibrium outside $[\underline{\pi}, \overline{\pi}]$; that is, if $\pi \notin [\underline{\pi}, \overline{\pi}]$ is part of an equilibrium trajectory, c is constant while the economy moves along the relevant vertical portion of the $\dot{\pi} = 0$ curve.

The intersection of the two curves is the zero-inflation steady state, denoted by $(c^\star, 0)$, which is the target of the monetary policy. This requires the Taylor rule intercept $\bar{\iota}$ to be set to $\bar{\iota} = r_\star(b^o)$, so that the bond market clears at the zero-inflation steady state.[16] Note that achieving the zero inflation outcome requires an intercept that depends on the level of debt, which reflects the non-Ricardian environment, and anticipates our discussions of fiscal and monetary policy coordination. The zero inflation steady state is unstable. In particular, the eigenvalues of the linearized system evaluated at the steady state both have real parts strictly greater than zero.

It is possible that in our model the ELB binds. In the next section, we will however abstract from this possibility and postpone the discussion of the ELB and the implications for fiscal and monetary policy interactions until Section 5.2.

4   Positive and Normative Implications of Government Debt

This section considers changes in the stock of government debt affects the economy, both from a positive and normative viewpoint, under alternative monetary policy rules.

## 4.1   The Economic Dynamics of Anticipated Deficits

Our main policy experiment involves an anticipated debt-financed tax cut (or transfer increase). Many politicians run on such a platform, including US Presidents Reagan and George W. Bush, as well as, most recently, the mini-budget of UK Prime Minister Elizabeth Truss. Presidents Trump and Biden also implemented some combination of tax cuts and large transfer programs, particularly after the onset of the COVID pandemic. We show that anticipated deficits may increase inflation and may increase or decrease consumption on impact, depending on the horizon and the response of monetary policy.

We initialize the current period as $t = 0$ and assume we are at the zero inflation steady state with some level of debt b^o for $t < 0$. At $t = 0$, there is an unanticipated announcement that at time $t' > 0$ the government will increase debt to $b' > b^o$ and rebate the proceeds to workers.

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[16]Note that $\bar{\iota} = r_\star(b^o) > \rho + g - \alpha$ for $b^o > 0$, and thus the $\dot{c} = 0$ lines crosses the horizontal axes. In addition, the plot as drawn requires that the ELB does not bind at the intersection, that is, $\bar{\iota} = r_\star(b^o) > 0$. We will discuss the case where the ELB binds at the zero-inflation steady state later in Section 5.2.

Figure 2: Response to an Anticipated Bond Issue
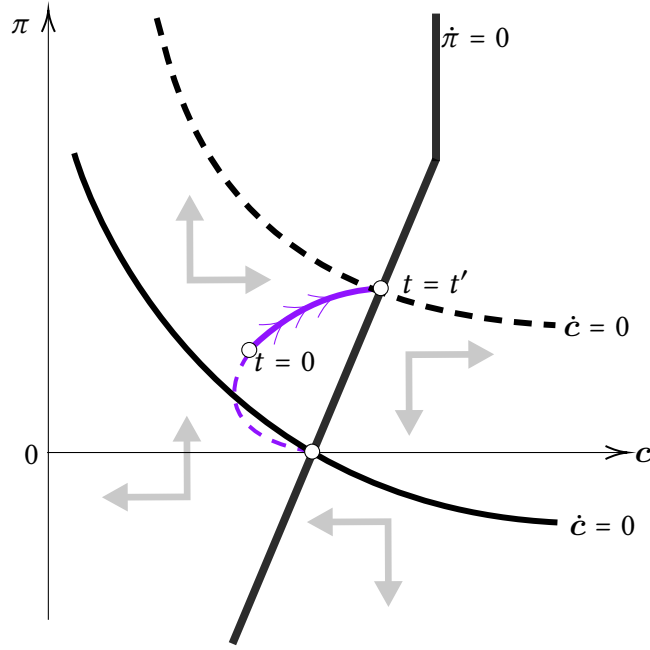
We trace out the path of inflation and worker consumption until the economy reaches the new steady state. In doing so, we focus on the dynamics away from the ELB.

In Figure 2, we replicate the phase diagram from Figure 1 and we first study the case when $\theta_b = 0$. At t', the "$\dot{c} = 0$" shifts out due to the change in b, which is the dashed line labeled $\dot{c} = 0$. The size of this shift for a given b' depends on parameters, in particular λ, ρ, and α. Note that in the Ricardian case of $\alpha = \lambda = 0$, there is no shift. Prior to t', the economy is still subject to the dynamics governed by the original b^o, which are depicted by solid lines and the arrows. The Phillips curve does not depend on b, and hence the "$\dot{\pi} = 0$" remains stable.

The economy jumps to the trajectory at $t = 0$ and travels along that path until it reaches the new steady state at exactly $t = t'$. For larger t', the announcement effect places the economy closer to the original steady state; for smaller t', the economy jumps closer to the eventual steady state. Depending on parameters, the eigenvalues may be complex or real, and the resultant path may cycle or not, respectively.

We depict a thick solid portion of the trajectory as an example path. The initial point involves higher inflation and lower worker consumption. This is combined with positive $\dot{c}$ and $\dot{\pi}$. The economy's response can be understood through the logic of the bond market (which is the flip side of the goods market), working backwards in time from t'. The first thing to note is that for $t \geq t'$, there is a steady state at the intersection of the dashed $\dot{c} = 0$ line and the Phillips curve. Due to the Taylor principle, this is an unstable steady state, and hence the economy must be at this point at t'; otherwise, the explosive dynamics would generate paths that are not consistent

with the equilibrium conditions. At this steady state, inflation and consumption are higher. The higher inflation is needed to generate a higher real interest due to the monetary policy rule: $r = \bar{\imath} + (\theta_\pi - 1)\pi$, with $\theta_\pi > 1$. The higher consumption accompanies the higher inflation due to the Phillips curve. Both elements work to restore equilibrium in the bond market after the issuance of the additional debt.

Dynamics prior to t' are driven by the anticipation of the bond issuance. In particular, consumers anticipate higher consumption and fiscal transfers at $t = t'$. A higher eventual worker consumption lowers the demand for the initial (fixed) stock of bonds for standard inter-temporal substitution reasons. The bond market clears at $t \in [0, t')$ with a higher real interest rate, restoring equilibrium at b^o. Again due to the monetary policy rule, the higher real interest rate must be accompanied by higher inflation. Whether the jump in π is associated with an increase or decrease in the $t = 0$ worker consumption depends on the time horizon and the parameters of the model.

For the experiment of Figure 2, we considered the case of a monetary rule with $\theta_b = 0$, which holds the intercept constant at $r_\star(b^o)$. The increased b requires an increase in the real interest rate, but given the interest rate rule, this must be associated with higher inflation. If the monetary authority increased its intercept at t', the shift in the $\dot{c} = 0$ curve would be dampened. In particular, the central bank can keep the economy at the zero inflation initial steady state for all $t > 0$ by promising to increase the intercept (and hence the nominal interest rate) one-for-one with the necessary increase in the real interest rate to absorb the new bonds at the initial level of income. In particular, this outcome is possible by setting $\theta_b = \theta_b^\star$ in the rule. The economy is at the zero inflation steady state for all $t > 0$, and worker consumption is unchanged. The only effect of the fiscal expansion is to increase the level of debt and a rotation of the $\dot{c} = 0$ at time $t = t'$. We depict this case in Figure 3. At $t = t'$, the monetary policy rule implements a real interest rate increase that rotates the $\dot{c} = 0$ locus through the point $(c^\star, 0)$. Thus, there is no change in consumption at $t = t'$, and hence no anticipatory change in consumption for $t < t'$.

The primary conclusion from this analysis is that in a non-Ricardian environment, the economic consequences of anticipated deficits hinge on the response of the monetary authority's rule to the increase in debt. If the central bank follows the policy advice obtained from the Ricardian benchmark that the long-run target real interest rate is invariant to fiscal policy, the consequences are higher inflation and avoidable fluctuations in consumption. To an outsider well versed in the Ricardian literature, the central bank appears to be doing exactly as prescribed; namely, following a set monetary rule that leans against lax fiscal policy. However, the rigidity of the rule is cause rather than cure for the inflation observed in equilibrium.

Figure 3: Response to an Anticipated Bond Issue: Alternative Taylor Rule
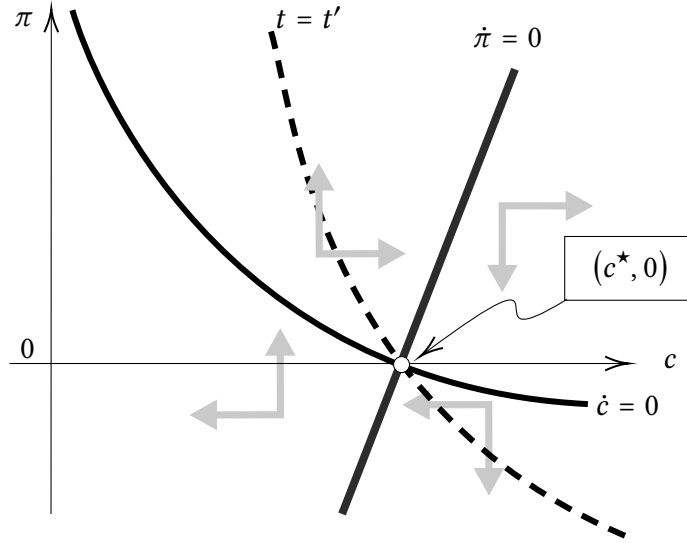
4.2 A Discount Rate Shock

A standard approach to conceptualizing a "demand shock" in the New Keynesian model is a transitory increase in patience.[17] We now show an interesting parallel between such a shock and the analysis above.

Suppose the discount rate of workers, ρ, unexpectedly declines at $t = 0$, but is expected (with perfect foresight) to return to its initial level at $t = t'$. Specifically, indexing ρ by time, we have:

$$\rho(t) = \begin{cases} \overline{\rho} & \text{for } t \notin [0, t') \\ \underline{\rho} & \text{for } t \in [0, t'), \end{cases}$$

with $\underline{\rho} < \overline{\rho}$. The discount rate ($\hat{\rho}$) of the entrepreneurs remains constant throughout.

We explore this scenario in Figure 4. The initial (and final) steady state is depicted by the intersection of the solid "$\dot{c} = 0$" and "$\dot{\pi} = 0$" lines. The dashed $\dot{c} = 0$ locus is drawn for $\rho = \underline{\rho}$, when we maintain the interest rate rule intercept constant at ι. From (26), a decline in ρ shifts the locus down in $\pi \times c$ space. The Phillips curve does not shift, as we maintain the $\hat{\rho}$ constant in this scenario.

This figure is the mirror image of Figure 2. Recall in Figure 2, the shift occurs because the future flexible price real interest rate (after the debt issuance) is higher than the initial rate. In the current experiment, the flexible price interest rate declines at $t = 0$. Hence, the curves shift similarly, but in opposite direction.

In both scenarios, the flexible price real interest rate is temporarily low, and hence the dy-

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[17]For example, see Krugman (1998), Eggertsson and Woodford (2003), and Jung, Teranishi, and Watanabe (2005).

Figure 4: Temporary Decline in ρ: ELB does not bind
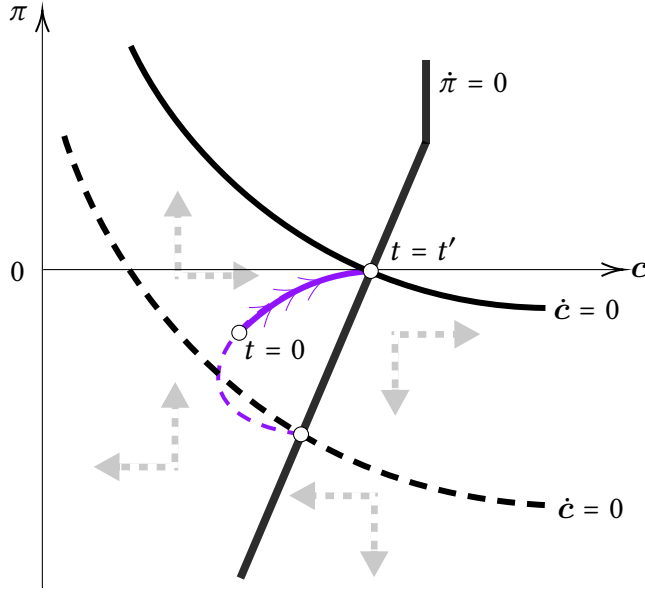
namics are similar, albeit shifted in $\pi \times c$ space. Specifically, the dashed arrows in Figure 4 depict the dynamics that hold for $t \in [0, t')$; that is, relative to the dashed $\dot{c} = 0$ locus. Perfect foresight and worker and firm optimization imply that the equilibrium must be back at the initial steady state at $t = t'$. The purple line depicts trajectories that depart and return to that steady state. The precise point on the trajectory that holds at $t = 0$ depends on t'.

As in the case of Figure 2, the potential trajectories correspond to combinations that clear the bond market. In this experiment, b^o is held fixed, but, all else equal, more patient workers desire to hold more bonds at a given real wage and interest rate. The market clears by a combination of lower c and higher $\dot{c}$, both of which reduced demand. The counterpart is a change in the path of π that ensures the worker consumption dynamics are consistent with firm optimization, which corresponds to lower π and higher $\dot{\pi}$. The inflation path in turn generates a path of the real interest rate via the interest rate rule. A temporary decline in the discount rate, therefore, generates the familiar temporary declines in inflation and consumption.

As before, the monetary authority could keep the equilibrium at the desired zero-inflation steady state by having $\bar{\imath}$ move one-to-one with $\rho(t)$, temporarily declining before returning to its longrun value. From (20), the appropriate $\bar{\imath}$ that clears the bond market at $c^\star$ and zero inflation is

$$\bar{\imath}(t) = \rho(t) - \alpha + (\rho(t) + \lambda)(\lambda + \alpha)b^o/c^\star. \tag{27}$$

Interestingly, a temporary fiscal expansion would also stabilize the economy. That is, at $t = 0$ the fiscal authority statiates workers' increased patience by issuing more bonds, which it will reverse at $t = t'$. From the analysis associated with Figure 2, this would keep the $\dot{c} = 0$ locus at

its original (and longrun) location. This discussion highlights the symmetry between monetary and fiscal policy, a point recently emphasized by Wolf (2021).

One important caveat is that the decline in ρ was assumed to be small enough that the monetary authority does not run afoul of the ELB. We will take that possibility up in Section 5.2. However, the preceding paragraph already anticipates one result of that section; namely, that fiscal policy can substitute for monetary policy at the ELB and deliver the economy away from the ELB.

4.3 Welfare Implications of Government Debt

We now turn to the question of whether and how increasing the stock of government debt affects the distribution of consumption and welfare in our New Keynesian environment with heterogeneous agents.

We begin by noticing that the only aggregate store of value for workers in this economy is government bonds. To understand how the OLG structure breaks Ricardian equivalence, consider a balanced growth path on which $B > 0$ and $r > g + n$. In this case, a bond is a claim on future taxes, some of which is paid by future generations, and hence perceived as net wealth by those currently alive. That is, the newborn cohort pays taxes but holds no bonds, which represents a net transfer to older cohorts. This is the intuition that is perhaps the most familiar from the literature on Ricardian non-equivlanece.

A more useful intuition for our puproses is to consider the fact that newborn agents that wish to save must purchase a bond from older cohorts. The value of this perspective is it makes clear that the same logic holds even if $r < g + n$. In this case, the government earns money from outstanding debt, which it lumpsum transfers to those currently alive. Nevertheless, if an owner of a bond perceives that they will be able to sell the bond at a positive price to future to generations not yet alive, it will be considered net wealth. This future bond demand from generations not yet born represents the "social contrivance" of Samuelson (1958), and encapsulates the main inefficiency of the OLG environment.

4.3.1 Pareto Improvements: Revisiting Samuelson

Suppose we start from an initial BGP with $b(0) = b^o \geq 0$ and a corresponding real rate r^o. We will consider a simple policy change: at $t = 0$, the government issues new debt, moving the total stock of debt to b' from b^o, and adjusts taxes on (or transfers to) workers to satisfy its budget constraint. We will assume the economy is away from the ELB, but consider the alternative in Section 5.2.

Along a BGP, we can compute the welfare of the newborn cohort in closed form, which we

present in Appendix B. Intuitively, welfare is increasing in the real wage, which represents the share of output going to workers. However, the key implication for our purposes is that welfare depends non-monotonically on r. A greater r implies lower discounted lifetime wealth, lowering initial consumption, but a higher return to saving and a faster growth rate of consumption going forward. As shown in Appendix B, ror $r \in [\rho - \alpha + g, \rho + \lambda + g + n)$, the range of rates consistent with a BGP according to (21), U^t is a strictly concave function in r with a maximum at $r = g + n$:

> **Lemma 4.** *For a given wage, the welfare of a newborn worker in a BGP is maximized when* $r = g + n$.

In an equilibrium, individual workers may see a net return to bonds different than $g + n$. For example, the autarkic interest rate (when $b = 0$) is $\rho + g - \alpha$ which may be less than the value of $g + n$ for n or α large enough. From their Euler equation, an $r \neq g + n$ distorts the inter-temporal path of consumption relative to the social optimum, which in turn distorts the cross-sectional allocation of the exogenous amount of resources. This allows for an increase in the supply of bonds to improve upon the equilibrium allocation by raising the interest rate and facilitating inter-generational trades.

Newborn welfare is maximized at $r = g + n$, but a Pareto improvement must also account for existing cohorts at the time of the policy change. If $b^o = 0$, then existing cohorts have zero wealth and are thus identical to newborns in regard to the welfare consequences of a change in r, but in addition get the initial distribution $b' - b^o$. However, if $b^o > 0$, then existing cohorts have positive wealth and are thus sensitive to changes in r beyond the effects on newborn cohorts. The positive savings give them an extra benefit from higher real interest rates. For a given level of $a(s, t) \geq 0$, utility is strictly increasing in the interest rate for $r < g + n$. Contrary to newborns, if $a > 0$, utility peaks at r strictly greater than $g + n$, due to the presence of positive assets. This restricts attention to policies that increase r starting from $r^o < g + n$.

Turning to the entrepreneurs, they are indifferent to equilibria with the same real wage and rate of inflation. In particular, profits are declining in w (the mirror of worker welfare), but non-monotonic in inflation, as inflation changes the markup as well as represents a deadweight loss. Thus, if $r^o < g + n$, the government can implement a Pareto improvement by issuing debt and moving r closer to $g + n$, while ensuring entreprenuer profits do not decrease. This provides a motivation for our experiment discussed in Section 4.1. That anlaysis made clear that it is imperative that the monetary authority accomodate this expansion of debt with a higher nominal interest rate target. Otherwise, the economy experiences inflation, reducing the welfare of entrepreneurs.

The following lemma summarizes this result.

Lemma 5. *Suppose the economy is in an initial BGP with $r^0 < g + n$. Then there exists a new BGP with a combination of fiscal and monetary policies that is a Pareto improvement.*

We want to highlight that the key inefficiency in the model behind Lemma 5 is the presence of a never-ending flow of new generations of workers (or the "infinite hotel" of Shell, 1971). This is the seminal insight of Samuelson (1958), and there is a simple way to see it in our current environment. Set $\alpha = 0$, and consider a situation without new cohorts being born. This corresponds to the case where $n = -\lambda$, that is, population shrinks at the death rate λ. In that case, $A(t)$ drops out from the aggregate Euler equation (10).[18] There is then a unique real rate consistent with a BGP, $r = \rho + g > g + n$, and thus, there is no role for government bonds in improving upon (or even affecting) the market equilibrium.

4.3.2 Robust Pareto Improvements

To implement the above Pareto improvement the government must alter taxes and transfers, T. Given the knowledge of the utility function, it knows how much agents are willing to trade off higher taxes (or lower transfers) in exchange for a higher return to saving. This assumes a level of knowledge that is not realistic. For example, if there were a negligible fraction of consumers that are "hand-to-mouth" consumers, the change in taxes may not be worth the higher interest rate. Such concerns motivated our use of a "robust" criteria in our earlier work (Aguiar, Amador, and Arellano, 2022). In this subsection, we revisit "Robust Pareto Improvements" (RPI) in a New Keynesian setting.

Aguiar, Amador, and Arellano (2022) defines an RPI to be a policy that induces a change in prices and taxes such that the budget set of any agent is guaranteed to be weakly expanded at any state and time. In our current environment:

Definition 1 (RPI)**.** *We say that the fiscal policy of increasing debt from b^o to $b' > b^o$ generates an <u>Robust Pareto Improvement (RPI)</u> if (i) $T' \leq T^o$, (ii) for any (s,t), $r'a^o(s,t) \geq r^o a^o(s,t)$, where $a^o(s,t)$ is the original equilibrium choice of assets for cohort s at time t, (iii) $w' \geq w^o$, and (iv) $\Pi' - f(\pi') \geq \Pi^o - f(\pi^o)$.*

The first condition is that taxes weakly decrease; the second states that financial income weakly increases; the third condition is that real wages weakly increase; and the final condition requires that the profits do not fall. Condition (iv) ensures the entrepreneurs are no worse off in the new equilibrium.

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[18]Buiter (1988) referred to this as the debt neutrality case. Note that our equations do not work in the limit of $n = -\lambda$ when $\alpha > 0$, as there is no effective aggregate amount of labor left in the long-run in this case.

From the workers' perspective, the first three conditions ensure that their budget set weakly increases. Given that the initial transfer to workers is strictly positive ($b' > b^o$), generations alive at $t = 0$ have a strict increase in their budget sets. In aggregate, worker consumption may not increase, but nevertheless individual workers prefer the new allocation due to the better inter-generational distribution. An important element here is that it is not necessary to detail how aggregate consumption is reallocated across workers.[19]

In the appendix (see the proof of Lemma 6), we show that in the BGP, $a(s, t) \geq 0$ for all s and t. That is, no cohort has a negative asset position in the initial equilibrium, and thus condition (ii) requires $r' \geq r^o$.[20]

One advantage of the RPI criteria is that it ensures a Pareto improvement regardless of how agents tradeoff consumption intertemporally (or across uncertain states in a richer environment). In an RPI, with weakly greater flow income at all dates, any agent's initial equilibrium consumption path remains affordable. Thus their welfare cannot fall and must increase if the budget set expands strictly. A second advantage of the RPI criteria is that, to check for the existence of an RPI, only knowledge about how aggregate private savings respond to real interest rate changes is needed. The potentially rich household heterogeneity at the micro level only affects the conditions for the existence of an RPI through the shape and interest rate elasticity of this aggregate savings response.

In the present model, that elasticity is endogenous and partially controlled by the monetary policy response to fiscal policy. This raises the question of whether monetary policy introduces an additional tool to achieve an RPI with respect to a real model. In this section, we show that the answer is no.

First, suppose the monetary authority implmements the flexible price equilibrium's response to a debt issuance.[21] In particular, suppose we begin on a balanced growth path with $\pi^o = 0$ and worker wages at $w^o = w^\star$. From the Phillips curve, aggregate worker consumption is $c^\star$. Let initial debt be $b(0) = b^o \geq 0$, with a corresponding real rate $r^o = r_\star(b^o)$. At $t = 0$, the government issues new debt, moving the total stock of debt (in efficiency units) to b' from b^o, and adjusts taxes on workers to satisfy its budget constraint.[22]

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[19]This was our main motivation when introducing the RPI in the context of a Bewley-Aiyagari model. It is not necessary to specify how agents trade off consumption across states and time. As will be true here as well, only the aggregate savings supply is needed to determine the feasibility of an RPI.

[20]Every cohort has a consumption profile that changes at a constant rate, $r^o - \rho$, and a labor earnings profile (net of taxes) that changes at a constant but weakly lower rate, $g - \alpha \leq r^o - \rho$. The latter is a requirement for the economy to sustain non-negative levels of government debt in the stationary equilibrium, see equation (20). Given that newborns have zero assets, it follows that the asset holdings of every cohort must remain non-negative over their lifecycle.

[21]We assume that the ELB is not binding and characterize the new BGP that arises as the result of the policy. For the ELB not to bind at the initial BGP, it must be that $r_\star(b^o) \geq 0$. Given that we are assuming that the ELB is not binding after the policy change, the economy must transfer to a new BGP, given the unstable dynamics.

[22]As noted above, the case of a bond issuance is equivalent to the government distributing bonds directly to the

After $t = 0$, there are no additional policy changes, debt remains constant at b', and taxes are adjusted to satisfy the government budget constraint. From equation (17), it follows that $r(t)$ must also be constant, at a value we denote by r'. To implement the flexible price equilibrium response, the monetary authority simultaneously increases the nominal interest rate from $i = r^o$ to $i = r'$. From equation (18) it follows then that transfers remain constant at a value T'. Taken together, this means that after the policy change, the economy jumps to a new BGP, and there are no transitional dynamics.[23]

We can combine the government budget constraint at the new level of debt, b', and the aggregate Euler equation, using the definition of $b_\star$ in (23), to obtain the following relation between taxes, debt, and the real rate

$$T' = (r' - (g + n))b_\star(r').$$ (28)

where $b_\star(r') = b'$. Note that $b_\star(r')$ is the aggregate savings supply schedule of the economy in the new BGP, and uniquely determines r', given b'.

Now, a local increase in b from b^o must be matched by an increase in r from r^o. For this to generate a *decrease* in taxes, T, it suffices that

$$-b'_\star(r^o)\frac{r^o - (g + n)}{b^o} > 1.$$ (29)

Thus, if *the aggregate savings elasticity with respect to the real interest rate* (net of $n + g$) is higher than unity, then an RPI exist. Given that $b'_\star(r^o) > 0$, it follows from (29) that $r^o < g + n$. This is not sufficient however, $b'_\star(r^o)$ must also be large enough.

The elasticity condition (29) is related to the debt Laffer curve: it guarantees that the economy starts from a region below the peak of the curve, and thus it is possible to increase government revenue by increasing debt.[24] Figure 5 shows the debt Laffer curve and its relationship with the existence of an RPI. In our current model, we can go a bit farther, as we can characterize the peak of the Laffer curve in closed form:

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existing worker cohorts, increasing aggregate savings by $b' - b^o$.

[23] The lack of transitional dynamics is in our view a main difference with respect to Bewley-Aiyagari type models, where the distribution of wealth needs time to converge to a new ergodic state after a policy change. In our model, the distribution of wealth does not need to adjust after the policy change, a feature driven by the linearity of the policy functions. A potential extension of the present model, that will generate transitory dynamics, is to introduce additional heterogeneity on the workers' side as in Gertler (1999).

[24] As mentioned in the introduction, several other papers have studied the debt Laffer curve in the context of $r < g$. See for example Bassetto and Sargent (2020) and Mian, Straub, and Sufi (2022). The first is very related. In particular, their discussion in their Section 4.1 represents an example of an RPI within an OLG model, although focused on steady states.

Figure 5: The Debt Laffer Curve and RPIs in the Flex Price Equilibrium
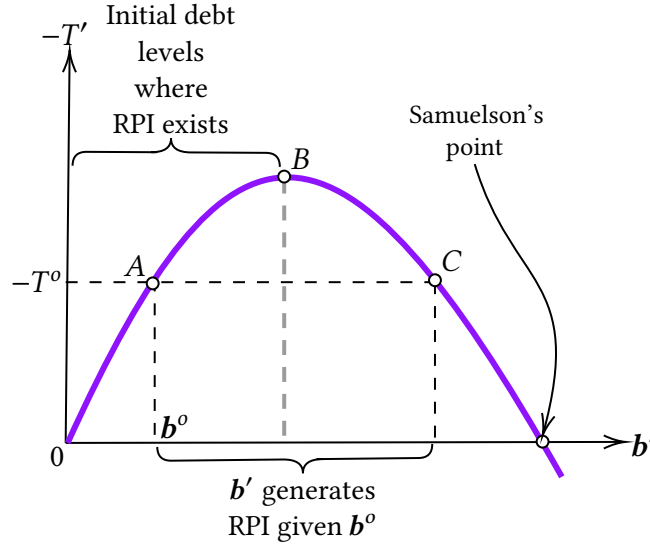
Notes: The horizontal axis represents the new levels of debt, b', and the vertical axis represents the negative of the corresponding tax levels, T'. For any initial value of b^o that lies from the origin to point B, there exists an RPI. Given the b^o plotted, the ranges of b' from point A to C represent a corresponding RPI.

Lemma 6 (Existence of an RPI in the Flexible Price Equilibrium). *Suppose the economy is in an initial BGP. There exists a debt issuance at $t = 0$ that generates an RPI if and only if $r^o - g - n < (\rho - \alpha - n)/2 < 0$.*

In Figure 5 we denote the point where $r = g + n$ as the "Samuelson's point". At this point, the economy is at a Pareto optimum with respect to the allocation of resources among workers. For values of debt below this point, the equilibrium allocation is Pareto dominated by Samuelson's point. However, if the economy started from a lower level of debt and policy moved it towards this point, although interest rates would increase towards $g + n$, transfers do not uniformly increase. As the figure shows, transfers decrease with debt for levels of debt to the right of the peak of the transfer curve. Such a decline in transfers is the reason increasing debt up to the Samuelson's point does not constitute an RPI.

The debt Laffer curve in Figure 5 was drawn assuming the monetary policy rule implemented the flexible price equilibrium. We now turn to the question of how the curve changes under alternative monetary policy rules, and whether such changes expand the scope for RPIs. Suppose the monetary policy rule is given by

$$i' = \bar{\imath} + \theta_\pi \pi' + \theta_b(b' - b^o).$$

The parameter θ_b controls the response of monetary policy to government debt. Using the above, we can solve for the level of taxes at the new BGP as a function of the aggregate savings schedule as follows

$$T' = (r' - (g + n))\boldsymbol{b}_m(r'), \tag{30}$$

where $\boldsymbol{b}_m(r')$, the aggregate savings supply schedule in the monetary economy, is now given by:

$$\boldsymbol{b}_m(r') = \boldsymbol{b}_\star(r') \left[1 + \frac{\left(\theta_b^\star - \theta_b \right)(r' - r^o)}{\theta_b(r' - r^o) + \theta_b^\star \left(\gamma + \theta_b \boldsymbol{b}^o \right)} \right]$$

with $\theta_b^\star \equiv \frac{\mu}{c^\star}$ and $\gamma \equiv \frac{(\theta_\pi - 1)\kappa}{\hat{\rho} c^\star}$.

A comparison of T' in (30) with T' in the flex-price model, (28), reveals an important distinction: the shape of the aggregate savings supply schedule is now a function of the monetary policy rule coefficients, as well as the Phillips curve parameters. When $\theta_b = \theta_b^\star$, the monetary rule implements the flexible price equilibrium. However, alternative rules trace out different Laffer curves.

For any value of $\theta_b \in [0, \theta_b^\star)$, we have that $\boldsymbol{b}_m(r') > \boldsymbol{b}_\star(r')$ and $\boldsymbol{b}_m'(r^o) > \boldsymbol{b}_\star'(r^o)$. In this case, the monetary model features a total savings schedule that is more elastic to interest rates than the real model. Thus, the range of debt levels for which debt issuances reduce taxes has expanded in comparison.

In Figure 6, we illustrate two distinct debt Laffer curves under two distinct values of θ_b. The solid line corresponds to $\theta_b = \theta_b^\star$, and it is the same Laffer curve as in Figure 5. A monetary policy rule with a $\theta_b = 0$, increases the elasticity of the aggregate savings schedule at $\boldsymbol{b}^o$ to the interest rate, and thus, the peak of the Laffer curve shifts to the right.

However, to achieve this, the monetary policy necessarily boosts inflation after the fiscal expansion, and as a result, the share of output that is allocated to labor increases: $w' > w^o$ and $\pi' > 0$. Both of these *reduce* entrepreneurial profits. Thus, *the fiscal policy does not generate an RPI* unless the monetary policy sets $\theta_b = \theta_b^\star$.

To summarize, in our model, monetary policy does not create a free lunch, since it inevitably redistributes resources between workers and entrepreneurs. We did not let the government compensate entrepreneurs, however, through, for example, subsidies for potential profit losses due to a fiscal expansion. And a more accommodative monetary policy rule could generate additional fiscal revenue to fund such subsidies, and thereby facilitate the existence of an RPI. Allowing transfers from workers to entrepreneurs makes the model less tractable, and, arguarbly, less realistic. For this reason, we leave this question for future work.

Figure 6: The Debt Laffer Curve and RPIs in the Monetary Model
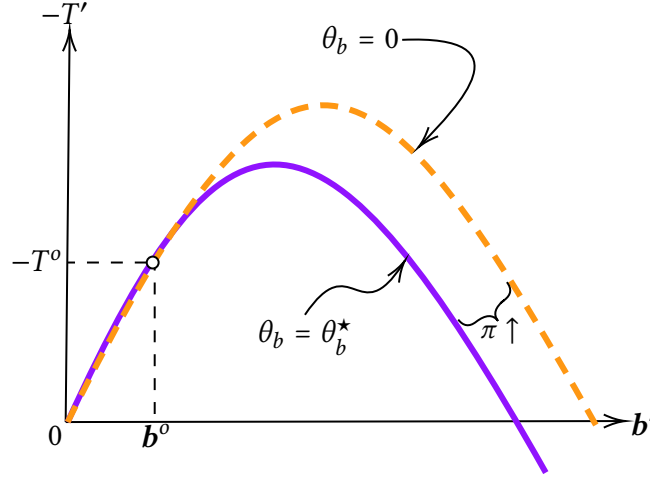
Notes: The horizontal axis represents the new levels of debt, b', and the vertical axis represents the negative of the corresponding tax levels, T'. The curves represent the levels of transfers in the new BGP for two different monetary policy rules. The solid curve is the case where $\theta_b = \theta_b^\star$ which replicates the real economy. The dashed curve is the case with $\theta_b = 0$, where the monetary policy rule facilitates larger transfers, but at the cost of higher inflation and lower profits.

This subsection extends the insight of Samuelson to a New Keynesian environment.[25] The crucial lesson learned here is that expanding the stock of safe assets is not sufficient to improve welfare. In the New Keynesian model, this will have potentially unappealing inflationary and distributional consequences. However, the pairing of debt issuance with an appropriate monetary rule that responds to the fiscal expansion rescues the traditional insight that increasing the quantity of safe assets may improve the distribution of a fixed amount of income. As noted above, this insight may have implications for the broader HANK literature.

5 Additional Monetary and Fiscal Policy Interactions

In this section, we use the phase diagram to explore equilibrium dynamics for various additional scenarios that feature monetary and fiscal interactions. In doing so, we use the phase diagram to clarify and/or reconcile several results in the literature. In all of these scenarios, we set $g = n = 0$; that is, no technological or population growth. We do this for simplicity.

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[25]Balasko and Shell (1980) extended Samuelson (1958) to provide precise conditions on the existence of Pareto improvements in OLG models. Balasko and Shell show that a stationary equilibrium is Pareto efficient if and only if $r \geq g + n$. This subsection extends that result to the standard New Keynesian framework, including the necessary adjustment of the monetary policy rule. Moreover, these improvements can be made with simple policies that do not rely on complex changes to the tax schedule.

5.1 The Forward Guidance Puzzle

The phase diagram is also a transparent analytical tool to understand the "forward guidance puzzle" of Del Negro, Giannoni, and Patterson (2023) and McKay, Nakamura, and Steinsson (2016). In a standard representative agent New Keynesian model, Del Negro, Giannoni, and Patterson (2023) showed that an announcement to temporarily reduce real interest rates at some point in the future had a large effect on consumption and inflation in the announcement period, and all periods leading up to the interest rate cut. Moreover, pushing the reduction arbitrarily far into the future does not dampen the economy's initial response to the announcement.

Using alternative quantitative models, Del Negro, Giannoni, and Patterson (2023), McKay, Nakamura, and Steinsson (2016), and Kaplan, Moll, and Violante (2018) showed that breaking Ricardian equivalence mitigates this puzzle. Del Negro, Giannoni, and Patterson (2023) builds on Blanchard-Yaari, and hence is closest to our framework. The other two papers build on Aiyagari (1994), but draw similar lessons.

As a counterpoint, Farhi and Werning (2019) uses a Blanchard-Yaari model and argues that incomplete markets (or the presence of new generations in our model) are not sufficient to resolve the forward guidance puzzle. We use our tractable framework to understand these approaches and why they differ in their conclusions. In particular, we show how the different conclusions can be mapped into different assumptions on the underlying fiscal policy.

We can adapt the McKay, Nakamura, and Steinsson (2016) experiment to our setting as follows. Let r^o be the stationary real interest rate $r_\star(b^o)$ that clears the bond market at zero inflation (equation 22). At $t = 0$, the monetary authority announces that at $t_0 > 0$ it will reduce the *real* interest rate below r^o for the interval $[t_0, t_1)$, before resuming its original policy.

This is equivalent to an interest rate rule with $\theta_\pi = 1$, $\theta_b = 0$, and an intercept that shifts down for $t \in [t_0, t_1)$. Specifically, for some $\Delta > 0$, monetary policy follows:

$$i(t) = \begin{cases} r^o + \pi(t) & \text{for } t \notin [t_0, t_1) \\ r^o - \Delta + \pi(t) & \text{for } t \in [t_0, t_1). \end{cases} \tag{31}$$

After t_1, we assume that the economy is back at the zero inflation steady state, which the monetary authority implements by following the corresponding Taylor rule.

We can replicate the representative agent "forward guidance puzzle" by considering the case with zero government bonds. From (10), we recover an aggregate Euler Equation that is identical to that of a representative agent economy:

$$\dot{c} = (r(t) - \rho + \alpha)c(t). \tag{32}$$

Figure 7: Forward Guidance Puzzle

(a) Representative Agent Scenario (b) Non-Ricardian Scenario
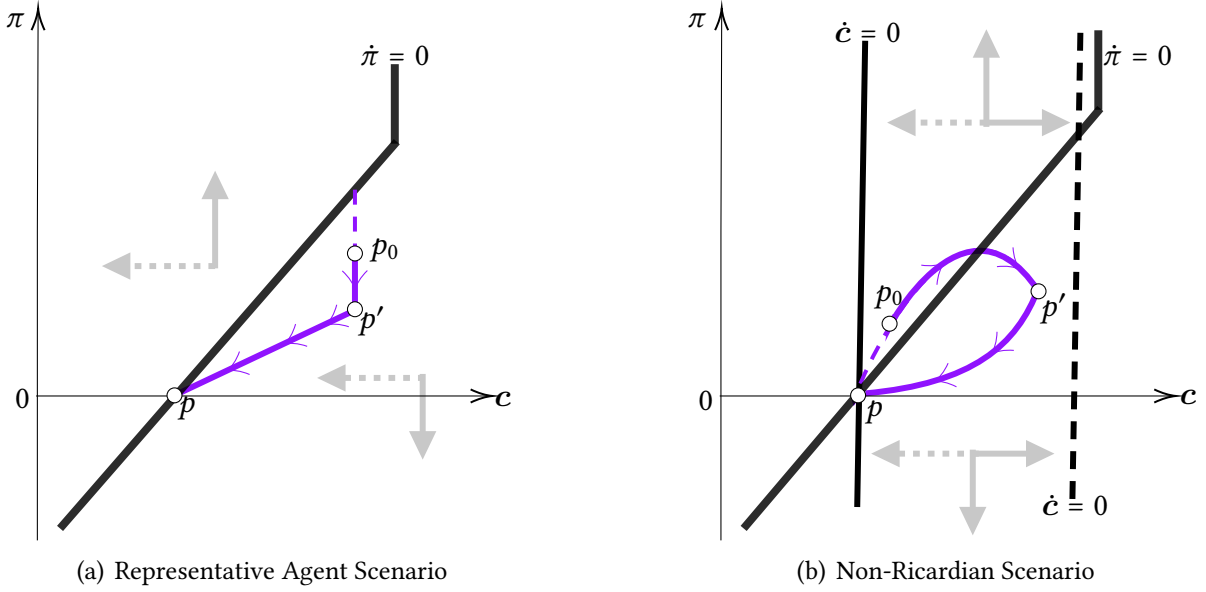
The stationary real interest rate is $\rho - \alpha$, which for the current experiment will be r^o. Panel (a) of Figure 7 depicts the associated phase diagram. The $\dot{\pi} = 0$ curve remains unchanged from the previous scenarios. However, for the current experiment that mimics the representative agent, any level of c is stationary for $t < t_0$, and hence we drop the $\dot{c} = 0$ locus.[26] . When $t \in [t_0, t_1)$ and $r(t) = r^o - \Delta < \rho - \alpha$, we have $\dot{c} < 0$, for all c. The consumption dynamics for $t \in [t_0, t_1)$ are depicted with a dashed arrow.

The equilibrium can be solved backwards from t_1. At $t = t_1$, the real interest rate returns to $r^o = \rho - \alpha$, and the economy must be back at its zero inflation steady state, labeled p. For $t \in [t_0, t_1)$, we can solve (32) with $r(t) = r^o - \Delta$ backwards in time from the boundary condition $c(t_1) = c^\star$. Associated with this path for $c(t)$ there is a path for $\pi(t)$ that satisfies (14) with the boundary condition $\pi(t_1) = 0$. As $\dot{c} < 0$ over this time interval for any c, we know the saddle path lies to the right of p. Moreover, given the dynamics associated with the Phillips curve, the higher c is associated with a higher π. Hence, the saddle path lies to the north-east of p and below the $\dot{p} = 0$ locus. This trajectory is depicted as the path leading from the point labeled p' to the zero-inflation steady state p.

For $t \in [0, t_0)$, the monetary authority sets $r(t) = r^o = \rho - \alpha$, and $\dot{c} = 0$. The economy at $t = 0$ thus jumps to a point *directly above* p' and follows a vertical trajectory that reaches p' at $t = t_0$. This is depicted as point p_0. Note that at announcement, the economy jumps to a point with higher inflation and higher c. Moreover, the further in the future is t_0, keeping the interval

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[26]There will be a $\dot{c} = 0$ locus again after t', given the assumed return to the zero inflation steady state and change in the monetary rule.

of low rates $t_1 - t_0$ constant, the longer time is spent on the vertical trajectory. As $t_0 \to \infty$, the economy starts closer and closer to the $\dot{\pi} = 0$ line and spends longer in the high inflation-high consumption situation. The puzzle has two parts: first, the economy immediately responds to the promise of a future decline in real rates, *no matter* how arbitrarily far in the distant future this decline occurs; and second, the *further* in the future the reduction is, the *larger* the initial effect on inflation and the longer the economy experiences high inflation and high consumption.

We now show why breaking Ricardian equivalence can resolve/remove this puzzling feature of the standard model. To do so, we assume the economy starts with a positive level of government bonds $b^o > 0$. The monetary policy continues to be given by (31), but now $r^o = r_\star(b^o)$, the stationary real interest rate given by (22).

The phase diagram is depicted in Panel (b) of Figure 7. The main difference between the two panels concern the dynamics of consumption. When $b > 0$, for each r there is a *unique c* that keeps $\dot{c} = 0$. The solid vertical line depicts the $\dot{c} = 0$ line associated with r^o and is relevant for $t \in [0, t_0)$. The dashed vertical line is the case for $r^o - \Delta$. From (20), this lies to the right of the solid line.[27] The solid and dashed arrows correspond to the dynamics over the two intervals, respectively.

To explore the equilibrium response to forward guidance, we again work backwards through time. The trajectory for $t \in [t_0, t_1)$ behaves in a qualitatively similar fashion as in the representative agent case. As $\dot{c} < 0$ over the low-interest rate interval in both, we have a similar trajectory from high inflation/high consumption back towards the zero-inflation steady state. Thus, during the low interest rate period, the two economies operate similarly.

However, the anticipatory behavior prior to t_0 is quite different. In panel (b), consumption cannot remain constant for $t \in [0, t_0)$ without a discontinuity in the anticipated path of consumption (which is inconsistent with equilibrium). To avoid a discontinuity, consumption must follow the "solid line" dynamics, which implies a path that leaves from p and heads northeast toward p'. At the announcement, the economy jumps to a point p_0 on this trajectory that ensures arrival at p' at $t = t_0$. The further in the future is t_0, keeping $t_1 - t_0$ constant, the longer the span spent on this trajectory, and the closer to the original (and final) steady state p the economy starts at $t = 0$. Hence, there is no longer a "forward guidance puzzle": The further in the future is the planned interest rate cut, the less the initial economy reacts.

The crucial difference between panels (a) and (b) is that the representative agent economy can spend an indefinite amount of time at *any* level of consumption, as long as $r(t) = \rho - \alpha$. Conversely, in the non-Ricardian scenario, the stationary interest rate depends on b/c, and hence there is a unique $c = c^\star$ that is stationary, given b^o. This is a property shared by the HANK models of

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[27]Here, we assume that $0 < \Delta < r^o - \rho + \alpha$; otherwise there is not a well defined stationary locus for the low-interest environment.

McKay, Nakamura, and Steinsson (2016) and Kaplan, Moll, and Violante (2018). Moreover, given the unstable dynamics around the zero-inflation steady state, the dynamics pick up speed as we move away from p, and hence long trajectories must start close to that steady state.

5.1.1 Alternative Fiscal Policies and Forward Guidance

As noted above, Farhi and Werning (2019) uses a Blanchard-Yaari model to argue that incomplete markets (or the presence of new generations in our model) are not sufficient to resolve the forward guidance puzzle. In this subsection, we present a formulation insipired by their analysis. We use this to show the role that fiscal policy plays in determining the efficacy of forward guidance.

In the analysis associated with Figure 7, we kept the stock of debt fixed. As debt is the only store of value for workers, this implied aggregate wealth was constant, and the ratio of wealth to consumption varied with c. In the model of Farhi and Werning (2019), the equilibrium ratio of wealth to output is constant.[28] In our environment, this is akin to the ratio $b(t)/c(t)$ being constant. More precisely, suppose fiscal policy sets $b(t)/c(t) = \delta$ for all t, where $\delta > 0$ is the target ratio of debt to worker consumption. We now explore how does this alternative fiscal policy affects the forward guidance puzzle.

The policy associated with δ can be implemented by ensuring aggregate tax revenues are proportional to c; for example $T(t) = \tau c(t)$.[29] The government budget constraint becomes

$$\dot{b}(t) = r(t) - g - n)b(t) - \tau c(t). \tag{33}$$

Together with the Euler equation (17), and setting $\dot{b}/b = \dot{c}/c$ we see that for a given target ration of b/c there is a unique tax policy τ that implements it (regardless of the path of $r(t)$). Specifically, $\tau = \mu \delta^2 + (\rho - n - \alpha)\delta$, where μ is defined in equation (17).

We can now revisit the forward guidance puzzle. To use the phase diagram, we follow Farhi and Werning (2019) and assume prices cannot adjust at all; that is, $\pi(t) = 0$ regardless of monetary policy and the Phillips curve is completely flat. This eliminates $\pi(t)$ as a state variable, and we can rewrite the dynamic system in terms of $b(t)$ and $c(t)$ and analyze the equilibrium using a 2-dimensional phase diagram.

The dynamics, and hence the phase diagram, depends on the choice of $r(t)$. We first solve for a balanced growth path along which $b = \delta c$. Setting $\dot{c} = 0$ in equation (17) and $b/c = \delta$, we see that c is stationary as long as

$$r(t) = \bar{r} \equiv \rho + g - \alpha + \mu \delta.$$

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[28]In Farhi and Werning (2019), the constant wealth to income ratio is obtained in an equilibrium in which savers hold an asset whose dividends are proportional to output.

[29]We maintain our assumption that taxes are lump-sum. This avoids introducing a distortion in labor supply. But we could also assume that the government sets a proportional tax on labor income.

Figure 8: Forward Guidance Puzzle: An Alternative Fiscal Policy

(a) $r = \bar{r}$

(b) Dynamics with Forward Guidance
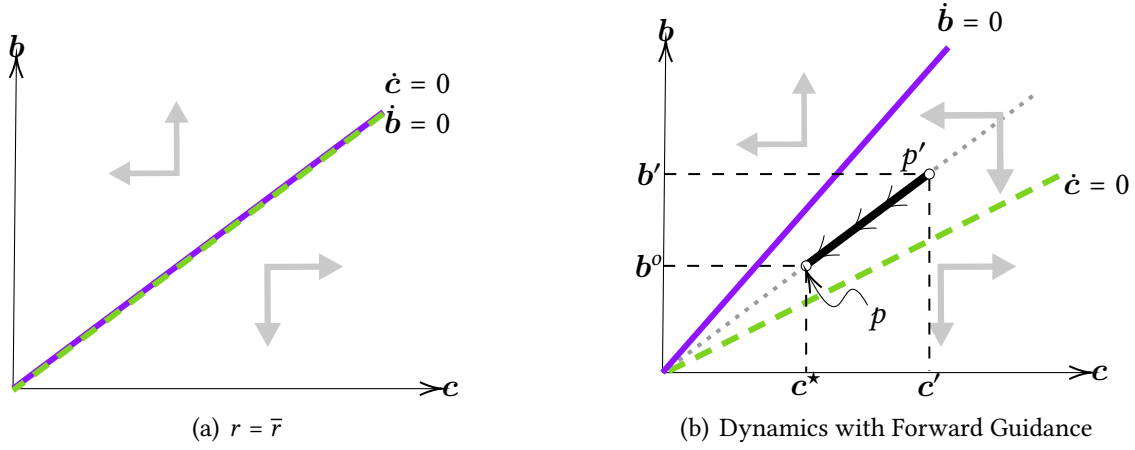
Similarly, from (33), and substituing for τ given δ, we see that b is also stationary as long as $r(t) = \bar{r}$. We depict this case in Panel (a) of Figure 8. The stationary points lie on a ray from the origin with slope δ, the targeted value of b/c. For a given c, moving above this locus puts the government in the situation of $\tau c < \bar{r} b$, and hence $\dot{b} > 0$. The reverse is true below the stationary line. From the Euler equation, an decrease in c relative to the $\dot{c} = 0$ line decreases $\dot{c} < 0$. This gives the dynamics depicted by the arrows to the northwest of the $\dot{c} = \dot{b} = 0$. The dynamics to the southeast are reversed.

The key eliment induced by this fiscal policy is that *any* level of consumption is consistent with a steady state when $r(t) = \bar{r}$. That is, fiscal policy adjusts to ensure that *any* c can be sustained indefinitely. This indeterminacy of steady-state consumption is exactly the key feature of the representative agent economy in Figure 7 Panel (a) that generated the forward guidance puzzle. It will be no surprise, therefore, that we can replicate the puzzle in the current environment under this alternative fiscal policy, which we do next.

Suppose the economy starts at a BGP on which $c = c^\star$ and $r = \bar{r}$. As before, the monetary policy is announced at $t = 0$: the real interest rate will be reduced at t_0 to $\bar{r} - \Delta$ for $t \in [t_0, t_1)$, after which it returns to $\bar{r}$. As before, we assume the economy returns $c^\star$ after t_1.[30]

Panel (b) of Figure 8 depicts the phase diagram for $r(t) < \bar{r}$. From (33), the $\dot{b} = 0$ locus becomes: $b = \frac{\tau c}{\bar{r} - \Delta} > \frac{\tau c}{\bar{r}} = \delta$, where the last equality is by definition of $\bar{r}$. Thus, the $\dot{b} = 0$ locus rotates up relative to δ. The Euler equation evaluated with $r < \bar{r}$ implies that the $\dot{c} = 0$ rotates down relative to the ray with slope δ. Panel plots these stationary lines, along with a dashed ray from the origin with slope δ for reference. Note that the dashed ray also contains the equilibrium trajectory for the economy, due to the fiscal rule keeping $b(t) = \delta c(t)$ for all t.

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[30]This is harder to justify in an environment with a flat Phillips curve, but keeps the analysis simple and keeps us close to the assumption made in the literature.

We can use Panel (b) of Figure 8 to describe the dynamics of the economy. Working backwards through time, at $t = t_1$, the economy must be at its stationary point, depicted as $(c^\star, b^o = \delta c^\star)$. For $t \in [t_0, t_1)$, the economy moves along the dashed line in a southwesterly direction. Thus, at $t = t_0$, the economy must be at a point with $c(t_0) > c^\star$, which we label p'. Government debt is accordingly $b' = \delta c'$.

Prior to t_0, all points along the dashed ray are stationary points. Thus, to avoid a discontinuity in consumption at t_0, the economy is at p' for $t \in (0, t_0)$, as well. This also implies that $b(t)$ jumps to b' on announcement.[31] As in the representative agent case, there are no consumption dynamics leading up to $t = t_0$. The entire response happens on announcement, regardless of how far in the future is the interest rate decline.

The dynamics shown in panel (b) are similar to the dynamics that would arise in a representative agent model. That is, c jumps on announcement, remains constant while the interest rate remains unchanged, then slowly decreases back to $c^\star$ starting at $t = t_0$ (following the Euler equation with a lower real rate). At time $t = t_1$ the economy is back to its starting point. The horizon of the change in interest rates does not affect the change in consumption at announcement (as in the analysis in Panel (a) of Figure 7). Hence, despite the non-Ricardian agents, the foward guidance puzzle remains as strong as it was in the corresponding representative agent model.

An interesting lesson arises: the strength of forward guidance depends on the associated fiscal policy. In particular, an expansionary fiscal policy makes forward guidance more effective. The lesson should not be a surprise as, for example, the effects of monetary policy depend on the underlying assumptions about fiscal policy in HANK models (Kaplan, Moll, and Violante, 2018; Kaplan, Moll, and Violante, 2016).[32]

The use of forward guidance as policy tool arose out of the need to enhance the toolkit of central banks when facing the zero lower bound constraint on nominal interest rates. The exercise in this last section highlights that fiscal policy can help make forward guidance more effective. But also hints that fiscal policy on its own may be sufficient to boost the economy out a liquidity trap. We proceed to study this case in the next section.

5.2 The Effective Lower Bound

In the previous sections, we have abstracted from the possibility of the zero lower bound on nominal interest being binding. In this section, we complete the model description by stating the dynamics of the economy including the ELB, and discussing the monetary and fiscal policy

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[31]In Farhi and Werning (2019), the increase in b is effected by a capital gain on financial assets.

[32]Farhi and Werning (2019) discuss how having liquidity (in our model, $A(t)$) that co-moves with income (in our model, workers labor income) is important in their result. We reinterpret this from the point of view of our model by noting that the amount of liquidity is a policy choice, driven by fiscal policy.

interactions at the bound.

The phase diagram in Figure 1 depicts equilibrium trajectories when the monetary authority is unencumbered by the ELB. If the monetary authority sets $i(t) = \bar{\imath} + \theta_\pi \pi(t)$, this will be the case for $\pi(t) > -\bar{\imath}/\theta_\pi$. For $\pi(t) < -\bar{\imath}/\theta_\pi$, the monetary rule cannot be followed without running afoul of the ELB.

When the ELB binds, the nominal interest rate is zero, and the Fisher relation implies $r(t) = -\pi(t)$. Substituting this into (17), we have

$$\dot{c}(t) = (-\pi(t) - \rho - g + \alpha) \, c(t) - \mu b^o. \tag{34}$$

Hence, for $\pi(t) < -\bar{\imath}/\theta_\pi$, the stationary points for c are given by:

$$\pi = -(\rho + g - \alpha) - \mu \frac{b^o}{c}. \tag{35}$$

When the ELB binds, we have a positive relationship between π and c that keeps $c(t)$ constant. A lower real interest rate is obtained by higher inflation under the ELB, and hence bond-market clearing requires a higher c for a higher π when $\dot{c} = 0$. Note that equation (34) does not depend on any aspects of monetary policy but is sensitive to the amount of government debt. We return to this below when we discuss how to use fiscal policy to escape the ELB.

In Figure 9 we add this additional locus to that of Figure 1. The horizontal line labeled *ELB* demarcates the threshold $\pi = -\bar{\imath}/\theta_\pi$. Note that if $b^o = 0$, both schedules are independent of c, which would be the case in a Ricardian environment in which the stationary real interest is independent of the level of debt and income.

From (34), we see that as π increases for a given c relative to the $\dot{c} = 0$ locus, $\dot{c} < 0$. Thus, the direction of change for c and π are the same between the two $\dot{c} = 0$ loci, regardless of whether we are above or below the ELB threshold. Hence, there is no discontinuity in trajectories at the ELB threshold.

Figure 9 depicts a situation where there are two stationary points. The first steady state is that depicted already in Figure 1, at which the ELB does not bind. At the other steady state, the ELB is binding, and the dynamics around it are saddle path stable. The existence of this steady state, and the dynamics around it, follows the logic spelled out in Benhabib, Schmitt-Grohé, and Uribe (2001).[33]

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[33] Note that it is possible to have further steady states in the ELB region. If the lower part of the $\dot{c} = 0$ curve had intersected the $\dot{\pi} = 0$ curve twice, we would have a third steady state, which would be unstable, and if three times (which is possible due to the vertical section of the Phillips curve), then there is an additional stable point, as well. The fact that there could be multiple possible steady states in the ELB region is a consequence of the failure of Ricardian equivalence in the model.

Figure 9: Phase Diagram
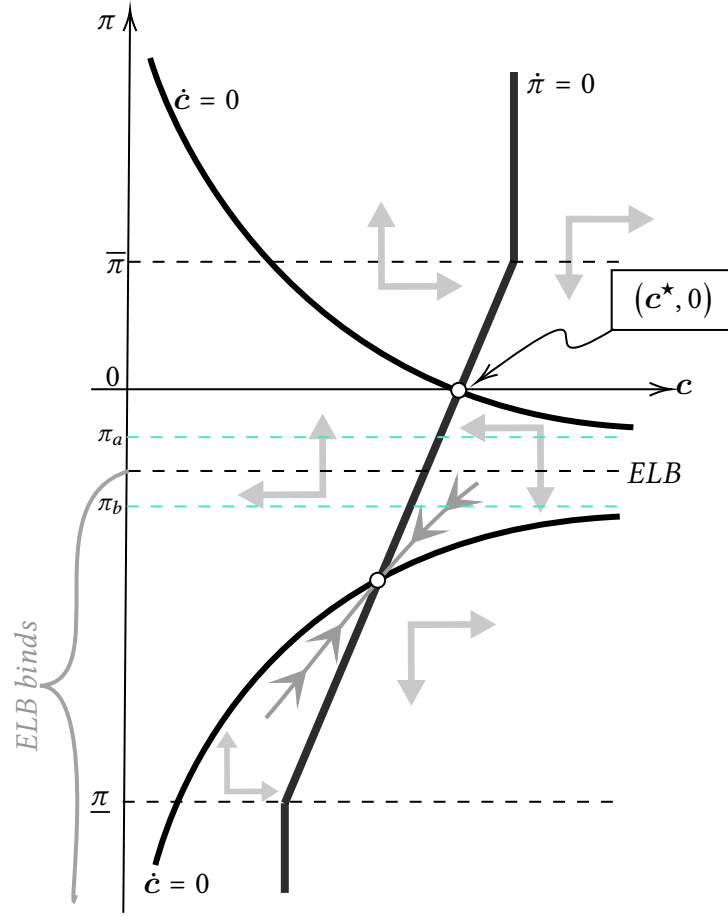
Notes: ELB represents the value $\pi = -\frac{\bar{\iota}}{\theta_\pi}$. For inflation levels below this value the ELB constraint is binding; while it is not above. The values of π_a and π_b represent the asymptotes of the respective $\dot{c} = 0$ lines. Their values are $\pi_a = \frac{\bar{\iota}-(\rho+g-\alpha)}{\theta_\pi-1}$ and $\pi_b = -(\rho+g-\alpha)$. The graph is drawn for the case where $\pi_a > -\frac{\bar{\iota}}{\theta_\pi} > \pi_b$.

Figure 10: Temporary Decline in ρ: Binding ELB and Fiscal Policy

(a) ELB Binds (b) Increase in b
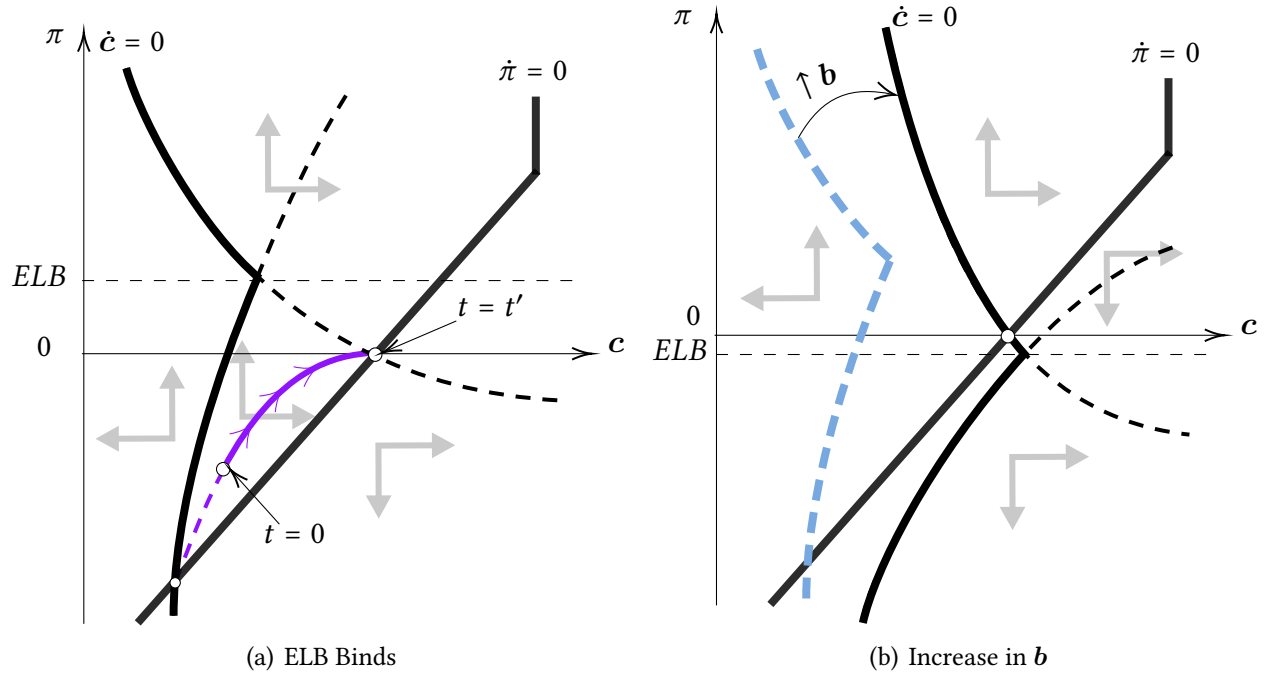
5.2.1 Discount Factor Shocks and Avoiding the ELB

We now revisit the transitory decline in workers' discounting analyzed in Section 4.2. However, we now assume that the decline in ρ is so severe that the monetary authority is constrained by the ELB. This will be the case if $\underline{\rho}$ is sufficiently low such that the implementing the flex price real interest rate at zero inflation requires a negative nominal interest rate. In particular, that $\bar{\imath}$ in equation (27) evaluated at $\rho(t) = \underline{\rho}$ is negative.

This case is depicted in Figure 10 panel (a). The downward and upward sloping $\dot{c} = 0$ lines represent equations (26) and (35), respectively, with ρ set to $\underline{\rho}$. The "ELB"' line is the inflation rate at which the monetary rule with intercept (27) hits the ELB; that is, $\pi = -\bar{\imath}/\theta_\pi$.

The decline in ρ makes the zero inflation steady state not achievable for $t \in [0, t')$, as $\pi = 0$ lies below the ELB point. The dynamics for $t \in [0, t')$ are governed by the upward sloping "$\dot{c} = 0$" locus, and the intersection of this curve with the $\dot{\pi} = 0$ Phillips curve is an unstable steady state (see footnote 33). The trajectory that leads away from this intersection and reaches the zero-inflation steady state at $t = t'$ is the equilibrium.[34] This trajectory follows the logic set out in Werning (2011) in his "no-commitment" policy, and echoes that of Krugman (1998), Eggertsson and Woodford (2003), and Jung, Teranishi, and Watanabe (2005). There is initially a large decline in worker consumption and inflation, and the economy slowly recovers anticipating a return to

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[34]Here, we ignore the possibility that there are additional deflationary steady states, which may be possible given the (omitted) vertical portion of the Phillips curve.

"normal" at $t = t'$.

Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), and Werning (2011) argue that commitment to future low interest rates (after the ELB no longer binds) can improve upon the no-commitment policy. Commitment to this post-t' inflationary policy "pulls up" the trajectory: at time t', the equilibrium has higher inflation and income than the zero-inflation steady state, and via anticipation, the entire trajectory adjusts. This logic underlies the now-standard "forward guidance" policy prescription when at the ELB.

As anticipated in Section 4.2, fiscal policy can provide an alternative solution that does not require the same type of commitment as forward guidance.[35] To see this, suppose that the monetary policy rule features $\theta_b = \theta_b^\star$, and the fiscal policy expands, $b' > b^o$. This shifts the upward-sloping portion of the "$\dot{c} = 0$" locus down. For modest increases in b', the intersection anchoring the trajectory in Figure 10 moves closer to the zero-inflation steady state. For large enough increases, the curve shifts enough that the ELB no longer binds, which occurs when the associated real rate at zero inflation is no longer negative. This case is shown in Figure 10 panel (b): fiscal policy is sufficiently large to render the ELB irrelevant, and the monetary authority can implement the target of $c = c^\star$ and $\pi = 0$. Note again the need for monetary and fiscal policy coordination as after the fiscal expansion, the monetary authority must adjust its interest rate rule as the underlying permanent real rate at zero inflation has increased.

It could be the case that the ELB binds indefinitely in our non-Ricardian environment for a given set of parameters; that is, $r_\star(b^o) < 0$. If the monetary authority targeted the zero-inflation steady state by setting $\iota = r_\star(b^o)$, in Figure 10 the economy would be at the intersection of the $\dot{\pi} = 0$ line and the lower $\dot{c} = 0$, and would have permanent deflation and $c < c^\star$. The monetary authority has the option to avoid this by raising the intercept of its interest rate rule, setting $\bar{\iota} > 0 > r_\star(b^o)$. This would shift the upper $\dot{c} = 0$ locus out, and the ELB threshold down, creating the opportunity for a positive inflation steady state with $c > c^\star$. At this steady state, $i > 0$ and the ELB would not bind, but inflation and worker consumption are higher than the original target of $c = c^\star$ and $\pi = 0$. Fiscal policy provides an alternative path to the zero-inflation steady state. An increase in government debt can increase the long-run real interest rate to the point that the zero-inflation steady state is attainable with $i > 0$.

6 Conclusion

This paper investigated the interaction of fiscal and monetary policy in a non-Ricardian New Keynesian model. We demonstrated how increasing government bond supply can generate Pareto

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[35]Correia et al. (2013), Mian, Straub, and Sufi (2022), and Wolf (2021) also discuss dealing with the ELB using alternative fiscal policy tools.

improvements, and characterized the mix of fiscal policy and monetary rule shifts needed to engineer them. We developed a phase diagram to characterize the global equilibrium dynamics. We used the diagram to analyze anticipated deficit effects and revisited how the global dynamics of the "forward premium puzzle" are sensitive to the knife-edge Ricardian equivalence case. We also explored using fiscal policy to escape a liquidity trap. We think that the insights from this tractable non-Ricardian framework provide lessons for more general heterogeneous agent models, including quantitative HANK models.

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A Proofs

Proof of Lemma 1

Proof. Given the log-log preferences, the inequality constraints will not bind at an optimum at almost all t, and hence we will ignore them in what follows.

Letting μ denote the (current value) co-state on assets, the Hamiltonian for the worker's problem is:

$$\mathcal{H}(s, t, c, n, \mu) =$$
$$\ln c(s, t) + \psi \ln(1 - l(s, t)) + \mu(s, t) \left(w(t)z(s, t)l(s, t) + (r(t) + \lambda)a(s, t) - c(s, t) - T(s, t) \right).$$

The first-order conditions for c and l are:

$$\frac{1}{c(s, t)} = \mu(s, t) \tag{36}$$

$$\frac{\psi}{1 - l(s, t)} = w(t)z(s, t)\mu(s, t). \tag{37}$$

Eliminating μ by combining (36) and (37) generates (7).

The evolution of the co-state is given by (suppressing s and t)

$$\dot{\mu} = (\rho + \lambda)\mu - \frac{\partial \mathcal{H}}{\partial a} = (\rho - r)\mu.$$

From this and (36), we obtain the familiar Euler Equation (6).

We can integrate the Euler Equation forward to obtain:

$$\int_t^\infty R(t, \tau)c(s, \tau)d\tau = c(s, t) \int_t^\infty R(t, \tau)e^{\int_t^\tau (r(m)-\rho)dm} d\tau$$
$$= c(s, t) \int_t^\infty e^{-(\rho+\lambda)(\tau-t)} d\tau = \frac{c(s, t)}{\rho + \lambda}. \tag{38}$$

Substituting this into the budget set (4), we obtain the "consumption function" that relates consumption at time t to financial assets and "human wealth," net of taxes, (8). □

Proof of Lemma 2

Proof. The static labor-consumption condition (7) can be integrated across cohorts to obtain (11). Aggregating the consumption function (8) gives (12).

Taking the time derivative of the aggregate consumption definition we have

$$\dot{C}^w(t) = c(t, t)\phi(t, t) + \int_{-\infty}^t \dot{c}(s, t)\phi(s, t)ds + \int_{-\infty}^t c(s, t)\dot{\phi}(s, t)ds$$
$$= c(t, t)(\lambda + n)e^{nt} + \int_{-\infty}^t (r(t) - \rho)c(s, t)\phi(s, t)ds - \lambda \int_{-\infty}^t c(s, t)\phi(s, t)ds$$
$$= (r(t) - \rho - \lambda)C^w(t) + (\lambda + n)c(t, t)e^{nt}$$

where the second line uses the first order condition for household consumption and the evolution of ϕ.

Note that using $a(t, t) = 0$, we have

$$c(t, t) = \left(\frac{\rho + \lambda}{1 + \psi}\right)(h(t, t) - \mathcal{T}(t, t)).$$

Note as well that using the process for z, we have

$$h(s, t) = e^{-\alpha(t-s)}h(t, t)$$

and thus

$$H(t) = h(t, t)\int_{-\infty}^{t} e^{-\alpha(t-s)}\phi(s, t)ds = \frac{(\lambda + n)e^{nt}}{\alpha + \lambda + n}h(t, t)$$

Similarly we obtain

$$\mathcal{T}(t) = \frac{(\lambda + n)e^{nt}}{\alpha + \lambda + n}\mathcal{T}(t, t).$$

Thus,

$$(\lambda + n)e^{nt}c(t, t) = (\lambda + n)e^{nt}\left(\frac{\rho + \lambda}{1 + \psi}\right)(h(t, t) - \mathcal{T}(t, t))$$

$$= \frac{\rho + \lambda}{1 + \psi}(\alpha + \lambda + n)(H(t) - \mathcal{T}(t))$$

$$= (\alpha + \lambda + n)C^w(t) - \frac{\rho + \lambda}{1 + \psi}(\alpha + \lambda + n)A(t).$$

Plugging this back into the aggregate Euler equation, we have:

$$\dot{C}^w(t) = (r(t) - \rho - \lambda)C^w(t) + (\lambda + n)c(t, t)e^{nt}$$

$$= (r(t) - \rho + \alpha + n)C^w(t) - \frac{\rho + \lambda}{1 + \psi}(\alpha + \lambda + n)A(t).$$

which delivers equation (10).

Using the definition of aggregate assets, and the first order condition for household labor, we get

$$\dot{A}(t) = a(t, t)\phi(t, t) + \int_{-\infty}^{t}(\dot{a}(s, t)\phi(s, t) + a(s, t)\dot{\phi}(s, t))ds$$

$$= r(t)A(t) + w(t)Z(t) - (1 + \psi)C^w(t) - \overline{T}(t)$$

$$= (r(t) - \rho - \lambda)A_t + w(t)Z(t) - \overline{T}(t) - (\rho + \lambda)(H(t) - \mathcal{T}(t)),$$

and using (11) delivers (13), and completes the proof.

□

Proof of Lemma 3

Proof. Given the CES aggregator across intermediates, we have demand for a good with price $p_j = p$ given by

$$y^D(p, t) = \left(\frac{p}{P}\right)^{-\eta}Y,$$

where $P = P(t)$ is the aggregate price index and $Y = Y(t)$ is final demand. Facing a real wage of $w(t)$, this implies

that real profits at time t are

$$\Pi(p, t) = \frac{p}{P(t)} y^D(p, t) - w(t) y^D(p, t)$$

$$= \left(\frac{p}{P(t)} - w(t) \right) \left(\frac{p}{P(t)} \right)^{-\eta} Y(t).$$

The firm's Hamiltonian is given by:

$$\mathcal{H}(t, p, x, \mu) = \Pi(p, t) - f(x) Y(t) + \mu(t) x(t) p(t),$$

where we repurpose $\mu(t)$ to be the co-state on the price adjustment equation $\dot{p}(t) = x(t) p(t)$. The first-order condition with respect to $x(t)$ is:

$$\mathcal{H}_x = 0 \implies f'(x) Y(t) = \mu(t) p(t)$$

Imposing symmetry across all firms, we have that $p(t) = P(t)$ and $x(t) = \dot{P}(t)/P(t) \equiv \pi(t)$. Then,

$$f'(\pi) Y(t) = \mu(t) P(t).$$

The first-order condition with respect to the state $p(t)$ is:

$$\dot{\mu}(t) = \hat{\rho} \mu(t) - \mathcal{H}_p(t, p(t), x(t), \mu(t))$$

$$= \hat{\rho} \mu(t) - \left(\frac{p}{P(t)} \right)^{-\eta} Y(t) \left[\frac{1}{P(t)} - \eta \frac{1}{p} \left(\frac{p}{P(t)} - w(t) \right) \right] - \mu(t) x(t)$$

$$= \hat{\rho} \mu(t) - \frac{Y(t)}{P(t)} (1 - \eta + \eta w(t)) - \mu(t) \pi(t),$$

where the last line imposes symmetry.

Differentiating the condition $\mathcal{H}_x = 0$ with respect to time and substituting we get

$$\frac{\dot{\pi}(t)}{\pi(t)} + \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{P}(t)}{P(t)} + \frac{\dot{\mu}(t)}{\mu(t)}.$$

Solving for $\dot{\pi}$ and using the previous equation for $\dot{\mu}$, we get:

$$\dot{\pi} = (\hat{\rho} - g_Y) \pi + \kappa \left[w^\star - w \right] \qquad\qquad \text{if } \pi \in [\underline{\pi}, \overline{\pi}]$$

For $\pi \notin [\underline{\pi}, \overline{\pi}]$, we have:

$$(\hat{\rho} - g_Y) \underline{\pi} = \kappa \left(w - w^\star \right) \qquad\qquad \text{if } \pi < \underline{\pi}$$

$$(\hat{\rho} - g_Y) \overline{\pi} = \kappa \left(w - w^\star \right) \qquad\qquad \text{if } \pi > \overline{\pi}.$$

$\square$

Proof of Lemma 6

Proof. The only missing element in the derivation of Lemma 6 is to establish that no cohort has a negative asset position in the original BGP. Given the premise of a stationary equilibrium, we drop time arguments when possible. The budget constraint for cohort s at time t is given by equation (3):

$$\dot{a}(s,t) = (r + \lambda)a(s,t) + wz(s,t)l(s,t) - c(s,t) - T(s,t).$$

We have from (7),

$$wz(s,t)l(s,t) = wz(s,t) - \psi c(s,t),$$

and from (2) $T(s,t) = z(s,t)T(t)$ where $T(t) = T$, given that we are in a BGP. Finally, from (8), we have

$$c(s,t) = \left(\frac{\rho + \lambda}{1 + \psi}\right)(a(s,t) + h(s,t) - \mathcal{T}(s,t))$$

$$= \left(\frac{\rho + \lambda}{1 + \psi}\right)\left(a(s,t) + (w - T)\int_t^\infty R(t,\tau)z(s,\tau)d\tau\right),$$

where the second line uses the definition of h and $\mathcal{T}$. Also,

$$\int_t^\infty R(t,\tau)z(s,\tau)d\tau = \frac{z(s,t)}{r + \alpha + \lambda - g}$$

which requires $r + \alpha + \lambda - g > 0$ for bounded human wealth. Substituting the consumption into the $\dot{a}$ equation above, and rearranging, we have:

$$\dot{a}(s,t) = (r - \rho)a(s,t) + z(s,t)(w - T)\left(\frac{r - \rho - g + \alpha}{r + \alpha + \lambda - g}\right)$$

In a BGP with $b^o \geq 0$, (20) implies $r \geq \rho + g - \alpha$. Non-negative consumption and zero assets at birth imply $w - T \geq 0$. Thus, the last term is positive. Starting from zero assets at birth, this indicates assets can never go below zero, as $\dot{a} \geq 0$ when $a = 0$, which completes the proof. □

B   Worker Welfare on the BGP

In this appendix, we derive in closed form the value function of a worker along a BGP.

Lemma 7. *On a BGP with real interest rate $r \in [\rho - \alpha, \rho + \lambda + g + n)$ and real wage $w > 0$, a newborn worker at time t has utility*

$$U^t(r,w) = \frac{1 + \psi}{\rho + \lambda}\left[\log(\rho + \lambda + g + n - r) + \frac{r - \rho}{\rho + \lambda} + \frac{\log w}{1 + \psi} + \frac{gt}{1 + \psi} + u_0\right],$$

where u_0 is a combination of parameters.

Before proving this lemma, note that the proof of Lemma 4 is immediately obtained by setting $\partial U^t/\partial r = 0$ and verifying that $\partial^2 U^t/\partial r^2 < 0$.

Proof. Utility of cohort s at time t is given by:

$$U(s, t) = \int_t^\infty e^{-(\rho+\lambda)(\tau-t)} \left[\log c(s, \tau) + \psi \log(1 - l(s, \tau))\right] d\tau.$$

In a BGP, the Euler Equation implies $c(s, \tau) = c(s, t)e^{(r-\rho)(\tau-t)}$, and the static first-order condition

$$1 - l(s, \tau) = \frac{\psi c(s, \tau)}{wz(s, \tau)} = \frac{\psi c(s, t)e^{(r-\rho)(\tau-t)}}{wz(s, \tau)}.$$

Substituting in, we have

$$\frac{\rho + \lambda}{1 + \psi} U(s, t) = \left(\log c(s, t) + \frac{r - \rho}{\rho + \lambda} - \frac{\psi \log w}{1 + \psi}\right) - \zeta_0(s, t)$$

where

$$\zeta_0(s, t) \equiv \psi \int_t^\infty e^{-(\rho+\lambda)(\tau-t)} \log z(s, \tau) d\tau$$
$$= \frac{\psi(g - \alpha)}{\rho + \lambda} t + \frac{\psi\alpha}{\rho + \lambda} s + \frac{\psi(g - \alpha)}{(\rho + \lambda)^2} + \frac{\psi \log z_0}{\rho + \lambda}.$$

In a BGP,

$$h(s, t) = \frac{z_0 w e^{gt-\alpha(t-s)}}{r + \lambda + \alpha - g}$$

$$\mathcal{T}(s, t) = \frac{z_0 T e^{-\alpha(t-s)}}{r + \lambda + \alpha - g},$$

This implies

$$(1 + \psi)c(s, t) = (\rho + \lambda)\left(a(s, t) + \frac{z_0(w - T)e^{gt-\alpha(t-s)}}{r + \lambda + \alpha - g}\right).$$

Substituting into the expression for $U(s, t)$ we obtain

$$\left(\frac{\rho + \lambda}{1 + \psi}\right) U(s, t) = \log\left(a(s, t) + \frac{z_0(w - T)e^{gt-\alpha(t-s)}}{r + \lambda + \alpha - g}\right) + \frac{r - \rho}{\rho + \lambda} - \frac{\psi \log w}{1 + \psi} + \log\left(\frac{\rho + \lambda}{1 + \psi}\right) - \frac{\rho + \lambda}{1 + \psi}\zeta_0(s, t),$$

Using that $w = (1 + \psi)c$ and that $T = (r - g - n)b$ together with (20) we have

$$\left(\frac{\rho + \lambda}{1 + \psi}\right) U(s, t) = \log\left(\frac{a(s, t)}{w} + \frac{\rho + \lambda + g + n - r}{(\alpha + \lambda + n)(\rho + \lambda)} z_0 e^{gt-\alpha(t-s)}\right) + \frac{r - \rho}{\rho + \lambda} + \frac{\log w}{1 + \psi} + \log\left(\frac{\rho + \lambda}{1 + \psi}\right) - \frac{\rho + \lambda}{1 + \psi}\zeta_0(s, t).$$

Using the value of $\zeta_0(s, t)$, we obtain after some manipulation:

$$\left(\frac{\rho + \lambda}{1 + \psi}\right) U(s, t) = \log\left(\frac{a(s, t)(\rho + \lambda)(\alpha + \lambda + n)}{wz_0 e^{gt}} + (\rho + \lambda + g + n - r)z_0 e^{-\alpha(t-s)}\right) +$$
$$+ \frac{r - \rho}{\rho + \lambda} + \frac{\log w}{1 + \psi} + \frac{1}{1 + \psi}gt + \frac{\psi}{1 + \psi}\alpha(t - s) + u_0$$

where

$$u_0 \equiv -\log((\alpha + \lambda + n)(1 + \psi)) - \frac{\psi}{1 + \psi}\frac{g - \alpha}{\rho + \lambda} + \frac{1}{1 + \psi} \log z_0$$

Evaluating at $s = t$, we obtain:

$$\left(\frac{\rho + \lambda}{1 + \psi}\right) U(t, t) = \log\left(\rho + \lambda + g + n - r\right) + \frac{\log w}{1 + \psi} + \frac{r - \rho}{\rho + \lambda} + \frac{1}{1 + \psi} gt + u_0$$

□